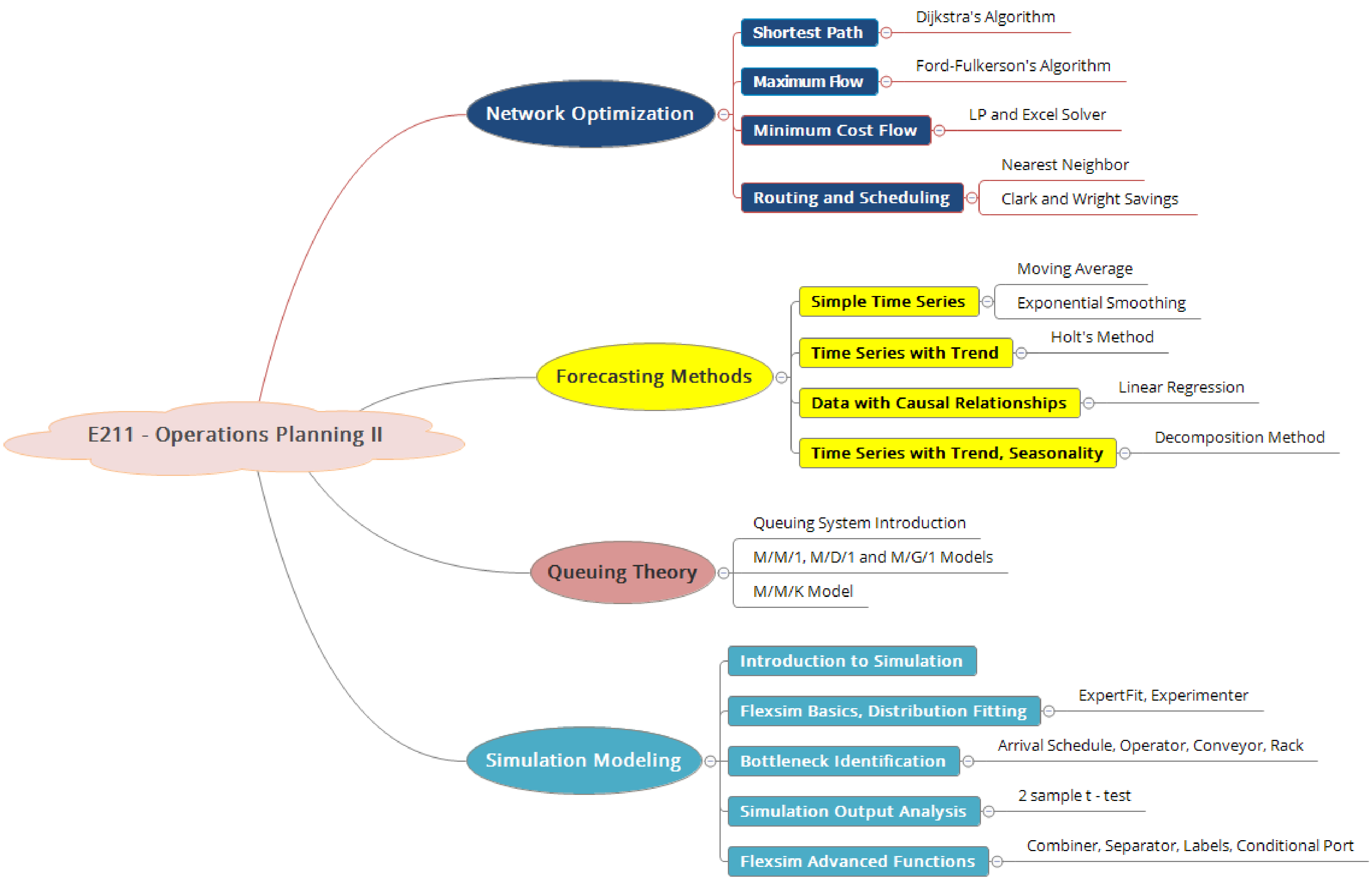
Problem 03
The Right Delivery Sequence

E211 – Operations Planning II

SCHOOL OF ENGINEERING

Module Coverage: E211 Topic Tree



Routing and Scheduling Problems

- The scheduling of customer service and the routing of service vehicles are at the heart of many service operations.
  - Service routing: dispatching of installation or repair technicians
  - Passenger routing: routing and scheduling of school buses, public buses, ambulances
  - Freight routing: pickup and delivery service of logistics firms; postal and parcel delivery
- Scheduling involves planning **the timing** for each location to be visited.
- Routing involves forming **the sequence** in which locations are to be visited.

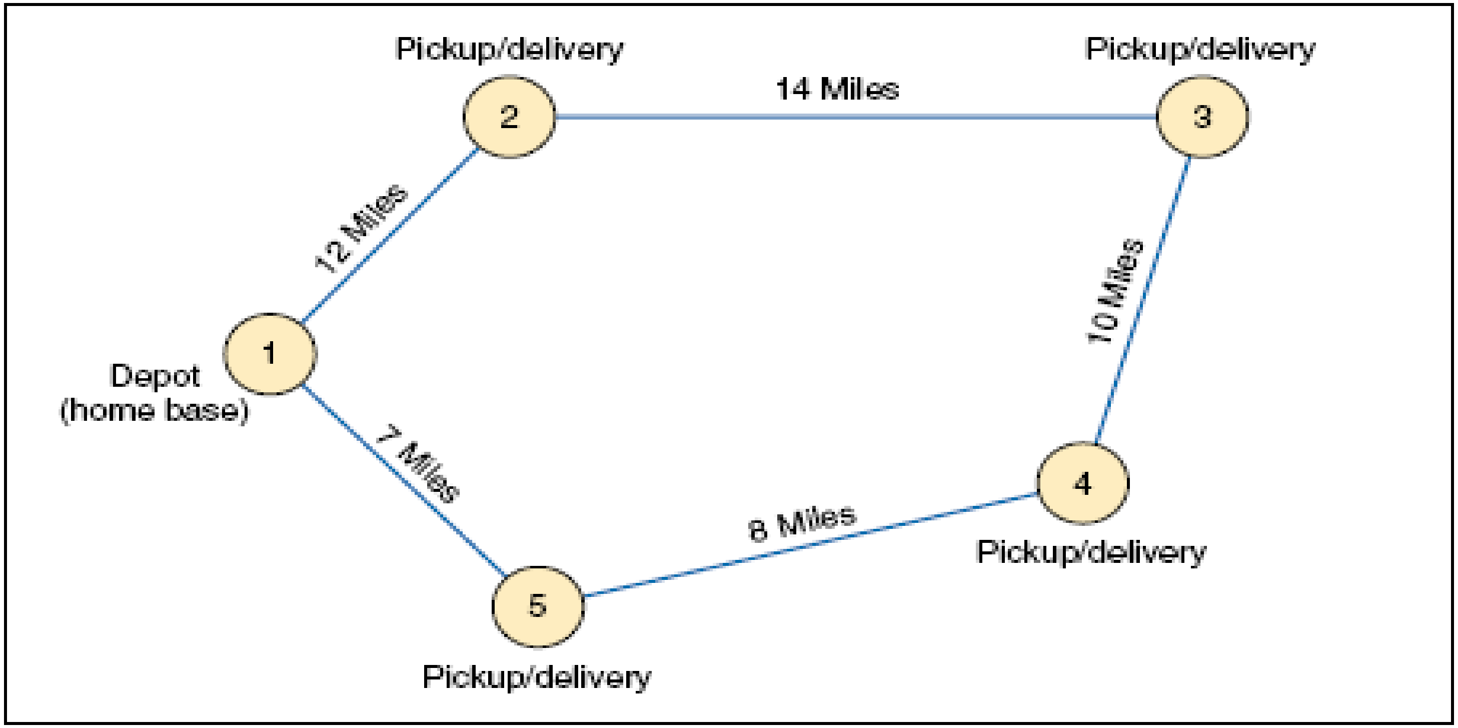
Objectives and Constraints of Routing and Scheduling Problems

