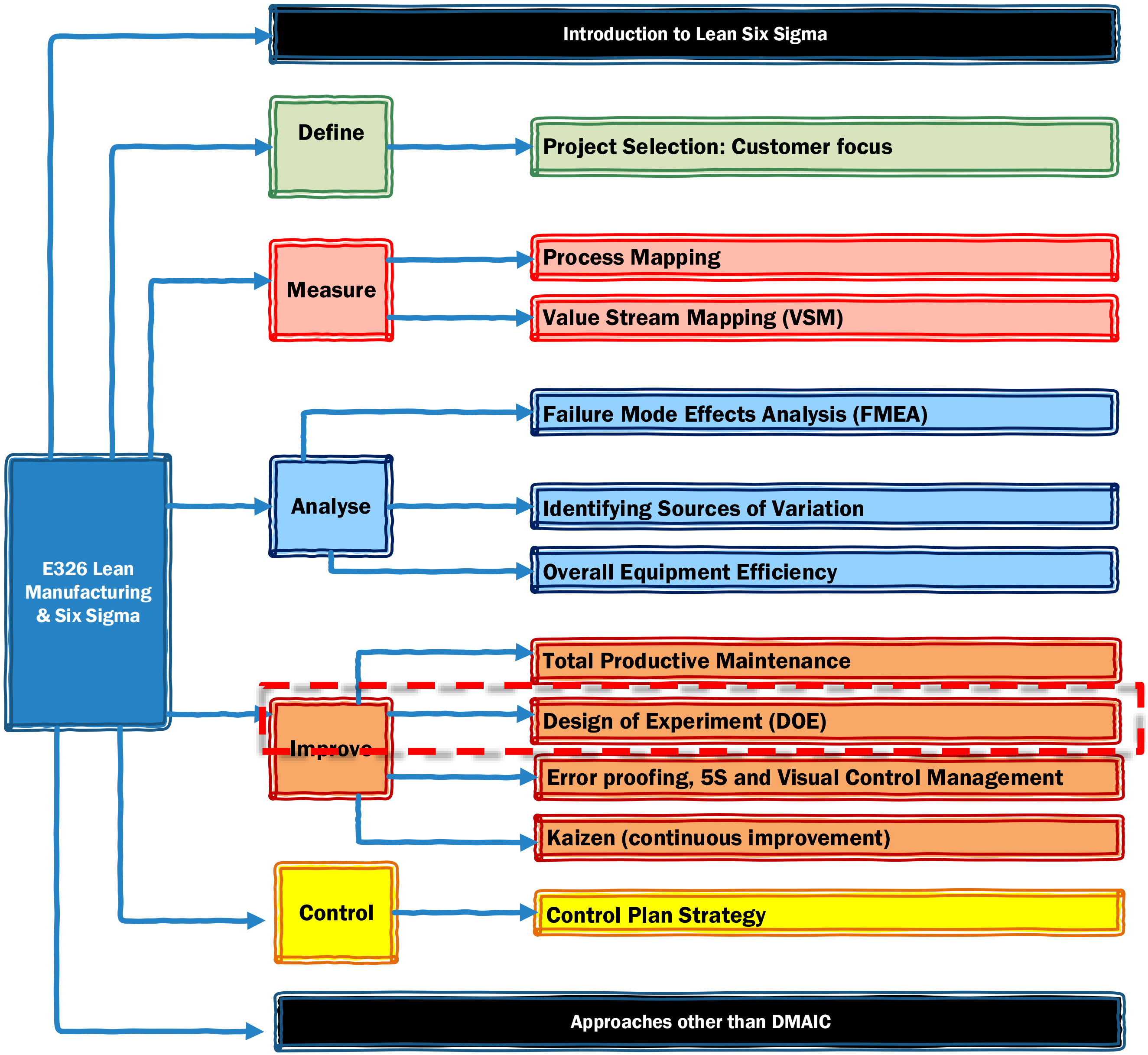
Problem 9

Design of Experiments (DOE) Interactive Seminar

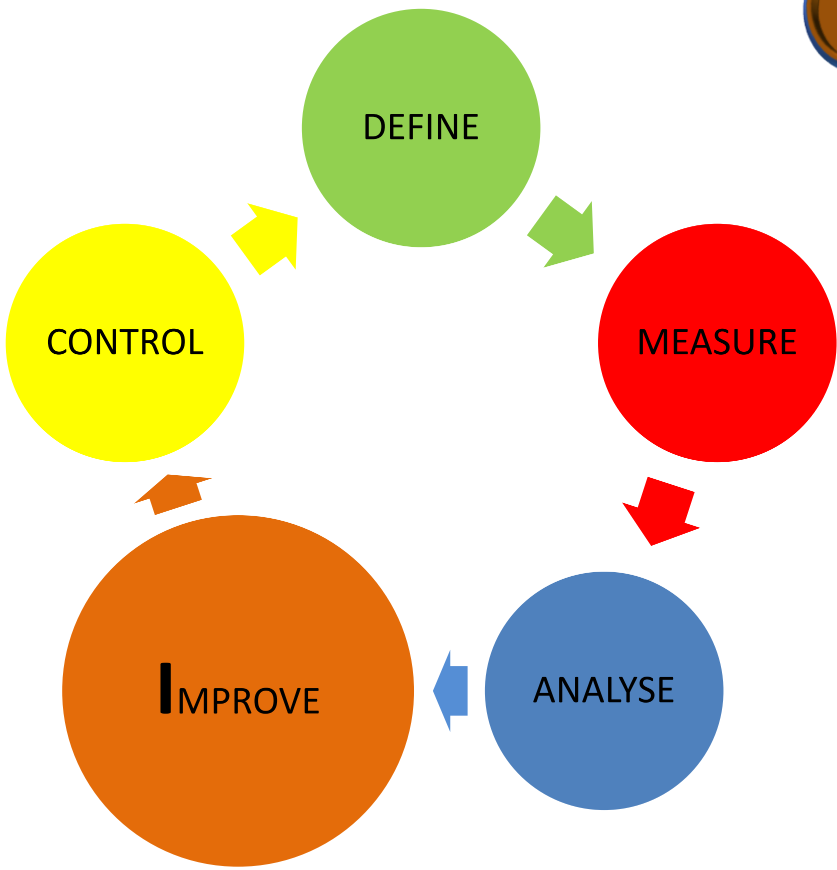
E326 – Lean Manufacturing & Six Sigma

SCHOOL OF ENGINEERING

E326 Lean Manufacturing and Six Sigma Topic Tree



DMAIC

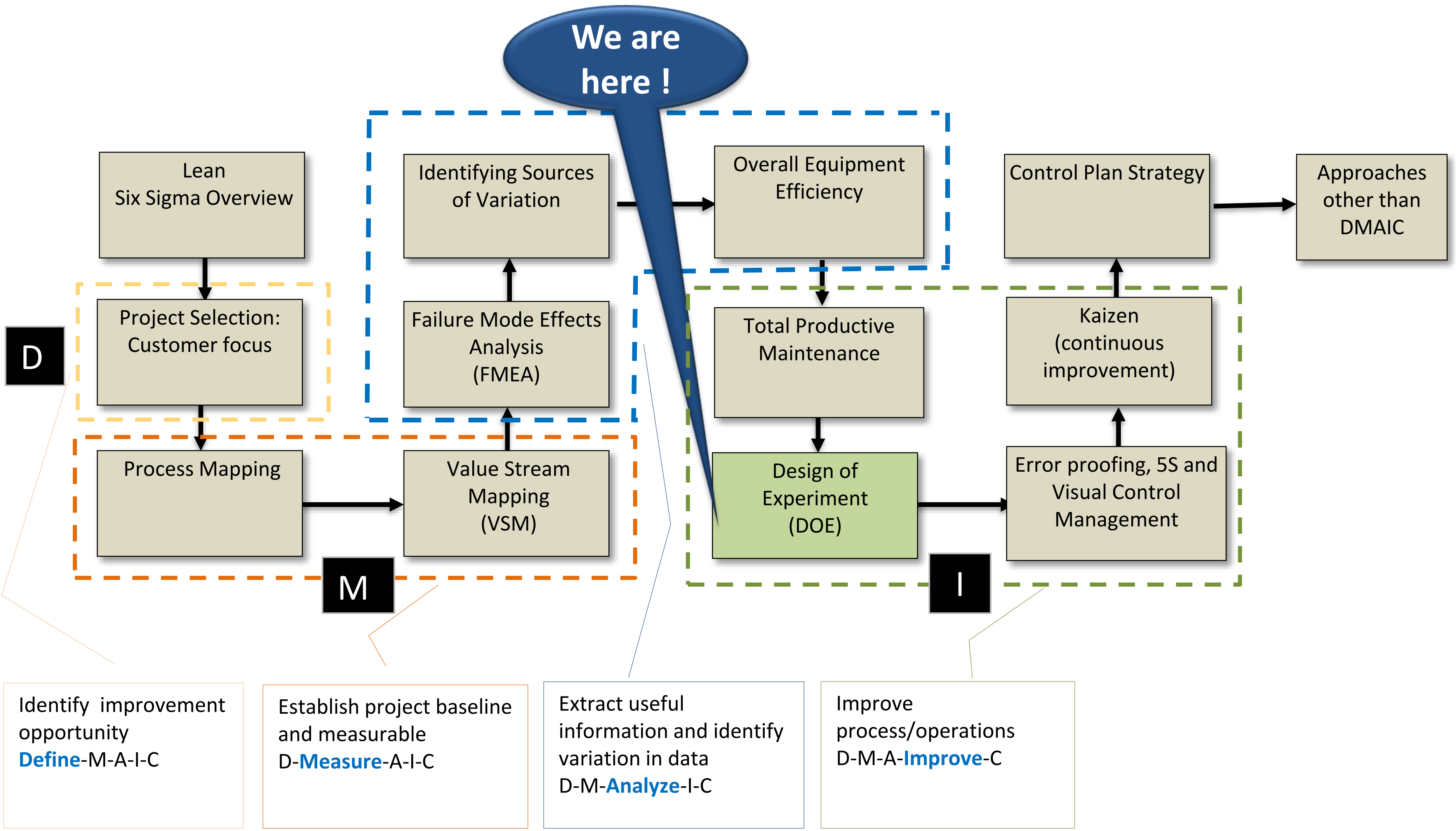


- Design of Experiment
- One-Factor-At-A-Time Experiment
- Factorial Experiment

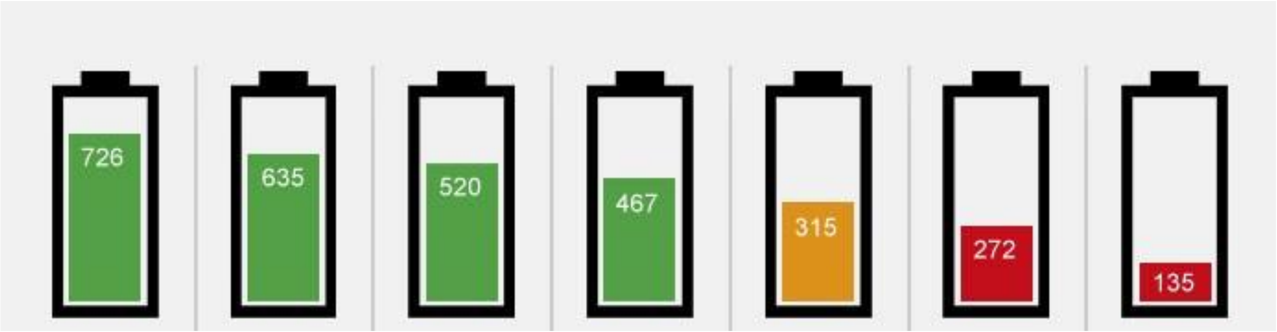
Objectives of Improve phase:

- Improve the process by identifying the vital "Xs" and implementable solutions.

Overview of E326 Lean Manufacturing and Six Sigma



Scenario – Right on Target



- Mobile phone manufacturer T has recently declared that they have developed new battery technology which would speed up the charging process of batteries by multiple times current standards.

- You, as a learner of Lean Six Sigma, have learnt about using ANOVA to identify significant factors affecting response. You are interested to know if there are systematic methods for you to conduct experiments to predict the desired response (output) with varying variables (inputs) so that you can optimized efforts.

