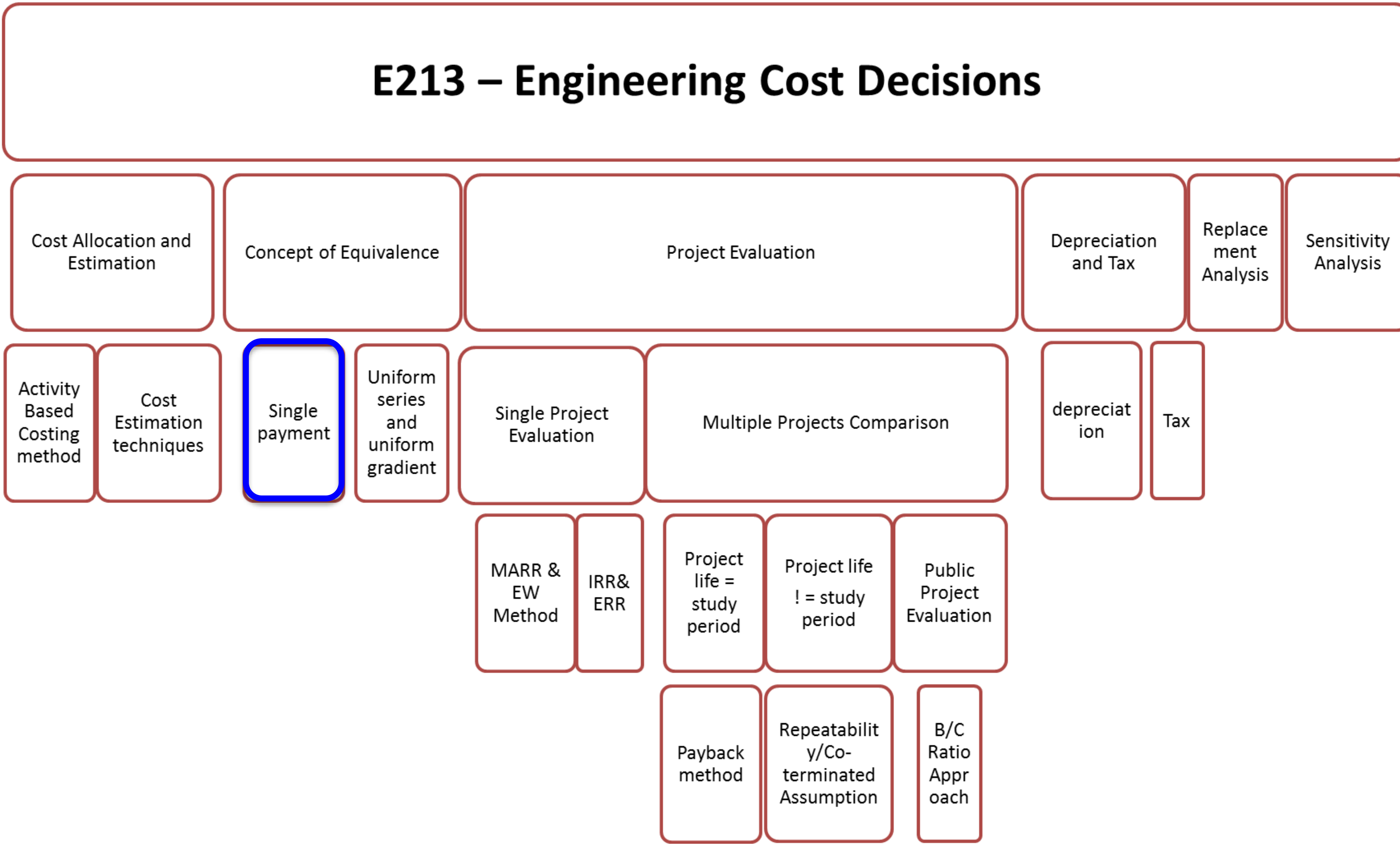
Problem 03

Which one is the best?

E213 – Engineering Cost Decisions

SCHOOL OF ENGINEERING

Module Coverage: Topic Tree



Time Value of Money

- The value of money changes over time
- A dollar that you have now is different from a dollar that you have 10 years later
- Due to factors such as inflation, bank interest, etc.
  - E.g. $1.20 can buy you a plate of chicken rice 20 years ago but not now. The value of the $1.20 has decreased due to the inflation
  - E.g. You put $100 in a bank and you may get $101 from the bank 1 year later. The value of your $100 has increased due to the bank interest.

