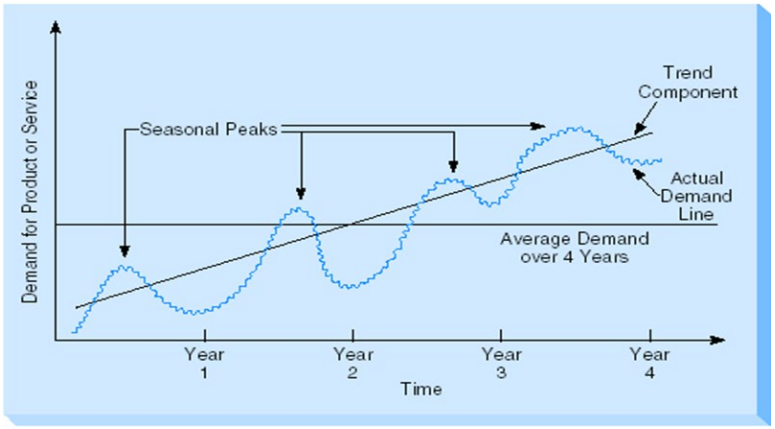
Problem 06

The Art of Forecasting (Part 3)

E211 – Operations Planning II



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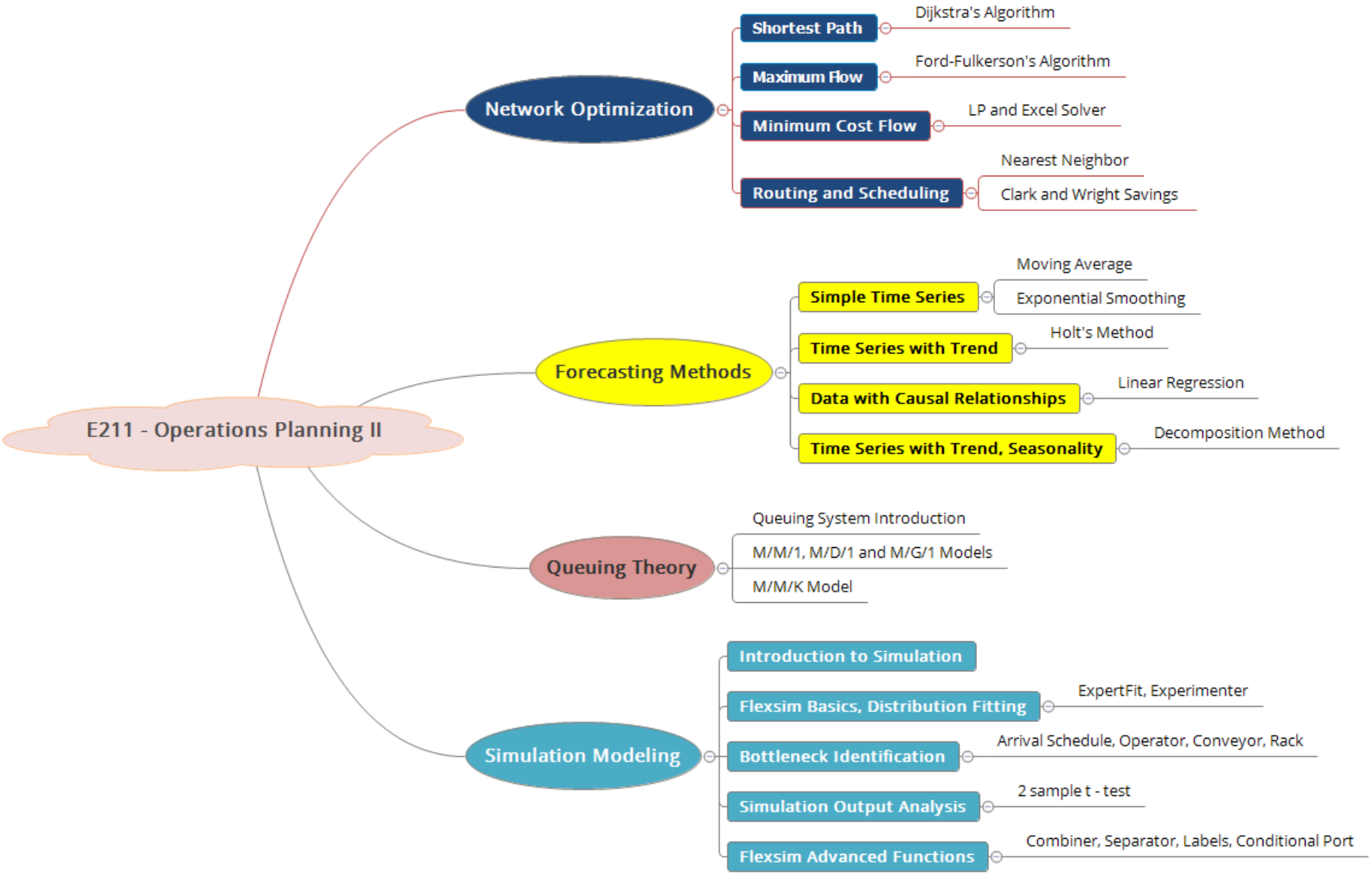
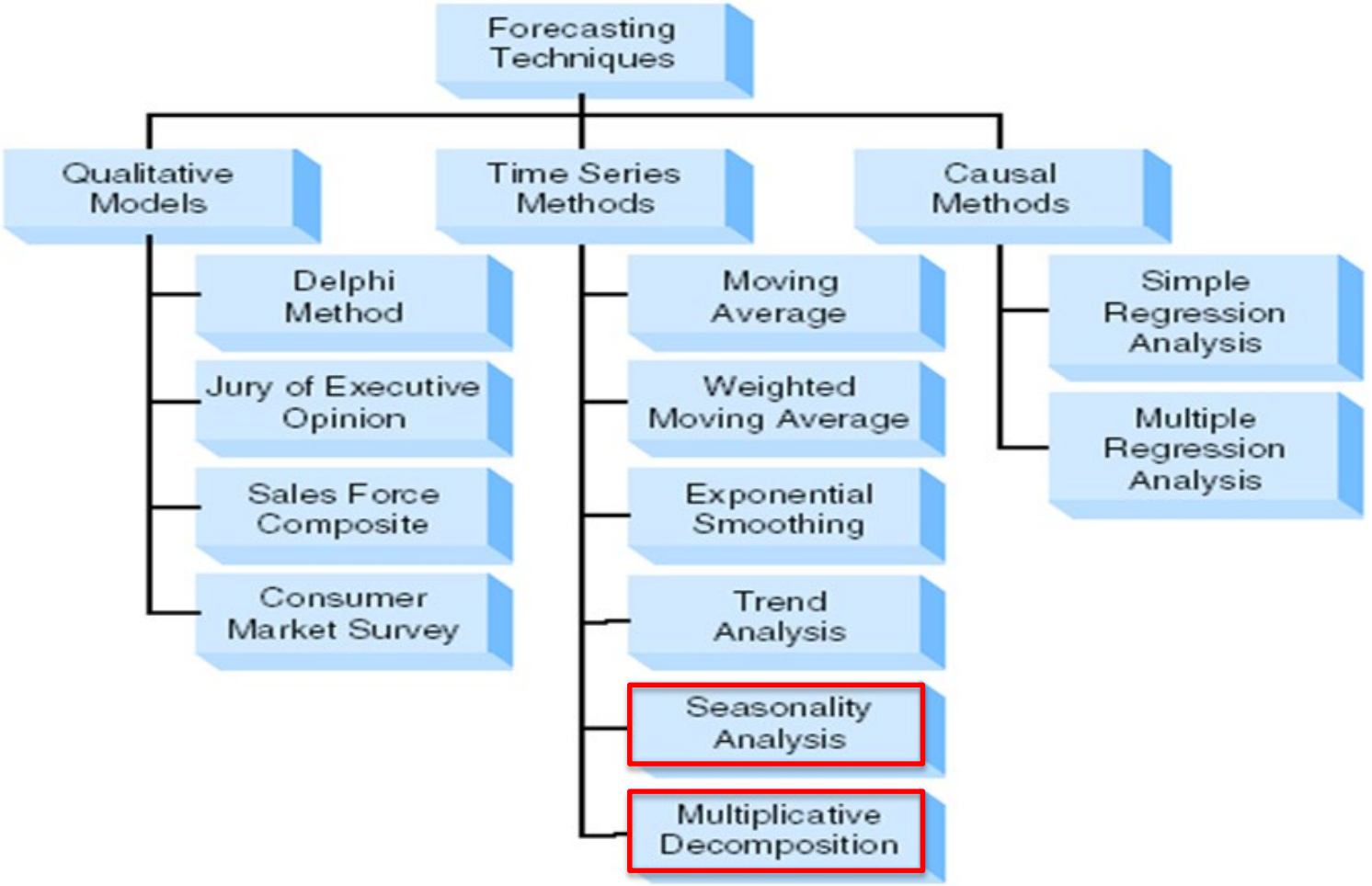
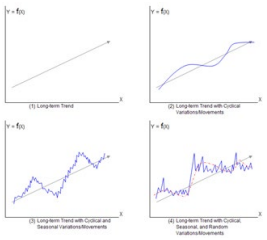
Module Coverage: E211 Topic Tree