Scenario Tasks of the day

- Using the catapult experimentation tool provided, decide on a suitable plan a systematic experiment and propose the significant factors that will factor the response being studied.

- You will present the charts that will assist in predicting the significant factors with their settings that will allow desired response's setting to be obtained.

Scenario Definition Template

- What do we know?

- What do we not know?

- What do we need to find out?

Design of Experiment (DOE)

- DOE is the scientific process of planning an experiment that will yield statistically useful results.

- It is a structured, organized method that is used to determine the relationship between the different factors (Xs) affecting a process and the response of that process (Y).

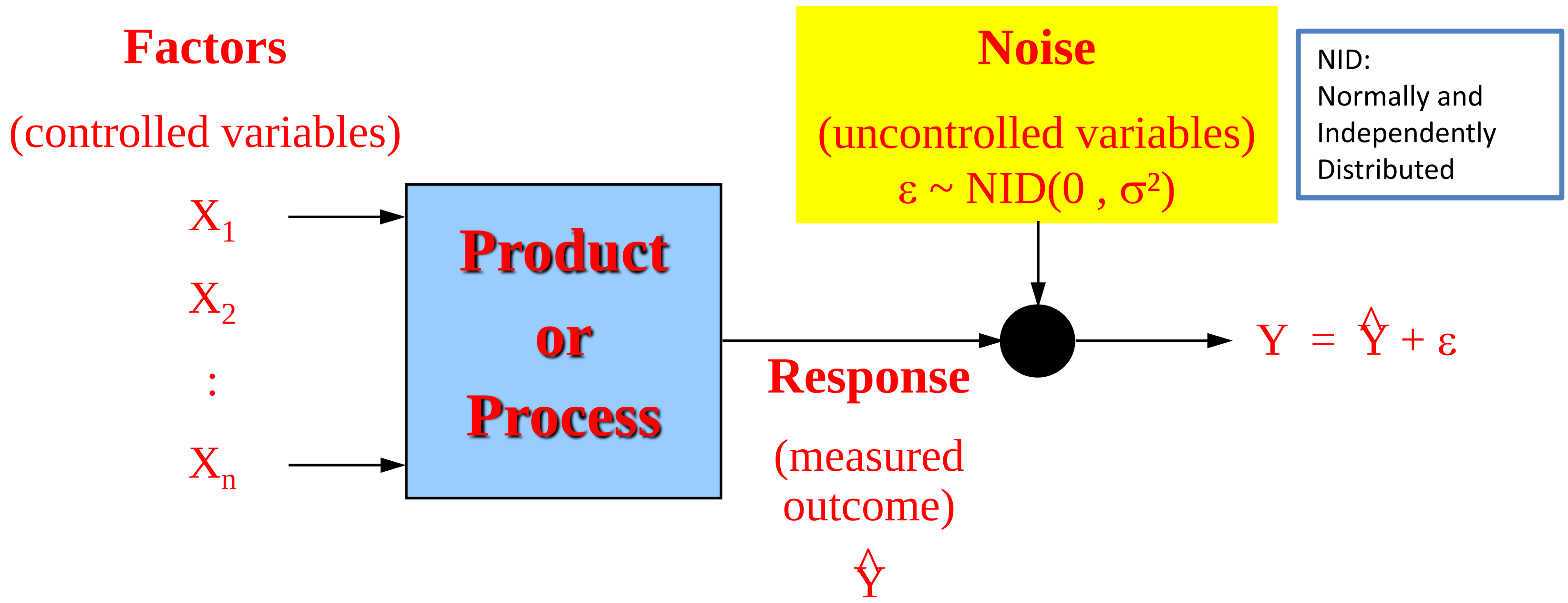
- The general objective/application of a designed experiment is to find a design that will produce a specific, desired amount of information at a minimum cost to the company. A designed experiment involves varying input factors (Xs) to find out which variables, or combination of variables, deliver optimised output (Y).

