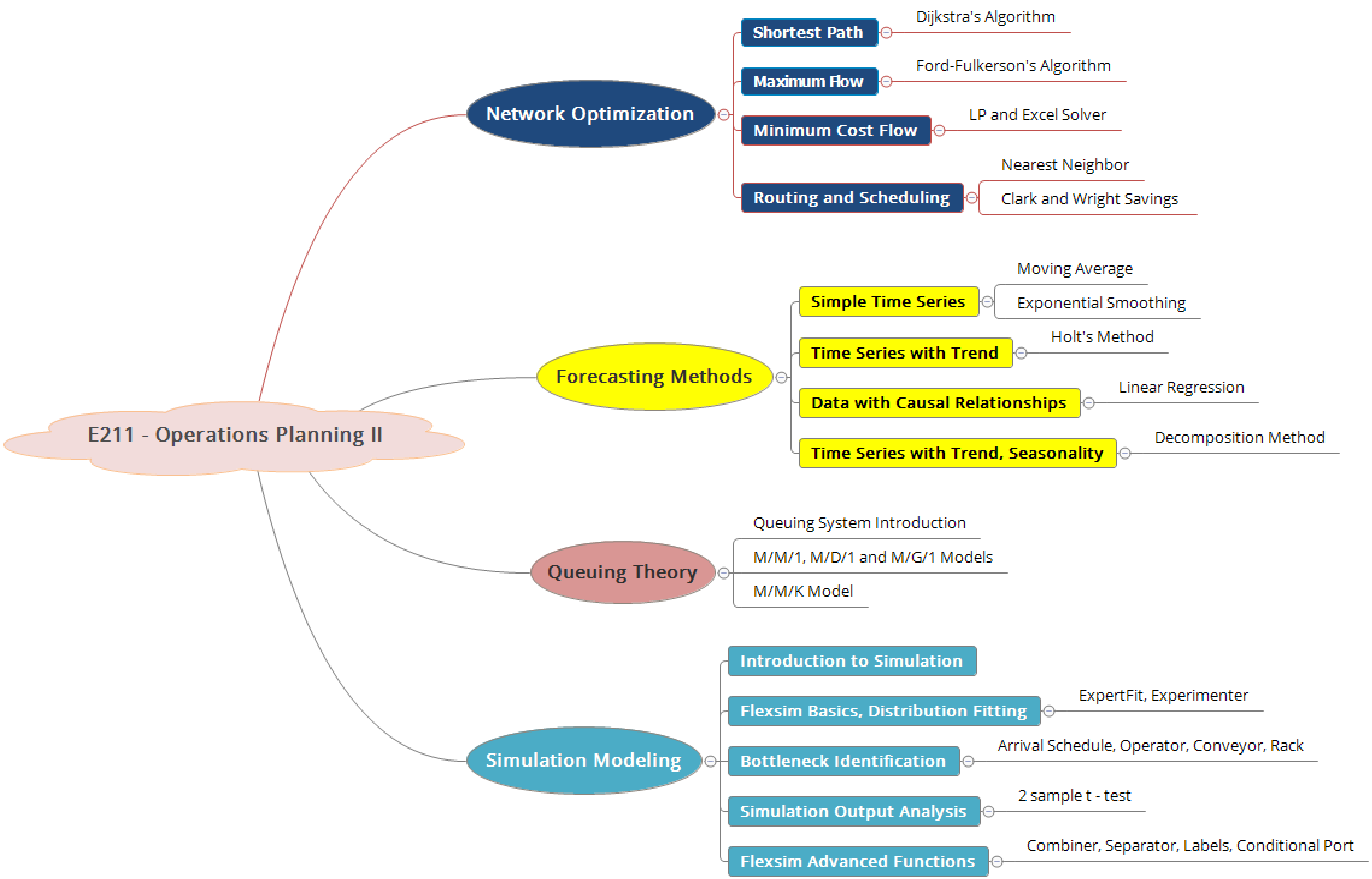
Problem 09

Berthing Capacity Analysis

E211 – Operations Planning II

SCHOOL OF ENGINEERING

Module Coverage: E211 Topic Tree



Recall: Factors Influencing Waiting

- Line configuration (one long line or several shorter ones)
- Jockeying (switching between queues)
- Balking (not joining queue if too long)
- Priority (service order) Queue discipline
- Tandem queues (second service needed?)
- Homogeneity (all customers require same service?)

