REPUBLIC POLYTECHNIC

A113 End-Semester Examination Revision Worksheet

Give your answers in 2 decimal places, where applicable.

Numerical Operations and Algebra

1. Given that $\dfrac{14-5a}{4} = \dfrac{a}{2}$, find the value of a.

2. Solve $5 + 4\,(x - 3) = 29$

3. Determine the value of a in the equation: $3^a = 729$.

4. Find the value of h, given that $\dfrac{c^{30} \times c^{-50}}{c^h} = c^{-100}$.

Set Theory

5. Which option in Table 1 below represents the correct shaded area shown in the Venn diagram in Figure 1 below?
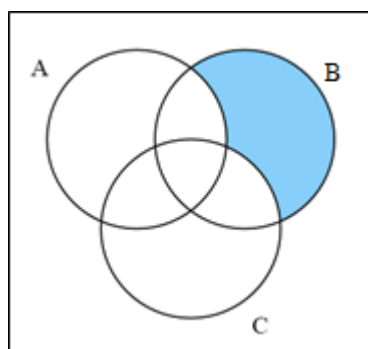
Table 1

Option	Set Notation
W	$(A'\cap B)\cap C\,'$
X	$(A'\cup B)\cup C\,'$
Y	$(A'\cap B)\cup C\,'$
Z	$(A'\cup B)\cap C\,'$

Figure 1(Not drawn to scale)

6. Determine the value of $n(A)$ for set $A = \{x : x$ is an multiple of 3 **and** $25 < x \le 35\}$.

7. Given two **disjoint sets**, G and F with $n(G) = 10$, $n(F) = 12$, and $n(G \cup F)\,' = 43$. Find the value of $n(\varepsilon)$.

8. Given the Truth table in Table 2 below. What are the truth values of q and v?

Table 2

A	B	$A \cap B$
T	T	T
F	T	F
F	F	q
T	F	v

Number System

9. Convert 110110_2 to decimal number form.

10. Find the sum of 56_8 and 47_8, expressing your answer in octal number form.

11. Convert 123 to base 5.

12. The number 22_3 is added to 54_6. Convert the result to a decimal number.

Linear Equation

13. The coordinates of Point P, Q and R are given as (5, -10), (10, -2), (15, k) respectively.
    a) Find the gradient of the line that passes through point P and Q.

    b) Given that P, Q and R are collinear, find the value of k.

14. The equation of another line MN is given as $2y = 8x + 4$.
    a) Find the gradient of the line MN.

    b) Find the x-intercept of the line MN.

    c) Find the y-intercept of the line MN.

    d) Find the gradient of the line AB which is perpendicular to line MN.

Quadratic Equations

15. Determine the coordinates of the minimum point of the quadratic equation $y = 2x^2 - x - 3$.

16. Given that one of the roots for the quadratic curve $y = ax^2 - 6x + 2$ is 2.75. Find the value of a.

17. The equation of a quadratic function is given as $y = 2x^2 - 2x - 12$.

    a) Find the values of the x-intercept.

    b) Find the y-intercept.

Systems of Linear Equations

18. The cost of two purchases involving 4 types of stationery, are summarised in Table 3 as shown below, in which the price for each of the type of stationery is the same in each purchase.

Table 3

Pen	Pencil	Ruler	Notebook	Total Cost ($)
5	3	2	6	60
8	7	3	9	107.50

    a) By observing the rows, what is the total cost of buying 18 pens, 13 pencils, 7 rulers and 21 notebooks?

    b) By observing the rows, what is the total cost of buying 11 pens, 11 pencils, 4 rulers and 12 notebooks?

    c) If John paid $108 for 10 pens, 6 pencils and 4 rulers, how much is the price of a notebook?