Design Of Experiments (DOE)

The objectives of DOE are to determine:

- Which variables (e.g. time, temperature, pressure) are most influential on a response (yield, strength).
- The settings of influential variables (e.g. high temperature, low pressure) that result in
  - a response near a desired value (e.g. maximum yield, 26 lbs. seal strength).
  - variability of the response is small.
  - minimize the impact of uncontrollable variables (e.g. room temperature, humidity).

General Model of DOE



<u>Objectives of DOE</u>

- Which factors are most influential on the response
- Best settings of the influential factors for maximum response
- Best settings of the influential factors for minimizing variability
- Best settings of the influential factors for robustness vs. the noise factors

Common Terminologies

Factors
- Input variables of interest in the experiment that affect the response variable

Levels
- Settings for the factors that are investigated in the experiment

Response variable
- output variable of the process that is to be optimized

Observation
- measured value of the response variable

Example:

A DOE was conducted on 2 different battery types (high cost and low cost) with either gold-plated or standard connector type placed in either ambient or cold temperature had their battery discharge time measured to predict battery discharge time when the influential factors were varied.

Solution:

Factors studied: 1. Battery type 2. Connector type 3. Battery temperature

Levels of each factor:

1. Battery type – High cost and Low cost
2. Connector type – Gold-plated and standard
3. Battery temperature – Ambient and cold

Response variable: battery discharge time

Common Terminologies

Treatments
- Combinations of the various factor levels used in the experiment

Experimental run
- an experimental trial corresponding to a particular treatment, under which the response is measured

Example:

A DOE was conducted on 2 different battery types (high cost and low cost) with either gold-plated or standard connector type placed in either ambient or cold temperature had their battery discharge time measured to predict battery discharge time when the influential factors were varied.

Solution:

Possible treatments: High cost Battery type gold-plated Connector type at ambient Battery temperature

Experimental runs:

Experimental run	Battery Type	Connector Type	Battery Temperature
1	High cost	Gold-plated	Ambient
2	High cost	Gold-plated	Cold
3	High cost	Standard	Ambient
4	High cost	Standard	Cold
5	Low cost	Gold-plated	Ambient
6	Low cost	Gold-plated	Cold
7	Low cost	Standard	Ambient
8	Low cost	Standard	Cold

Common Terminologies

Treatments
- Combinations of the various factor levels used in the experiment

Experimental run
- an experimental trial corresponding to a particular treatment, under which the response is measured

Response variable
- output variable of the process that is to be optimized

Example:

A DOE was conducted on 2 different battery types (high cost and low cost) with either gold-plated or standard connector type placed in either ambient or cold temperature had their battery discharge time measured to predict battery discharge time when the influential factors were varied.

Solution:

Possible treatments: High cost Battery type gold-plated Connector type at ambient Battery temperature

Experimental runs:

Experimental run	Battery Type	Connector Type	Battery Temperature
1	High cost	Gold-plated	Ambient
2	High cost	Gold-plated	Cold
3	High cost	Standard	Ambient
4	High cost	Standard	Cold
5	Low cost	Gold-plated	Ambient
6	Low cost	Gold-plated	Cold
7	Low cost	Standard	Ambient
8	Low cost	Standard	Cold

TEST Yourself

Keith, a design engineer, has prototyped a miniature Multi-Terrain Vehicle and is analysing the vehicle braking reaction time by conducting a Design of Experiment (DOE) study. The DOE test matrix and the respective braking reaction time (in seconds) are tabulated in the table below. Identify the factors and the number of levels. What is the response being studied?

Moving Speed (m/s)	Load (kg)	Braking Reaction Time (sec)
5	6	1.2
20	6	2.2
5	10	5
20	10	7.6
5	6	1.3
20	6	2.2
5	10	6
20	10	7.5

Solution-

2 factors: Moving Speed & Load

2 levels each: Moving Speed (5 and 20) & Load (6 and 10)

Response: Braking Reaction time

Main Principles of DOE

- Randomization

  Randomly assign the treatments to the experimental runs so as to control for unknown sources of variation such as human bias.

- Replication

  Measurements are usually subject to variation and uncertainty. Measurements are repeated and full experiments are replicated to help identify the sources of variation and to better estimate the true effects of treatments

- Blocking

  Blocking is the arrangement of experimental units into groups (blocks) that are similar to one another. Blocking reduces known but irrelevant sources of variation between units and thus allows greater precision in the estimation of the source of variation under study.

One-Factor-At-a-Time (OFAT) Experiment vs. Factorial Experiment

OFAT Experiment	Factorial Experiment
Perform one factor at a time while other factors remain unchanged	The effect of many factors (variables) can be studied simultaneously
Estimation of the effect of each factor is based on individual observation	The impact of two or more factors on a response can be determined
Unable to estimate interaction between factors	Interaction between factors can be estimated systematically
Unable to predict the optimization well	Improves the prediction of the response resulting in a more efficient process optimization

Factorial Designs

What are Factorial Designs?

They allow us to simultaneously study the effects of two or more factors in a process simultaneously rather than individually with each factor studied at two or more levels:

- Saves time and testing expense
- Reveals the main effect as well as interactions between the factors

When to use Factorial Designs?

Use factorial designs to:

- Efficiently estimate the effect of each factor on the response
- Estimate the effects of interactions between one or more factors on the response
- Test for curvature in the response by including center points in the design

Why use Factorial Designs?

Use factorial designs to answer questions such as:

- Which variables most strongly influence the response?
- What factor settings optimize the response?

Types of Factorial Designs

- Full factorial design
- Fractional factorial design

In this module, we will emphasize on the 2-level full factorial designs

Full Factorial Designs

- A full factorial design experiment measures **ALL** combinations of the experimental factor levels. The entire set of runs is called the “design”.

Example: Three factors with 2-level:

- ○ Pressure (10,20)
- ○ Speed (50,100)
- ○ Time (45,65)

RunOrder	Pressure	Speed	Temp
1	10	50	45
2	20	50	45
3	10	100	45
4	20	100	45
5	10	50	65
6	20	50	65
7	10	100	65
8	20	100	65

- Minitab provides two types of full factorial designs:
  - Use a *two-level*, full factorial design (2^k design) when each experimental factor has only two levels
  - Use a general full factorial design when any experimental factor has *more than two levels*. For e.g. Factor A may have more than two levels, Factor B may have three levels, and Factor C may have five levels.

Yates Order

- **Purpose:** To place the experimental factor levels of total factorial effects in a 2^n factorial design in a standard order prior to running an Experimental Design.

- The convention of alternating the -'s and +'s produces an standard order where the first column consists of alternating low and high settings, the second column consists of alternating pairs of low and high settings, the third column consists of four low followed by four high settings, etc. in the experimental design. It is called the Yates Order which is useful in the design and analysis.

- Observe the pattern in the columns of the table shown.
  - The number of like signs in a set = 2^{n-1}, where n is the column number in the matrix.
  - For the table shown of n = 3, $2^{(3-1)} = 2^2 = 4$ that is the number of possible combinations between two coded levels of two factors.

