Problem 9

Design of Experiments (DOE) Interactive Seminar

E326 – Lean Manufacturing & Six Sigma

SCHOOL OF ENGINEERING

Now you try it…

Today's Catapult

Steps:

1. Define the objective of the experiment
   - To determine the optimum settings to use in order to hit the Target distance, e.g. **58 +/- 3 cm**

2. Identify the response and input factors
   - Response factor = Distance travelled by the ball
   - Input factors = (i) Start Angle, (ii) Stop Position, (iii) Hook Position.

3. Determine the resources required
   - Catapult set, ball, measuring tape
   - Three replications of experiment

Today's Problem

4. Select experiments design matrix and analysis strategy
   - 2-level, 3 factors full factorial design (2^3):
     - 3 factors (Start Angle, Stop Position, Hook Position)
     - 2 Levels (160/180, 2/4, 2/4)

5. Conduct the experiments and record data
   - Use Minitab to generate all the experimental runs in the worksheet in a randomized manner.
   - Conduct the experiments to get the observation value at each treatment.

6. Analyze data, draw conclusions and perform confirmation runs.
   - Analyze Custom Factorial Design

2^k Factorial Level

You may use either the Coded or Uncoded Design Matrix to solve this problem. However, in this problem, we will focus on using the Uncoded Design Matrix .

Uncoded levels

Factors	Low Setting	High Setting
Hook Position	2	4
Stop Position	2	4
Start Angle	160	180

Coded levels

Factors	Low setting	High setting
Hook Position	-1	+1
Stop Position	-1	+1
Start Angle	-1	+1

Note:

Coded: Choose if the worksheet data are in coded form(-1 = low; +1 = high).

Uncoded: Choose if the worksheet data are in uncoded form. That is, the worksheet values are in units of the actual measurements.

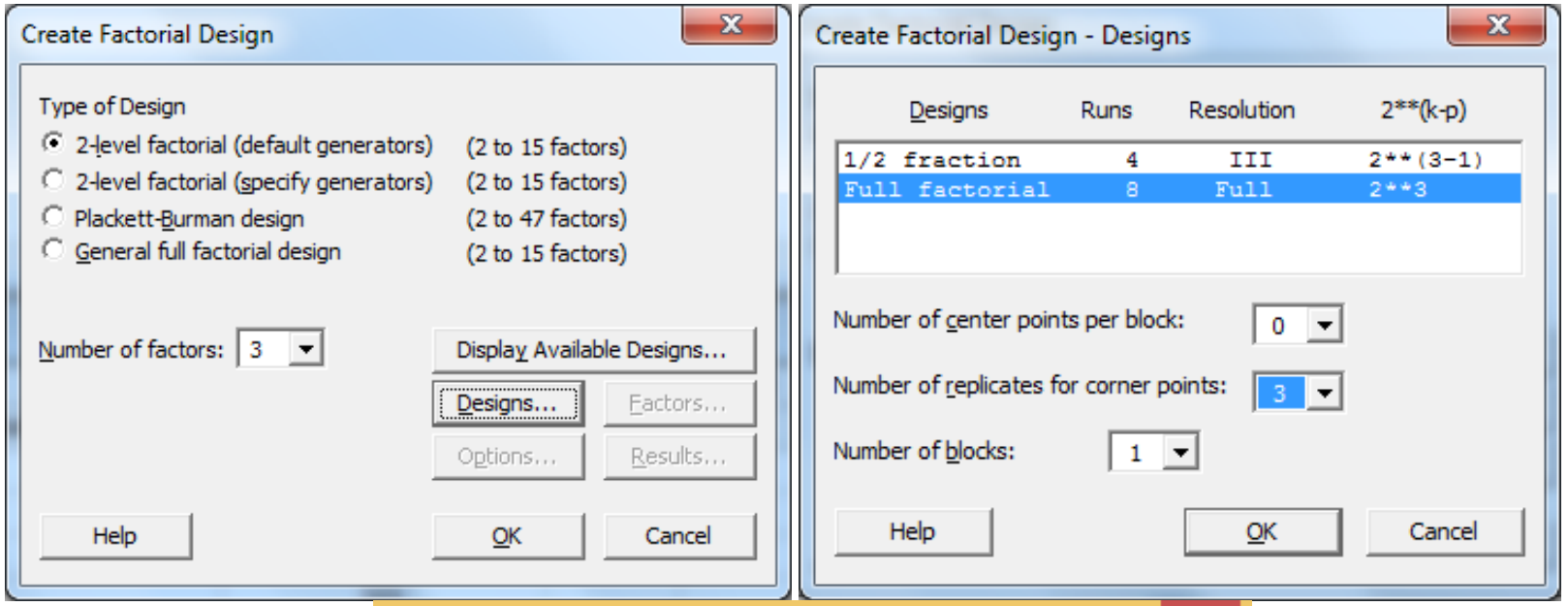
2^k Factorial Test Matrix (in Yates Order)

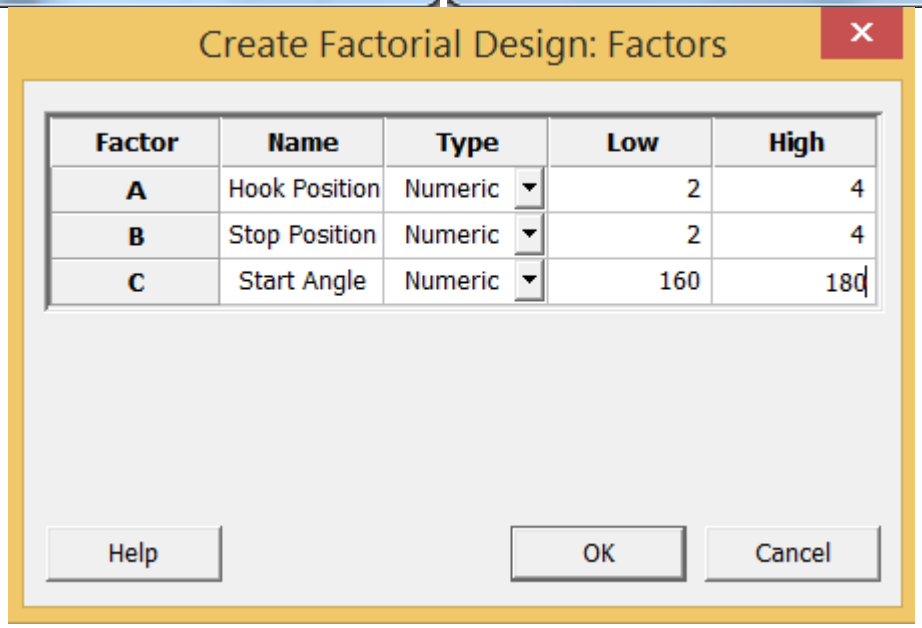
Un-coded Design Matrix		
Hook Position (level)	Stop Position (level)	Start Angle (degree)
2	2	160
4	2	160
2	4	160
4	4	160
2	2	180
4	2	180
2	4	180
4	4	180