Types of Forecasts



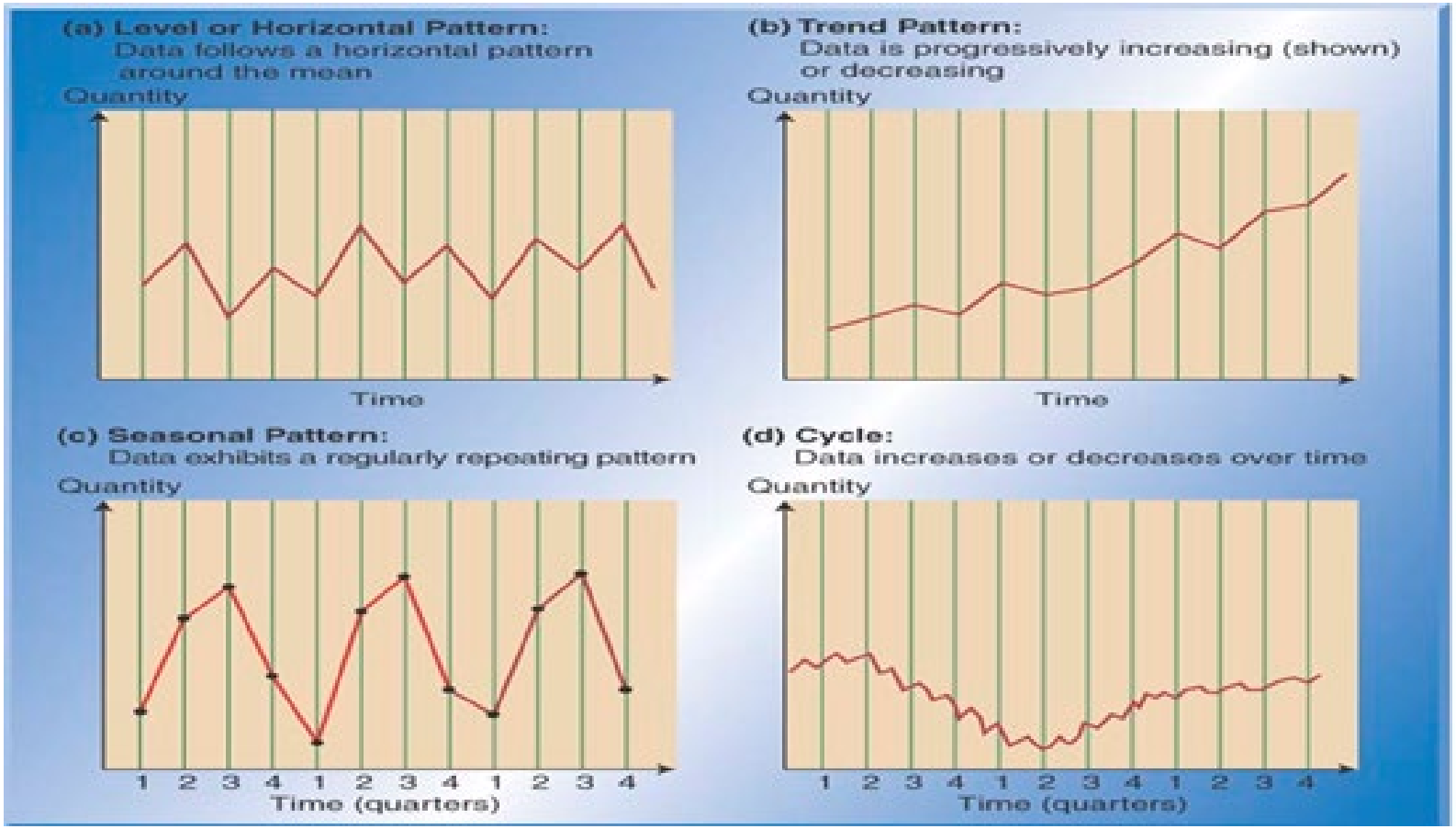
Time Series Models



- Factors in a Time Series:
  - Trend, Seasonality, Cyclicity, Irregular Variation (Random)
- We've examined Time Series using:
  - Moving Average, Weighted Moving Average, Exponential Smoothing – time series with only irregular (random) variation;
  - Holt's method (Double Exponential Smoothing), Linear Regression Analysis – time series with irregular (random) variation and **linear trend**.
- However, did we touch Seasonality before?
  - Seasonality: Any cyclical or periodic fluctuation in a time-series that recurs or repeats itself at the same phase of the cycle or period.
  - For example, toy sales at Christmas, ice cream sales in the summer.

