Lesson 04

Payment Plans

E213 – Engineering Cost Decisions

Interactive Seminar

SCHOOL OF ENGINEERING

Module Coverage: Topic Tree

E213 – Engineering Cost Decisions

- Cost Allocation and Estimation
  - Activity Based Costing method
  - Cost Estimation techniques
- Concept of Equivalence
  - Single payment
  - Uniform series and uniform gradient
- Project Evaluation
  - Single Project Evaluation
    - MARR & EW Method
    - IRR& ERR
  - Multiple Projects Comparison
    - Project life = study period
      - Payback method
    - Project life ! = study period
      - Repeatability/Co-terminated Assumption
    - Public Project Evaluation
      - B/C Ratio Approach
- Depreciation and Tax
  - depreciation
  - Tax
- Replacement Analysis
- Sensitivity Analysis

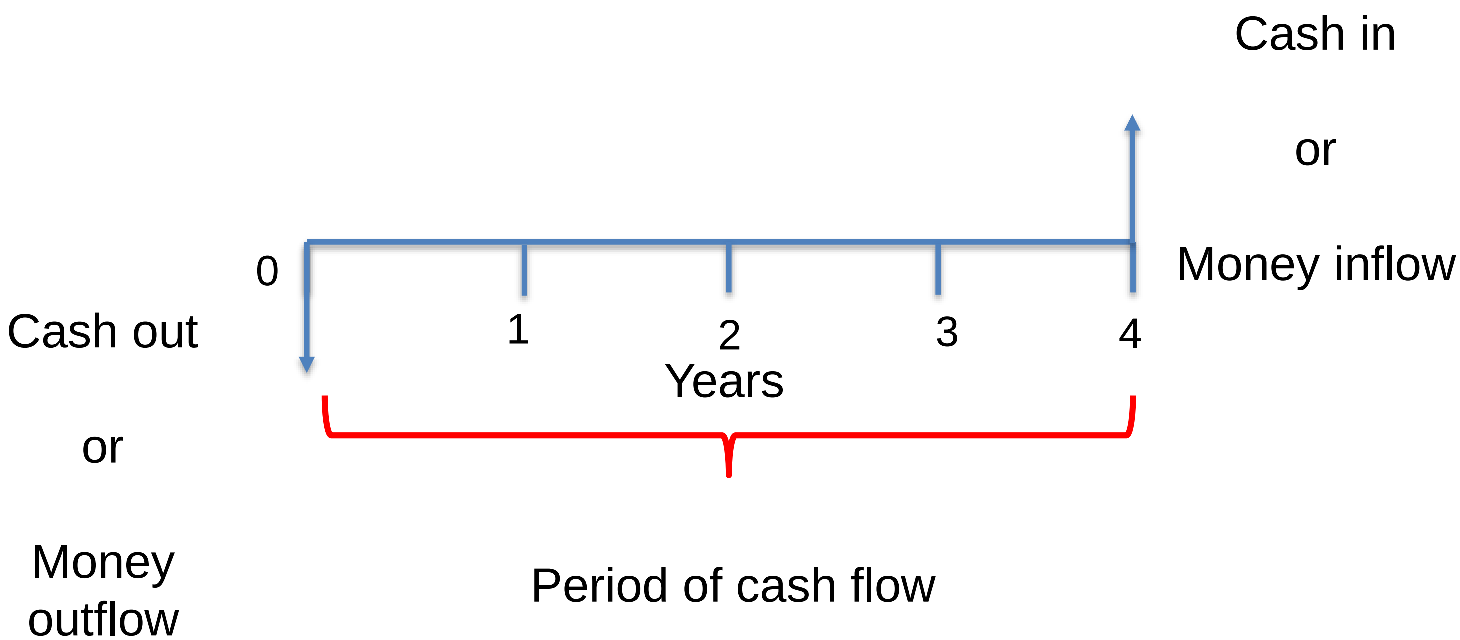
Scenario

Loan	Interest Rate	Payment
A	Effective annual interest of 5%	Pay $22,152.40 (2020); $27,152.40 (2021); $32,152.40 (2022)
B	Annual nominal interest of 5% compounded monthly	Pay $19,950 yearly from 2019 to 2022
C	Annual nominal interest of 5% compounded quarterly	Pay $28,456 (2019); $50456 (2021); $756 (2022)