Coded Matrix		
A	B	C
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1

Note : 2^k means Two levels, K factors

Setting up the DOE in Minitab





2^3 Full Factorial Design (Uncoded) with 3 Replicates

Key in all the responses collected

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	Hook Position	Stop Position	Start Angle	Dist-Heavy-Ball	Dist-Light-Ball
1	1	1	1	1	2	2	160	33.0	40.0
2	2	2	1	1	4	2	160	16.5	24.0
3	3	3	1	1	2	4	160	62.0	73.0
4	4	4	1	1	4	4	160	42.5	55.0
5	5	5	1	1	2	2	180	73.0	89.0
6	6	6	1	1	4	2	180	37.0	53.0
7	7	7	1	1	2	4	180	89.0	93.0
8	8	8	1	1	4	4	180	50.0	64.0
9	9	9	1	1	2	2	160	32.0	42.0
10	10	10	1	1	4	2	160	16.0	24.5
11	11	11	1	1	2	4	160	62.0	71.0
12	12	12	1	1	4	4	160	43.0	54.0
13	13	13	1	1	2	2	180	74.0	89.0
14	14	14	1	1	4	2	180	37.0	53.0
15	15	15	1	1	2	4	180	89.0	94.0
16	16	16	1	1	4	4	180	51.0	64.5
17	17	17	1	1	2	2	160	33.0	41.0
18	18	18	1	1	4	2	160	17.0	25.0
19	19	19	1	1	2	4	160	62.5	72.0
20	20	20	1	1	4	4	160	42.5	53.5
21	21	21	1	1	2	2	180	74.0	89.0
22	22	22	1	1	4	2	180	37.5	53.0
23	23	23	1	1	2	4	180	89.5	93.5
24	24	24	1	1	4	4	180	50.0	65.0

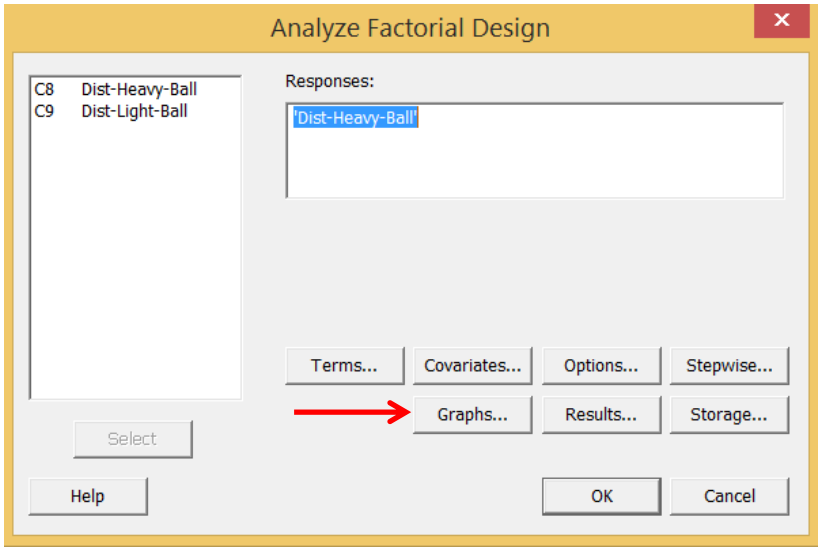
1st replicate

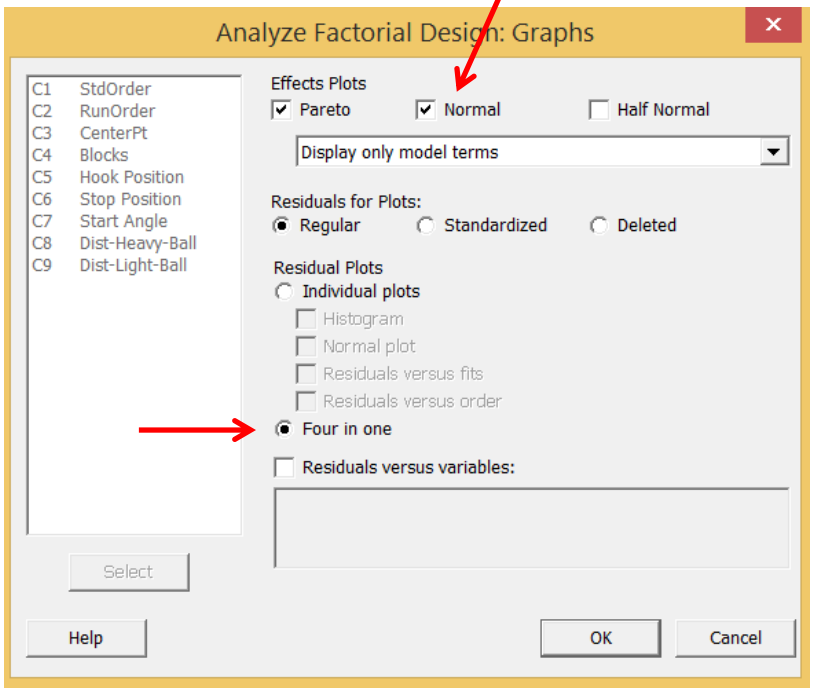
2nd replicate

3rd replicate

Results and Analysis (Identifying insignificant terms)

Minitab > Stat > DOE > Factorial > Analyse Factorial Design…>Graphs)





Results and Analysis (Identifying insignificant terms)

Factorial Regression: Dist-Heavy-Ball versus Hook Position, Stop Position, Start Angle

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	11643.3	1663.33	8404.18	0.000
Linear	3	10767.5	3589.15	18134.67	0.000
Hook Position	1	4620.4	4620.38	23345.05	0.000
Stop Position	1	2667.0	2667.04	13475.58	0.000
Start Angle	1	3480.0	3480.04	17583.37	0.000
2-Way Interactions	3	875.5	291.82	1474.46	0.000
Hook Position*Stop Position	1	12.0	12.04	60.84	0.000
Hook Position*Start Angle	1	590.0	590.04	2981.26	0.000
Stop Position*Start Angle	1	273.4	273.38	1381.26	0.000
3-Way Interactions	1	0.4	0.37	1.89	0.188
Hook Position*Stop Position*Start Angle	1	0.4	0.37	1.89	0.188
Error	16	3.2	0.20		
Total	23	11646.5			

Significant (P-value < 0.05)

Not significant (P value > 0.05)

Model Summary

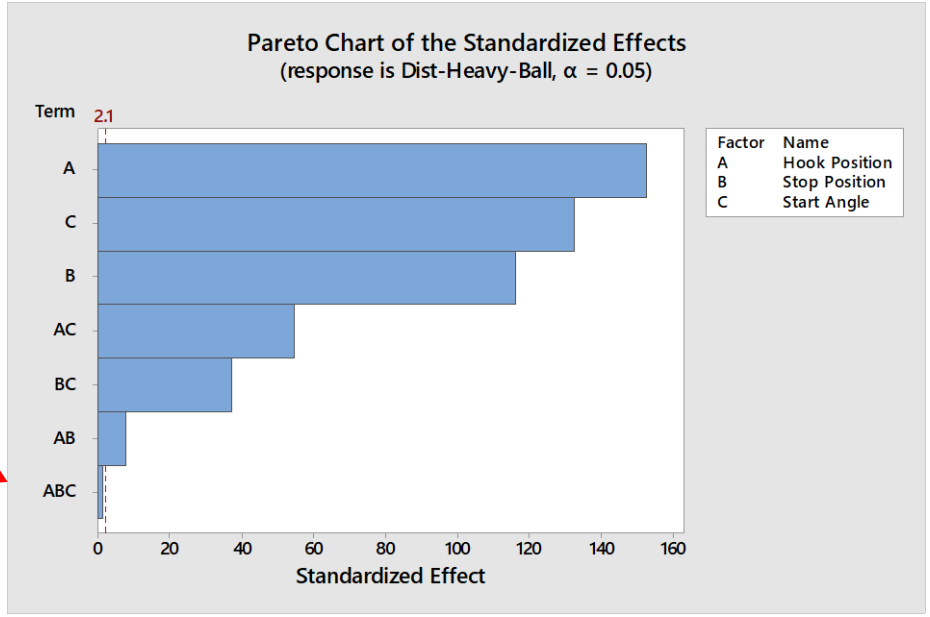
S	R-sq	R-sq(adj)	R-sq(pred)
0.444878	99.97%	99.96%	99.94%