Engineering projects often involve capital commitment over certain periods. Therefore time value of money must be considered.

Interest and Interest Rate

- Interest is a fee that is charged for the use of someone else's money. The size of the fee will depend upon the **total amount** of money borrowed and the **length of time** over which it is borrowed.

- If a given amount of money is borrowed for a specified period of time (typically one year), a certain percentage of the money is charged as interest. This percentage is called the interest rate.

- Two ways of calculating interest:
  1. Simple Interest
  2. Compound Interest

Simple Interest

- The total interest earned or charged is linearly proportional to the initial amount of the loan (principal sum), the interest rate and the number of interest periods for which the principal is committed.

- When applied, total interest "I" may be calculated by

$$I = (P) (N) (i)$$

where P = principal amount lent or borrowed

N = number of interest periods (e.g. years, months, etc.)

i = interest rate per interest period

- Usually nothing is repaid until the end of the loan period; the principal and the accumulated interest are repaid at the end of loan period. The total amount due, F, may be calculated by

$$F = P + I = P(1 + Ni)$$

Compound Interest

- Compound interest is paid on the original principal **and** on the accumulated past interest.

- The total time period is subdivided into several periods (e.g. one year, three months, one month) when interest is compounded.

- The interest charge for any interest period is based on the remaining principal amount plus any accumulated interest charges up to the beginning of that period.

- If there are N interest periods, we have

$$F = P(1+i)^N$$ *--- Law of compound interest*

where **F** = Total amount of money accumulated

P = principal amount lent or borrowed

N = number of interest periods (e.g. years , months, etc.)

i = interest rate per interest period

Compound Interest

Year (y)	Amount Owed at Beginning of Year y = F_{y-1}	Interest for the Year I_y	Total Amount Accumulated End of the Year F_y
1	P	i*P	P+iP = P*(1+i)
2	P*(1+i)	i*P(1+i)	P(1+i) +iP(1+i) =P(1+i)(1+i) $=P(1+i)^2$
3	$P(1+i)^2$	$i*P(1+i)^2$	$P(1+i)^2+iP(1+i)^2$ $=P(1+i)^3$
:	:	:	:
n	$P(1+i)^{N-1}$	$i*P(1+i)^{N-1}$	$=\mathbf{P(1+i)^N}$