A	B	C
-	-	-
+	-	-
-	+	-
+	+	-
-	-	+
+	-	+
-	+	+
+	+	+

Table of n = 3

Yates Order - explained

Description	Factor A	Factor B	Factor C
n =	1	2	3
Sub n into 2^{n-1}	$2^{n-1} = 2^{(1)-1} = 2^0 = 1$	$2^{n-1} = 2^{(2)-1} = 2^1 = 2$	$2^{n-1} = 2^{(3)-1} = 2^2 = 4$
This means:	1 low level (-) sign before alternate to 1 high level (+) sign	2 low level (-) sign before alternate to 2 high level (+) sign	4 low level (-) sign before alternate to 4 high level (+) sign

TEST Yourself

Keith, a design engineer, has prototyped a miniature Multi-Terrain Vehicle and is analysing the vehicle braking reaction time by conducting a Design of Experiment (DOE) study. How many treatments will be required using 2^k design? Change the table below to coded design.

Moving Speed (m/s)	Load (kg)	Braking Reaction Time (sec)
5	6	1.2
20	6	2.2
5	10	5
20	10	7.6
5	6	1.3
20	6	2.2
5	10	6
20	10	7.5

Solution-

As k=2, number of treatments = 2^2 = 4

Moving Speed (m/s)	Load (kg)	Braking Reaction Time (sec)
-	-	1.2
+	-	2.2
-	+	5
+	+	7.6
-	-	1.3
+	-	2.2
-	+	6
+	+	7.5

DOE Example

Using the following example, DOE will be guided in the next few slides.

Mixing Speed	Temperature	Concentration	Reaction Time
10	40	25	39.5
30	40	25	271.2
30	40	12	286.2
30	50	25	45.1
30	50	12	48.3
10	50	25	275.1
10	40	12	36.7
10	50	12	265.6

Setting up the DOE in Minitab

Create Factorial Design

Type of Design

- (•) 2-level factorial (default generators) (2 to 15 factors)
- ( ) 2-level factorial (specify generators) (2 to 15 factors)
- ( ) 2-level split-plot (hard-to-change factors) (2 to 7 factors)
- ( ) Plackett-Burman design (2 to 47 factors)
- ( ) General full factorial design (2 to 15 factors)

Number of 3

Display Available Designs...

Designs... Factors...

Options... Results...

Help OK Cancel

Create Factorial Design: Display Availa

For Full factorial 2^k designs, when k=3, 8 runs will be required.

Available Factorial D

	Factors													
Run	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	Full	III												
8		Full	IV	III	III	III								
16			Full	V	IV	IV	IV	III	III	III	III	III	III	III
32				Full	VI	IV	IV	IV	IV	IV	IV	IV	IV	IV
64					Full	VII	V	IV	IV	IV	IV	IV	IV	IV
128						Full	VIII	VI	V	V	IV	IV	IV	IV

Available Resolution III Plackett-Burman Designs

Factors	Runs	Factors	Runs	Factors	Runs
2-7	12,20,24,28,...,48	20-23	24,28,32,36,...,48	36-39	40,44,48
8-11	12,20,24,28,...,48	24-27	28,32,36,40,44,48	40-43	44,48
12-15	20,24,28,36,...,48	28-31	32,36,40,44,48	44-47	48
16-19	20,24,28,32,...,48	32-35	36,40,44,48		

Help OK

Create Factorial Design: Designs

Designs	Runs	Resolution	2^(k-p)
1/2 fraction	4	III	2^(3-1)
Full factorial	8	Full	2^3

Number of center points per block: 0

Number of replicates for corner 1

Number of blocks: 1

Help OK Cancel

Create Factorial Design: Factors

Factor	Name	Type	Low	High
A	Mixing Spee	Numeric	10	30
B	Temperature	Numeric	40	50
C	Concentratio	Numeric	12	25

Help OK Cancel

Collecting the response data in Minitab

```
Welcome to Minitab, press F1 for help.
```

Full Factorial Design

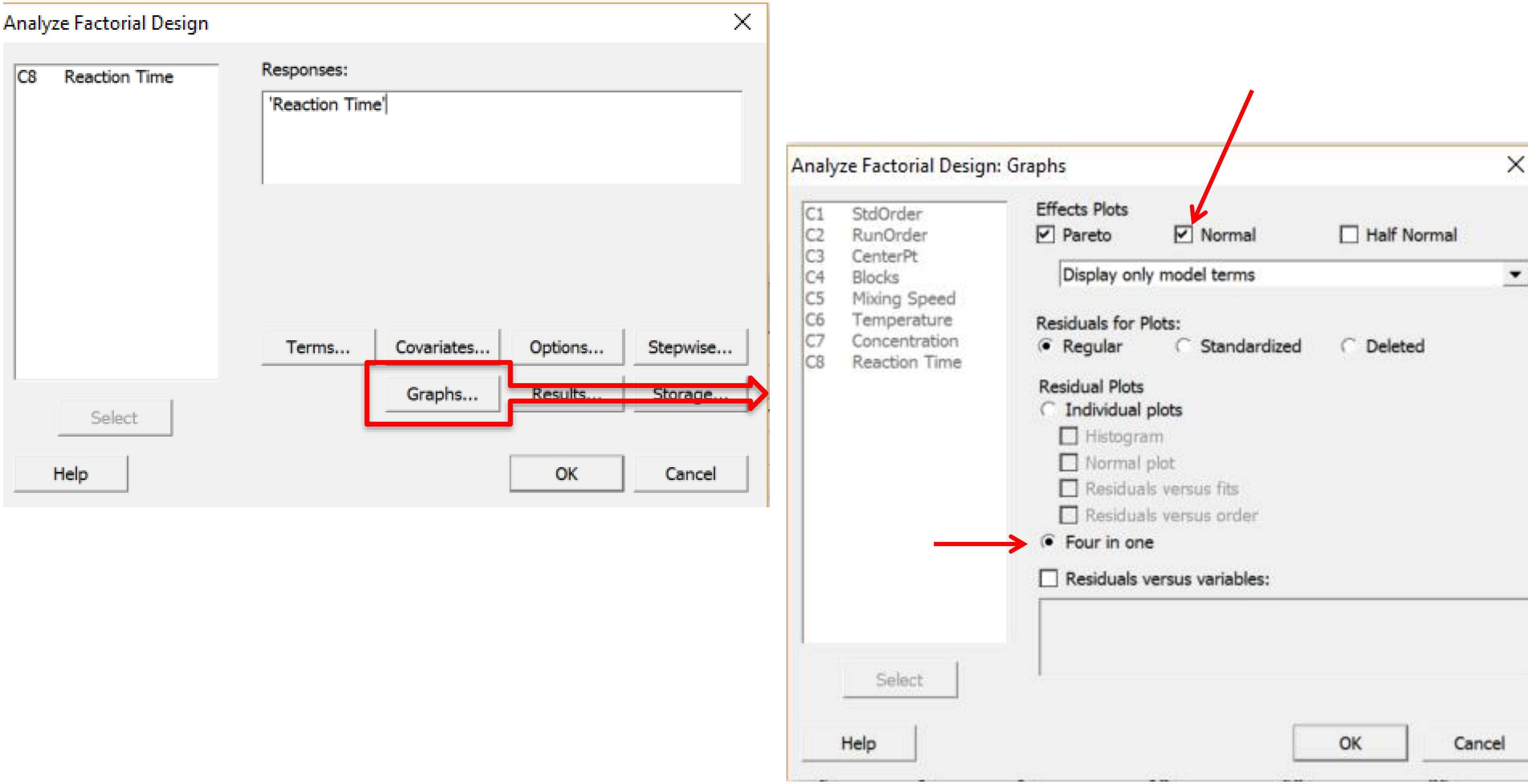
```
Factors:  3   Base Design:         3, 8
Runs:     8   Replicates:             1
Blocks:   1   Center pts (total):     0


All terms are free from aliasing.
```