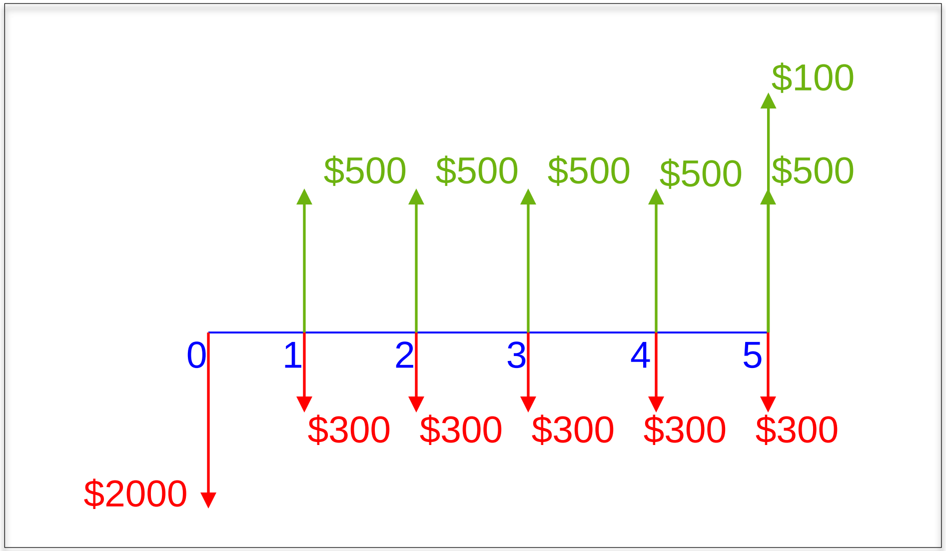
Dave wants to borrow $70,000 now and found 3 loans. Use Future Value to compare which loan type will result in the minimum amount of future value of the payments

What is a Cash Flow Diagram?

A cash flow diagram allows you to graphically show the timing of the cash flows as well as their nature as either inflows or outflows.



How to draw a Cash Flow Diagram?



i = 2% per year

N = 5

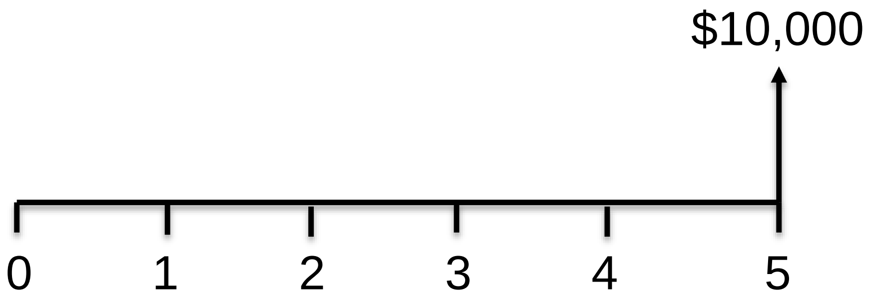
Cash Flow Diagram

- **Downward Arrow** – Negative cash flow (money outflow)
- **Upward Arrow** – Positive cash flow (money inflow)

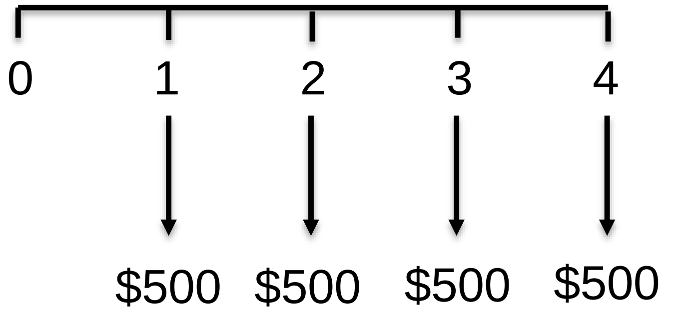
- N: number of compounding periods (refers to numbers on the horizontal line or time scale)
- i: interest rate per interest period

Test Yourself !

Draw your cash flow diagram if in 5 years time, you will receive $10,000.



Draw your cash flow diagram if you have to pay $500 every year for 4 years.



What is Interest Rate?

The interest rate is the cost of debt for the borrower and the rate of return for the lender.

Individuals borrow money to purchase homes, fund projects, start businesses, pay college tuition, etc. Businesses take loans to fund capital projects and expand their operations.

The money that is lent has to be repaid either in lump sum at some pre-determined date or in monthly instalments.

What is Interest Rate?

Simple Interest Rate

= Principal x Interest Rate x No. of Periods

An individual takes out a two year loan of $300,000 from a bank and the loan agreement stipulates that the interest rate on the loan is 15% per year.

This means that the borrower will have to pay the bank in 2 years time :

= Original loan amount of $300,000 + (15% x $300,000 X 2)
= $300,000 + $90,000
= $390,000

Nominal Interest Rate

- Interest rates are normally quoted on an annual basis. However, it may be compounded per year, monthly, quarterly, semi-annually, etc.
- Nominal interest (*r*) is the annual interest rate without considering the effect of any compounding.

$$r = M*i$$

i = interest rate per interest period

M = number of compounding periods per year

- If a year is divided into 4 quarters with interest at 2% per quarter, then such a scheme is usually quoted as “Annual (nominal) interest of 8% compounded quarterly”

Principal sum		= $1000.00
End of 3 months	1000(1+0.02)	= $1020.00
End of 6 months	1020(1+0.02)	= $1040.40
End of 9 months	1040.40(1+0.02)	= $1061.21
End of 12 months	1061.21(1+0.02)	= $1082.43

- The 8% is called the *nominal annual interest rate* and is denoted by *r = M*i =4 * 2% = 8%,* where i = 2% is the interest rate per quarter and M = 4 is the number of quarters per year. Here the interest period is quarter.