Logarithms

19. Find the value of x.

    a) $\log_4 x = 3$

    b) $3 = \log_2 2^x$

    c) $\ln x = 5$

    d) $5^x = 3$

20. Simplify the following expressions.

    a) $\log_3 3^5 + \log_3 3^{-\frac{1}{3}}$

    b) $\log_4 \sqrt{2} + \log_4 1 + \log_4 3$

21. Evaluate the following:

    a) $\log_4 100$

    b) $\log_2 e \times \log_e 7 \times \log_7 8$

    c) $\log_2 k \times \log_k 4$, where k is positive

22. Suppose $\ln x = 8$ and $\ln y = 4$, where x and y are positive. Evaluate the following expressions:

    a) $\ln\left(\dfrac{x}{y}\right)$

    b) $\ln(xy^4)$

    c) $\log_y x$

Displacement-Time and Speed-Time Graphs

23. A car travels at a constant speed of 80 km/h for half an hour along a straight road. Then it stopped for a 20 minutes break and continues at a constant speed of 120km/h for 45 minutes.

    a) Find the distance the car travelled in the first 10 minutes.

    b) Find the total distance the car travelled.

    c) Find the average speed for the whole journey.

24. Figure 2 below shows a displacement-time graph of the journey of a particle travelling along a straight path.
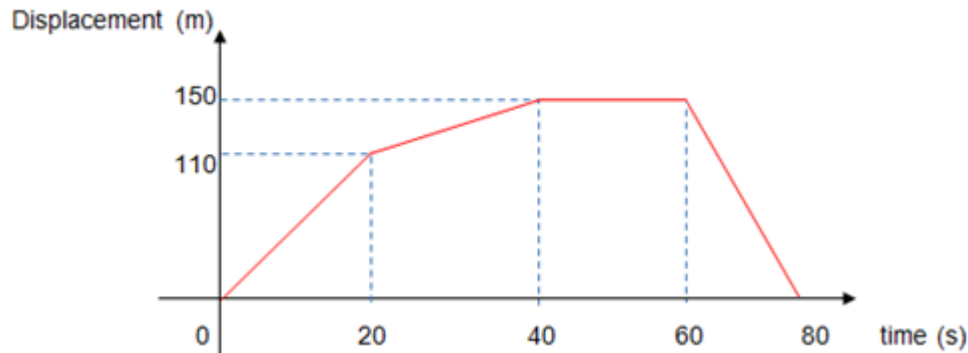

Figure 2 (Not drawn to scale)

a) What is the instantaneous velocity at time $t = 55$ s?

b) What is the instantaneous velocity at time $t = 30$ s?

c) What is the displacement at time $t = 30$ s?

d) What is the displacement at time $t = 65$ s?

e) What is the total distance travelled between $t = 0$ s and $t = 80$ s?

f) What is the average velocity travelled between $t = 0$ s and $t = 40$ s?

g) What is the average velocity travelled between $t = 40$ s and $t = 80$ s?

h) What is the average speed travelled between $t = 0$ s and $t = 80$ s?

25. Figure 3 below shows the download speed-time graph when Alice is downloading some data over the internet.
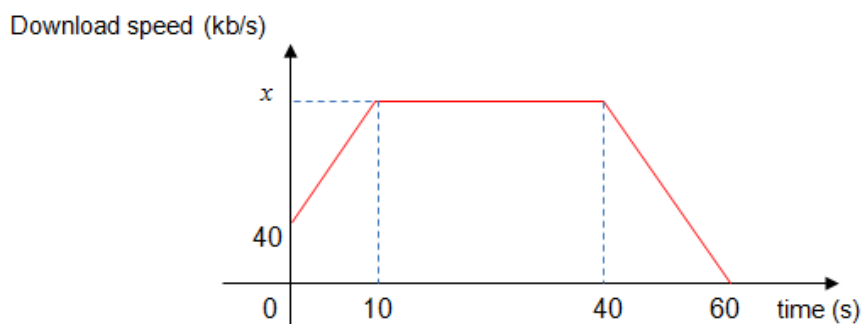

Figure 3 (Not drawn to scale)

a) Given that the rate of change of download speed for the first 10 seconds is 9 kb/s^2, find the value of x.

b) What is the total amount of data downloaded by Alice?