Worksheet 1 ***

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	Mixing Speed	Temperature	Concentration	Reaction Time
1	5	1	1	1	10	40	25	121.5
2	6	2	1	1	30	40	25	64.5
3	2	3	1	1	30	40	12	87.0
4	8	4	1	1	30	50	25	105.0
5	4	5	1	1	30	50	12	78.0
6	7	6	1	1	10	50	25	117.0
7	1	7	1	1	10	40	12	99.0
8	3	8	1	1	10	50	12	54.0
9								

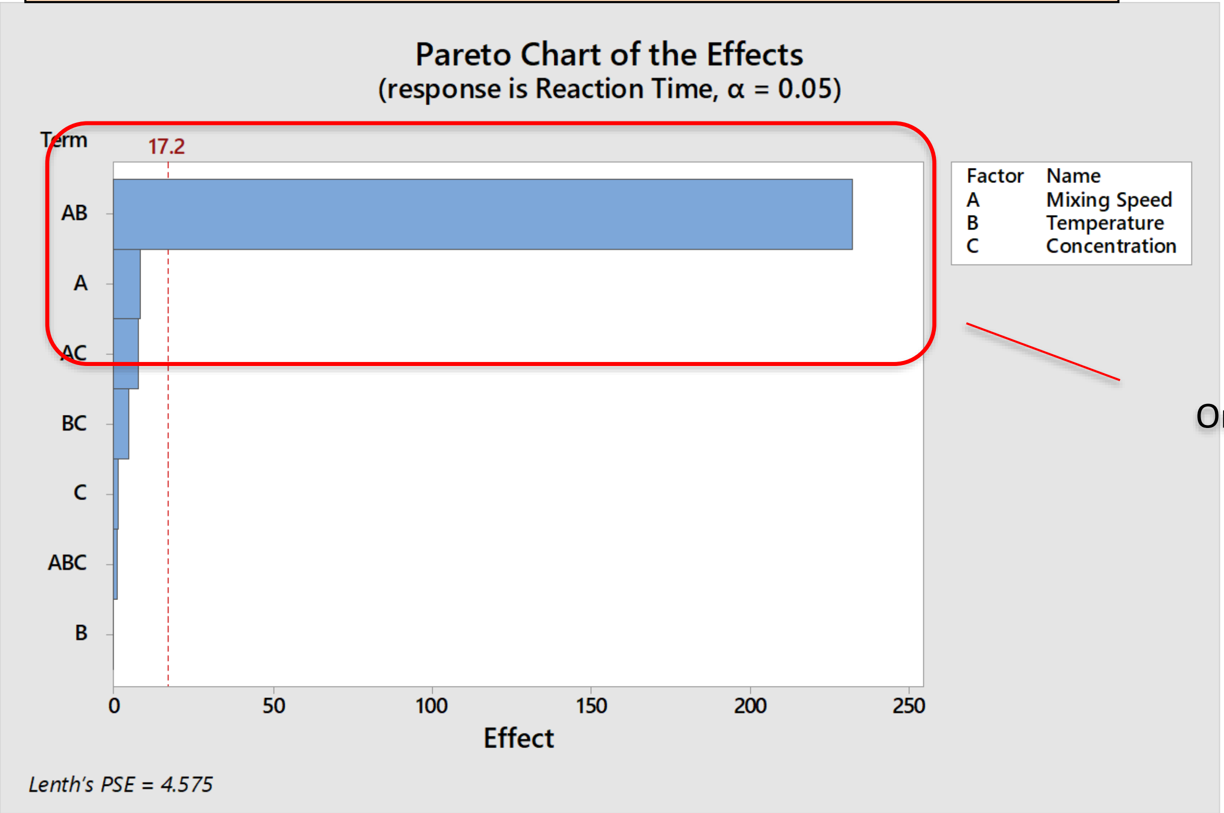
Results and Analysis (Identifying insignificant terms)

Minitab > Stat > DOE > Factorial > Analyse Factorial Design...>Graphs)



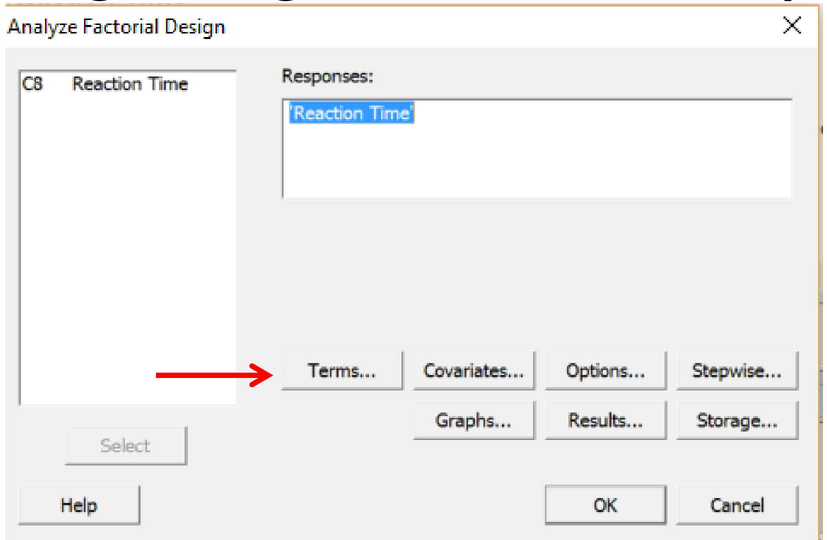
Results and Analysis (Identifying insignificant terms)

Minitab > Stat > DOE > Factorial > Analyse Factorial Design...>Graphs)