Results and Analysis (Identifying insignificant terms)

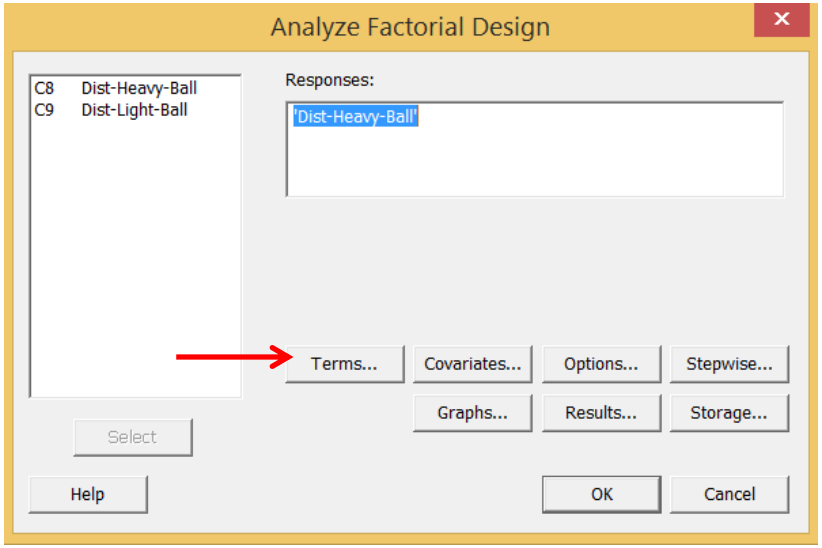
Not significant Term:

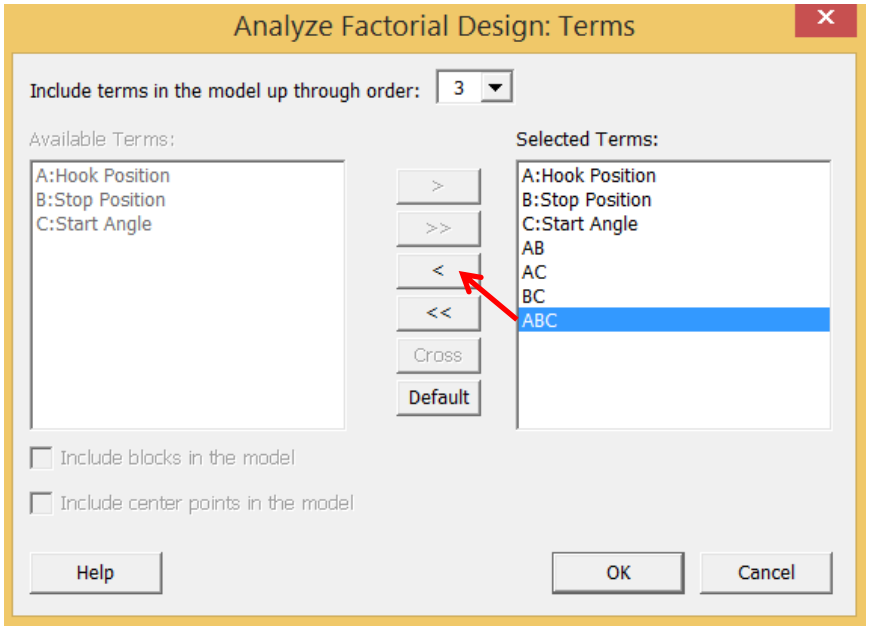
Hook Position * Stop Position * Start Angle

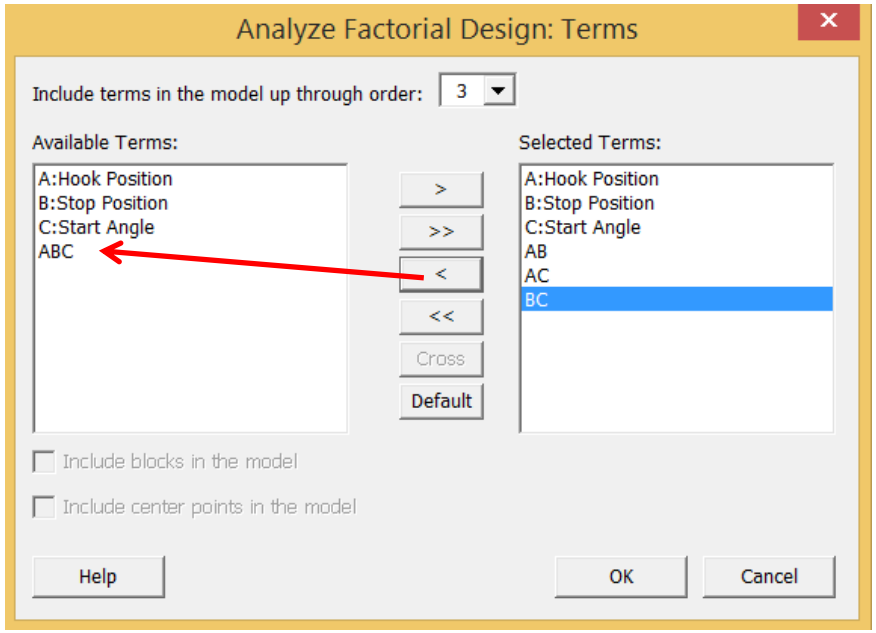
(P value > 0.05)



Results and Analysis (Removing insignificant terms)







Results and Analysis
(After removing insignificant terms)

Factorial Regression: Dist-Heavy-Ball versus Hook Position, Stop Position, Start Angle

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	6	11642.9	1940.49	9314.33	0.000
Linear	3	10767.5	3589.15	17227.93	0.000
Hook Position	1	4620.4	4620.37	22177.80	0.000
Stop Position	1	2667.0	2667.04	12801.80	0.000
Start Angle	1	3480.0	3480.04	16704.20	0.000
2-Way Interactions	3	875.5	291.82	1400.73	0.000
Hook Position*Stop Position	1	12.0	12.04	57.80	0.000
Hook Position*Start Angle	1	590.0	590.04	2832.20	0.000
Stop Position*Start Angle	1	273.4	273.37	1312.20	0.000
Error	17	3.5	0.21		
Lack-of-Fit	1	0.4	0.37	1.89	0.188
Pure Error	16	3.2	0.20		
Total	23	11646.5			

The ANOVA table includes a lack-of-fit test. The null hypothesis is that this model fits the experimental response data adequately. Since p value > level of significance, we do not reject the null hypothesis. Model is therefore a good fit with the observed data.