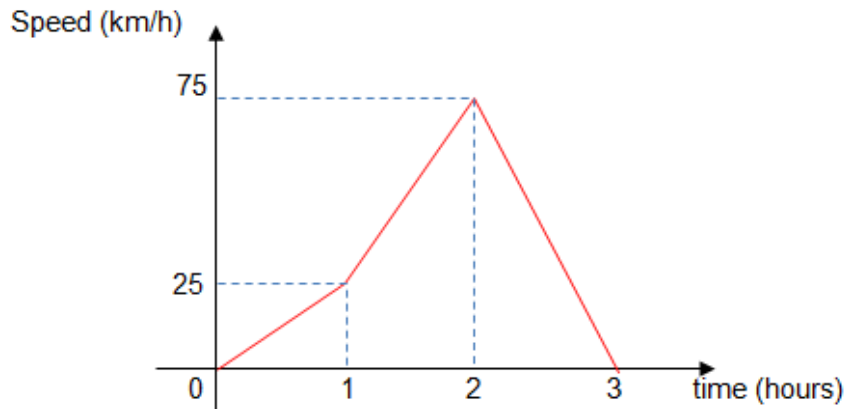26. Figure 4 below shows a speed-time graph of a particle travelling in a fixed direction.

Figure 4 (Not drawn to scale)

a) What is the speed at time t = 2 hours?

b) What is the speed at time t = 1.8 hours?

c) What is the rate of change of speed at time t = 1.2 hours?

d) What is the total distance travelled for the whole journey?

e) What is the average speed for the whole journey?

Statistics

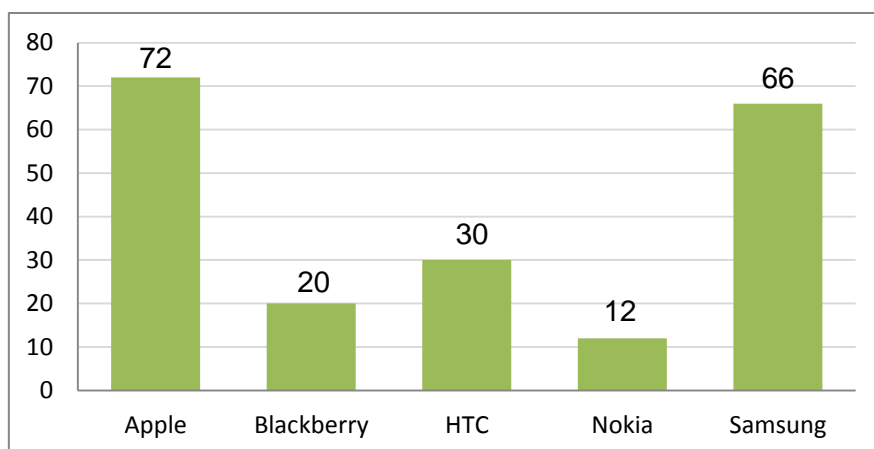27. Figure 5 below shows the records of a survey on student preferred mobile phone brands in a bar chart.

Figure 5

a) What is the total number of students who took part in the survey?

b) What is the **relative frequency** of students that preferred HTC?

c) Which is the most preferred brand by the students?

d) What is the percentage of students that preferred Samsung?

e) What is the percentage of students that preferred Apple or Samsung?

f) 45 female students responded in the survey preferred Apple. What is the percentage of male students preferred Apple, out of the total number of students?

28. The pie chart in Figure 6 shows the range of monthly salary of the employees in a company.
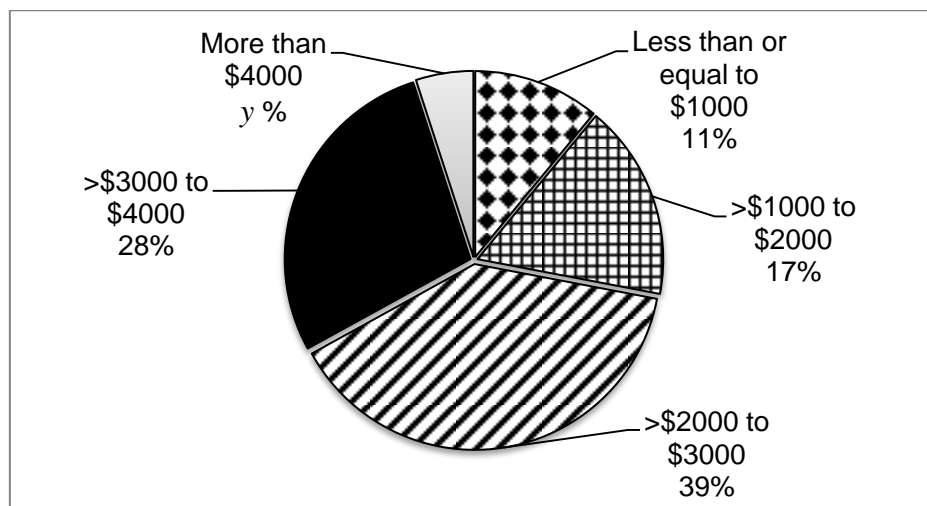

Figure 6

a) What is the value of y?