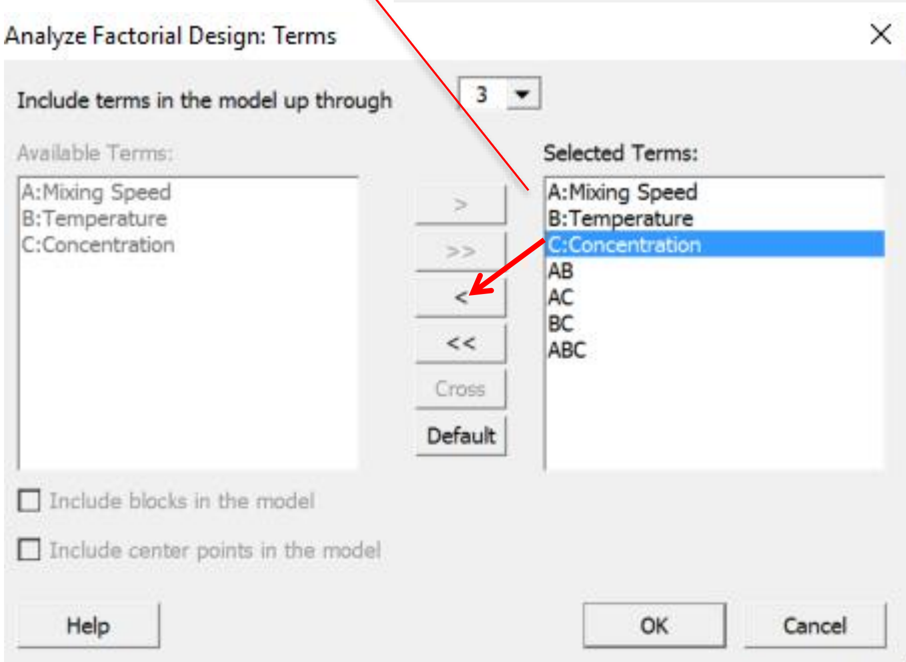


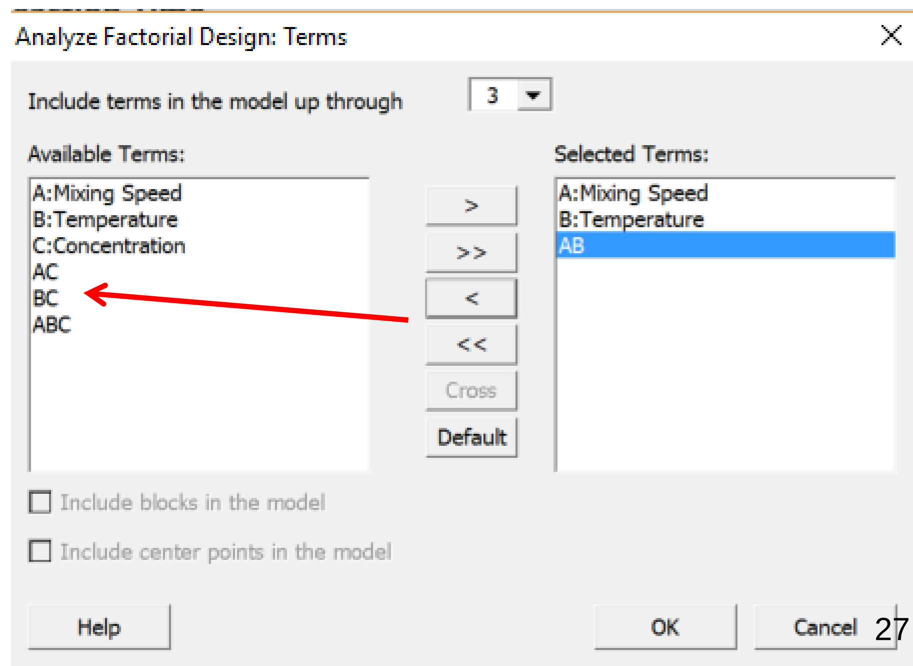
Only interaction AB is significant, as it crosses the test statistic red line.

Results and Analysis (Removing insignificant terms)



Although only interaction AB is significant, Factors A and B cannot be removed.





Results and Analysis
(After removing insignificant terms)

Factorial Regression: Reaction Time versus Mixing Speed, Temperature

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	107908	35969	863.27	0.000
Linear	2	144	72	1.72	0.288
Mixing Speed	1	144	144	3.45	0.137
Temperature	1	0	0	0.00	0.979
2-Way Interactions	1	107764	107764	2586.36	0.000
Mixing Speed*Temperature	1	107764	107764	2586.36	0.000
Error	4	167	42		
Total	7	108074			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
6.45494	99.85%	99.73%	99.38%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		158.46	2.28	69.44	0.000	
Mixing Speed	8.47	4.24	2.28	1.86	0.137	1.00
Temperature	0.13	0.06	2.28	0.03	0.979	1.00
Mixing Speed*Temperature	-232.13	-116.06	2.28	-50.86	0.000	1.00

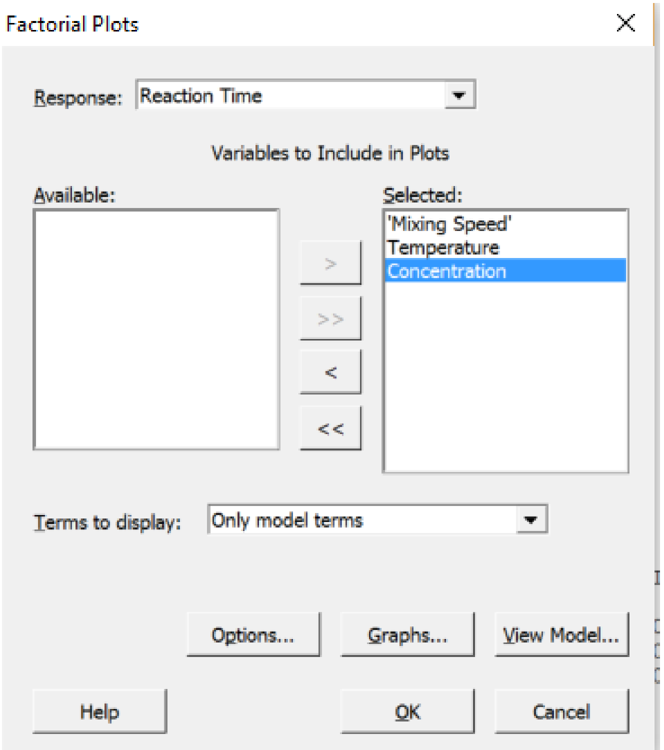
Regression Equation in Uncoded Units

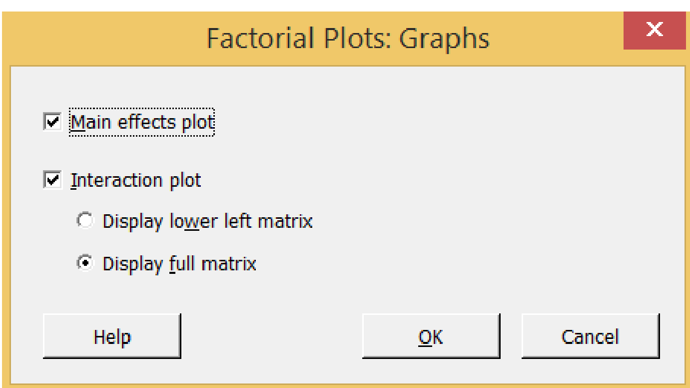
Reaction Time = -1939.7 + 104.88 Mixing Speed + 46.44 Temperature
- 2.3213 Mixing Speed*Temperature