Model Summary

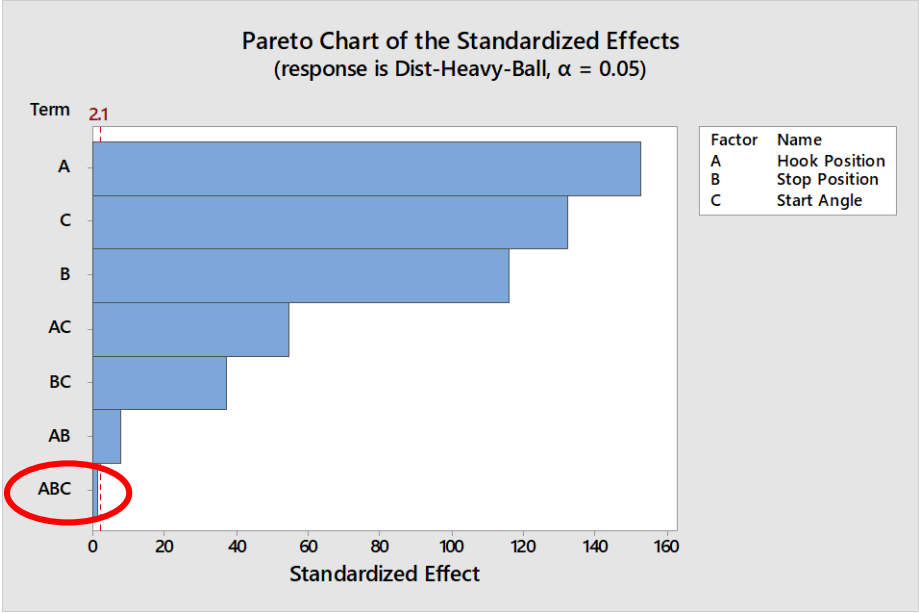
S	R-sq	R-sq(adj)	R-sq(pred)
0.456435	99.97%	99.96%	99.94%

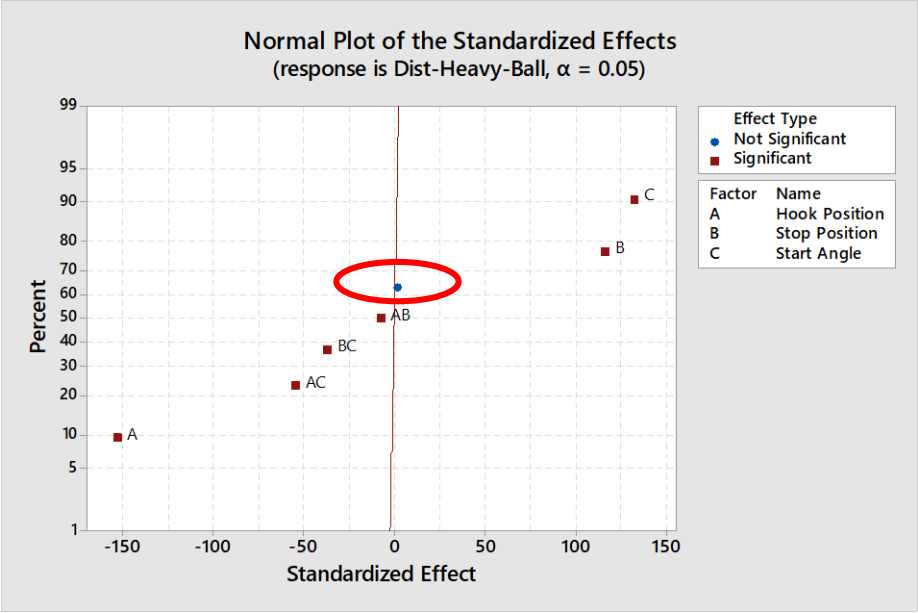
R^2 = proportion of variation in the response that can be explained by the model equation. It has 99.93% > 80% which indicates that the model is good.

Coded Coefficients

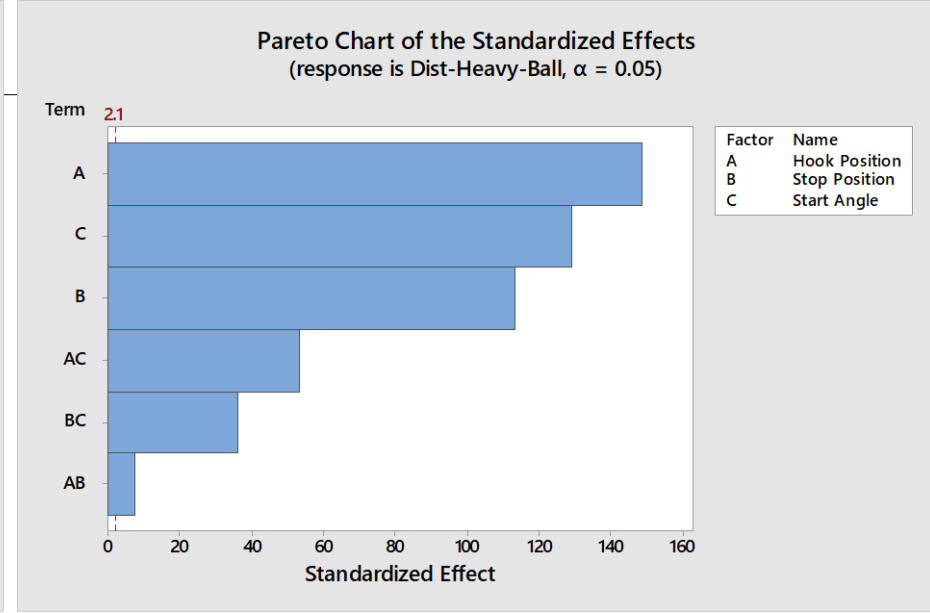
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		50.5417	0.0932	542.47	0.000	
Hook Position	-27.7500	-13.8750	0.0932	-148.92	0.000	1.00
Stop Position	21.0833	10.5417	0.0932	113.15	0.000	1.00
Start Angle	24.0833	12.0417	0.0932	129.24	0.000	1.00
Hook Position*Stop Position	-1.4167	-0.7083	0.0932	-7.60	0.000	1.00
Hook Position*Start Angle	-9.9167	-4.9583	0.0932	-53.22	0.000	1.00
Stop Position*Start Angle	-6.7500	-3.3750	0.0932	-36.22	0.000	1.00

