E114 MSA revision paper

SECTION A (MULTIPLE CHOICE QUESTIONS)
There is only one correct answer for each question. Circle the correct answer in the space provided.

Q1. With reference to the **geometric** sequence below, determine the value of h.

12, 3, 0.75, __, __, h, …

(A) 53/4
(B) 3/256
(C) 3/16
(D) 1/4

Q2. With reference to Figure A1 below, determine the length of AB.

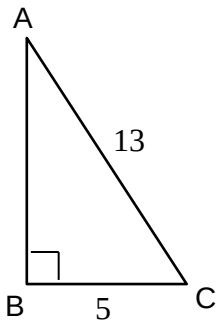

Figure A1 (Not drawn to scale)

(A) 12
(B) $\sqrt{194}$
(C) 144
(D) 25

Name:

Q3. With reference to Figure A2 below, determine the length of AC.

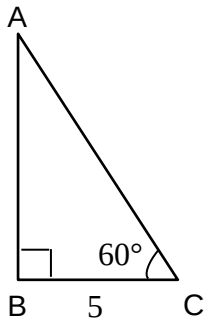

Figure A2 (Not drawn to scale)

(A) $5\sin 60°$

(B) $\dfrac{5}{\cos 60°}$

(C) $\dfrac{5}{\tan 60°}$

(D) $5\tan 60°$

Q4. With reference to Figure A3 below, which of the following cross product is pointing **out of the plane of the paper**?

Figure A3 (Not drawn to scale)

(A) $\mathbf{r}\times\mathbf{q}$

(B) $\mathbf{s}\times\mathbf{r}$

(C) $\mathbf{q}\times\mathbf{p}$

(D) None of the above

Q5. Points A, B and C has coordinates (-1, 2, 1), (1, 0, 2) and (1, 0, 3), respectively. Which point lies on the plane $-3x+y+z=0$?

(A) Point A

(B) Point B

(C) Point C

(D) None of the above

Name:

Q6. Which of the following matrix multiplications is **invalid**?

(A) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(B) $\begin{pmatrix} a & b & c \end{pmatrix}\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$

(C) $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$

Q7. What is the determinant of matrix $\begin{pmatrix} 1 & -2 \\ 3 & 7 \end{pmatrix}$?

(A) 13

(B) 1

(C) -13

(D) -42

Q8. Evaluate the following: $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$.

(A) $\begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$

(B) $\begin{pmatrix} 4 \\ -7 \\ 2 \end{pmatrix}$

(C) 1

(D) -1

Name:

Q9. Which of the following shows the correct values for variables A and C for the equation of the graph in Figure A4, given that $y = A\sin(x + c)$?

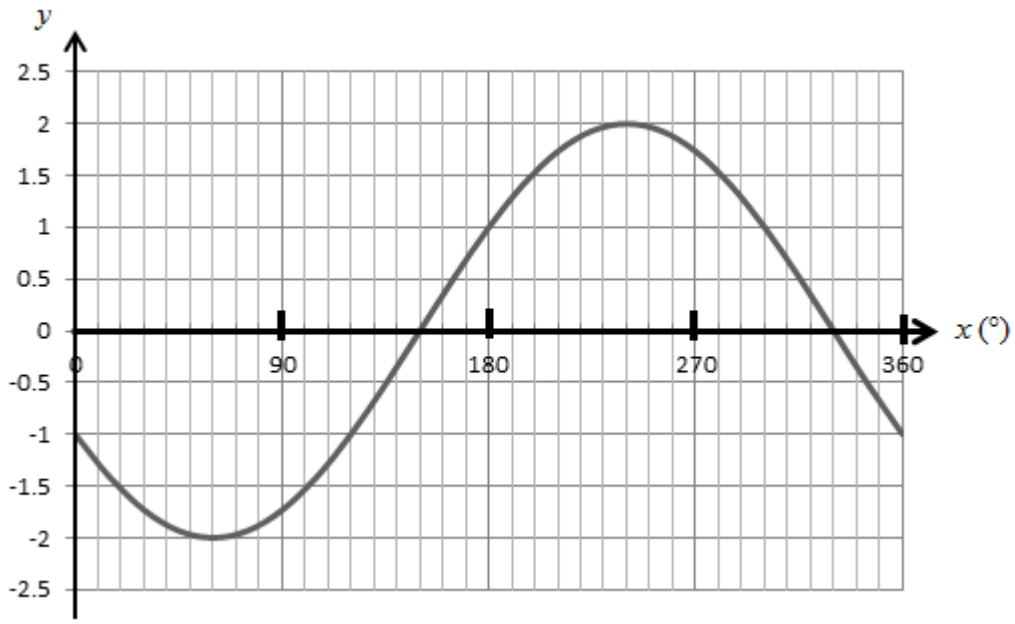

Figure A4

(A) $A = -2, C = 30°$
(B) $A = -2, C = -30°$
(C) $A = 2, C = 30°$
(D) $A = 2, C = -30°$

Q10. Determine the value of the unknown constant u given the scalar product

$$\begin{pmatrix} -2u \\ 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = -14$$

(A) 5
(B) 1/3
(C) -1/3
(D) -5

Q11. The multiplication of the matrices $\begin{pmatrix} 3 & 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 1 & 6 \end{pmatrix}$ gives a _____.