- Minimize total cost of service delivery or route by one or more of the following:
  - Minimize total distance traveled
  - Minimize total time traveled
  - Minimize number of vehicles utilized
  - Minimize number of personnel on duty
- Subject to the following considerations:
  - Vehicles
    - ✓ Capacities
    - ✓ Release Time, Maximum Time and Down Time
  - Customers
    - ✓ Time windows (Timing open for receiving of goods)
    - ✓ Priority
    - ✓ Pickup and delivery

Characteristics of Routing and Scheduling Problems

- Often can be represented using network diagrams
- Consist of nodes and arcs
  - ➢ Nodes represent locations to be visited. Depot node is the begin/end location
  - ➢ Arcs represent the possible paths among the locations. Arcs can be directed (one-way, represented by arrow) or undirected (two ways, represented by line)
- Schedule – time for each node to be visited
- Route – sequence in which nodes are to be visited

Feasibility of a Tour



Example of a tour for the above network: 1 → 2 → 3 → 4 → 5 → 1

For a tour to be feasible,

1. It must start and end with the depot node
2. All nodes must be included
3. A node must be visited only once

Classification of Routing and Scheduling Problems

- Most commonly seen routing and scheduling problems can be classified into the following categories:

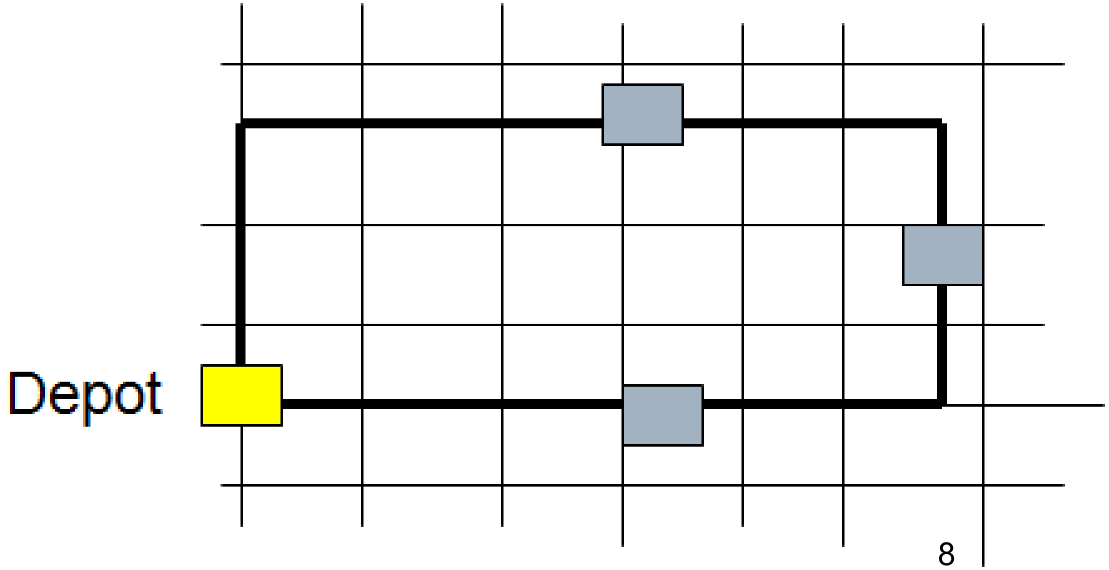
  - Traveling salesman problem (TSP) (Today's Problem)
  - Multiple traveling salesman problem (MTSP)
  - Vehicle routing problem (VRP) (Today's Problem)
  - Vehicle routing problem with time windows (VRPTW)
  - Pickup and delivery problem with time windows (PDPTW)

Travelling Salesman Problem

- Given a set of towns and the distances between them, the TSP determines the shortest tour starting from a given town, passing through all the other towns exactly once and returning to the first town.

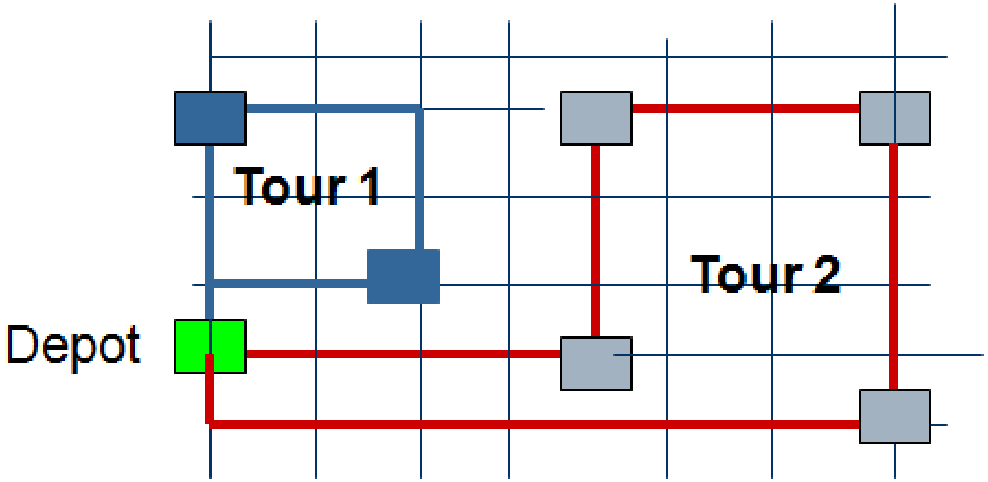
- Example - sales representative attending appointments at a number of different locations before returning to base.