Recall: Classification of Queuing Systems

- The **Kendall** classification of queuing systems (1953) exists in several modifications. In general, classification uses 6 symbols(parameters):

A/B/C/D/E/F

- **Parameter A -** the arrival pattern (distribution of intervals between arrivals).
  - M: Poisson (Markovian) process with exponential distribution of intervals
  - D: Deterministic(Constant) inter-arrival times
  - G: Inter-arrival time follows a general (any) distribution
- **Parameter B** – the service pattern (distribution of service duration).
  - M: Poisson (Markovian) process with exponential distribution of service duration.
  - G: Service duration follows a general (any) distribution
  - D: Deterministic(Constant) service duration
- **Parameter C** – the number of servers

Recall: Classification of Queuing Systems

A/B/C/D/E/F

- **Parameter D -** the queuing discipline (FIFO, LIFO, ...). Omitted for FIFO or if not specified.
- **Parameter E** – the system capacity. Omitted for unlimited queues.
- **Parameter F** – the population size (number of possible customers). Omitted for infinite (open) systems.

Examples

- **D/M/1/LIFO/10/50**: Deterministic (known) arrivals, one server with exponentially distributed service time, queue is limited by a maximum size of 10 with a last-in-first-out queueing discipline, and the total number of customers is 50.
- **D/G/3**: Multiple-server (3 servers) queuing model with deterministic (known) arrivals, service time following a general distribution (e.g.. Normal), FIFO queuing discipline, infinite number of waiting positions, and unlimited customer population.
- **M/M/1**: Single server queuing model with Poisson arrivals, exponentially distributed service time, FIFO queuing discipline, unlimited waiting positions, and unlimited customer population.

Recall: Examples of Queuing Models

- Single-channel Queuing Model (M/M/1)
  - Example: Information booth at shopping center
- Multi-channel Queuing Model (M/M/k)
  - Example: Airline ticket counter
- Constant Service Time (M/D/1)
  - Example: Automated car wash
- Limited Population
  - Example: Manufacturing site with only 10 machines that need repair service

What do these symbols mean?

Recall: M/M/1 Model Characteristics

- Type: Single-channel, single-phase system
- Input source: **Infinite; no balks, no reneging**
- Arrival distribution: Poisson
- Queue: Unlimited; single line
- Queue discipline: FIFO (FCFS)
- Service distribution: Negative exponential
- Relationship between arrival and service times: Service rate is independent of arrival rate
- Service rate is greater than arrival rate

Recall: M/M/1Queue Performance Measures

SINGLE SERVER QUEUE: M/M/1

Utilization Factor (a measure of how busy the system is) = $\rho = \lambda / \mu$

Prob {0 customers in the system} = Prob {system is idle} = $P_0 = 1 - \rho = 1 - \lambda / \mu$

Prob {customer has to wait for service} = $P_W = \lambda / \mu$

Prob {*n* customers in the system} = $P_n = P_0(\rho)^n = (1-\rho)\rho^n$ has a geometric distribution

L_s = mean number of customers in the system = $\dfrac{\lambda}{\mu - \lambda} = \dfrac{\rho}{1-\rho}$

L_q = mean number of customers in the waiting line (queue) = $\dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{\rho^2}{1-\rho}$

W_s = mean time a customer spends in the system = $\dfrac{1}{\mu - \lambda} = \dfrac{L_s}{\lambda}$

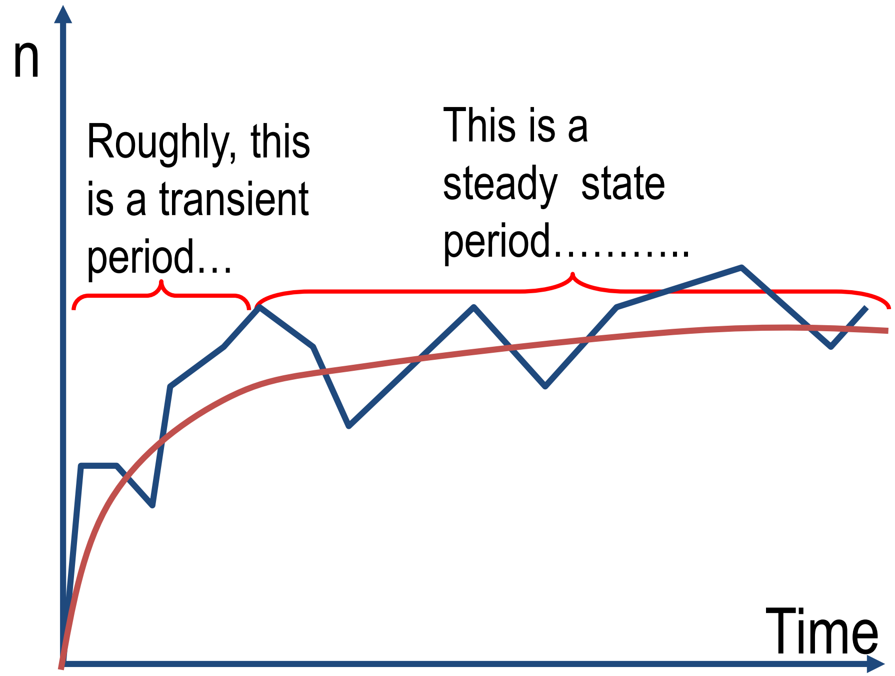
W_q = mean time a customer spends waiting (in the line/queue) = $\dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{L_q}{\lambda}$