Basic Time Series Components

Trend Seasonality Cycles Random Variations

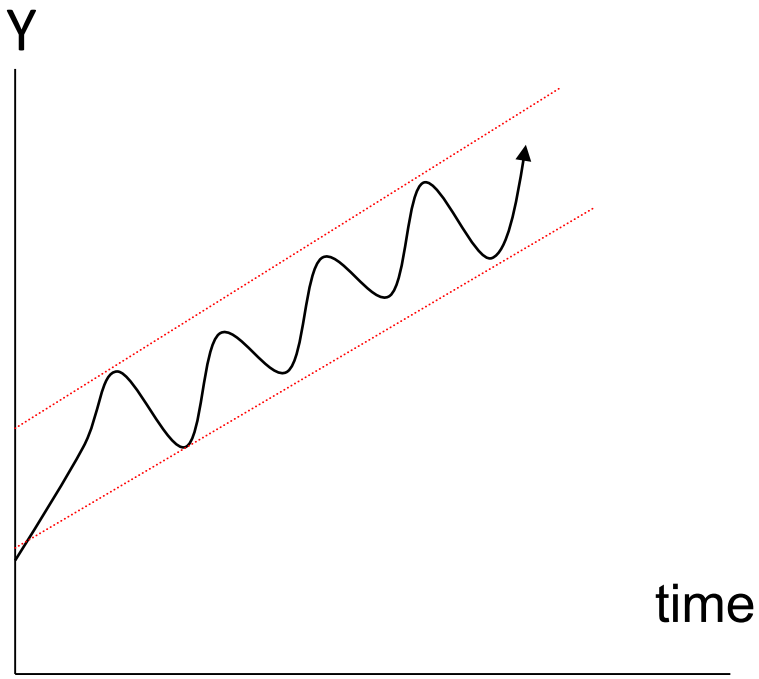


Decomposition Method

- Intuitive and has no theoretical basis
- Can be used to forecast time series that
  - Exhibits trend and seasonal effects
  - The parameters describing the time series are not changing over time
- Two general forms of time-series models
  - Multiplicative: Forecast = Trend x Seasonality x Cyclicity x Irregular Variations (Our Focus today)
  - Additive: Forecast = Trend + Seasonality + Cyclicity + Irregular Variations

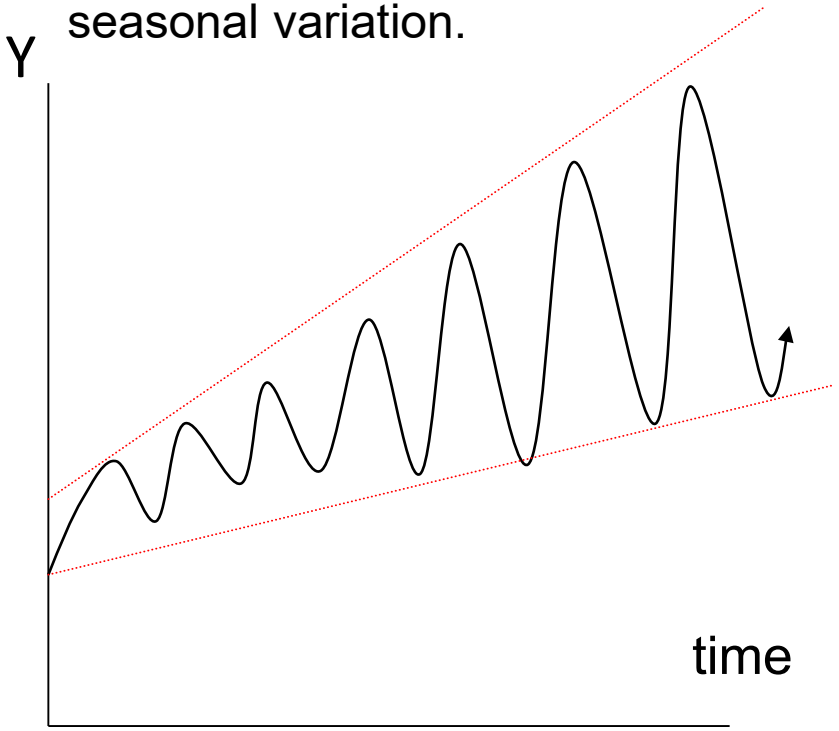
Decomposition Method

A time series exhibits constant seasonal variation.



Additive seasonality

A time series exhibits increasing seasonal variation.



Multiplicative seasonality

Additive Decomposition Method is used when a time series exhibits constant seasonal variation

Multiplicative Decomposition Method is used when a time series exhibits increasing or decreasing seasonal variation.

Multiplicative Decomposition Procedure

Step 1: Calculate the centred moving average, CMA, to remove the seasonal variation from data

- Let N be the number of periods (weeks, months, quarters etc.) in a repetitive cycle;
- Find the N-period moving average of the data;
- Calculate the CMA;
- Note that CMA is only needed when N is an even number.

Step 2: Estimate Seasonal Ratio by dividing the original data with CMA

Multiplicative Decomposition Procedure

Step 3: Calculate the Seasonal Index

- Group the values of Seasonal Ratio by like seasons, e.g. all Januaries, all Februaries;
- For each season, compute the average of Seasonal Ratio values for that season;
- Estimate the Seasonal Index:
  - Normalize the average of Seasonal Ratios so that normalized values sum to N;
  - This is done by multiplying each average Seasonal Ratio with a constant:

$$\frac{N}{\sum_{t=1}^{N}(Average\ Seasonal\ Ratio)_t}$$

Multiplicative Decomposition Procedure

Step 4: Compute the deseasonalized value by dividing the original data with their corresponding seasonal index;

Step 5: Compute the forecast with trend and seasonality

- With the deseasonalized value,
  - Apply the learnt Linear Trend analysis to get the best fit trend line;
  - Use the line equation, calculate the deseasonalized forecast.
- Get the forecast with trend and seasonality
  - Forecast with trend and seasonality is the seasonalized forecast obtained by multiplying the seasonal index with the deseasonalized forecast.
  - Here only Trend and Seasonal factors are used to forecast the future values without large amount of data.

Multiplicative Decomposition Procedure

Step 6: Evaluate cyclicity and random variations

- Since the multiplicative model is:
  - Forecast = Trend x Seasonality x Cyclicity x Random Variation
- Trend and Seasonal factors are already known, hence the (Cyclicity*Random Variation) can be calculated by dividing the original value with the Seasonalized forecast which has trend and seasonal factors.
- We can average the (Cyclicity*Random Variation) to estimate the Cyclicity factor by removing the random factor; then use (Cyclicity*Random Variation)/Cyclicity to get the Random factor.