- Type of decisions
  - Routing



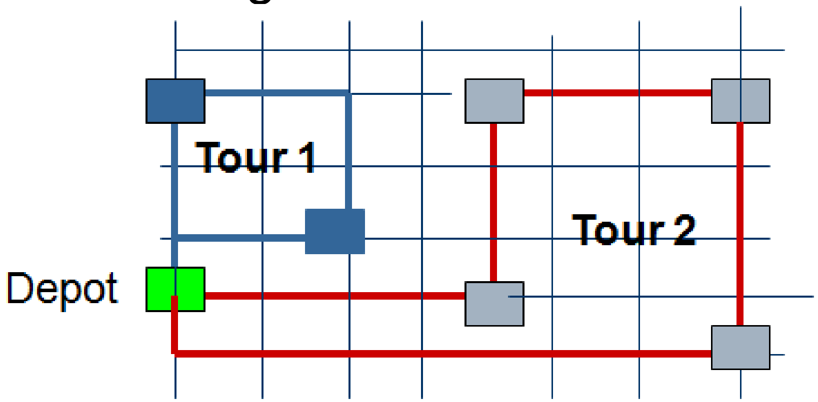
Multiple Travelling Salesman Problem

- The MTSP is a generalization of the TSP, where more than one salesman (vehicle) is needed to form the tours due to the limit on time (distance) travelled by each salesman.
- In MTSP, all the salesman (vehicle) are to leave from and return to a common depot.
- Typical applications of MTSP are in service routing
    - Example - dispatching of more than one installation/repair technician
- Type of decisions
    - Assigning
    - Routing



Vehicle Routing Problem

- In VRP, a fleet of vehicles located at a central depot has to serve a set of geographically dispersed customers and return to depot.
- Each vehicle has a given capacity and each customer has a given demand.
- The objective is to minimize the total cost (travelling distance) of serving the customers.
- Example - delivery of goods located at a central depot to customers who have placed orders for such goods.
- Type of decisions
  - Assigning
  - Routing

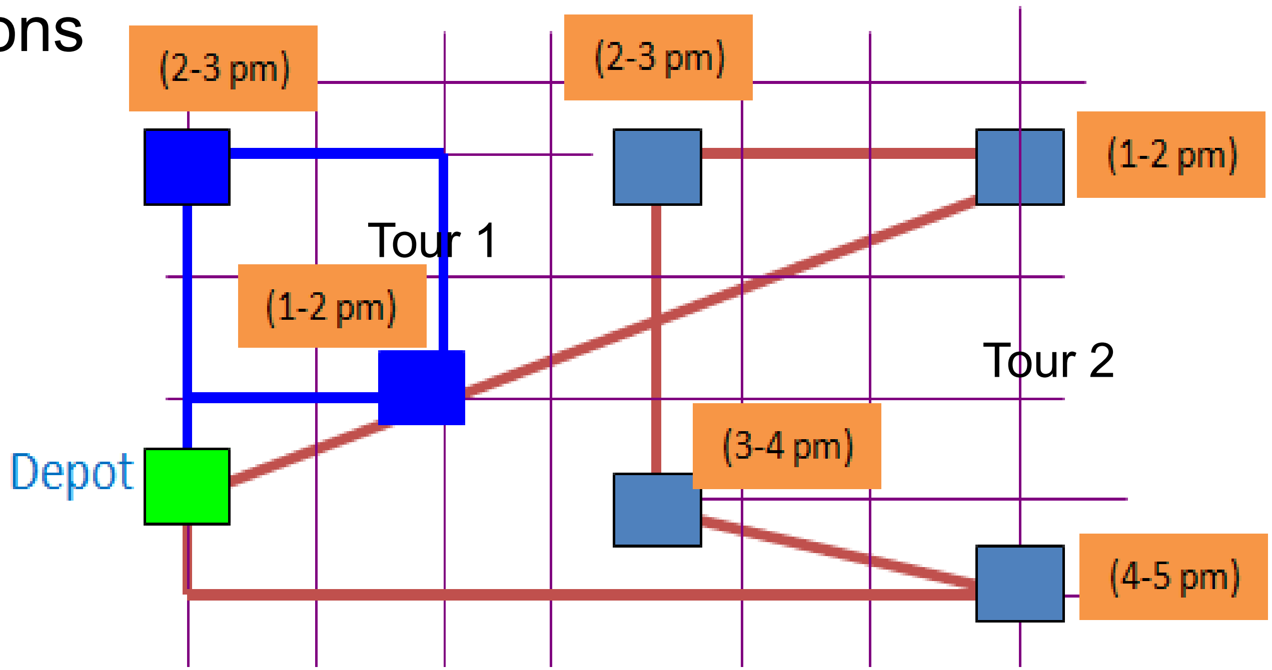


Vehicle Routing Problem with Time Windows

- The VRPTW is similar to the VRP except that, each customer may require to be served within a given time window.

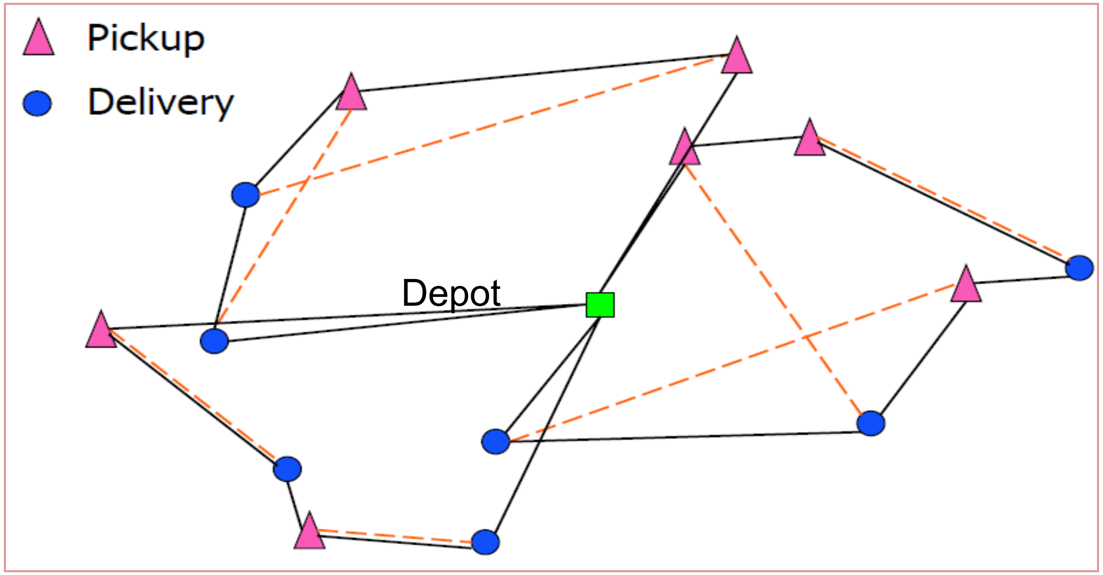
- Example - Delivery of perishable food supplies