Alias Structure

Factor	Name
A	Mixing Speed
B	Temperature
C	Concentration

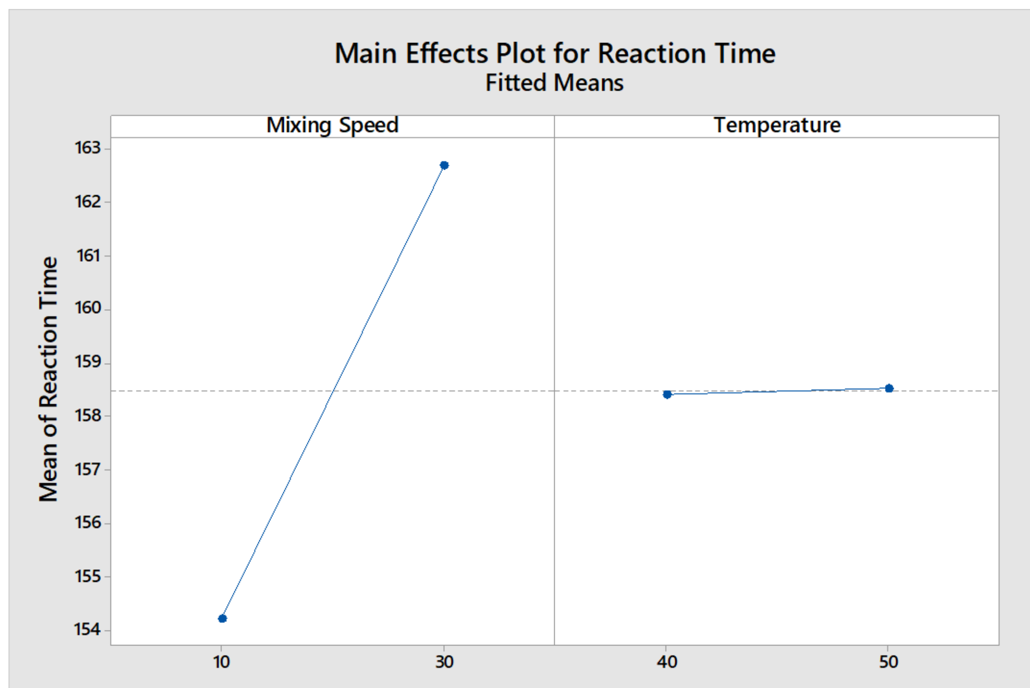
Main Effects Plots





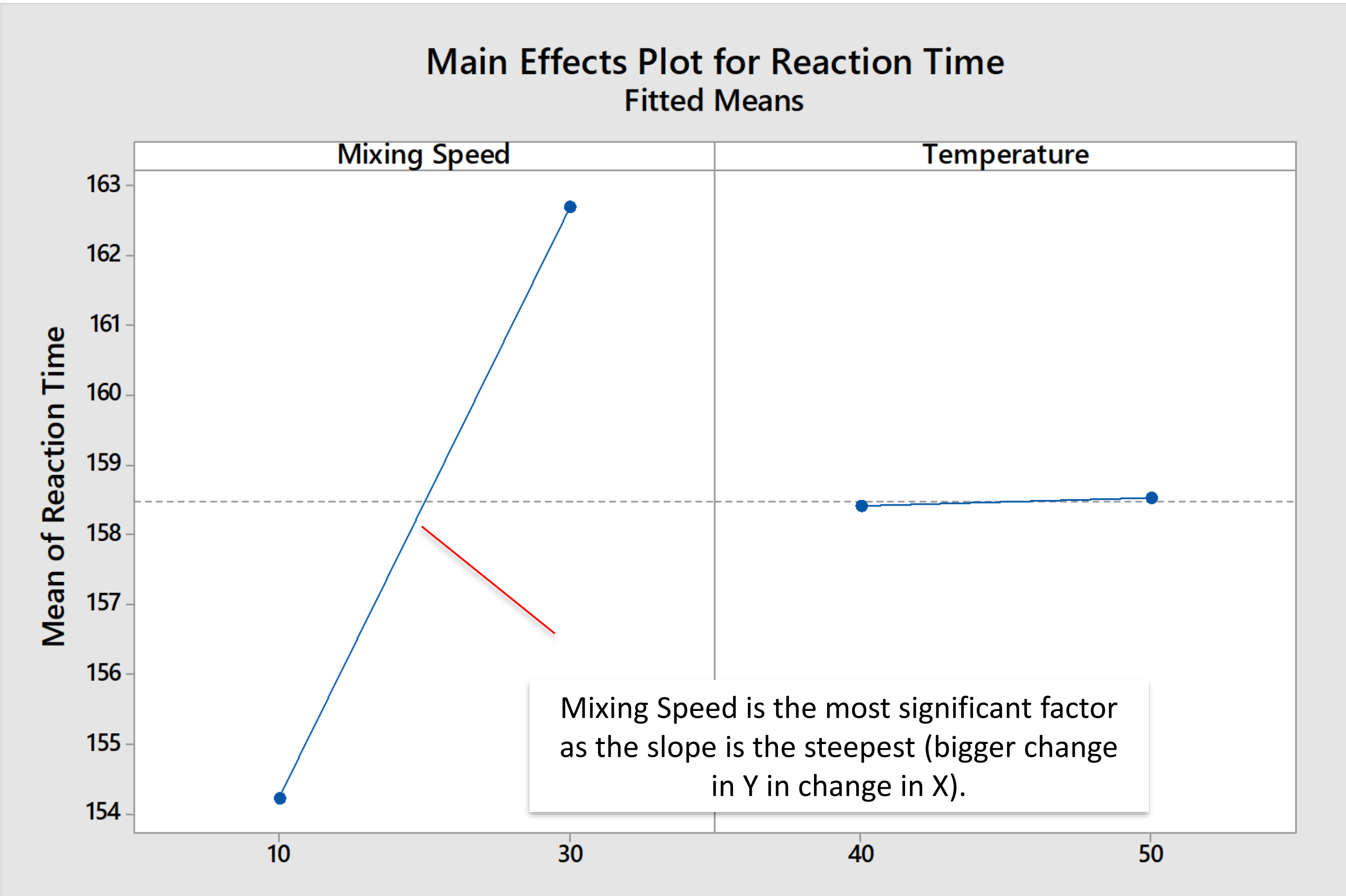
A main effect occurs when the mean response changes across the levels of a factor. You can use main effects plots to compare the relative strength of the effects across factors.

(MINITAB > Stat > DOE > Factorial > Factorial Plots...)



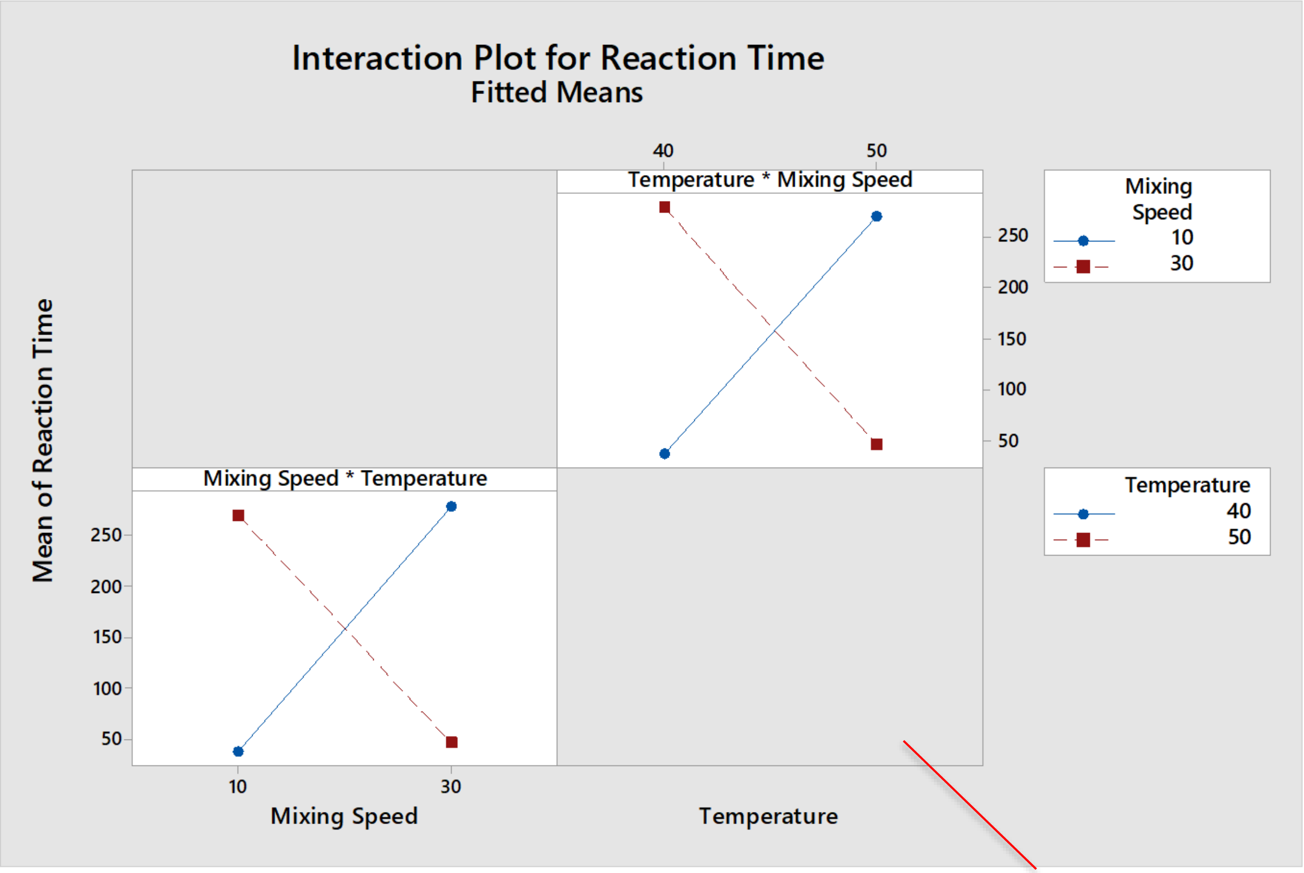
Results and Analysis (Identifying insignificant terms)

Minitab > Stat > DOE > Factorial > Analyse Factorial Design…>Graphs)



Results and Analysis (Identifying insignificant terms)

Minitab > Stat > DOE > Factorial > Analyse Factorial Design...>Graphs)