P06 Suggested Solution

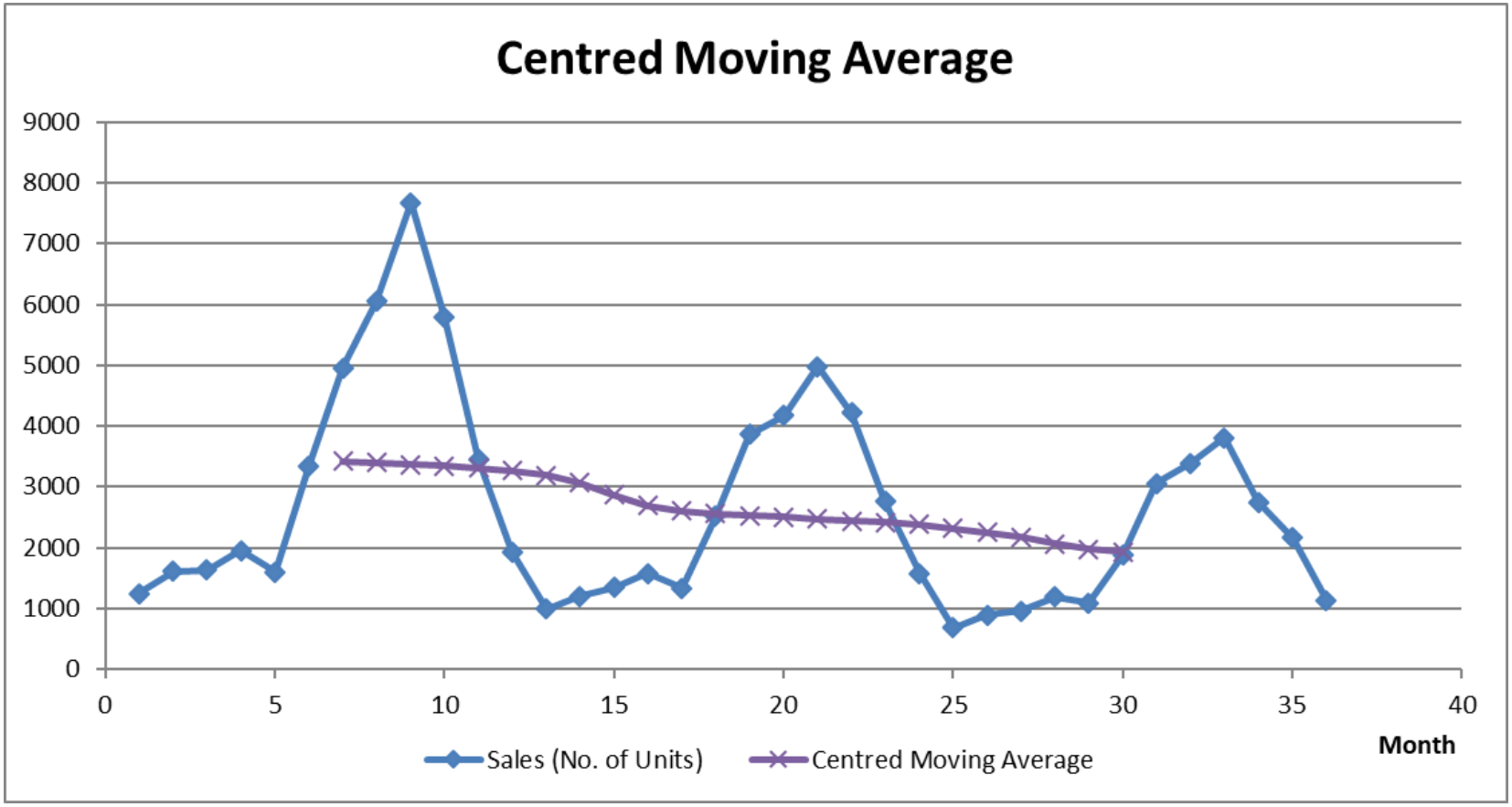
Refer to the link below for Multiplicative Decomposition e-learning video:

https://docs.google.com/open?id=0B9sGwZfXz0MkekxwNzhSSk4wWjg

Digital Camera Monthly Sales: CMA and Seasonal Ratio

- By observing the Digital Camera Monthly Sales data, the repetitive cycle is 12 months from October to September of the next year. So the number of averaging periods N should be 12.
- As N = 12 is an even number, CMA of Period 7 (Apr-16) will be:
  - (1245+1605+ … +3455+1935)/12 = 3436.250 (MA for Period 6.5 put in Period 6)
  - (1605+1635+ … +1935+990)/12 = 3415.000 (MA for Period 7.5 put in Period 7)
  - (3426.250 + 3415.000)/2 = 3425.625 (CMA for Period 7)
- Because the actual sales of Period 7 (Apr-16) is 4960, its Seasonal Ratio will be:
  - 4960 /3425.625= 1.448

Digital Camera Monthly Sales: CMA



- CMA removes seasonal variation from the actual sales data.

Digital Camera Monthly Sales: Seasonal Index

- Group the Seasonal Ratio for like seasons and get the average for each season, for example:
  - Season 3 (Dec): (0.466+0.439)/2 = 0.453
  - Season 9 (Jun): (2.278+2.010)/2 = 2.144

  The normalizing factor is calculated as:

  - 12/(0.303+0.394+ … +1.091+0.628) = 12/12.040 = 0.99667
  - In this case, the normalizing factor is very close to 1
  - Multiply the normalizing factor with the seasonal average to obtain the Normalized Seasonal Index:

Season	1 (Oct)	2 (Nov)	3 (Dec)	4 (Jan)	5 (Feb)	6 (Mar)	7 (Apr)	8 (May)	9 (Jun)	10 (Jul)	11 (Aug)	12 (Sep)
Normalized Average	0.302	0.393	0.451	0.579	0.529	0.975	1.482	1.717	2.137	1.722	1.088	0.626