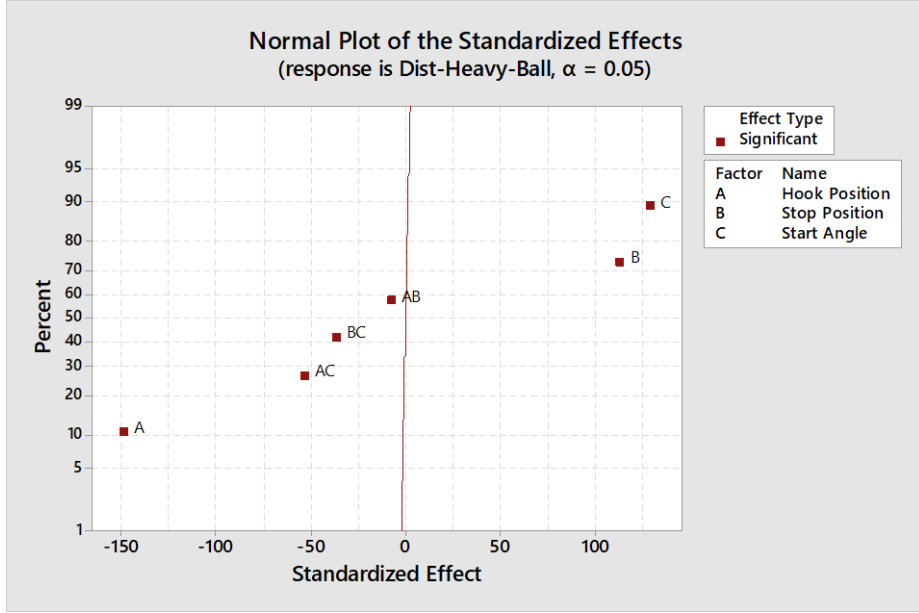
Before removing insignificant terms





After removing insignificant terms





Prediction Model

Before removing insignificant terms

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		50.5417	0.0908	556.56	0.000	
Hook Position	-27.7500	-13.8750	0.0908	-152.79	0.000	1.00
Stop Position	21.0833	10.5417	0.0908	116.08	0.000	1.00
Start Angle	24.0833	12.0417	0.0908	132.60	0.000	1.00
Hook Position*Stop Position	-1.4167	-0.7083	0.0908	-7.80	0.000	1.00
Hook Position*Start Angle	-9.9167	-4.9583	0.0908	-54.60	0.000	1.00
Stop Position*Start Angle	-6.7500	-3.3750	0.0908	-37.17	0.000	1.00
Hook Position*Stop Position*Start Angle	0.2500	0.1250	0.0908	1.38	0.188	1.00

Regression Equation in Uncoded Units

Distance = -594.7 + 78.92 Hook Position + 76.42 Stop Position + 3.8167 Start Angle
- 2.83 Hook Position*Stop Position - 0.5333 Hook Position*Start Angle
- 0.3750 Stop Position*Start Angle
+ 0.01250 Hook Position*Stop Position*Start Angle

After removing insignificant terms

Coded Coefficients

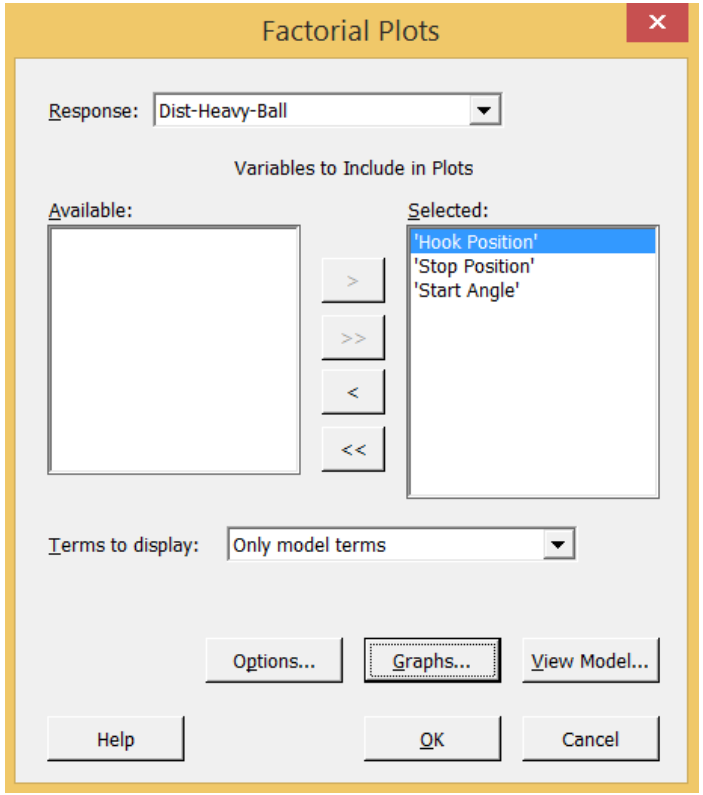
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		50.5417	0.0932	542.47	0.000	
Hook Position	-27.7500	-13.8750	0.0932	-148.92	0.000	1.00
Stop Position	21.0833	10.5417	0.0932	113.15	0.000	1.00
Start Angle	24.0833	12.0417	0.0932	129.24	0.000	1.00
Hook Position*Stop Position	-1.4167	-0.7083	0.0932	-7.60	0.000	1.00
Hook Position*Start Angle	-9.9167	-4.9583	0.0932	-53.22	0.000	1.00
Stop Position*Start Angle	-6.7500	-3.3750	0.0932	-36.22	0.000	1.00

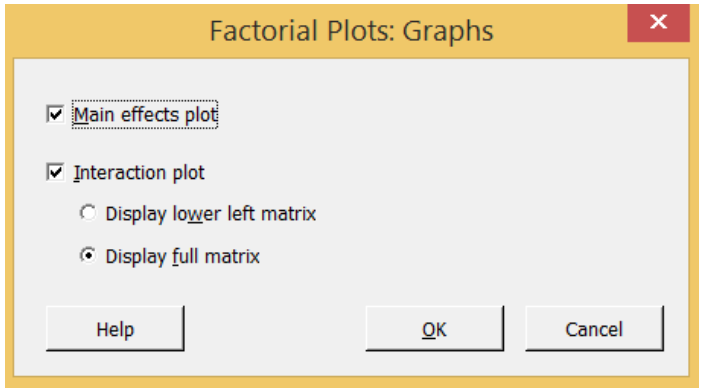
Regression Equation in Uncoded Units

Dist-Heavy-Ball = -575.54 + 72.54 Hook Position + 70.04 Stop Position + 3.7042 Start Angle
- 0.7083 Hook Position*Stop Position - 0.49583 Hook Position*Start Angle
- 0.33750 Stop Position*Start Angle

Prediction Model is based on 'uncoded units', i.e. hook position = 2, 4, instead of -1, 1