(A) 3 x 1 matrix
(B) 1 x 3 matrix
(C) 1 x 1 matrix
(D) 3 x 3 matrix

Name:

Q12. The values of x from $0° \le x \le 360°$ that satisfy the equation $\sin x = 0.5$ are _____.

(A) 30^o, 150^o
(B) 150^o, 210^o
(C) 210^o, 330^o
(D) 30^o, 330^o

Q13. Which of the following is a complex conjugate of $z = -2j - 4$?

(A) $-2j - 4$
(B) $-4j - 2$
(C) $2j - 4$
(D) $2j + 4$

Name:

SECTION B
Show your workings clearly. Give your answers to 2 decimal places, where applicable.

Answer parts (a) and (b) with reference to Figure B1.

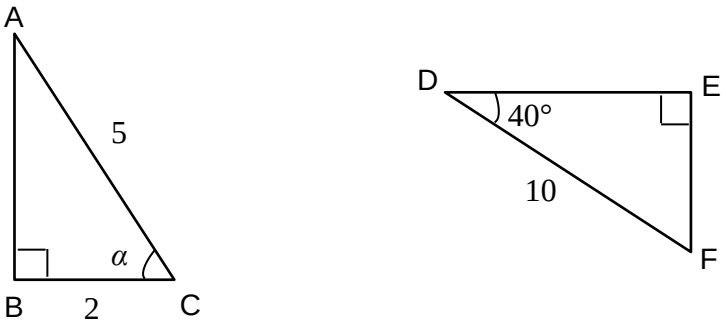

Figure B1 (Not drawn to scale)

a) Determine the value of α in degrees.

b) Determine the length EF.

c) With reference to Figure B2, determine the values of p, q and r.

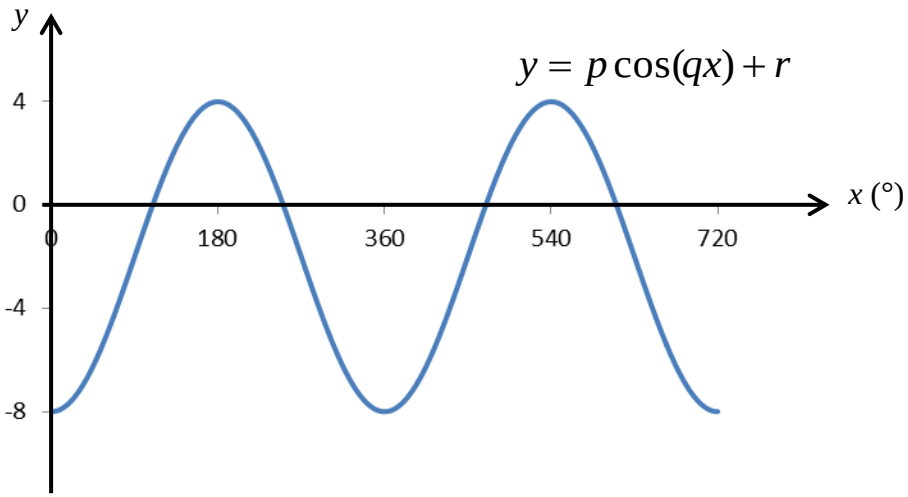

Figure B2 (Not drawn to scale)

d) Determine the maximum and minimum values of

$$y = -4 + 5\sin 3x.$$

e) Solve the following equation, where $0° \le x \le 360°$.

$$3\cos x + 2 = 0$$

Name:

SECTION C
Show your workings clearly. Give your answers to 2 decimal places, where applicable.

Answer parts (a) to (d) with reference to Figure C1.

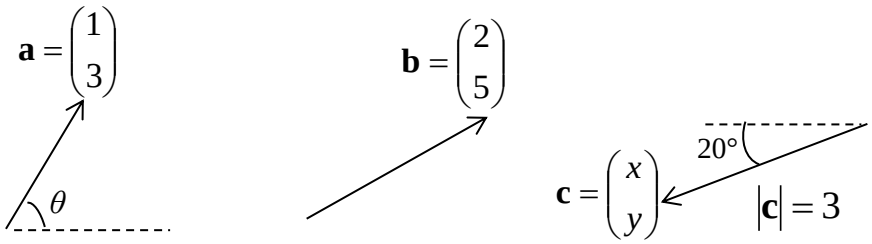

Figure C1 (Not drawn to scale)

a) Determine the value of θ in degrees.

b) Determine the magnitude of vector **b.**

c) Express vector **c** in column vector form (i.e. determine the values of x and y).

d) Determine -2**a** + 3**b** in column vector form.