Effective Interest Rate

Effective Interest Rate per year, i_{eff}, *is the annual interest rate taking into account the effect of any compounding during the year*

The relationship between the effective annual interest rate and the nominal annual interest rate is

$$i_{eff} = (1 + r/M)^{M} - 1$$

i_{eff}	= effective annual interest rate
r	= nominal interest rate per year
M	= number of compounding periods per year
r/M	= interest rate per interest period

Effective Interest Rate

If the nominal interest rate is 8% compounded quarterly, what is the effective annual interest rate?

The effective annual interest rate is :

$$i_{eff} = (1 + r/M)^M - 1$$

$$= (1 + 0.08/4)^4 - 1$$

$$= 0.0824 \text{ or } 8.24\%$$

i_{eff} = effective annual interest rate
r = nominal interest rate per year
M = number of compounding periods per year
r/M = interest rate per interest period

Note that :
M > 1 => $i_{eff} > r$ (eg: compound 4 times per year)
M = 1 => $i_{eff} = r$ (eg: compound 1 time per year)
M < 1 => $i_{eff} < r$ (eg: compound 1 time every 3 years)

Test Yourself !

If the nominal interest rate is 5% compounded once every 4 years, what is the effective annual interest rate?

The effective annual interest rate is :

$$i_{eff} = (1 + r/M)^{M} – 1$$

$$= (1 + 0.05/0.25)^{0.25} – 1$$

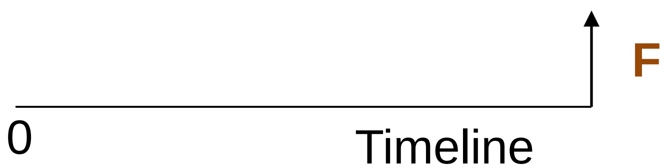
$$= 0.0466 \text{ or } 4.66\%$$

i_{eff} = effective annual interest rate
r = nominal interest rate per year
M = number of compounding periods per year
r/M = interest rate per interest period

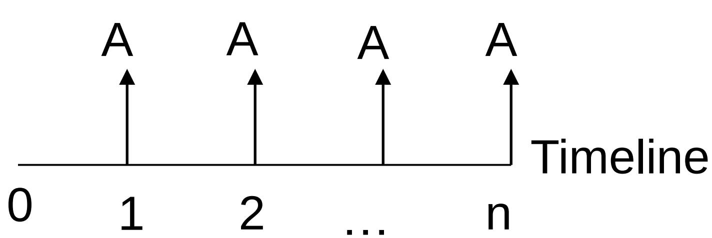
Types of Equivalent Values

Present Value

Future Value



Annual Value



<u>Note:</u>
The ***end-of-period cash flow*** *convention* is the standard assumption for this module (unless otherwise stated).

Finding Present Value

If you need $5,000 at the end of 5 years, how much money would you need to deposit in an account that pays an annual compound interest of 8%?
Use both Interest Factor Formula method and Interest Factor Notation method.

Single Payment

Given : F = $5,000, i =8% per year, N = 5 years
Find : P

Using Interest Factor Formula :
$P = F * [1/(1+i)^N]$
$= 5000 * [1/(1+0.08)^5]$
= $3402.92

Formula:

$$P = F\left(\frac{1}{(1+i)^N}\right)$$

Using Interest Factor Notation :
P = F * (P/F, 8%, 5)
= 5000 * (0.6806)
= $3403.00

Interest Factor Table (8%)

576 COMPOUND INTEREST TABLES

8% **Compound Interest Factors** 8%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.080	.9259	1.0000	1.0800	1.000	0.926	0	0	1
2	1.166	.8573	.4808	.5608	2.080	1.783	0.481	0.857	2
3	1.260	.7938	.3080	.3880	3.246	2.577	0.949	2.445	3
4	1.360	.7350	.2219	.3019	4.506	3.312	1.404	4.650	4
5	1.469	.6806	.1705	.2505	5.867	3.993	1.846	7.372	5
6	1.587	.6302	.1363	.2163	7.336	4.623	2.276	10.523	6
7	1.714	.5835	.1121	.1921	8.923	5.206	2.694	14.024	7
8	1.851	.5403	.0940	.1740	10.637	5.747	3.099	17.806	8
9	1.999	.5002	.0801	.1601	12.488	6.247	3.491	21.808	9
10	2.159	.4632	.0690	.1490	14.487	6.710	3.871	25.977	10
11	2.332	.4289	.0601	.1401	16.645	7.139	4.240	30.266	11
12	2.518	.3971	.0527	.1327	18.977	7.536	4.596	34.634	12
13	2.720	.3677	.0465	.1265	21.495	7.904	4.940	39.046	13
14	2.937	.3405	.0413	.1213	24.215	8.244	5.273	43.472	14
15	3.172	.3152	.0368	.1168	27.152	8.559	5.594	47.886	15

N=5

Finding Present Value

You wish to buy a car 4 years later. The payment is estimated to be $175,000 by that time. How much should you put in the bank now (at annual compound interest of 1.25%) so that you can have $175,000 in 4 years' time to buy the car?