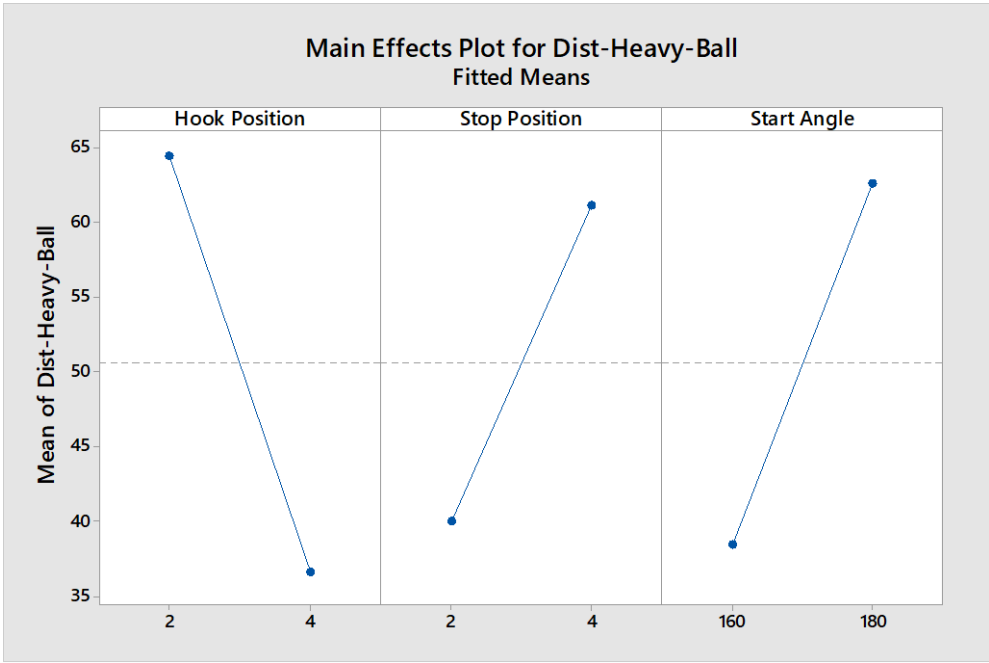
Main Effects Plots





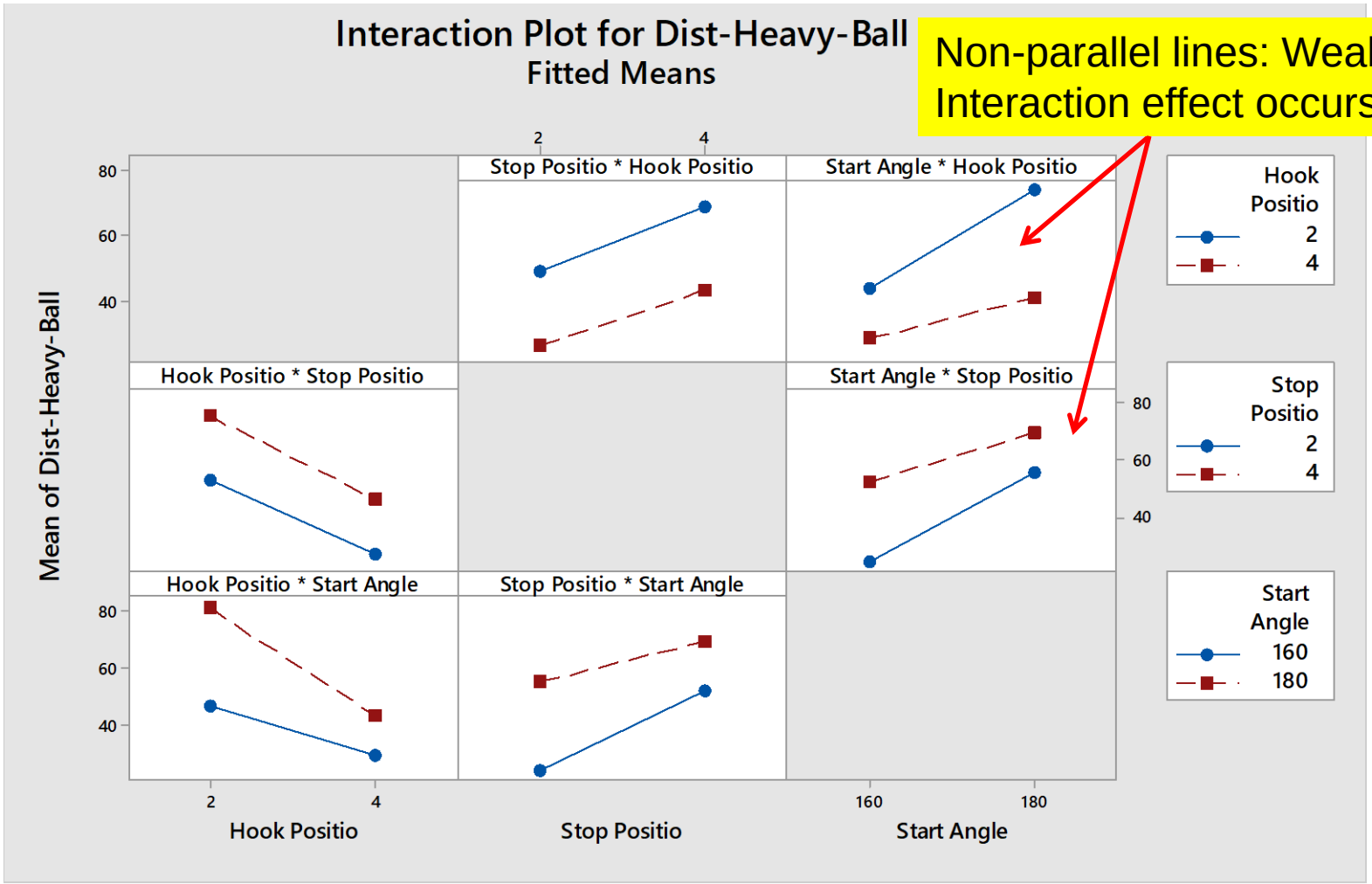
A main effect occurs when the mean response changes across the levels of a factor. You can use main effects plots to compare the relative strength of the effects across factors.

(MINITAB > Stat > DOE > Factorial > Factorial Plots...)



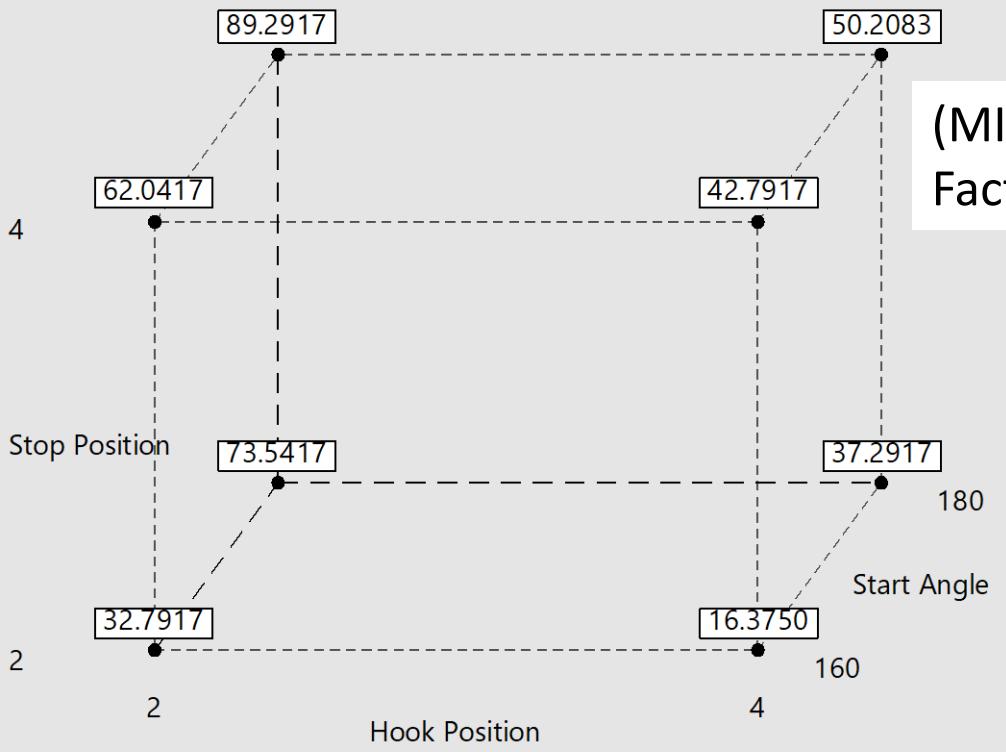
Interaction Plots

An interaction between factors occurs when the change in response from the low level to the high level of one factor is not the same as the change in response at the same two levels of a second factor. That is, the effect of one factor is dependent upon a second factor. You can use interactions plots to compare the relative strength of the effects across factors.