Suppose there are 51 employees earning a monthly salary of >$1000 to $2000.

b) How many employees are receiving a monthly salary of >$3000 to $4000?

c) How many employees are receiving a monthly salary of more than $2000?

d) How many employees does the company have?

29. The following data shows the test marks (1 – 10) obtained by a group of students:

| 6	7	3	6	5	9	7	10	9	6
8	4	7	5	1	7	8	7	8	6

a) Build a frequency table for the data.

b) Find the mean, median and mode(s) of the above data.

30. The table below shows the distribution of test score for a group of students:

Test Score	5	10	15	20	25	30	35	40
Frequency	4	10	14	25	37	48	5	2

   a) What is the relative frequency of students who scored 20 in the test?

   b) Determine the mean score for this group of students.

   c) Determine the mode(s) of the score for this group of students.

   d) Determine the median score for this group of students.

31. The histogram below shows the test scores of a group of students.
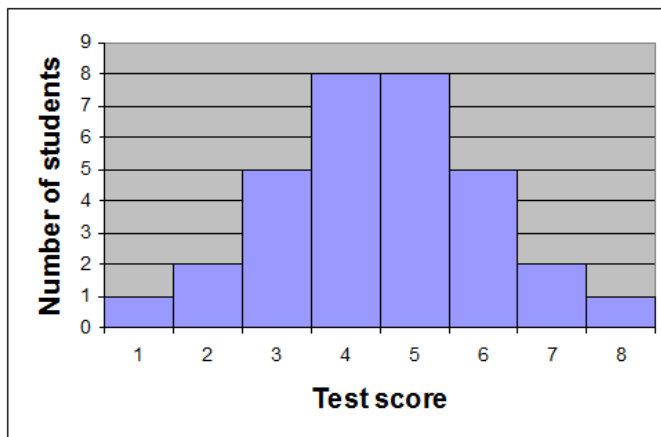
   a) What is the mean score?

   b) What is the median score?

   c) What is/are the modal score(s)?

Permutation and Combination

32. At a particular vehicle mart, there are 7 different car models, 8 different motorcycle models and 5 different minibus models.

   a) How many ways are there to choose 4 different car models?

   b) How many ways are there to choose 3 different vehicle models?

   c) How many ways are there to choose 4 vehicles with 1 car, 1 motorcycle and 2 different minibus models?

33. How many ways can all the letters of the word 'IMPOSSIBLE' be arranged?

34. There are 6 different jogging tracks between Location A and Location B on a hill. In how many ways can Benny jog from Location A to Location B and come back to Location A without using the same track in the return journey?

35. 8 people (represented by A, B, C, D, E, F, G and H) were arranged in a line.
    a) How many different possible arrangements are there?
    b) Suppose that person A has to stand at the 3rd position in a line, what will be the number of possible arrangements?
    c) If a basketball coach wants to choose 5 from these 8 people to form a team, how many different combinations are there?
    d) If the coach randomly picks 3 from these 8 people, and then assigned them to 3 different teams, Team A, B and C, how many possible ways are there?

36. A musical band consists of 8 female members and 6 male members.
    a) If all the 6 males are to be arranged in a single row, how many ways can they be arranged?
    b) If any 2 members out of the 14 members are to be selected and arranged in a single row, how many ways can they be arranged?
    c) All the 14 members are to be arranged in two rows instead. If the females are to be arranged in the front row and the males are to be arranged in the back row, how many ways can they be arranged?
    d) If 3 members out of the 14 members are to be selected,
        i) how many ways can they be selected if there are no constraints on the selection of members?
        ii) how many ways can they be selected if the selected members are to be all females?
        iii) how many ways can they be selected if the selected members are to consist of 1 female and 2 males?