Single Payment

Given : F = $175,000, i =1.25% per year, N = 4 years

Find : P

Using **Interest Factor Formula**

P = $F(1+i)^{-N}$

= $175000 * (1/(1+0.0125)^4)$

= $166,516.75

Using **Interest Factor Notation**

P = 175,000 * (P/F, 1.25%, 4)

= 175000 * (0.9515)

= $166,512.50

Using **Excel Financial Function**

P = 175,000 * PV(1.25%,4,0,-1,0)

= $166,516.75

Formula:

$$P = F\left(\frac{1}{(1+i)^{N}}\right)$$

Excel Financial Functions

Excel Financial Functions	Purpose
NPV (rate, value1, value2, …)	Calculates the **net present value** of an investment by using a discount rate and a **series of future payments** (negative values) and income (positive values).
PMT (rate, nper, pv, fv, type) PMT(10%,5,-200,,0)	Calculates the payment for a loan based on constant payments and a constant interest rate.
FV (rate, nper, pmt, pv, type) FV(10%,5,-20,,0)	Returns the **future value** of an investment based on periodic, constant payments and a constant interest rate.
PV (rate, nper, pmt, fv, type) PV(10%,5,-20,,0)	Returns the **present value** of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender.
type 0	Default, **end** of period cash flow
type 1	**Beginning** of period cash flow

The *end-of-period cash flow convention* is the standard assumption for this module

Excel Financial Functions

Assumptions:

- The per period interest rate, i, shall remain constant.
- There is exactly one period between the cash flows
- The period length shall remain constant
- The default end-of-period cash flow convention (type=0) is used
- The first cash flow is in a range at the end of the first period

Finding Future Value

You have $50,000 in the bank now. How much can you get back 7 years later with annual compound interest rate of 5%.

Given : P = $50,000, i = 5% per year, N = 7 years

Find : F

Single Payment

Using **Interest Factor Formula**

F $= P * (1+i)^N$

$= 50,000 * (1+0.05)^7$

= $70,355

Using **Interest Factor Notation**

F = 50,000 * (F/P, 5%, 7)

= 50,000 * (1.407)

= $70,350

Using **Excel Financial Function**

F = 50,000 * FV(5%, 7, 0, -1,0))

= $70,355

Formula:

$$F = P(1 + i)^N$$

where

F: Future value

P: Present value

i : Interest rate

N: Number of compounding periods (years, months, etc.)

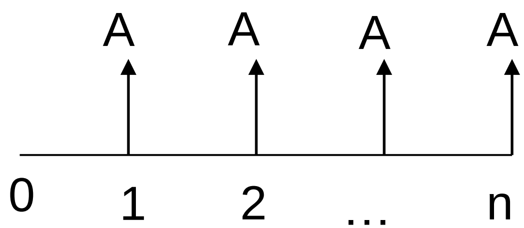
Interest Factor Table (5%)

5% Compound Interest Factors 5%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find F Given P F/P	Find P Given F P/F	Find A Given F A/F	Find A Given P A/P	Find F Given A F/A	Find P Given A P/A	Find A Given G A/G	Find P Given G P/G	n
1	1.050	.9524	1.0000	1.0500	1.000	0.952	0	0	1
2	1.102	.9070	.4878	.5378	2.050	1.859	0.488	0.907	2
3	1.158	.8638	.3172	.3672	3.152	2.723	0.967	2.635	3
4	1.216	.8227	.2320	.2820	4.310	3.546	1.439	5.103	4
5	1.276	.7835	.1810	.2310	5.526	4.329	1.902	8.237	5
6	1.340	.7462	.1470	.1970	6.802	5.076	2.358	11.968	6
N=7 7	1.407	.7107	.1228	.1728	8.142	5.786	2.805	16.232	7
8	1.477	.6768	.1047	.1547	9.549	6.463	3.244	20.970	8
9	1.551	.6446	.0907	.1407	11.027	7.108	3.676	26.127	9
10	1.629	.6139	.0795	.1295	12.578	7.722	4.099	31.652	10
11	1.710	.5847	.0704	.1204	14.207	8.306	4.514	37.499	11
12	1.796	.5568	.0628	.1128	15.917	8.863	4.922	43.624	12
13	1.886	.5303	.0565	.1065	17.713	9.394	5.321	49.988	13
14	1.980	.5051	.0510	.1010	19.599	9.899	5.713	56.553	14
15	2.079	.4810	.0463	.0963	21.579	10.380	6.097	63.288	15

What is Uniform Series?

Suppose that there is a series of uniform payments for ‘n’ number of years. The payment is ‘A’ amount and is uniform in amount and uniformly spaced.



Timeline

Uniform Series of payments

Uniform Payment in Sinking Fund

Instead of forking out a lump sum now in the bank, you want to put some money into the bank every year (at annual compound interest of 1.25%) to save up for a car. How much should you put in the bank every year so that you have $175,000 in 4 years' time to buy the car?