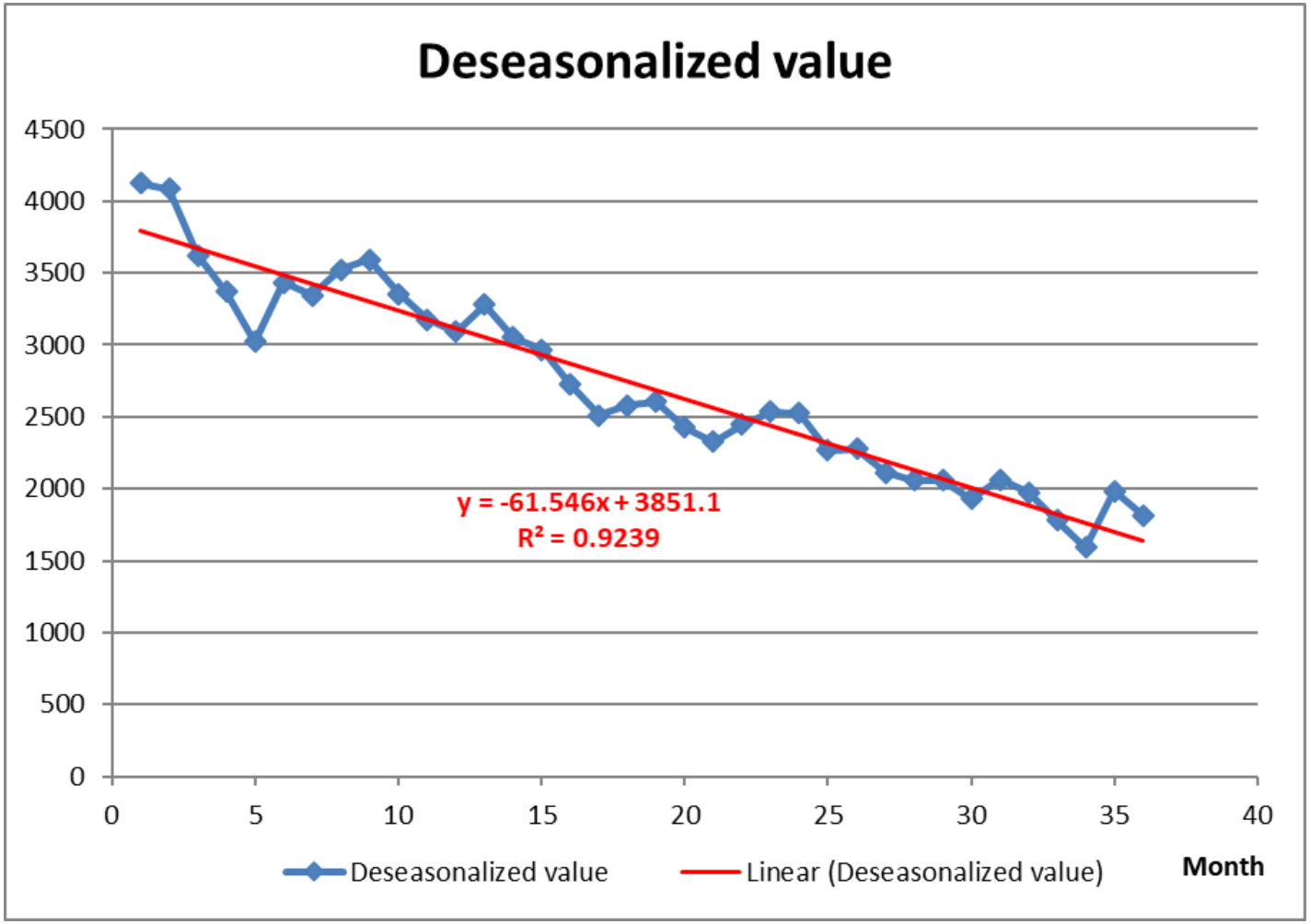
Digital Camera Monthly Sales: Deseasonalized Values

Input Data			Seasonal Index Computation				
Month	Sales (No. of Units) (Y)	Time Period (X)	Moving Average (e.g. 12 months)	Centred Moving Average ($=(MA_{i-1}+MA_i)/2$)	Seasonal Ratio ($=Y_i/CMA_i$)	Seasonal Index (= normalized average of seasonal ratio)	Deseasonalized value ($= Y_i$ / Seasonal $Index_i$)
Oct-15	1245	1				0.302	4124.674
Nov-15	1605	2				0.393	4084.241
Dec-15	1635	3				0.451	3622.294
Jan-16	1950	4				0.579	3370.419
Feb-16	1600	5				0.529	3021.774
Mar-16	3345	6	3436.250			0.975	3432.523
Apr-16	4960	7	3415.000	3425.625	1.448	1.482	3346.738
May-16	6050	8	3381.250	3398.125	1.780	1.717	3523.293
Jun-16	7675	9	3356.667	3368.958	2.278	2.137	3591.643
Jul-16	5780	10	3325.417	3341.042	1.730	1.722	3356.808
Aug-16	3455	11	3302.917	3314.167	1.042	1.088	3176.962
Sep-16	1935	12	3233.750	3268.333	0.592	0.626	3092.150

Deseasonalised value = $\frac{1605}{0.393}$

* Deseasonalized value = Y_i / Seasonal $Index_i$

Digital Camera Monthly Sales: Deseasonalized Value



After removing the seasonal effect, the data exhibits an observable linear trend with random variations.

Digital Camera Monthly Sales: Deseasonalized Forecast /Seasonalized Forecast

Time Period (X)	Seasonal Index (= normalized average of seasonal ratio)	Deseasonalized value (= Y_i / Seasonal $Index_i$)	Deseasonalized forecast (=slope*X_i + intercept)	Seasonalized forecast (= Seasonal $Index_i$ *Deseasonalized $forecast_i$)
1	0.302	4124.674	3789.524	1143.838
2	0.393	4084.241	3727.978	1464.998
3	0.451	3622.294	3666.431	1654.923
4	0.579	3370.419	3604.885	2085.653
5	0.529	3021.774	3543.339	1876.164
6	0.975	3432.523	3481.793	3393.013
7	1.482	3346.738	3420.247	5068.943
8	1.717	3523.293	3358.701	5767.371
9	2.137	3591.643	3297.154	7045.706
10	1.722	3356.808	3235.608	5571.309
11	1.088	3176.962	3174.062	3451.846
12	0.626	3092.150	3112.516	1947.744