Probability

37. A man throws a <u>fair eight-sided dice</u>.

    a) What is the probability that an odd number is obtained from a throw?

    b) What is the probability that a number greater than 5 is obtained from a throw?

    c) The fair eight-sided dice is thrown 200 times. Determine the expected frequency that the number 8 is obtained.

38. A bag contains 8 red balls, 12 blue balls and 16 green balls. Suppose Alex randomly picks one ball from the bag.

    a) What is the probability of him getting a yellow ball?

    b) What is the probability of him getting a red ball?

    c) What is the probability of him not getting a red ball?

    d) What is the probability of him getting either a red or a blue ball?

    Suppose Alex randomly picks two balls from the bag (with 8 red, 12 blue and 16 green balls).

    e) If he **replaces** the first ball before picking the second ball from the bag, what is the probability of him getting two blue balls?

    f) If he **replaces** the first ball before picking the second ball from the bag, what is the probability of him getting one blue and one green ball?

    g) If he **does not replace** the first ball before picking the second ball from the bag, what is the probability of him getting two blue balls?

39. A man tossed an <u>unbiased</u> coin three times.

    a) What is the probability that the same result is obtained in all the three tosses?

    b) What is the probability that only one head is obtained?

    c) What is the probability that the second toss gives a head?

    d) **Given that** the first toss is a head, what is the probability that only one head is obtained?

    e) **Given that** only one head is obtained, what is the probability that the first toss is a head?

40. A Math teacher gave her class two tests. 58% of the class passed the first test. Only 26% of those who passed the first test, passed the second test too. 37% of those who failed the first test, failed the second test too.

   Using a probability tree diagram or otherwise,

   a) What is the probability of students who passed at least one test?

   b) What is the probability of the students who passed the first test **given that** they also passed the second test?

41. In a bolt factory, there are 3 machines that produce bolts. Machine 1 produces 50% of the total bolts, Machine 2 produces 35% of the total bolts while Machine 3 produces the remaining bolts. 8% of Machine 1's output are defective, 10% of Machine 2's output are defective while 10% of Machine 3's output are defective.

   a) What percentage of the total bolts is produced by Machine 3?

   Using a probability tree diagram or otherwise,

   b) Find the probability that a bolt is produced by Machine 1 and is defective.

   c) Find the probability that a bolt produced is defective.

   d) **Given that** a bolt is defective, find the probability that it is produced by Machine 2.

   e) **Given that** a bolt is not defective, find the probability that it is produced by Machine 2.

   f) **Given that** a bolt is produced by Machine 2, find the probability that it is defective.

Binomial Distribution

*For the following questions, round off your answers to **3 significant figures**.*

42. Based on statistical records, 90% of the rear seat passengers do not fasten their seat belts. A random sample of 12 rear seat passengers is selected. Assuming a Binomial distribution,

   a) Determine the probability that none of the rear seat passengers fastened their seat belts.

   b) Determine the probability that 2 of the rear seat passengers fastened their seat belts.

43. In the population of Singapore, the proportion of people having blood group A is 38%. Assume that the probability distribution follows a Binomial distribution.

   a) Out of 2 people who came to donate blood, find the probability that both of them have blood group A.

   b) Out of 8 people who came to donate blood, find the probability that there are 5 people with blood group A.

   c) Out of 7 people who came to donate blood, find the probability that there is at least 1 person with blood group A.

44. In a large population, 30% of the population is short-sighted. Samples consisting of 6 people are randomly chosen from the population. Assume that the probability distribution follows a Binomial distribution.

   a) Find the probability that more than 2 people in a sample are short-sighted.

   b) Three such samples of 6 people are taken.

      i. Find the probability that all of these three samples have more than 2 people who are short-sighted.

      ii. Find the probability that one of these three samples has exactly 1 person who is short-sighted.

45. Based on a recent survey conducted nationwide, it was found that 62% of teenagers spend an average of more than 4 hours daily on social media. Assume that the probability distribution follows a Binomial distribution.

   a) If 7 teenagers are randomly selected,

      i. What is the chance that 3 of them spend an average of more than 4 hours daily on social media?

      ii. What is the chance that **at least** 5 of them spend an average of more than 4 hours daily on social media?

      iii. What is the chance that **at most** 5 of them spend an average of more than 4 hours daily on social media?

   b) If 10 teenagers are randomly selected,

      i. What is the chance that 4 of them spend an average of not more than 4 hours daily on social media?

      ii. If two groups comprising of 10 teenagers each are randomly selected. What is the chance that **at least** one group has 4 teenagers who spend an average of not more than 4 hours daily on social media?