- Type of decisions
  - ➢ Assigning
  - ➢ Routing
  - ➢ Scheduling



Pickup and Delivery Problem with Time Windows

- The PDPTW is similar to the VRP except that, in addition to the size of the load to be transported, each customer service also specifies
  - ➢ The location where the load is to be picked up and the pickup time window;
  - ➢ The location where the load is to be delivered and the delivery time window;
- Example – Public Bus



- Type of decisions
  - ➢ Assigning
  - ➢ Routing
  - ➢ Scheduling

The PDPTW determines how to form the routes and schedule the pickup and delivery.

Solution approaches to the Routing and Scheduling Problems

- Exact algorithms such as integer programming algorithms
  - For a problem with *n* drop-off points, there are *n*! possible routings. E.g. 8 drop-off points → 40320 possible routings!
  - The solution space increases extremely fast as the number of drop-off points increases, and therefore exact algorithms are often not suitable for large size problems.

- Heuristic algorithms
  - Rule-of-thumb methods
  - May not achieve optimality but solutions are often reasonably good.

- Meta-heuristics
  - Top-level heuristics guiding other heuristics to search for better feasible solutions in the solution space, such as Genetic Algorithms, Ant Colony Optimization, Simulated Annealing, Tabu search.

  - They are very commonly used in solving complex vehicle routing and scheduling problems.

Solution approaches to the Routing and Scheduling Problems

- Commonly used vehicle routing heuristics
  - **Construction algorithm:** an algorithm that determines a tour according to some construction rules, but does not try to improve upon this tour. Examples are Nearest Neighbor method and Clarke and Wright Savings method.

  - **Improvement algorithm:** an algorithm that performs a sequence of edge or vertex exchanges within or between vehicle routes to improve the current tour. Examples are **2-opt, 3-opt, 1-relocate, 2-swap.**

  - **Two-phase algorithm:** an algorithm that constructs vehicle routes in two phases
    - ✓ **Cluster-first-route-second method:** customers are first organized into feasible clusters, and a vehicle route is constructed for each of the clusters.
    - ✓ **Route -first-cluster-second method:** a tour is first built on all customers and the tour is then segmented into feasible vehicle routes.

Tour construction—Nearest Neighbor Method

- Select a node 1 (depot) to start with
- Add the closest node to node 1
- Repeat by adding to the last node the closest unvisited node until no more nodes are available
- Connect the last node with the first node

Tour construction—Clarke and Wright Savings Method

- Select a node 1 (depot)
- Compute the savings S_{ij} , for linking nodes *i* and *j*

$$S_{ij} = c_{1i} + c_{1j} - c_{ij} \text{ for } i \text{ and } j = \text{nodes } 2,\ 3,\ \ldots,\ n.$$

where c_{ij} = the cost of travelling from node i to node j.

- Rank the savings from the largest to the smallest
- Starting from the top of the list, form larger subtours by linking **'appropriate'** nodes *i* and *j* (so that *j* is visited immediately after *i* on the resulting route)
- Stop when a complete tour is formed

'appropriate' means:

➢Not violating route constraints such as vehicle capacity or time window constraints;

AND

➢*i* and *j* fulfil one of the following conditions:

Elearning Videos:
https://docs.google.com/open?id=0B9sGwZfXz0MkTjJYODczRFIzcFE
https://docs.google.com/open?id=0B9sGwZfXz0MkSjlxMzVYUTMtbGs

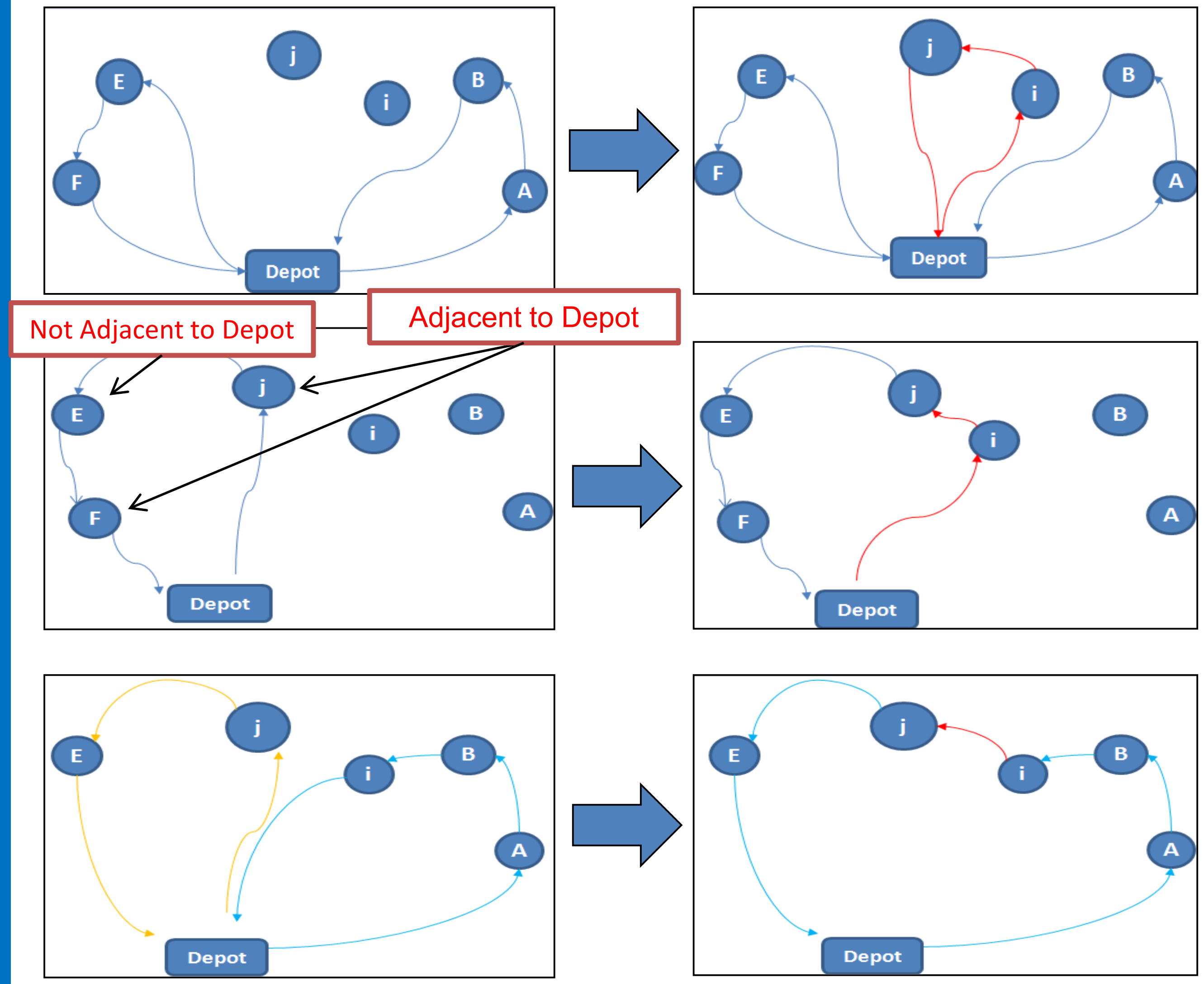
Tour construction—Clarke and Wright Savings Method