Intercept	**3851.070**
Slope	**-61.546**
Equation Y(X)	**967.225+52.501*X**

Then calculate seasonalized forecast

$$\text{Seasonalized Forecast} = 0.302 * 3789.524$$

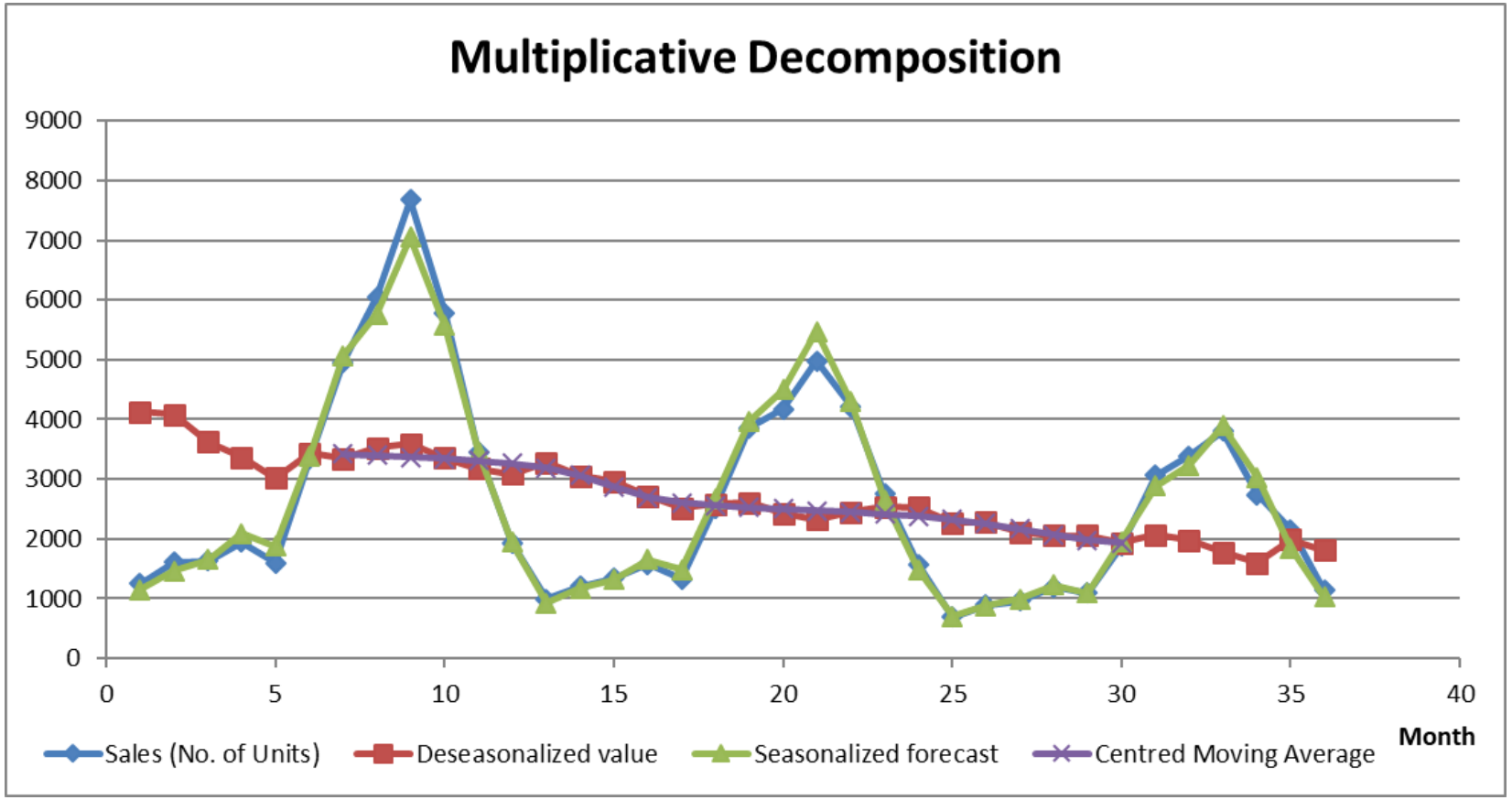
First calculate deseasonalized forecast

$$\text{Deseasonalized Forecast} = 3851.070 - 61.546 * 1$$

Deseasonalized forecast = slope*Xi + intercept

Seasonalized forecast = Seasonal $Index_i$ * Deseasonalized $Forecast_i$

Digital Camera Monthly Sales: Actual Value/Deseasonalized Value /Seasonalized Forecast



The seasonalized forecast and the actual values are quite close, so the multiplicative model built is valid to be used to do the forecasting.

Digital Camera Monthly Sales: Cyclical and Random Components

Input Data			Forecast & Error	Further decomposition for Cyclicity and Irregular Variations		
Month	Sales (No. of Units) (Y)	Time Period (X)	Seasonalized forecast (= Seasonal Index$_i$ *Deseasonalized forecast$_i$)	Cyclicity*Irregular (=Yi/Seasonalized forecasti)	Cyclicity (3 months MA)	Irregular Variations (=Cyclicity*Irregular$_i$ /Cyclicity$_i$)
Oct-15	1245	1	1143.838	1.088		
Nov-15	1605	2	1464.998	1.096	1.057	1.036
Dec-15	1635	3	1654.923	0.988	1.006	0.982
Jan-16	1950	4	2085.653	0.935	0.925	1.011
Feb-16	1600	5	1876.164	0.853	0.925	0.922
Mar-16	3345	6	3393.013	0.986	0.939	1.050
Apr-16	4960	7	5068.943	0.979	1.004	0.974
May-16	6050	8	5767.371	1.049	1.039	1.010
Jun-16	7675	9	7045.706	1.089	1.059	1.029
Jul-16	5780	10	5571.309	1.037	1.043	0.995
Aug-16	3455	11	3451.846	1.001	1.011	0.990
Sep-16	1935	12	1947.744	0.993	1.023	0.971