Simple Vs. Compound Interest

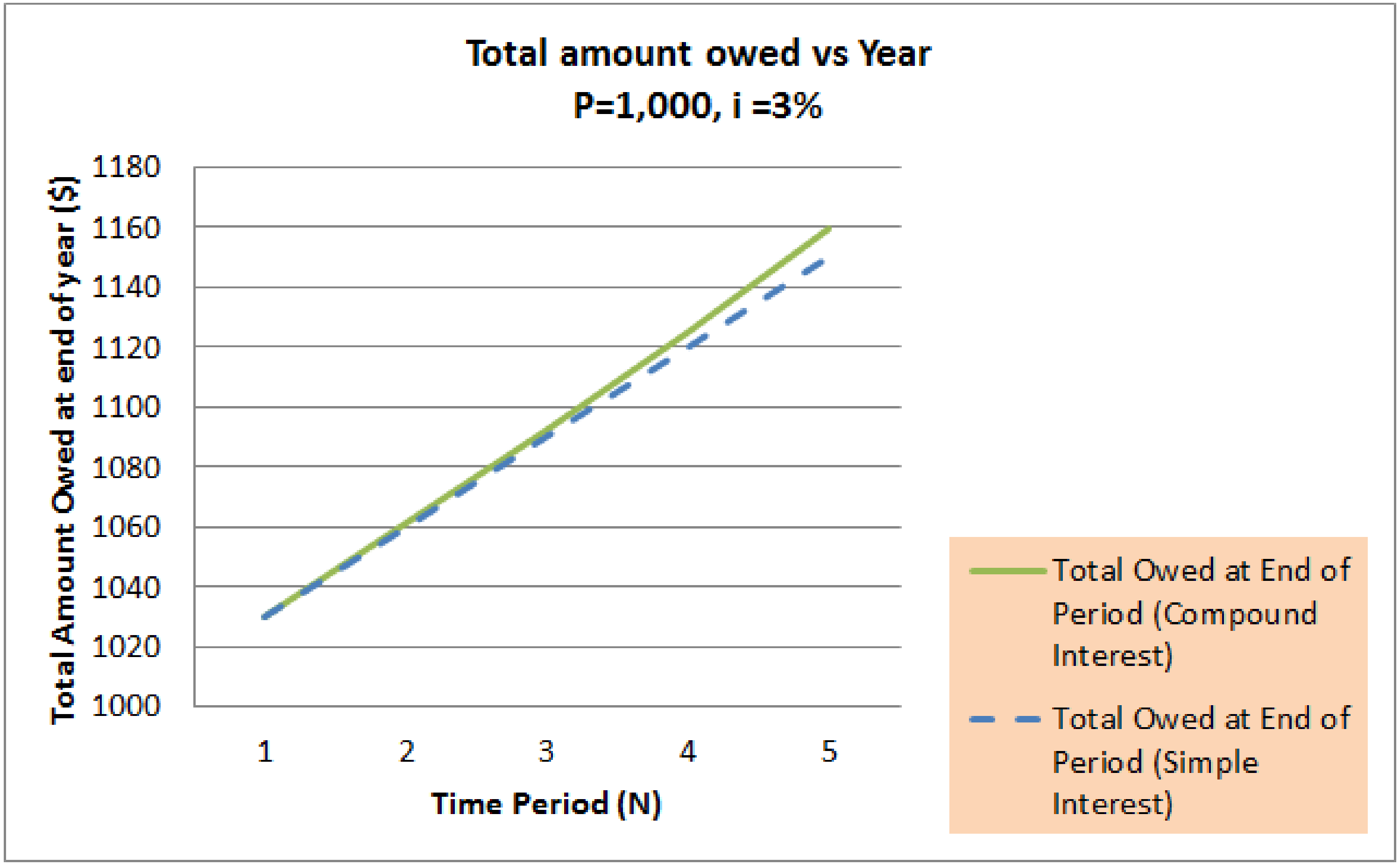
Period	(A) Amount Owed at beginning of the year ($)	(B) Simple Interest 3% Owed for that year ($)	(C)=(A)+(B) Total Owed at End of Year ($)
1	1000	30	1030
2	1030	30	1060
3	1060	30	1090
4	1090	30	1120
5	1120	30	1150

Simple Interest @ 3%

Period	(A) Amount Owed at beginning of the year ($)	(B) Compound Interest 3% Owed for that year ($)	(C)=(A)+(B) Total Owed at End of Year ($)
1	1000	30	1030
2	1030	30.9	1060.9
3	1060.9	31.827	1092.727
4	1092.727	32.78181	1125.50881
5	1125.50881	33.7652643	1159.274074

Compound Interest @ 3%

Simple Vs. Compound Interest



- The difference is due to the **effect of compounding** where interest is charged on **previously earned interest**.
- Compound interest is **much more common** in practice than simple interest. In this module, compound interest is considered unless simple interest is specified.

Concept of Equivalence

- Established when we are *economically indifferent* between two financial proposals (e.g. a series of future payments, and a present sum of money).
  - For example, receiving $10 now is equivalent to receiving $11 one year from now if the interest rate is 10%.
- Always compare the equivalent value of money instead of their face value
  - It will be incorrect to compare $1 you have now with $2 that you may have 2 years later
  - Should convert the value of that $2 to its equivalent value now in order to compare it with the $1 now
- Considers the comparison of alternative options by reducing them to an equivalent basis, depending on:
  - interest rate;
  - amounts of money involved;
  - timing of the affected monetary receipts and/or expenditures;

Concept of Equivalence

Consider a loan of $5,000 at an interest rate of 8% annually to be serviced in 5 years.