Given : F = $175,000, i =1.25% per year, N = 4 years

Find : A

Uniform Series

Using **Interest Factor Formula**

A = 175,000 *(0.0125/[(1+0.0125)^4 -1])

= $42,938

Using **Interest Factor Notation**

A = 175,000 * (A/F, 1.25%, 4)

= 175000 * (0.2454) = $42,945

Using **Excel Financial Function**

A = 175,000 *PMT(1.25%, 4, 0, -1, 0)

= $42,938

Formula:

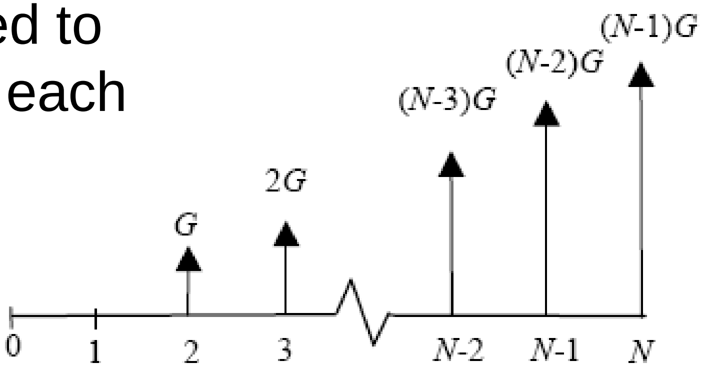
$$A = F\left(\frac{i}{(1+i)^N - 1}\right)$$

Uniform Gradient Cash Flow

What if we have cash flow that are projected to increase or decrease by a uniform amount each time period?

- We call this uniform gradient series (G)

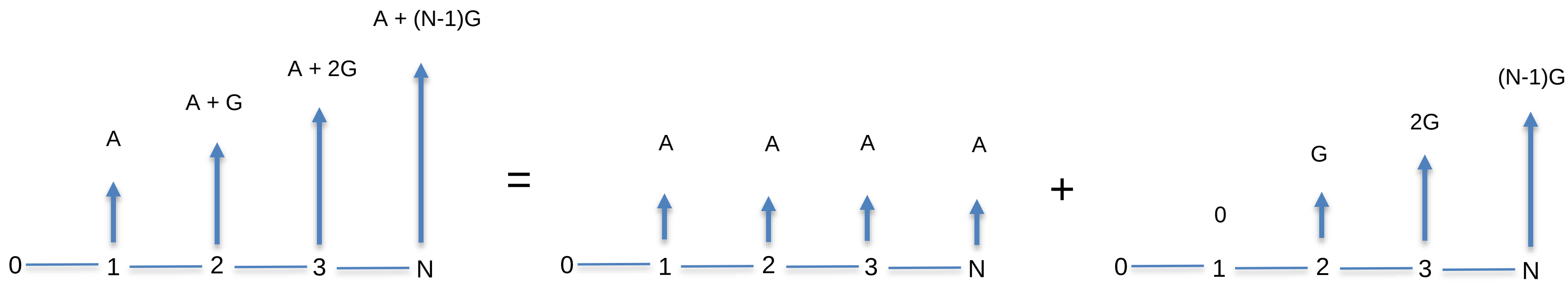
- We can have positive or negative G



To Find	Given	Symbol	Formula
F	G	[F/G, i%, N]	$\frac{G}{i}[F/A, i\%, N] - \frac{NG}{i}$
A	G	[A/G, i%, N]	$G\left(\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right)$
P	G	[P/G, i%, N]	$G\left\{\frac{1}{i}\left(\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right)\right\}$

Uniform Gradient Cash Flow

Determine the Present Value (PV) that is equivalent to the cash-flow pattern given in the table below where interest rate is 7% per year. (*Apply Arithmetic Gradient Cash Flow*).



End of Year	Amount ($)
0	0
1	230
2	290
3	350
4	410
5	470
6	530
7	590

Given : A = $230, G = $60, i =7% per year, N = 7 years

Find : P

Using **Factor Notation**

PV = $230 * [P/A, 7%, 7] + $60 * [P/G, 7%, 7]

= $230 * 5.389 + $60 * 14.715

= $2,122.43

Interest Factor Table (7%)

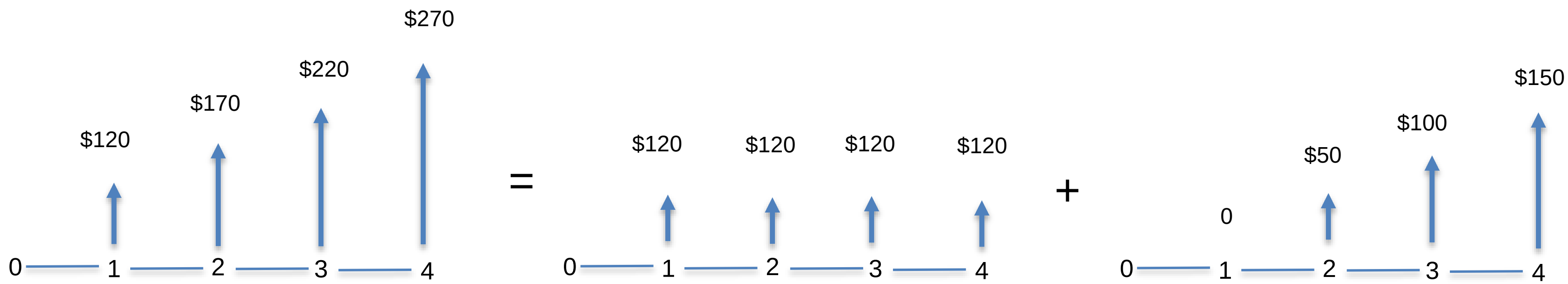
7% **Compound Interest Factors** 7%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.070	.9346	1.0000	1.0700	1.000	0.935	0	0	1
2	1.145	.8734	.4831	.5531	2.070	1.808	0.483	0.873	2
3	1.225	.8163	.3111	.3811	3.215	2.624	0.955	2.506	3
4	1.311	.7629	.2252	.2952	4.440	3.387	1.416	4.795	4
5	1.403	.7130	.1739	.2439	5.751	4.100	1.865	7.647	5
6	1.501	.6663	.1398	.2098	7.153	4.767	2.303	10.978	6
7	1.606	.6227	.1156	.1856	8.654	5.389	2.730	14.715	7
8	1.718	.5820	.0975	.1675	10.260	5.971	3.147	18.789	8
9	1.838	.5439	.0835	.1535	11.978	6.515	3.552	23.140	9
10	1.967	.5083	.0724	.1424	13.816	7.024	3.946	27.716	10
11	2.105	.4751	.0634	.1334	15.784	7.499	4.330	32.467	11
12	2.252	.4440	.0559	.1259	17.888	7.943	4.703	37.351	12
13	2.410	.4150	.0497	.1197	20.141	8.358	5.065	42.330	13
14	2.579	.3878	.0443	.1143	22.551	8.745	5.417	47.372	14
15	2.759	.3624	.0398	.1098	25.129	9.108	5.758	52.446	15