$$I = \frac{1.096}{1.057}$$

Lastly, estimate the Irregular Variations

$$C * I = \frac{1245}{1143.838}$$

First estimate the Cyclicity*Irregular

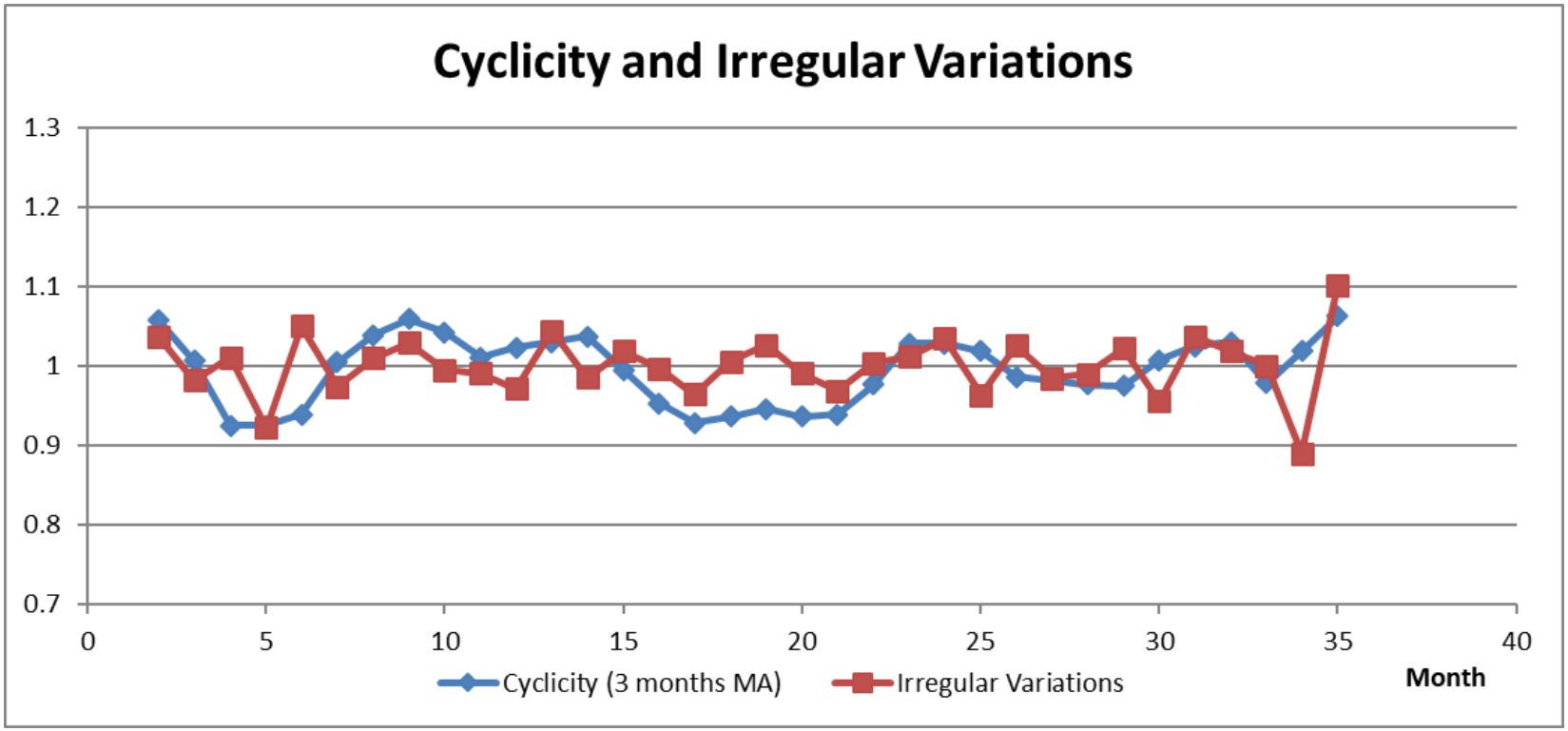
$$MA = \frac{1.088 + 1.096 + 0.988}{3}$$

Then estimate the Cyclicity factor

Cyclicity * Irregular$_i$ = Y_i / Seasonalized forecast$_i$

Irregular Variations = (Cyclicity * Irregular$_i$) / Cyclicity$_i$

Digital Camera Monthly Sales: Cyclical and Random Components

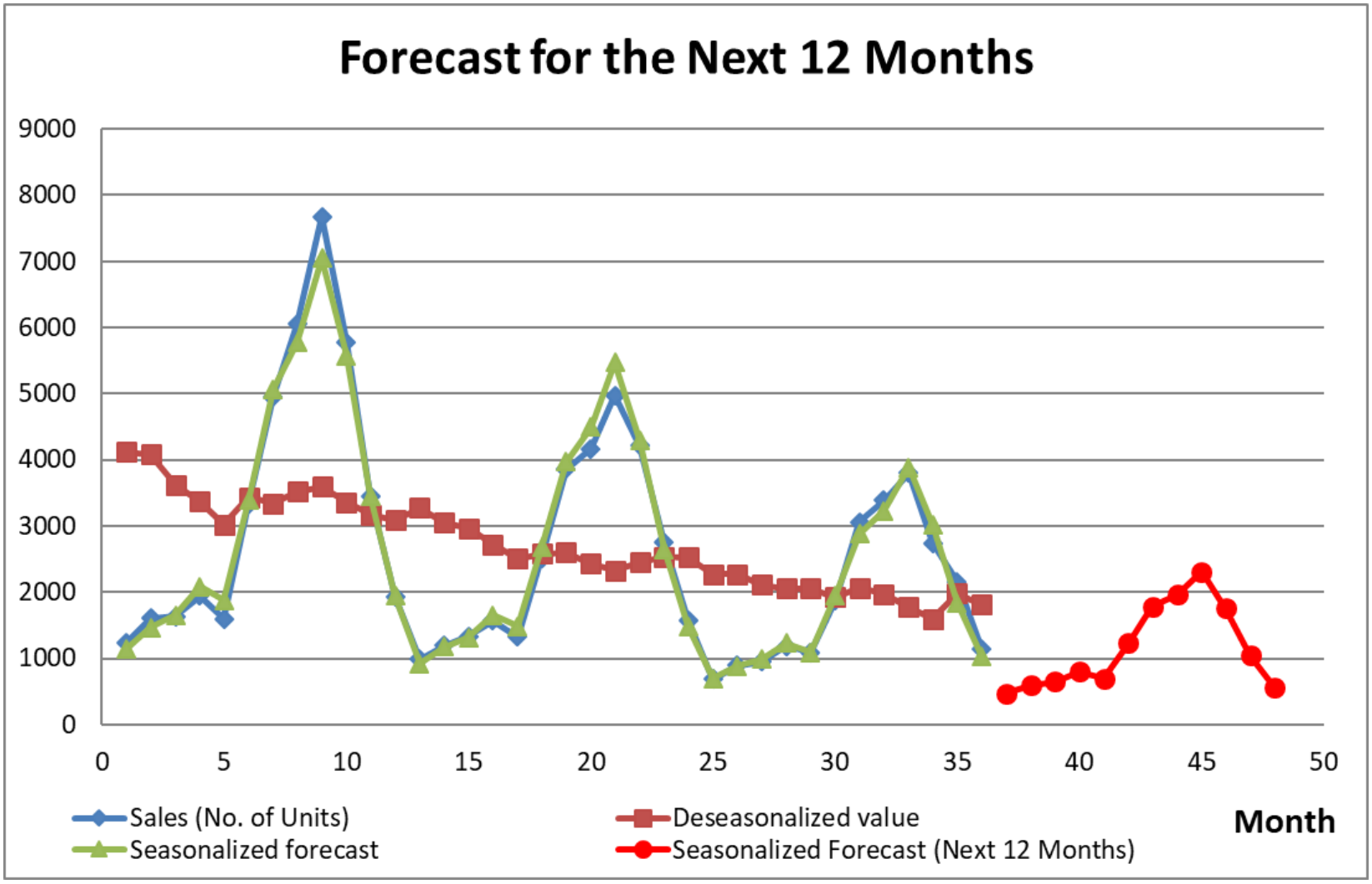


Most of the cyclical and random factors are between 0.9 and 1.1, which can be considered as minor. Therefore they can be ignored for the forecast of the Digital Camera Monthly Sales in the next 12 months.