Note: $W_s = W_q + \dfrac{1}{\mu}$

Transient and Steady State Periods

- Queue Performance is measured for steady state

Initial (transient) behaviour not representative of long run performance



EQUILIBRIUM CONDITION:

In order to achieve steady state, effective arrival rate (λ) must be less than sum of effective service rates (μ).

$\lambda < \mu$ (for single server)

$\lambda < K\mu$ (for K servers)

Transient and Steady State Periods

- In transient period (start-up bias), initial system behavior is not representative of long-run performance.
- After transient period, the system settles into steady state where long-run probabilities remain constant over time.
- For a queuing system to reach steady state, effective arrival rate of customers must be **<u>lesser</u>** than sum of effective service rates of all servers.

Queuing System Performance Measures

- Average queue time, *Wq*
- Average queue length, *Lq*
- Average time in system, *Ws*
- Average number in system, *Ls*
- Probability of idle service facility, P_0
- System utilization ρ
- Probability of *n* units in system, P_n

Little's Formulas

- Little's Formulas represent important relationships between L_s or L , L_q, W_s or W, and W_q.
- Provided:
  - System has <u>Single Queue</u>,
  - Customers arrive at a finite arrival rate λ, and
  - System operates in steady state
- **No assumptions on arrival or service time distributions or queue limits**

$$\boxed{W_q = \frac{L_q}{\lambda}} \qquad \boxed{L_s = L_q + \lambda/\mu} \qquad \boxed{W_s = \frac{L_s}{\lambda}}$$

λ (lambda) – Arrival rate

μ (Mu) – Service rate

Assumptions of the Basic Simple Queuing Model with k servers (M/M/k)

- **Arrival**
  - ➢ Arrivals are served on a first come, first served basis **FCFS**
  - ➢ Arrivals are independent of preceding arrivals. **Memoryless**
  - ➢ Arrivals are described by the **Poisson probability distribution**, and customers come from a **very large population**
- **Service time**
  - ➢ Service times vary from one customer to another, and are independent of one another; the average service time is known. **Random and Memoryless**
  - ➢ Service times are described by the negative exponential probability distribution
  - ➢ The sum of service rate of all servers is greater than the arrival rate

Multi-Channel (M/M/k) Model Characteristics

- Type: Multi-Channel system
- Input source: Infinite; no balks, no reneging
- Arrival process: Number of arrivals follows Poisson distribution
- Queue: Unlimited; single line
- Queue discipline: FIFO (FCFS)
- Service time: Negative exponential distribution
- Relationship between arrival and service rate: Service rate is independent of arrival rate
- The sum of service rate of all servers is greater than arrival rate

M/M/k Performance Measures

MULTIPLE SERVER QUEUE: M/M/k

k = number of channels or servers

Utilization Factor = $\lambda / k\mu$

$$P_0 = \frac{1}{\left[\sum_{n=0}^{k-1} \frac{(\lambda/\mu)^n}{n!}\right] + \frac{(\lambda/\mu)^k}{k!}\left(\frac{k\mu}{k\mu - \lambda}\right)}$$

For example if k = 3: $$P_0 = \frac{1}{1 + \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2}{2!} + \frac{(\lambda/\mu)^3}{3!}\left(\frac{3\mu}{3\mu - \lambda}\right)}$$

Continue in this order:

$$L_q = \frac{(\lambda/\mu)^k \lambda\mu}{(k-1)!(k\mu - \lambda)^2} P_0$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{k!k^{n-k}} P_0 & \text{for } n > k \\ \frac{(\lambda/\mu)^n}{n!} P_0 & \text{for } 0 \le n \le k \end{cases}$$

$$P_w = \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{k\mu}{k\mu - \lambda}\right) P_0$$