N=7

Test Yourself !

Determine the Present Value (PV) that is equivalent to the cash-flow pattern given in the table below where interest rate is 5% per year. (*Apply Arithmetic Gradient Cash Flow*).



End of Year	Amount ($)
0	0
1	120
2	170
3	220
4	270

Given : A = $120, G = $50, i =5% per year, N = 4 years

Find : P

Using **Factor Notation**

PV = $120 * [P/A, 5%, 4] + $50 * [P/G, 5%, 4]

= $120 * 3.546 + $50 * 5.1028

= $680.66

Uniform Series Present Worth

My father gives me \$200 for my monthly allowance. What is the equivalent amount if he gave me at the start of the year? (Assume monthly compound interest =2%).

Given : A = \$200, i =2% per month, N = 12 months

Find : P

Uniform Series

Using **Interest Factor Formula**

$P = A * [((1+i)^N-1)/(i(1+i)^N]$

$= 200 *[((1+0.02)^{12}-1)/(0.02(1+0.02)^{12})]$

= \$2,115

Formula:

$$P = A\left(\frac{(1+i)^N - 1}{i(1+i)^N}\right)$$

Using **Interest Factor Notation**

P = 200 * (P/A, 2%, 12)

= 200 * (10.575)

= \$2,115

Using **Excel Financial Function**

P = 200 * PV(2%, 12, -200, 0, 0)

= \$2,115

Test Yourself !

If you save $100 monthly for 24 months, what is the equivalent amount at the start of the year? (Assume monthly compound interest =1%).

Given : A = $100, i =1% per month, N = 24 months

Find : P

Using **Interest Factor Formula**

$P = A * [((1+i)^N-1)/(i(1+i)^N]$

$= 100 *[((1+0.01)^{24}-1)/(0.01(1+0.01)^{24})]$

= $2124.34

Using **Interest Factor Notation**

P = 100 * (P/A, 1%, 24)

= 100 * (21.2434)

= $2124.34

Using **Excel Financial Function**

P = 100 * PV(1%, 24, -100, 0, 0)

= $2,124.34

Scenario

Loan	Interest Rate	Payment
A	Effective annual interest of 5%	Pay $22,152.40 (2020); $27,152.40 (2021); $32,152.40 (2022)
B	Annual nominal interest of 5% compounded monthly	Pay $19,950 yearly from 2019 to 2022
C	Annual nominal interest of 5% compounded quarterly	Pay $28,456 (2019); $50456 (2021); $756 (2022)

Dave wants to borrow $70,000 now and found 3 loans. Use Future Value to compare which loan type will result in the minimum amount of future value of the payments

Suggested Answer for Scenario

Scenario

Loan	Interest Rate	Payment
A	Effective annual interest of 5%	Pay $22,152.40 (2020); $27,152.40 (2021); $32,152.40 (2022)
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Dave wants to borrow $70,000 now and found 3 loans. Use Future Value to compare which loan type will result in the minimum amount of future value of the payments

Scenario

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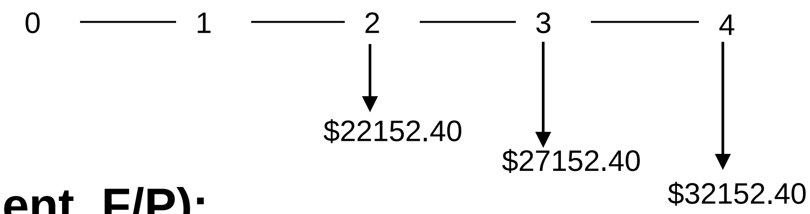
Year	2018	2019	2020	2021	2022
Period	0	1	2	3	4
Loan A	0	0.00	22152.70	27152.70	32152.70
Loan B	0	19950.00	19950.00	19950.00	19950.00
Loan C	0	28456.00	0.00	50456.00	756.00