✓**Plan 1: At the end of each year, pay $1252.28.**

Year	Repayment
1	1252.28
2	1252.28
3	1252.28
4	1252.28
5	1252.28

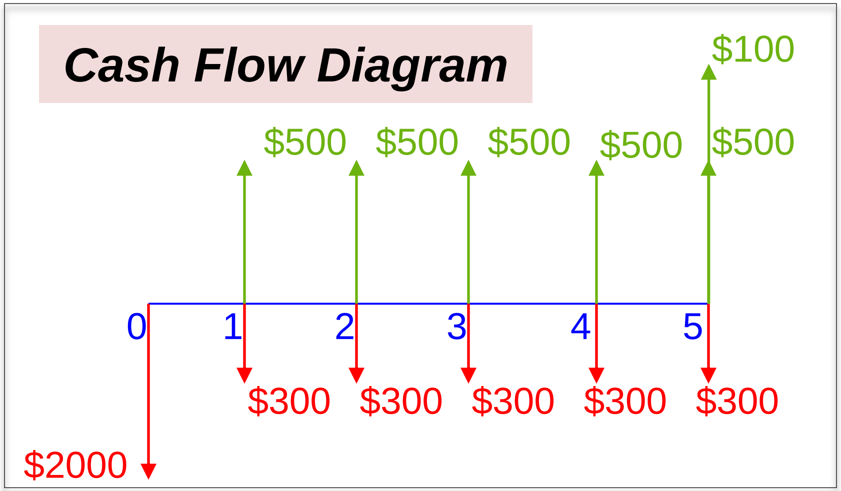
✓ **Plan 2: Pay interest due at the end of each year, and the principal at the end of 5 years.**

Year	Repayment
1	400
2	400
3	400
4	400
5	5400

- ✓ **Which loan plan should you opt for?**
- ✓ **Future Value (FV) for both Plan 1 and Plan 2 = $7,345**

-> Even though both plans require significantly different repayment patterns and even different total amounts of repayment, these plans are EQUIVALENT (economically indifferent).

Discrete Cash Flow Methods



i = 2% per year; interest rate per interest period

N = 5; Number of compounding periods (numbers on the horizontal line or time scale)

Downward Arrow

Negative cash flow(money outflow)

Upward Arrow

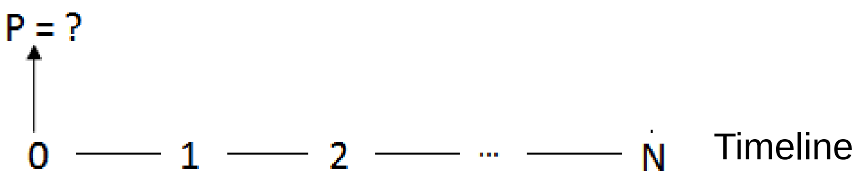
Positive cash flow (money inflow)

- In most business transactions and economy studies, interest is compounded at the end of discrete periods of time.
- Cash flows are assumed to occur in discrete amounts at the end of periods.

It should be noted that one can consider Continuous Compounding Cash Flow Methods (not within the scope of this module) when available cash can be used profitably; creating opportunities for very frequent compounding of interest earned.

Present and Future Value

- Present Value

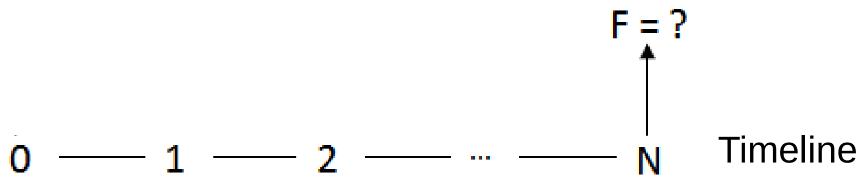


Interest Factor Formula Method

$$P = F\left(\frac{1}{(1+i)^N}\right)$$

$$F = P(1+i)^N$$

- Future Value



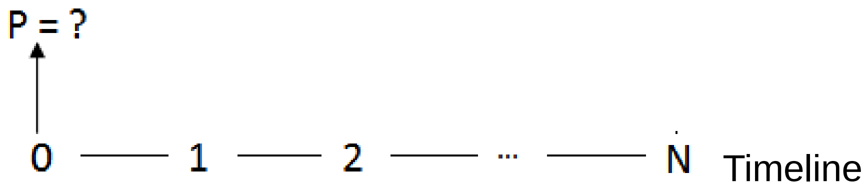
Note:

The *end-of-period cash flow convention* is the standard assumption for this module (unless otherwise stated).