Cube Plot

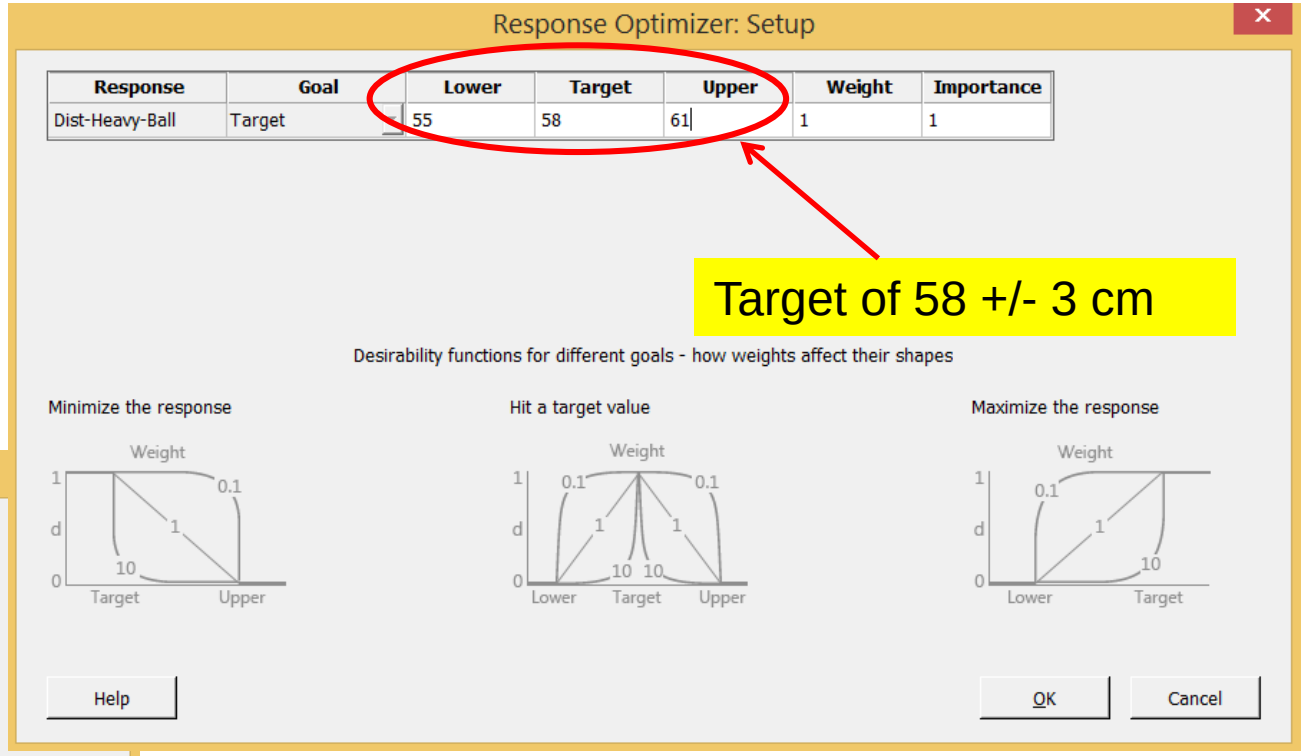
Showing the ***Mean Distance data*** collected from experiment at each treatment (level combination)

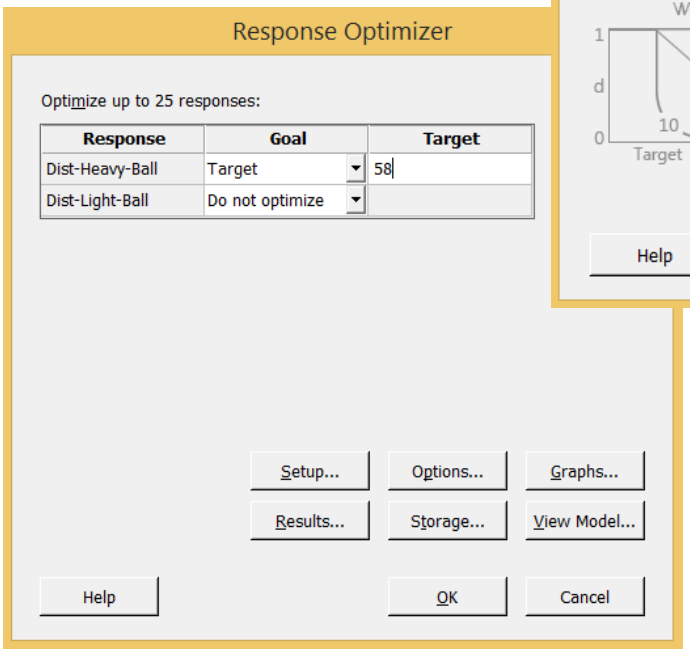


(MINITAB > Stat > DOE > Factorial > Cube Plots…)

Response Optimizer: Target at 58cm

Minitab>Stat>DOE>Factorial>Response Optimizer





Response Optimizer: Target at 58cm

Response Optimization: Dist-Heavy-Ball

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Dist-Heavy-Ball	Target	55	58	61	1	1

Solution

Solution	Hook Position	Stop Position	Start Angle	Dist-Heavy-Ball Fit	Composite Desirability
1	2.03614	3.74699	160	58	1

Multiple Response Prediction

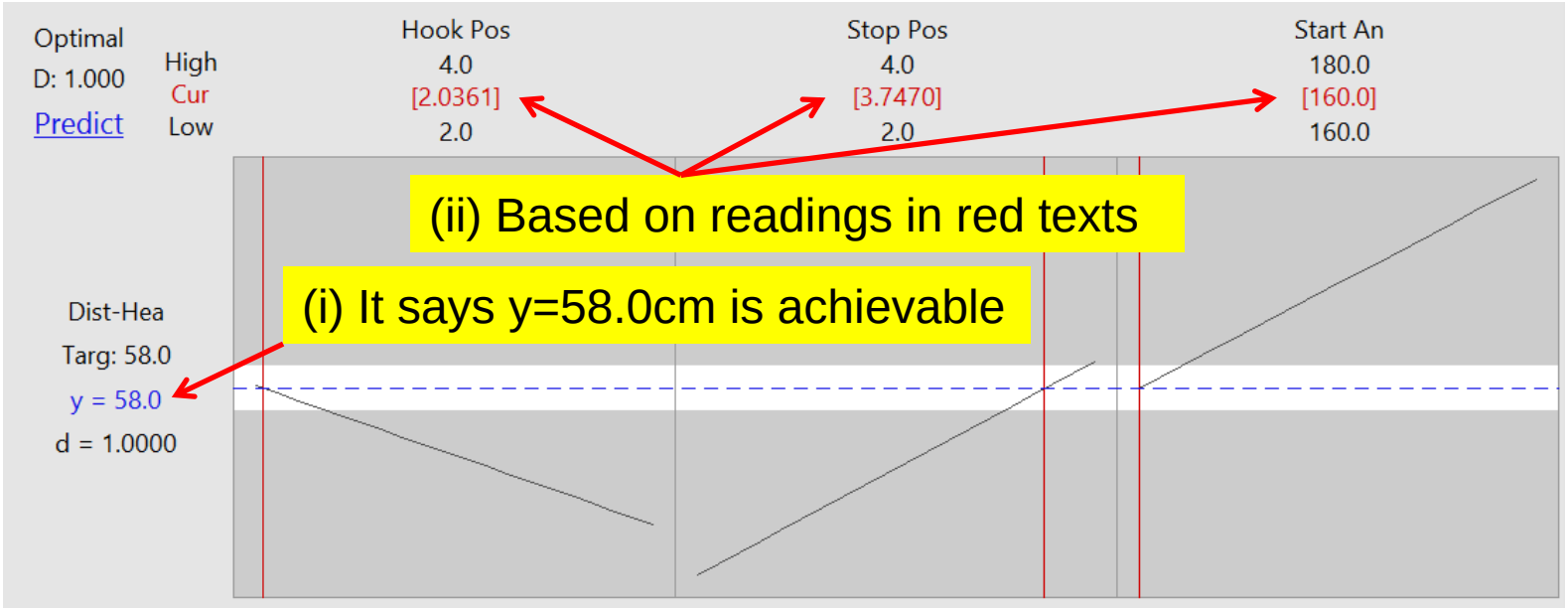
Variable	Setting
Hook Position	2.03614
Stop Position	3.74699
Start Angle	160