Loan A : Effective Annual Interest of 5%

Cash Flow Diagram:



Solution (using Single Payment, F/P):

Given : P_2 = \$22152.40, P_3 = \$27152.40, P_4 = \$32152.40

i_{eff} = 5% per year, N = 4 years

Find : F

$$i_{eff} = (1 + \frac{0.05}{1})^1 - 1$$
$= 0.05$
$= 5\%$

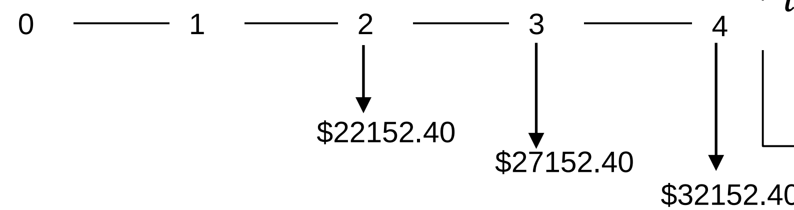
Future Value of the payments (at Year 2022)
= 22152.4*(F/P, 5%,2) + 27152.4*(F/P, 5%,1) + 32152.4
= 22152.4*(1.1025) + 27152.4*(1.05) + 32152.4
= $85,085 (Round off to the nearest dollar)

Future Value of the loan at the end of 2022 based on interest given in Loan A
= 70000*(F/P, 5%,4)
= 70000*(1.2155)
= $85,085

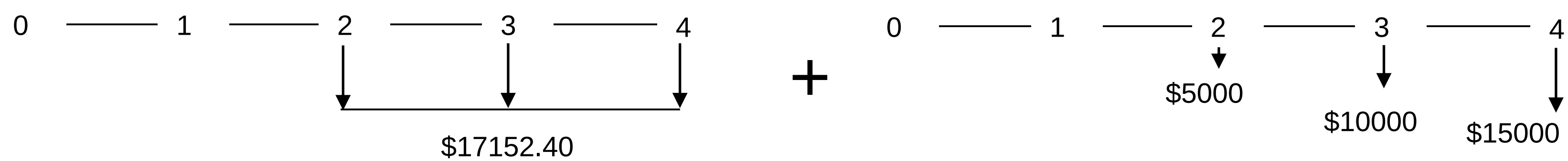
- The future value of the required payment plan (***$85085***) is the same as the future value of the principal amount borrowed (***$85085***).
- The payment plan demands an interest rate that is as specified in the plan.

Loan A : Effective Annual Interest of 5%

Cash Flow Diagram:



$$i_{eff} = \left(1 + \frac{0.05}{1}\right)^1 - 1$$
$$= 0.05$$
$$= 5\%$$



Solution (using Uniform Series, F/A & Uniform Gradient, F/G):

Using : A = $17152.40, G = $5000.00

Are there any other possible values for A and G?

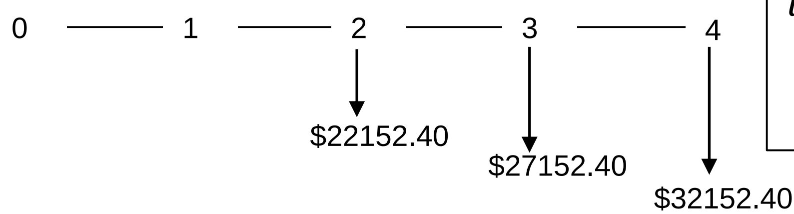
Find : F

Future Value of the payments (at Year 2022)
= 17152.4*(F/A, 5%,3) + 5000*(F/G, 5%,4)
= 17152.4*(3.152) + 5000*(6.2025)
= $85085 (Round off to the nearest dollar)

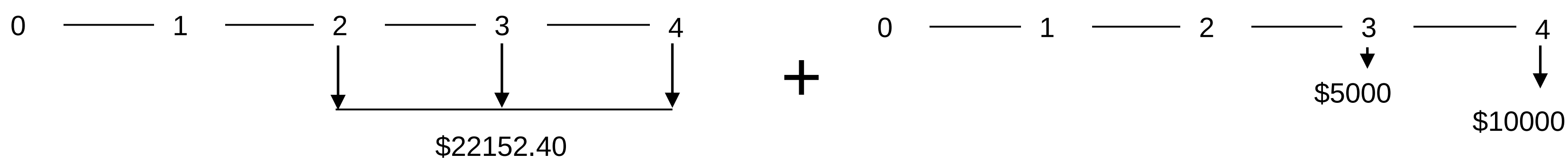
- The future value of the required payment plan (***$85085***) is the same as the future value of the principal amount calculated using Single Payment method (***$85085***).