➢ ***i* and *j* fulfil one of the following conditions:**

A. Neither i nor j has already been assigned to a route. Initiate a new route including both i and j

B. Exactly *one* of the two points (i or j) has already been included in an existing route and that point is adjacent to the depot. Add link (i, j) to that same route.

C. Both i and j have already been included in two different existing routes and both of them are adjacent to the depot. Merge the two routes.



Clarke and Wright Savings Method – An Example

Distance Table

From	To (Distance in km)						
	A (Depot)	B	C	D	E	F	G
A (Depot)	0	22	19	15	7	12	26
B	22	0	5	16	18	12	30
C	19	5	0	6	14	8	16
D	15	16	6	0	8	3	21
E	7	18	14	8	0	6	25
F	12	12	8	3	6	0	28
G	26	30	16	21	25	28	0

Clarke and Wright Savings Table

$S_{BD} = D_{AB} + D_{AD} – D_{BD}$ --> $S_{BD} = 22 + 15 – 16 = 21$

From	Savings						
	A (Depot)	B	C	D	E	F	G
A (Depot)	0	0	0	0	0	0	0
B	0	0	36	21	11	22	18
C	0	36	0	28	12	23	29
D	0	21	28	0	14	24	20
E	0	11	12	14	0	13	8
F	0	22	23	24	13	0	10
G	0	18	29	20	8	10	0

Clarke and Wright Savings Method – An Example

Rank(Dist.)	From	To	Path
1(36)	B	C	A - B - C - A
2(29)	C	G	A - B - C - G - A
3(28)	C	D	-
4(24)	D	F	A - D - F - A, A - B - C - G - A,
5(23)	C	F	-
6(22)	B	F	A - D - F - B - C - G - A,
7(21)	B	D	-
8(20)	D	G	-
9(18)	B	G	-
10(14)	D	E	A - E - D - F - B - C - G - A, End.
11(13)	E	F	
12(12)	C	E	
13(11)	B	E	
14(10)	F	G	
15(8)	E	G	

C is not adjacent to A (Depot)

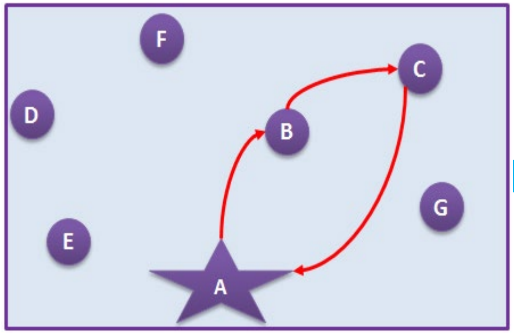
Both C and F are assigned to two different existing routes but C is not adjacent to A (Depot)

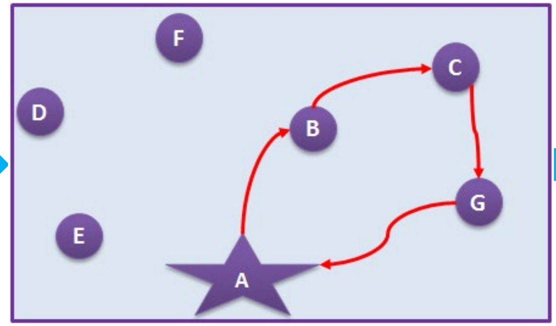
B, D and G are already assigned to the same route

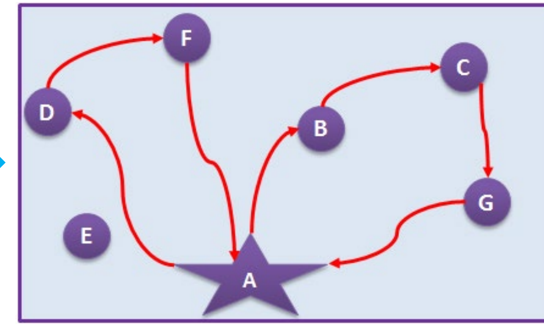
Route Constructed: A - E - D - F - B - C - G - A