Response	Fit	SE Fit	95% CI	95% PI
Dist-Heavy-Ball	58.000	0.218	(57.539, 58.461)	(56.932, 59.068)

In order to hit a target at 58cm distance, one optimized catapult setting is:

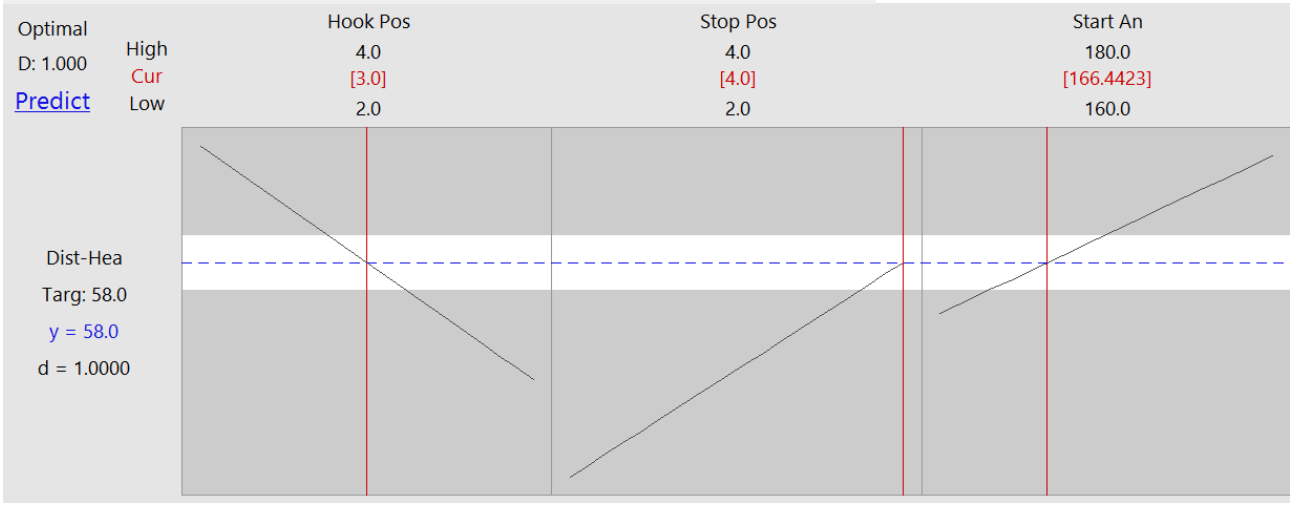
- Hook Position 2.03614
- Stop Position 3.74699
- Start Angle 160 deg

It's however not possible to set Hook Position and Stop Position at decimal places. So go Response Optimizer / Options… to add constraints to 'Hook Position', 'Stop Position'.



Constraints

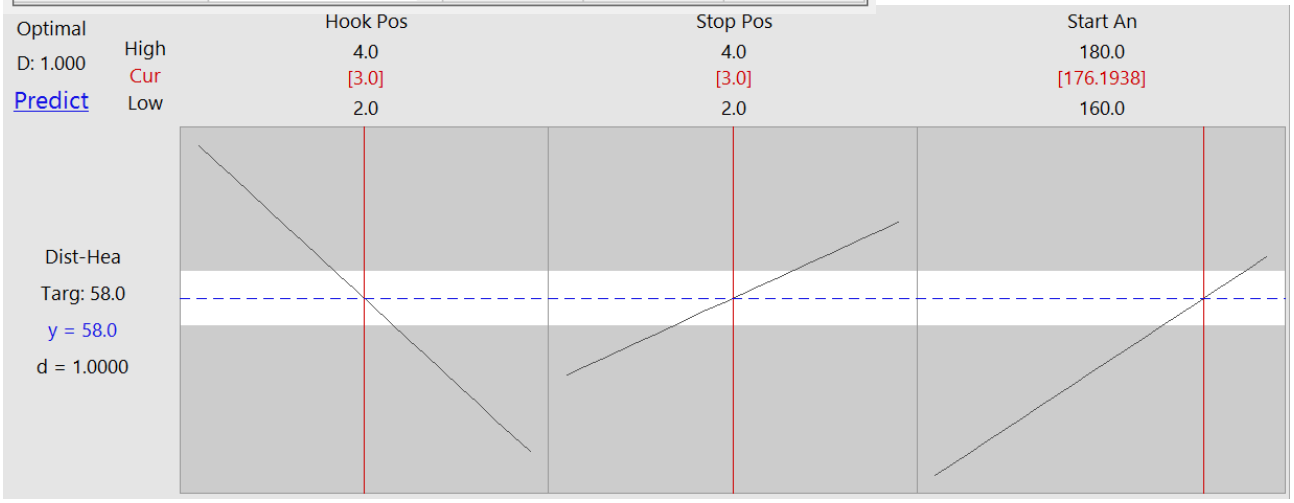
Variable	Constraint	Hold Value	Lower	Upper
'Hook Position'	Hold at value	3		
'Stop Position'	Hold at value	4		
'Start Angle'	No constraints			



Variable	Setting
Hook Position	3
Stop Position	4
Start Angle	166.442deg

Constraints

Variable	Constraint	Hold Value	Lower	Upper
'Hook Position'	Hold at value	3		
'Stop Position'	Hold at value	3		
'Start Angle'	No constraints			



Variable	Setting
Hook Position	3
Stop Position	3
Start Angle	176.194deg

Limitations of Prediction Model

The prediction model does not take into account other factors, such as:

- Stability of the frame of the catapult set
- Direction of cup releasing the ball
- Energy losses due to friction
- Presence of strong wind
- Surface friction

Conclusions

- From the DOE, all 3 main effects, Hook Position, Stop Position, Start Angle are significant factors.

- In addition, the interaction between “Hook Position” & “Stop Position”, “Hook Position” & “Start Angle”, “Stop Position” & “Start Angle” are also significant.

- 3-way interaction, Hook Position * Stop Position * Start Angle is NOT significant.

- To reach a target distance, e.g. 58cm, one of the optimized settings are Hook Position 3, Stop Position 3, Start Angle 176deg.

Learning Objectives

- Explain Design of Experiments (DOE) and their uses in Improve Phase of Lean Six Sigma project
- Apply factorial experiments to practical problem solving
- Distinguish between One-Factor-At-a-Time (OFAT) and Factorial experiment

Going Further: Fractional Factorial Designs

- A fractional factorial design is an experimental design consisting of a subset (fraction) of the factorial design.

- Typically, the fraction is simple proportional of the full set of possible treatment combinations. For example, half-fractions, quarter-fractions, and so forth are common. While fractional factorial designs require fewer runs, certain information cannot be separated.

  Example: Fractional factorial for a three factors, with 2-level experiment :

  - Pressure (10,20)
  - Speed (50,100)
  - Time (45,65)

RunOrder	Pressure	Speed	Temp
1	20	100	65
2	10	100	45
3	10	50	65
4	20	50	45

Overview of E326 Lean Manufacturing and Six Sigma