Digital Camera Monthly Sales: Forecast for the Next 12 Months using the Model Built

Forecast for the next 12 months				
Month	Time Period	Deseasonalized Forecast	Seasonal Index	Seasonalized Forecast (Next 12 Months)
Oct-18	37	1573.861	0.302	475.057
Nov-18	38	1512.315	0.393	594.300
Dec-18	39	1450.769	0.451	654.836
Jan-19	40	1389.223	0.579	803.753
Feb-19	41	1327.676	0.529	702.992
Mar-19	42	1266.130	0.975	1233.846
Apr-19	43	1204.584	1.482	1785.242
May-19	44	1143.038	1.717	1962.760
Jun-19	45	1081.492	2.137	2311.045
Jul-19	46	1019.945	1.722	1756.217
Aug-19	47	958.399	1.088	1042.275
Sep-19	48	896.853	0.626	561.231

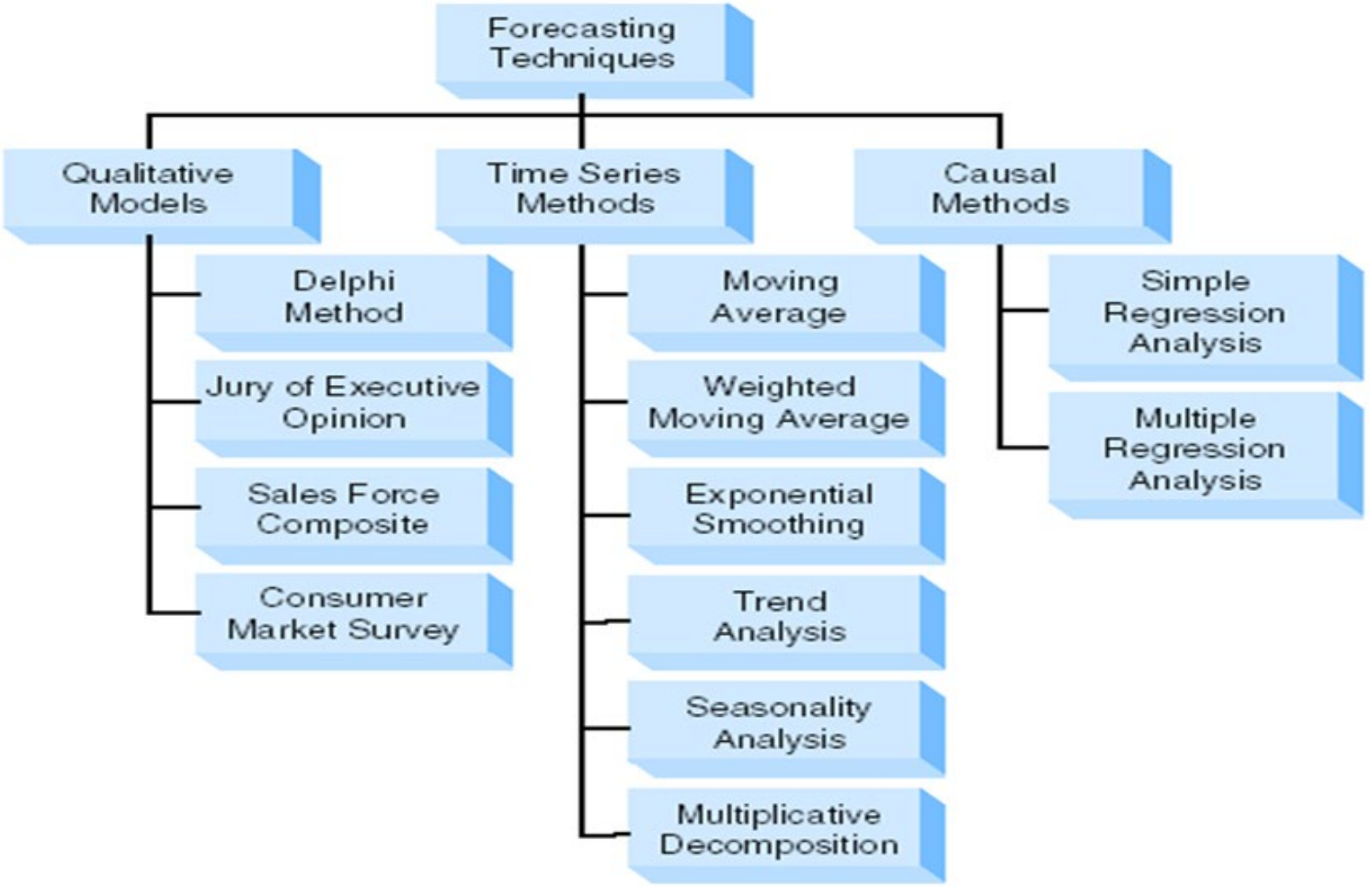
Digital Camera Monthly Sales: Seasonalized Forecast for the Next 12 Months



Summary

- Multiplicative Decomposition is an intuitive method for forecasting, which is useful for time series analysis with fixed describing parameters.
- Forecasting can be done with the Seasonal and the Trend component.
- There’re various forecasting methods, none is perfect under all conditions.

Summary: Types of Forecasts



Summary: Selection of Forecasting Methods

1. When the data exhibits correlation between two variables,
   - Forecast of a dependant variable using one independent variable can be done through Linear Regression.
   - For example, use Mobile Phone Monthly Sales (independent variable) to forecast Distribution Expense of Mobile Phone (dependent variable).
2. When the data is a time series,

a) If the data exhibit seasonality (cyclical or periodic fluctuation),
   - Apply the multiplicative (increasing/decreasing seasonal variation) or additive (constant seasonal variation) decomposition methods.
   - For example, the forecast for the Digital Camera Monthly Sales.

b) Conduct linear regression and conclude whether the trend is significant based on the P-value of slope.
   - If there is no trend, moving average and exponential smoothing methods can be used.
     - For example, the forecast for the Mobile Phone Monthly Sales.
   - If there is a trend, choose between linear regression and Holt's method.
     - If recent data's trend is significant, Holt's method may be better. Linear regression treats all data points in time series equally.
     - For example, the forecast for the Fitness Tracker Monthly Sales.

3. Forecasting is still only a method of predicting future events and thus, validation should be performed with/when actual data is available. Only when actual data is available, then we can compute the forecasting errors for the methods to decide which is better.

Learning Objectives

- Perform forecasting on time series data with trend and seasonality.
- Apply the procedure of multiplicative decomposition method using concepts of Centred Moving Average (CMA), Seasonal Ratio, Seasonal Index and deseasonalized forecast to derive the forecast for time series data with trend and seasonal components.
- Investigate the impact of the cyclical and irregular components.

Overview of E211 Operations Planning II Module