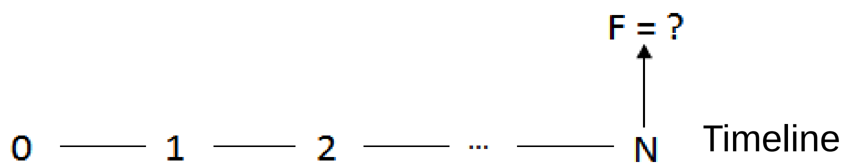
Factor	To Find	Given	Symbol	Formula
Single payment compound amount	F	P	$[F/P, i\%, N]$	$(1+i)^N$
Single payment present worth	P	F	$[P/F, i\%, N]$	$\frac{1}{(1+i)^N}$

Present and Future Value

- Present Value



- Future Value



Interest Factor Notation Method

P = F(P/F, i%, N) →[Find P given F]

F = P(F/P, i%, N) →[Find F given P]

Example:
If $1000 is to be received in 5 years, at an annual interest rate of 12
find the present worth of this amount.

Solution:
Find : P
Given: F = $1,000, i = 12% per year, N = 5 years

Need to find P given F :
Use P = F(P/F, i%, N)

Using Interest Factor Table on the right (interest rate of 12%),

P = F(P/F, i%, N)
= 1,000 (P/F, 12%, 5)
= 1,000*0.5674
= $567.40

12%	Single Payment	
	Compound Amount Factor	Present Worth Factor
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*
1	1.120	.8929
2	1.254	.7972
3	1.405	.7118
4	1.574	.6355
5	1.762	.5674

(P/F, 12%, 5)=0.5674

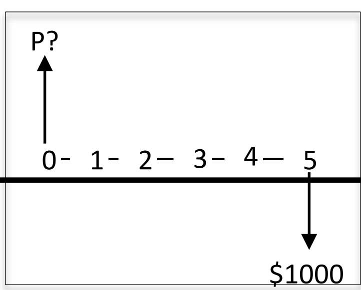
Present and Future Value

Example 1:
If Ally needs to pay $1,000 at the end of year 5 given an interest rate at 3% compounded annually. How much loan is she taking up **now**?
Find : P
Given: F = $1,000, i = 3% per year, N = 5 years

Method I: Use Interest Factor Formula

$P = 1{,}000 * 1/(1+0.03)^5$
$\underline{= \$862.61}$

Method II: Use Interest Factor Notation
P = 1,000 * (P/F, 3%,5)
= 1,000 * (0.8626)
<u>**= $862.6**</u>



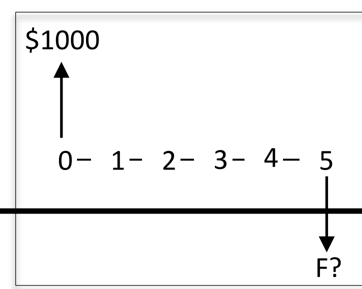
3%

	Single Payment	
	Compound Amount Factor	Present Worth Factor
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*
1	1.030	.9709
2	1.061	.9426
3	1.093	.9151
4	1.126	.8885
5	1.159	.8626

Example 2:
If Ally takes up a loan of $1,000 now at an interest rate of 3% compounded annually. How much does she need to pay **at the end** of year 5?
Find : F
Given: P = $1,000, i = 3% per year, N = 5 years

Method I: Use Interest Factor Formula
$F = 1{,}000 * (1+0.03)^5$
$\underline{= \$1{,}159.27}$

Method II: Use Interest Factor Notation
F = 1,000 * (F/P, 3%,5)
= 1,000 * (1.159)
<u>**= $1,159**</u>



3%

	Single Payment	
	Compound Amount Factor	Present Worth Factor
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*
1	1.030	.9709
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Problem 03

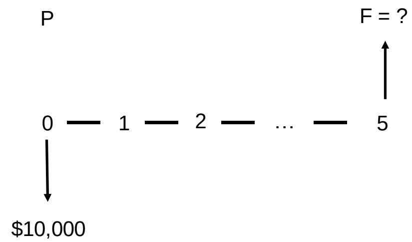
- Suggested Solution

Steps for economic decision making

1. Recognize the problem
   - To increase the value of money for Lionel through deposit
   - Which savings plan to choose?
2. Identify feasible alternative solutions
   - Savings Plan A with annual compound interest rate of 2.5%
   - Savings Plan B with annual simple interest rate of 2.6%
3. Depict the cash flows outcome for each alternative/option
4. Select a decision criteria
   - Select the option with the higher amount of interest over the time period
5. Compare and analyze the alternatives
   - Calculate the amount of money in the bank at the end of the stated time period for both savings plans
   - Compare both savings plans