P_0 =probability of no object in the queuing system

P_n =probability of n objects in the queuing system

P_w =probability of at least k objects in the queuing system; i.e., probability that an object needs to wait before getting the service.

elearning video on calculation of M/M/K model performance measures:
https://docs.google.com/file/d/0B9sGwZfXz0MkdFJGcjZLMWp5a2c/edit?usp=drive_web&pli=1

Application of Waiting Line Models

I. Design Service Operations

- **Arrival rates** - Adjust through advertising, promotions, pricing, appointments.
- **Number of service facilities** - Adjust service system capacity.
- **Number of phases** - Consider splitting service tasks.
- **Number of servers per facility** - Work force size.
- **Server efficiency** - Training, incentives, work methods, capital investment.
- **Priority rule** - Decide whether to allow pre-emption.
- **Line arrangement** - Single or multiple lines.

Application of Waiting Line Models

II. Analyse Service Operations

- **Balance costs** against benefits of improving service system. Also, consider the costs of not making improvements.
- **Line length** - Long lines indicate poor customer service, inefficient service, or inadequate capacity.
- **Number of customers in system** - A large number causes congestion and dissatisfaction.
- **Waiting time in line** - Long waiting times are associated with poor service.
- **Total time in system** - May indicate problems with customers, server efficiency, or capacity.
- **Service facility utilization** - Control costs without unacceptable reduction in service.

Problem 09
Suggested Solution

The Problem Statement
- Queuing Model Representation

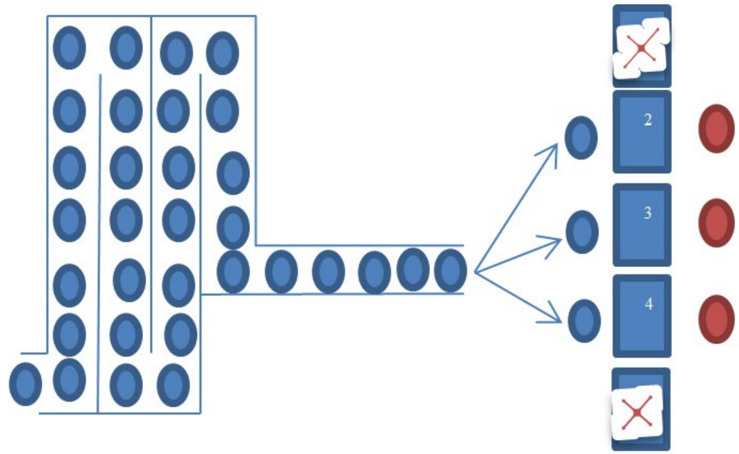
- The problem statement is to determine the right number of berths to meet the increasing customer demands. It is a berthing capacity expansion problem.
- Model the berthing capacity expansion problem as a queueing system.

Component	Characteristics
Arrival: Vessels arrive at the port	Infinite population; exponentially distributed inter-arrival time
Queue: Waiting line of Vessels before they are guided to the berths	Unlimited length (assuming no physical/technological constraint for queue); FCFS rule
Service facility: Berths and facilities needed in handling containers	Multi-server, single phase, exponentially distributed service time

The Problem Statement
- Queuing Model Representation

- M/M/k

λ Arrival - Poisson



μ Service time – negative exponential

K = 27 berths

- Mr Lee observed that on average, 36 vessels arrived every 24 hours. Mean arrival rate (λ) = 36/24 = 1.5 per hour
- The average port stay time at each berth is estimated to be 15.6 hours. Mean service rate (μ) = 1/15.6 = 0.0641 per hour
- Since $\lambda \leq k\mu$, the queuing system will reach steady state after the initial transient period. System behaviour will be representative of long-run performance.

The Problem Statement
- M/M/k Model Assumptions

- Type: Multi-Channel system (Since currently the wharf has 27 berths)
- Input source: Infinite; all vessels heading to BlueSky Port will proceed to join the queue. Assume no balking and reneging.
- Arrival process: Number of vessel arrivals follows Poisson distribution (no batch arrivals, arrivals are independent of each other)
- Queue: Unlimited length; single line
- Queue discipline: FIFO (FCFS)
- Service time: Negative exponential distribution. Assume that service starts when vessel is guided by the pilot to the berth allocated to it.
- Relationship between arrival and service rate: Service rate is independent of arrival rate. The sum of service rate of all servers is greater than arrival rate

The Problem Statement
- Analysis of Performance Measures

- Management will be concerned with
  - Berth (and the necessary facilities) utilisation
  - Average number of vessels in queue
  - Average number of vessels in system (at the wharf)
  - Average waiting time in queue
  - Average stay time in system (at the wharf)
  - Safety and Security issues