Total Distance
= 7 + 8 + 3 + 12 + 5 + 16 + 26 = 77 km

Clarke and Wright Savings Method – An Example



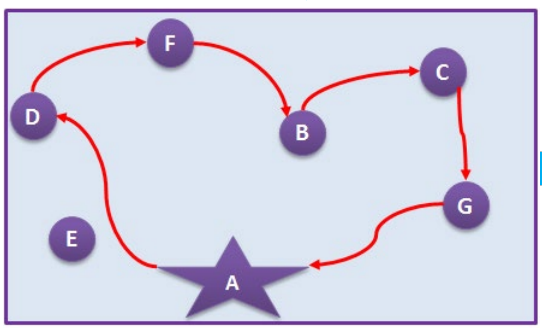


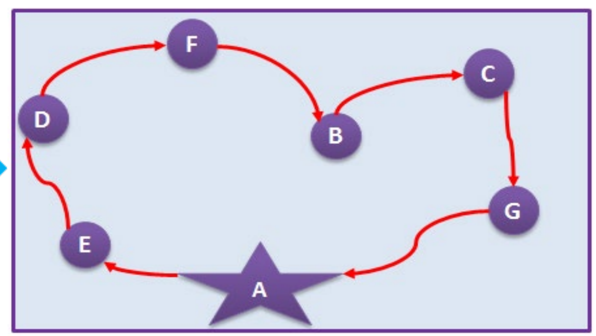


Initiate a new route: A-B-C-A

Link G to C: A-B-C-G-A

Initiate another route: A-D-F-A





Join both routes by linking B and F: A-D-F-B-C-G-A

Link E to D: A-E-D-F-B-C-G-A

Total Distance = 77 km

Clarke and Wright Savings Method – An Example Considering Vehicle Capacity

Customer demand:

Location	B	C	D	E	F	G
Demand	20	19	15	25	18	22

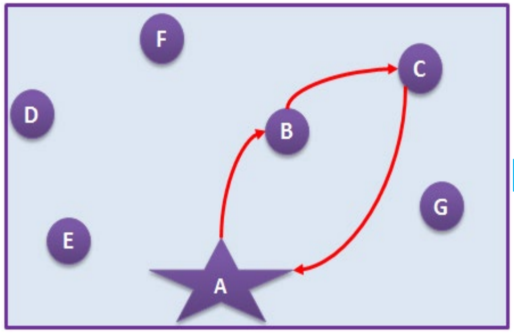
Rank (Dist.)	From	To	Current Tour	Cumulative Load	Remark
1(36)	B	C	A - B - C - A	20 + 19 = 39	Initiate a new route: A-B-C-A
2(29)	C	G	A - B - C - G - A	20 + 19 + 22 = 61	Link G with C
3(28)	C	D	-	-	C is not adjacent to A (Depot)
4(24)	D	F	A - D - F - A, A - B - C - G - A	15 + 18 = 33, 20 + 19 + 22 = 61	Initiate another route: A-D-F-A
5(23)	C	F	-	-	C is not adjacent to A (Depot)
6(22)	B	F	A - D - F - B - C - G - A	15 + 18 + 20 + 19 + 22 = 94	Link B and F.
7(21)	B	D	-	-	Both B and D are already in a route
8(20)	D	G	-	-	Both D and G are already in a route
9(18)	B	G	-	-	Both B and G are already in a route
10(14)	D	E	A - D - F - B - C - G - A, A - E - A	15 + 18 + 20 + 19 + 22 = 94, 25	Cannot link D and E, because truck capacity (100 boxes) will be violated, start new route for E. End
11(13)	E	F			
12(12)	C	E			
13(11)	B	E			
14(10)	F	G			
15(8)	E	G			
Truck 1: A - D - F - B - C - G - A with load 94 boxes and total distance 77km Truck 2: A - E - A with load 25 boxes and total distance 14km					

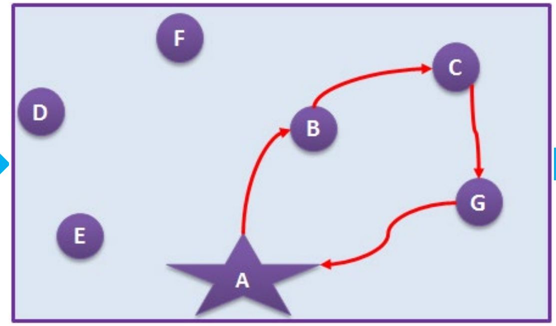
Vehicle Capacity:

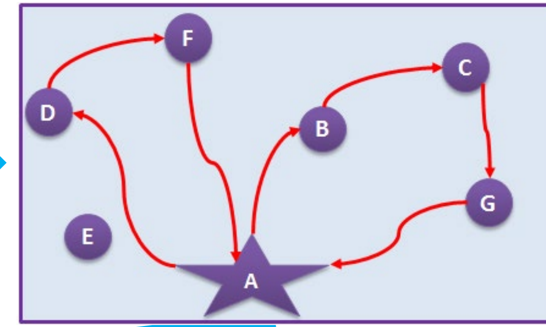
100 boxes

Clarke and Wright Savings Method – An Example

Tour 1 Construction





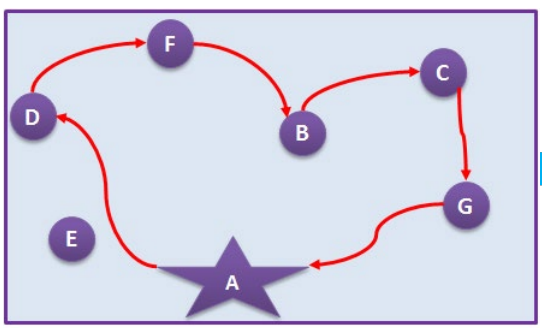


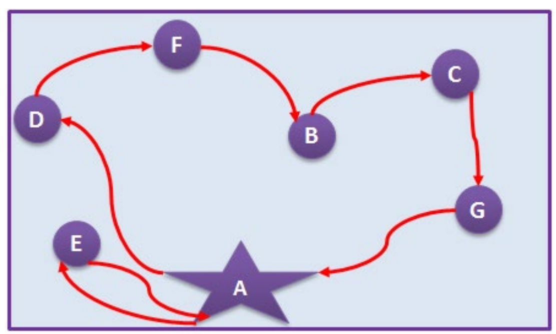
Initiate a new route: A-B-C-A

Link G to C: A-B-C-G-A

Initiate another route: A-D-F-A

Tour 2 Construction





Route (s)	Demand (boxes)	Distance
A - D - F - B - C - G - A	94	77
A - E - A	25	14
	Total Distance	**91** km

Join both routes by linking B and F: A-D-F-B-C-G-A

Initiate new route: A-E-A

Total Distance = 91 km

P03 Suggested Solution

Today's Problem: Nearest Neighbor Method

From	To (Traveling time in minutes)						
	FS	**A**	**B**	**C**	**D**	**E**	**F**
FS	0	32	38	40	26	47	52
A	32	0	42	48	42	54	66
B	38	42	0	30	44	48	64
C	40	48	30	0	36	46	56
D	26	42	44	36	0	39	45
E	47	54	48	46	39	0	28
F	52	66	64	56	45	28	0

Nearest Neighbor Method

From	Nearest Neighbor	Traveling Time
FS (Depot)	D	26
D	C	36
C	B	30
B	A	42
A	E	54
E	F	28
F	FS (Depot)	52
	Total time	**268** Mins

Total Distance =
26+36+30+42+54+28+52 = 268 minutes

Since the total demand = 48 Boxes is less than the truck capacity of 50 boxes, one delivery truck is sufficient.