Steps for economic decision making - Steps 3, 4 and 5

- Lionel only deposits current savings of $10,000 inside the bank



Plan A: Future Value of Lionel's money: compound interest of 2.5% annually

<u>Interest Factor Formula Method:</u>

F =10,000*(1+2.5%)^5 = $11,314.08

<u>Interest Factor Notation Method:</u>

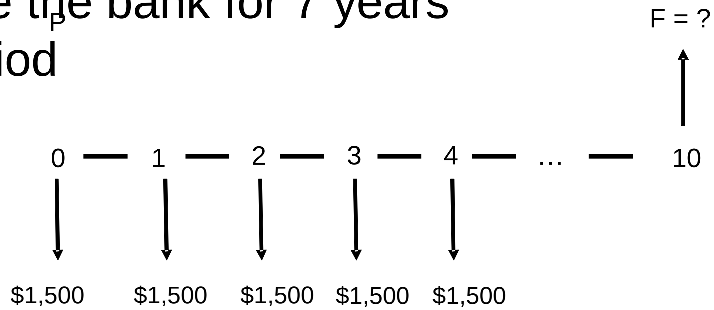
F =10,000*(F/P,2.5%,5) = 10,000*1.131 = $11,310.00

Plan B: Future Value of Lionel's money: simple interest of 2.6% annually

F =10,000*(1+5*2.6%) = $11,300.00

Steps for economic decision making - Steps 3, 4 and 5

- Lionel deposits $1,500 each year inside the bank for 7 years starting from this year over 10 year period



Plan A: Future Value of Lionel's deposit: compound interest of 2.5% annually

<u>Interest Factor Formula Method:</u>

F =1,500*(1+2.5%)^10 + 1,500*(1+2.5%)^9 + 1,500*(1+2.5%)^8 + 1,500*(1+2.5%)^7 + 1,500*(1+2.5%)^6 + 1,500*(1+2.5%)^5 + 1,500*(1+2.5%)^4 + 1,500*(1+2.5%)^3
= **$14,111.76**

<u>Interest Factor Notation Method:</u>

F = 1,500*(F/P,2.5%,10) + 1,500*(F/P,2.5%,9) + 1,500*(F/P,2.5%,8) + 1,500*(F/P,2.5%,7) + 1,500*(F/P,2.5%,6) + 1,500*(F/P,2.5%,5) + 1,500*(F/P,2.5%,4) + 1,500*(F/P,2.5%,3)
= 1,500*1.280 + 1,500*1.249 + 1,500*1.218 + 1,500*1.189 + 1,500*1.160 + 1,500*1.131 + 1,500*1.104 + 1,500*1.077
= **$14,112.00**

Plan B: Future Value of Lionel's deposit: simple interest of 2.6% annually

F =1,500*(1+10*2.6%) + 1,500*(1+9*2.6%) +1,500*(1+8*2.6%) + 1,500*(1+7*2.6%)

+1,500*(1+6*2.6%) + 1,500*(1+5*2.6%) + 1,500*(1+4*2.6%) + 1,500*(1+3*2.6%)

= **$14,028.00**

Steps for economic decision making

6. Choose the best alternative
    - Make decision based on Steps 4 & 5
    - Which option would you implement?

Final Decision:

Savings Plan	Deposit $10000 this year for a duration of 5 years	Deposit $1500 each year for 7 years starting from this year for a duration of 10 years
Plan A	$11,314	$14,112
Plan B	$11,300	$14,028

Savings Plan A has higher amount of money than Plan B. Lionel should choose Plan A.

7. Monitor results and evaluate
    - Monitor results from decision and plan for future improvements

Learning Objectives

- Interpret the concepts of time value of money, interest and interest rate
- Compute simple and compound interest
- Apply the equivalence of present and future values
- Compare alternative options by applying the concept of equivalence

E213 Engineering Cost Decisions (Topic Flow)

Today's learning

Application of ABC costing method in cost management

Application of different cost estimating techniques

Comparison of alternatives using the concept of equivalence

Alternatives evaluation using single, uniform series and uniform gradient cash flows

Project evaluation using MARR and Equivalent Worth method

Project evaluation based on Internal Rate of Return and External Rate of Return

Evaluate alternatives of equal life spans using payback method

Evaluate alternatives with different life spans

Evaluate public projects through incremental B/C analysis

Depreciation estimation and consideration in economic analysis

Tax consideration in economic analysis

Replacement analysis application

Risk and uncertainties handling in economic analysis