- Shipping Lines' interests
  - Number of vessels in queue
  - Delays (Average waiting time in queue)
  - Average stay time in system (at the wharf)

The Problem Statement
- Scenario 1: Current Situation

- 36 vessels arrive per 24 hours
- Average port stay time (service time) of 15.6 hours
- 27 berths (servers)

Lambda	arrival rate	1.500	per hour
Mu	service rate	0.0641	per hour
k	no. of servers	27	
arrival rate/sum of service rate (Utilisation)	ρ	0.8667	
Prob (no vessels in system)	P_0	5.80E-11	
Average no. of vessels in queue	Lq	2.4153	
Average no. of vessels in system	Ls	25.8153	
Average waiting time in queue	Wq	1.6102	hours
Average time in system	Ws	17.2102	hours

- Current service level at BlueSky Port is acceptable with average waiting time within 2 hours as stipulated in the agreement with the shipping lines

The Problem Statement
Scenario 1: Current Situation

Calculate the probability of:

1) At most 25 vessels in the queuing system.
2) More than 30 vessels in the queueing system.

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0 \qquad \text{for } 0 \le n \le k. \qquad \text{-----(1)}$$

$$P_n = \frac{(\lambda/\mu)^n}{k!\,k^{n-k}} P_0 \qquad \text{for } n>k. \qquad \text{-----(2)}$$

1) Using equation (1) as n = 25< 27 (number of berths, k):
   P(n<=25) = P0+P1+…P25 = 0.5713

2) For P(n>30) = 1 – P(n<=30)
   = 1- P(n<=27) [using eqn. (1)] – P(28<=n<= 30) [using eqn. (2)]
   = 1 – 0.6780 - 0.1124
   = 0.2096

The Problem Statement
- Scenario 2: After Expansion

- 52 vessels arrive per 24 hours
- Average port stay time (service time) remains to be 15.6 hours

Lambda	arrival rate	2.167	per hour
Mu	service rate	0.0641	per hour
k	no. of servers	38	
arrival rate/sum of service rate (Utilisation)	ρ	0.8895	
Prob (no vessels in system)	P_0	1.75E-15	
Average no. of vessels in queue	Lq	3.0543	
Average no. of vessels in system	Ls	36.8543	
Average waiting time in queue	Wq	1.4097	hours
Average time in system	Ws	17.0097	hours

- Need at least 38 berths to ensure average waiting time is within 2 hours as stipulated in the agreement with the shipping lines.

The Problem Statement
- Scenario 2: After Expansion

- 52 vessels arrive per 24 hours
- Average port stay time (service time) reduces to 13.8 hours

Lambda	arrival rate	2.167	per hour
Mu	service rate	0.0725	per hour
k	no. of servers	34	
arrival rate/sum of service rate (Utilisation)	ρ	0.8794	
Prob (no vessels in system)	P_0	8.74E-14	
Average no. of vessels in queue	Lq	2.6662	
Average no. of vessels in system	Ls	32.5662	
Average waiting time in queue	Wq	1.2306	hours
Average time in system	Ws	15.0306	hours

- Need at least 34 berths to ensure average waiting time is within 2 hours as stipulated in the agreement with the shipping lines.

Recommendations

- To avoid uncontrollable queuing situation, BlueSky Port should ensure the effective service rate is greater than the arrival rate.
- To meet higher customer demand of 52 vessels per 24 hours, BlueSky Port should expand berthing capacity with more berths to ensure average waiting time is within 2 hours.
  - With the same average port stay time of 15.6 hours, 38 berths are required
  - With the average port stay time reduced to 13.8 hours, 34 berths are required
- BlueSky Port could consider reducing the port stay time by:
  - Simplify and streamline the vessel operations needed in the loading/unloading of the containers
  - Ensure enough capacity and high technology of the facilities involved in vessel operations
  - Train staff to increase productivity in operating facilities such as cranes, and in handling of containers
- Assumptions and limitations of the queuing model:
  - Estimation of arrival pattern using Poisson distribution
  - Estimation of service time using negative exponential distribution
  - Use of FCFS queue discipline

Learning Objectives

At the end of today's lesson, students should be able to:

- Explain the characteristics of an M/M/k queuing model.
- Calculate the queue performance measures of M/M/k queuing model with given data.
- Analyse the queue performance so as to optimally match available resource to meet customer expectations and service level.
- Discover the limitations of the queuing model.

Overview of E211 Operations Planning II Module