Route:
FS (Depot) - D - C - B - A - E - F - FS (Depot)

Today's Problem: Clarke and Wright Savings Method

Clarke and Wright Savings Table

From	Savings						
	FS(Depot)	A	B	C	D	E	F
FS(Depot)	0	0	0	0	0	0	0
A	0	0	28	24	16	25	18
B	0	28	0	48	20	37	26
C	0	24	48	0	30	41	36
D	0	16	20	30	0	34	33
E	0	25	37	41	34	0	71
F	0	18	26	36	33	71	0

Distance Table

From	To (Distance in km)						
	FS	A	B	C	D	E	F
FS	0	32	38	40	26	47	52
A	32	0	42	48	42	54	66
B	38	42	0	30	44	48	64
C	40	48	30	0	36	46	56
D	26	42	44	36	0	39	45
E	47	54	48	46	39	0	28
F	52	66	64	56	45	28	0

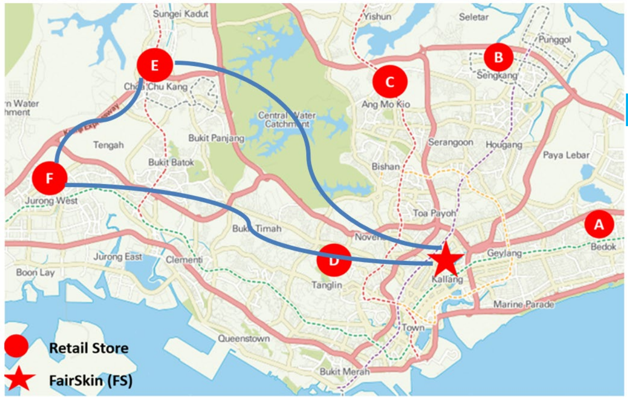
$$S_{BC} = 38 + 40 - 30 = 48$$

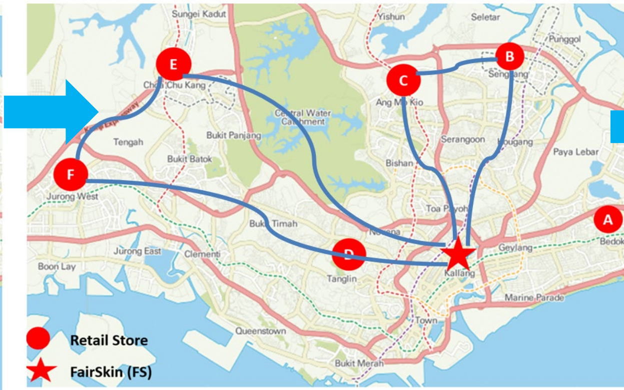
Rank(Minutes)	From	To	Current Tour	Remarks
71	E	F	FS - E - F - FS	Initiate a new tour
48	B	C	FS - B - C - FS	Initiate a new tour
41	C	E	FS - B - C - E - F - FS	Link C with E as both of them are adjacent to depot
37	B	E	-	Cannot link B with E as both of them are in the same tour
36	C	F	-	Cannot link C with F as both of them are in the same tour
34	D	E	-	Cannot link D with E as E is not adjacent to depot
33	D	F	FS - B - C - E - F - D - FS	Link D with F as F is adjacent to depot
30	C	D	-	Cannot link C with D as both of them are in the same tour
28	A	B	FS - A - B - C - E - F - D - FS	Link A with B as B is adjacent to depot
26	B	F		
25	A	E		
24	A	C		
20	B	D		
18	A	F		
16	A	D		

Route Constructed: FS - A - B - C - E - F - D - FS

Total Distance = 32 + 42 + 30 + 46 + 28 + 45 + 26 = 249 minutes

Today's Problem: Clarke and Wright Savings Method

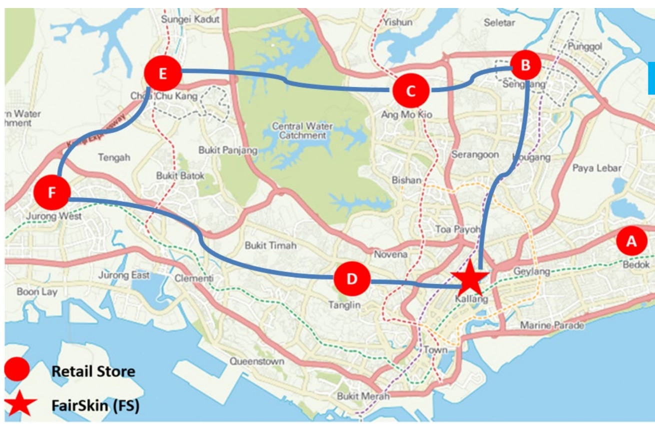




Initiate new route: FS-E-F-FS

Initiate another route: FS-B-C-FS

Link C to E: FS-B-C-E-F-FS



Link D to F: FS-B-C-E-F-D-FS

Link A to B: FS-A-B-C-E-F-D-FS

Total traveling time = 249 minutes [vs 268 minutes from the Nearest Neighbor method]

Today's Problem: Considering Vehicle Capacity

By Nearest Neighbor Method

From	To (Distance in km)						
	FS	A	B	C	D	E	F
FS	0	32	38	40	26	47	52
A	32	0	42	48	42	54	66
B	38	42	0	30	44	48	64
C	40	48	30	0	36	46	56
D	26	42	44	36	0	39	45
E	47	54	48	46	39	0	28
F	52	66	64	56	45	28	0

Location	A	B	C	D	E	F
Demand	5	8	7	6	9	13

Location	A	B	C	D	E	F
Demand	11	17	14	9	19	21

From	Nearest Neighbor	Distance	Boxes
FS (Depot)	D	26	9
D	C	36	14
C	B	30	17
B	FS (Depot)	38	
FS (Depot)	A	32	11
A	E	54	19
E	FS (Depot)	47	
FS (Depot)	F	52	21
F	FS (Depot)	52	

Tour (s)	Demand (box)	Traveling Time (minutes)
FS - D - C - B - FS	40	130
FS - A - E - FS	30	133
FS - F - FS	21	104

Total traveling time 367 minutes

- With partial tour FS-D-C-B-FS, any other customers can not be added due to the capacity limit.
- Another tour is formed by applying the nearest neighbour method for the rest customers, A, E and F. After forming the tour FS-A-E-FS, customer F cannot be added due to the capacity limit.
- Lastly, a tour FS-F-FS is formed.