Normal Distribution and Hypothesis Testing

46. It is known that the weight of apples in a big carton from a particular supermarket follows a normal distribution with a mean of 150 grams and a standard deviation of 18 grams. This distribution is illustrated below (not drawn to scale) with additional information.

0.21

0.02

135   186

Distribution of weights of apples
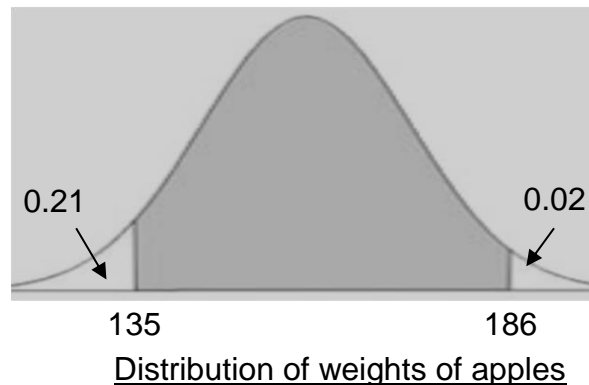
Find the probability that a randomly chosen apple from this carton weighs

a) lighter than 135 grams.

b) between 135 grams and 186 grams.

c) between 150 grams and 186 grams.

d) heavier than 114 grams.

47. In a student population, the mean weight is 75 kg and its variance is 120 kg^2. Assume that the weight of this population follow a normal distribution. For a sample size of 80,

a) Determine the mean of the distribution of sample means.

b) Determine the variance of the distribution of sample means.

c) Determine the standard deviation of the distribution of sample means.

48. The ABCTransit claimed that the average waiting time for a train is 2.5 minutes during peak hours. Sam feels that he waited longer than 2.5 minutes. He conducts a survey with 70 passengers and the average waiting time and standard deviation are 2.9 minutes 0.56 minutes respectively. Below is the graph of the distribution of the average waiting time based on sample size 70.
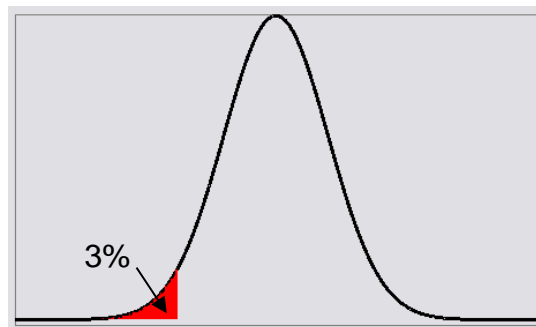
2.37
Distribution of the average waiting time

a) What are the null and alternative hypotheses?

b) What is the conclusion of the hypothesis test at 3% level of significance? Justify your answer.

49. The "Awesome" fireworks is claimed to burn for 50 seconds. The producer of the fireworks feels that the burning time of the fireworks is shorter than 50 seconds. A random sample of 40 "Awesome" fireworks is experimented and it is found that the burning time of the sample has a standard deviation of 5.6 seconds. Below is the graph of the distribution of the average burning times.
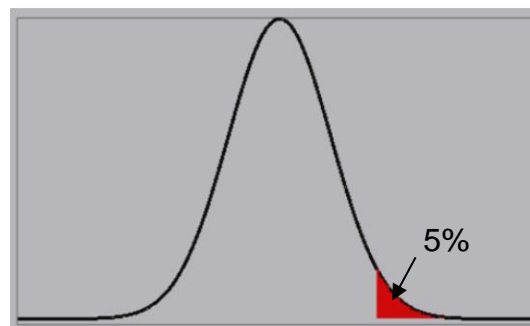
51.46
Distribution of the average burning times

a) State the null and alternative hypotheses.

b) Suppose the null hypothesis is **not** rejected at 5% level of significance, what is the range of the sample (size 40) mean time interval that the fireworks will burn out?