Answer parts (e) to (i) with reference to the following vectors.

$$\mathbf{p} = \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}$$

e) Determine the scalar product of vectors **p** and **r** (i.e. $\mathbf{p} \bullet \mathbf{r}$).

f) The cross product of vectors **q** and **r** (i.e. $\mathbf{q} \times \mathbf{r}$) is $\begin{pmatrix} c \\ d \\ 3 \end{pmatrix}$. Determine the values of *c* and *d***.**

g) Determine the scalar projection of vector **q** onto vector **r**.

h) Show whether the vectors **p** and **q** are perpendicular to each other.

Name:

SECTION D
Show your workings clearly. Give your answers to 2 decimal places, where applicable.

a) Given that $\begin{pmatrix} 1 & -2 & -4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ j \\ k \end{pmatrix} = \begin{pmatrix} -7 \\ 14 \\ -6 \end{pmatrix}$, state the values of *j* and *k*.

b) Determine the value of *f* from the following matrix equation.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ -1 & 9 & 4 \\ -5 & 3 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ d & e & f \\ -5 & 3 & 11 \end{pmatrix}$$

c) Determine matrix **M** from the following matrix equation.

$$\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathbf{M} = \begin{pmatrix} 7 & 4 & 5 \\ 2 & 3 & 6 \\ -2 & 0 & 1 \end{pmatrix}$$

d) Determine the inverse of $\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$.

e) Given that a point (1, 2) undergoes a 60° clockwise rotation about the origin to become point A, determine the coordinates of point A.

f) Given that a point (5, 3) undergoes a translation of 2 units to the right and 6 units downwards to become point B, determine the coordinates of point B.

g) Point C undergoes the following transformations to become (4, -7).

First: Translation of $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Second: 20° clockwise rotation about the origin

Third: Translation of $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

Determine the coordinates of point C.

Name:

SECTION E
Show your workings clearly. Give your answers to 2 decimal places, where applicable.

Given that an arithmetic sequence has first term 10 and common difference of -3,

a) determine the sum of the first 21 terms**.**

Given that a geometric sequence has a 3rd term of 100 and 5th term of 2500,

b) determine the common ratio.
c) determine the sum of the first 7 terms.

SAMSENG Inc. recently launched a new smartphone model. Based on a forecast made by the marketing department, the **projected** sales volume in a certain country 1 week after the launch is 20000. Subsequent **weekly** sales volume is projected to increase by 5% each week.

d) What is the projected sales volume for the 8^{th} week? Round your answer to the nearest whole number.

The **actual** sales volume of the new smartphone model 1 week after the launch is 15000.
Subsequent **weekly** sales volume increases by 1000 each week.

e) What is the actual sales volume for the 8^{th} week?
f) How many weeks after the launch would the **actual total** sales volume exceed 50000?

Name:

SECTION F
Show your workings clearly. Give your answers to 2 decimal places, where applicable.

Given the following complex numbers:

$$z_1 = 6 - 8j\,,\ z_2 = 3 + 4j$$

(a) Determine $\mathrm{Re}(z_1)$

(b) Determine $\mathrm{Im}(z_2)$

(c) Determine $3z_1$ in Cartesian form.

(d) Determine $\mathrm{Re}(z_1^2)$ in Cartesian form.

(e) Determine $Z_1 \times Z_2$ in Cartesian form

(f) Determine $\frac{z_1}{z_2}$ in Cartesian form.

(END OF MSA REVISION WORKSHEET)

Name:

Answers

Section A

1. B
2. A
3. B
4. A
5. C
6. C
7. A
8. D
9. A
10. A
11. B
12. A
13. C

Section B

a) $66.42°$
b) 6.43
c) $p = -6, q = 1, r = -2$
d) Maximum value = 1, Minimum value = -9
e) 131.81°, 228.19°

Section C

a) $71.57°$
b) 5.39
c) $x = -2.82, y = -1.03$
d) $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$
e) -30
f) $c = 23, d = 4$
g) 2.80
h) Not perpendicular to each other

Name:

Section D

a) $j = 8, k = -2$

b) $f = -12$

c) $\mathbf{M} = \begin{pmatrix} \mathbf{1} & \mathbf{4} & \mathbf{8} \\ \mathbf{2} & \mathbf{3} & \mathbf{6} \\ \mathbf{-2} & \mathbf{0} & \mathbf{1} \end{pmatrix}$

d) $\begin{pmatrix} 0.4 & -0.6 \\ 0.2 & 0.2 \end{pmatrix}$

e) 2.23, 0.13

f) (7, -3)

g) (4.74, -3.22)

Section E

a) -420

b) 5

c) 78124

d) 28142

e) 22000

f) 36 weeks

Section F

a) 6

b) 4

c) 18-24j

d) -28

e) 50

f) -0.56-1.92j