Today's Problem: Considering Vehicle Capacity

By Clarke and Wright Savings Method

Clarke & Wright Savings Table

From	Savings						
	FS(Depot)	A	B	C	D	E	F
FS (Depot)	0	0	0	0	0	0	0
A	0	0	28	24	16	25	18
B	0	28	0	48	20	37	26
C	0	24	48	0	30	41	36
D	0	16	20	30	0	34	33
E	0	25	37	41	34	0	71
F	0	18	26	36	33	71	0

Distance Table

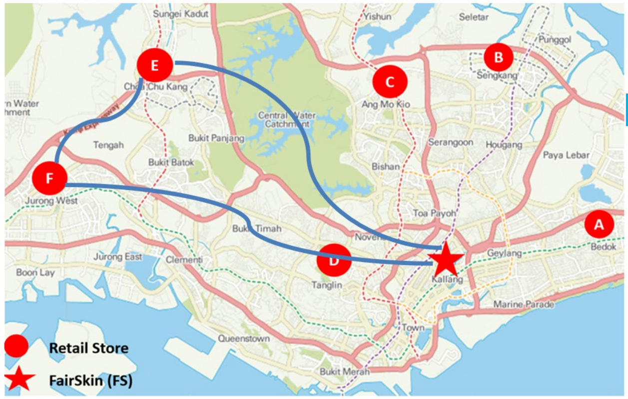
From	To (Distance in km)						
	FS	A	B	C	D	E	F
FS	0	32	38	40	26	47	52
A	32	0	42	48	42	54	66
B	38	42	0	30	44	48	64
C	40	48	30	0	36	46	56
D	26	42	44	36	0	39	45
E	47	54	48	46	39	0	28
F	52	66	64	56	45	28	0

Rank(Dist.)	From	To	Current Tour	Cumulative Load	Remark
71	E	F	FS - E - F - FS	19 + 21 = 40	Initiate a new tour: FS - E - F - FS
48	B	C	FS - B - C - FS	17 + 14 = 31	Initiate a new tour: FS - B - C - FS
41	C	E	-	40 + 31 = 71	Cannot link C and E because of capacity limit
37	B	E	-	40 + 31 = 71	Cannot link B and E because of capacity limit
36	C	F	-	40 + 31 = 71	Cannot link C and F because of capacity limit
34	D	E	FS - D - E - F - FS	40 + 9 =49	Link D with E because E is adjacent to depot
33	D	F	-		Cannot link D with F because both of them are already in the tour
30	C	D	-	49 + 31 = 80	Cannot link C and D because of capacity limit
28	A	B	FS - A - B - C - FS	31 + 11 = 42	Link A with B because B is adjacent to depot
26	B	F			
25	A	E			
24	A	C			
20	B	D			
18	A	F			
16	A	D			

Route (s)	Demand (box)	Traveling Time (minutes)
FS - A - B - C – FS	42	144
FS - D - E - F - FS	49	145

Total traveling time: **289** **minutes**

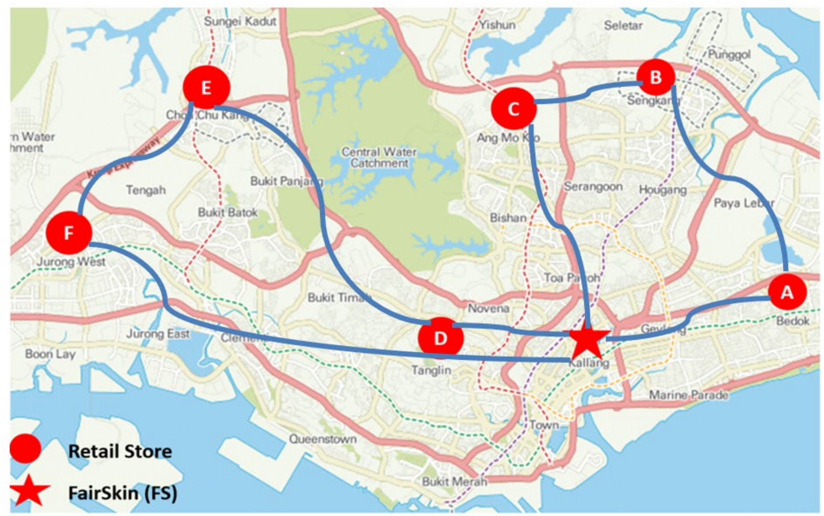
Today's Problem: Clarke and Wright Savings Method



Initiate new route: FS-E-F-FS

Initiate another route: FS-B-C-FS

Link D with E: FS-D-E-F-FS



Tour 1: FS-A-B-C-FS
Tour 2: FS-D-E-F-FS

<u>Total traveling time</u>
= 289 minutes [vs 367 minutes from Nearest Neighbour method]

Link A to B: FS-A-B-C-FS

Recommendations

- Linda should:
  - Apply Clarke & Wright Savings Method to construct the delivery routes.
  - Deploy two trucks to cater for the increase in demand.
  - When demand is less than vehicle capacity

Route (s)	**Traveling Time (minutes)**
FS - A - B - C - E - F - D - FS	249

  - When demand is more than the vehicle capacity:

Route (s)	**Demand (box)**	**Traveling Time (minutes)**
FS - A - B - C – FS	42	144
FS - D - E - F - FS	49	145

Total traveling time: **289** **minutes**

  - If certain customers require that delivery to be done within given time periods, Linda would need to consider vehicle routing with time windows. The delivery problem becomes a routing and scheduling problem.

Conclusion

- Routing and scheduling problems can be found in our daily lives and operations.

- The objective is to find the shortest tour that covers all the places (nodes) without a repeat visit and begins / ends at one node.

- General algorithms for solving the vehicle routing and scheduling problems are heuristics:

  - ➢ Tour construction algorithms such as nearest neighbor and Clarke and Wright Savings methods;

  - ➢ Tour improvement algorithms such as 2-opt, 3-opt, and 1-relocate.

Learning Objectives

- Identify what a Vehicle Routing and Scheduling problem is.
- Differentiate the different classifications of Routing and Scheduling Problems.
- Apply the general algorithms: Nearest Neighbour Method and Clarke and Wright Savings Method in obtaining a solution with feasible route(s).

Overview of E211 Operations Planning II Module