Loan A : Effective Annual Interest of 5%

Cash Flow Diagram:



$$i_{eff} = \left(1 + \frac{0.05}{1}\right)^{1} - 1$$
$$= 0.05$$
$$= 5.00\%$$



Solution (using Uniform Series, F/A & Uniform Gradient, F/G):

Using : A = $22152.40, G = $5000.00

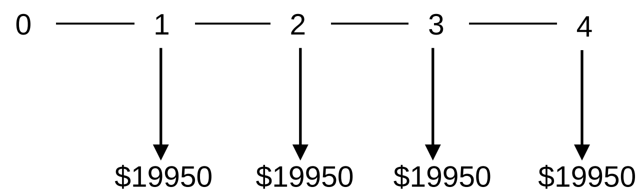
Find : F

Future Value of the payments (at Year 2022)
= 22152.4*(F/A, 5%,3) + 5000*(F/G, 5%,3)
= 22152.4*(3.1525) + 5000*(3.05)
= $85085 (Round off to the nearest dollar)

- The future value of the required payment plan (***$85085***) is the same as the future value of the principal amount calculated using Single Payment method (***$ 85085***).

Loan B : Annual Nominal Interest of 5% Compounded Monthly

Cash Flow Diagram:



$$i_{eff} = (1 + \frac{0.05}{12})^{12} - 1$$
$$= 0.05116$$
$$= 5.116\%$$

Given : $P_1 = P_2 = P_3 = P_4 = A = \$19{,}950$, $i_{nominal} = 5\%$ per year, N = 4 years
Find : F

Solution (using Single Payment, F/P):

Future Value of the payments (at Year 2022)
= 19950*(F/P, 5.116%,3) + 19950*(F/P, 5.116%,2) + 19950*(F/P, 5.116%,1) + 19950
= 19950*(1.1615) + 19950*(1.1049) + 19950*(1.052) + 19950
= $86152 (Round off to the nearest dollar)

Solution (using Uniform Series, F/A):

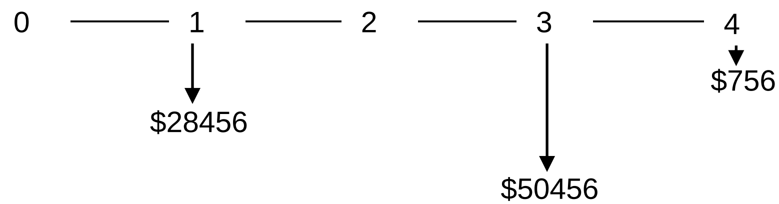
Future Value of the payments (at Year 2022)
= 19950*(F/A, 5.116%, 4)
= 19950*(4.3176)
= $86136 (Round off to the nearest dollar)

Future Value of the loan at the end of Year 2022 based on interest given in Loan A)
= 70000*(F/P, 5.116%,4)
= 70000*(1.2209)
= $85463

- The future value of the required payment plan (***$86136***) is higher than the future value of the principal amount calculated using Single Payment method (***$85463***).
- The payment plan demands an interest rate higher than specified in the plan.

Loan C : Annual Nominal Interest of 5% Compounded Quarterly

Cash Flow Diagram:



$$i_{eff} = (1 + \frac{0.05}{4})^{4} - 1$$
$$= 0.05095$$
$$= 5.095\%$$

Solution (using Single Payment, F/P):

Given : P_1 = \$28456, P_3 = \$50456, P_4 = \$756

i_{eff} = 5.095% per year, N = 4 years

Find : F

Future Value of the payments (at Year 2022)
= 28456*(F/P, 5.095%,3) + 50456*(F/P, 5.095%,1) + 756
= 28456*(1.1608) + 50456*(1.0510) + 756
= \$86816 (Round off to the nearest dollar)

Future Value of the loan at the end of Year 2022 based on interest given in Loan A
= 70000*(F/P, 5.095%,4)
= 70000*(1.2199)
= \$85393

- The future value of the required payment plan (***\$86816***) is higher than the future value of the principal amount calculated using Single Payment method (***\$85393***).
- The payment plan demands an interest rate higher than specified in the plan.

Comparison

Loan	Calculated Annual % (eff)	FV of principal	FV of payment	Actual Annual %
Loan A	5.000%	85085	85085	5.000%
Loan B	5.116%	85463	86136	5.323%
Loan C	5.095%	85393	86816	5.529%

Loan with the lowest interest rate and minimum future value of payments

The actual annual interest rate for Loan A is as per specified and it is the lowest annual interest rate among the 3 options. Thus, we recommend Dave to take up Loan A.

Learning Objectives

- ✓ Apply the equivalence of present, future and annual values
- ✓ Develop cash flow diagrams
- ✓ Apply the single, uniform series and uniform gradient cash flow for analysis
- ✓ Compute the nominal and effective interest rates
- ✓ Compute Present, Future and Annual values using formulas, interest factor tables

E213 Engineering Cost Decisions (Topic Flow)

Today's learning

- Application of ABC costing method in cost management
- Application of different cost estimating techniques
- Comparison of alternatives using the concept of equivalence
- Alternatives evaluation using single, uniform series and uniform gradient cash flows
- Project evaluation using MARR and Equivalent Worth method
- Project evaluation based on Internal Rate of Return and External Rate of Return
- Evaluate alternatives of equal life spans using payback method
- Evaluate alternatives with different life spans
- Evaluate public projects through incremental B/C analysis
- Depreciation estimation and consideration in economic analysis
- Tax consideration in economic analysis
- Replacement analysis application
- Risk and uncertainties handling in economic analysis