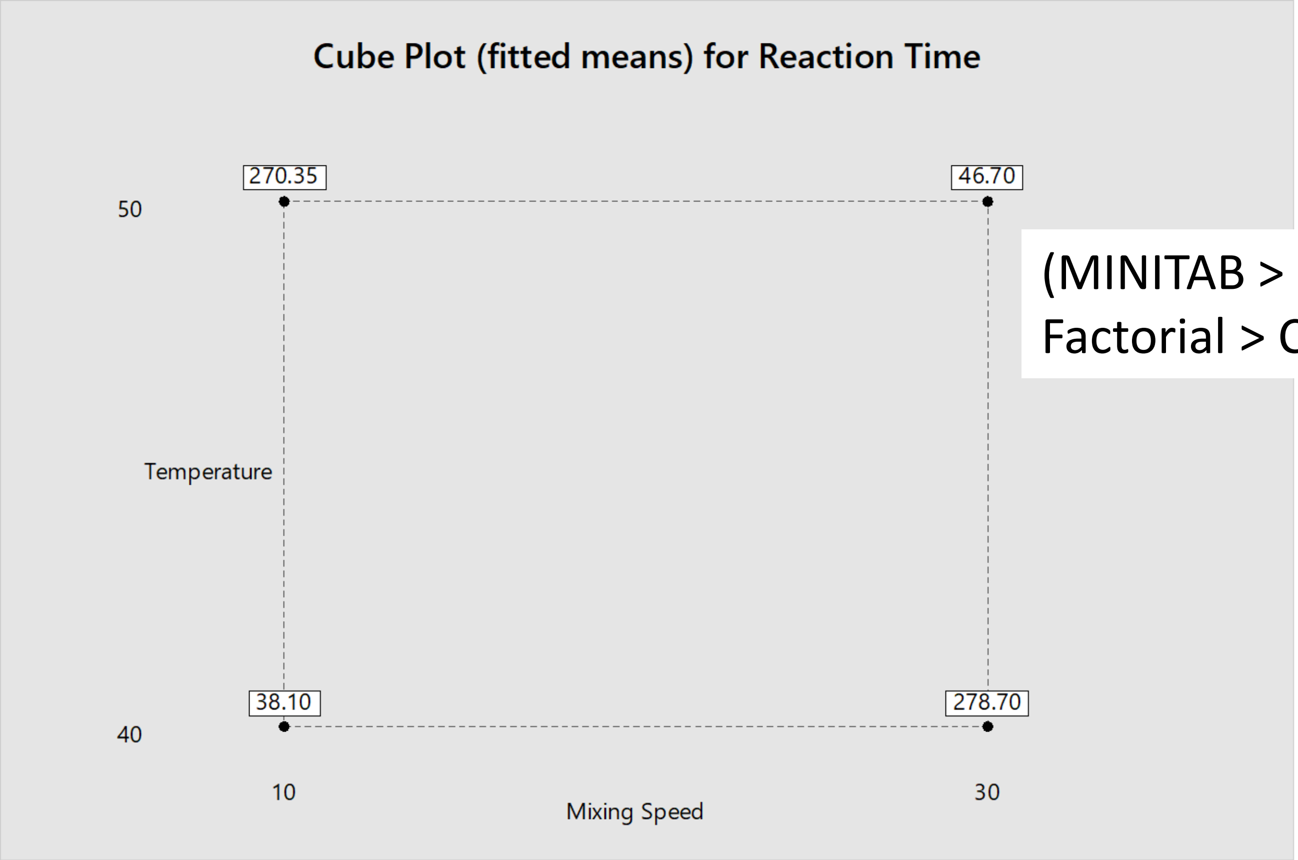


3 possible scenarios:
NO interaction – parallel lines
WEAK interaction – non parallel lines but lines don't intersect
STRONG interaction – intersect lines

There is STRONG interaction between the factors.

Cube Plot

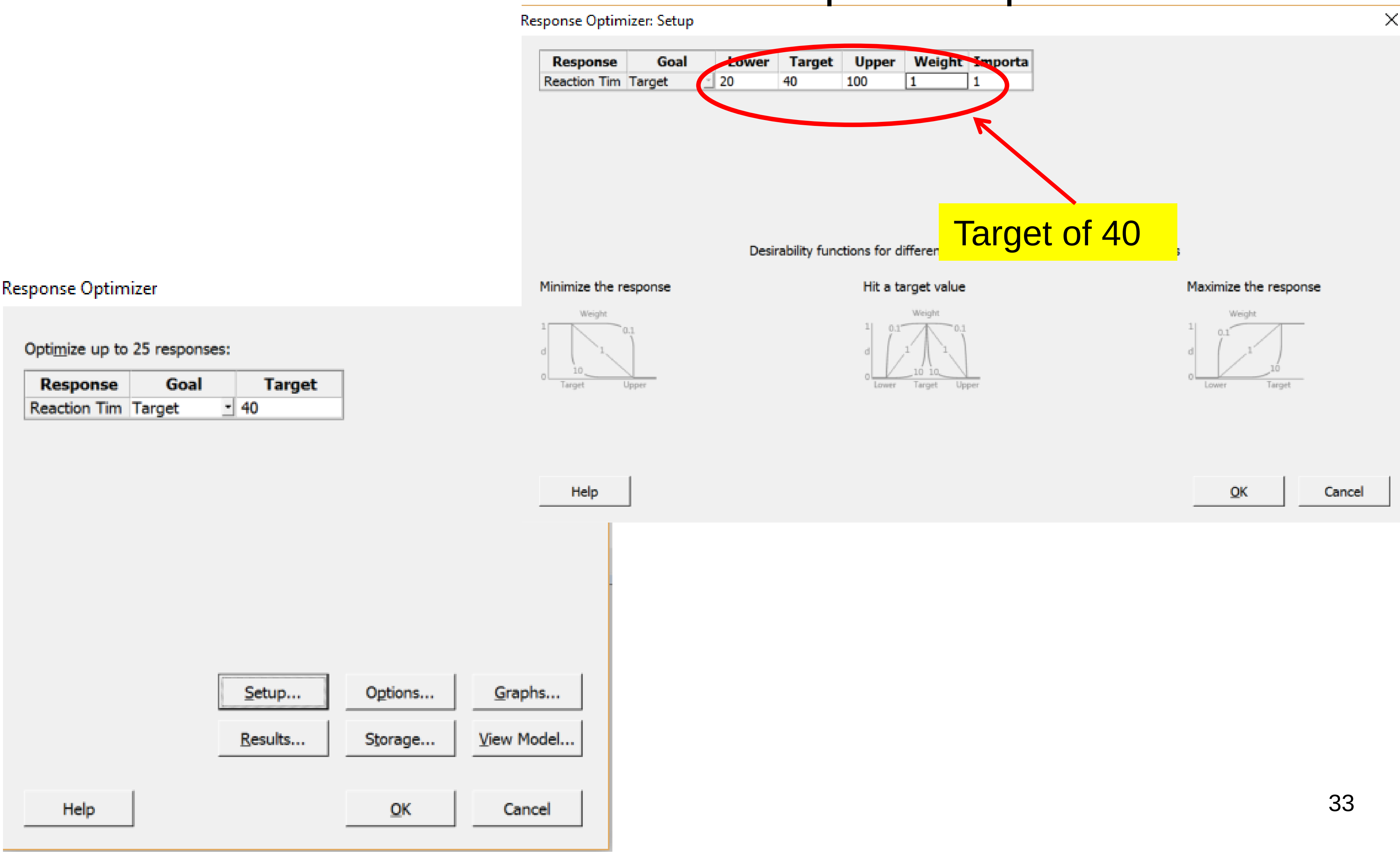
Showing the ***Mean Distance data*** collected from experiment at each treatment (level combination)



(MINITAB > Stat > DOE > Factorial > Cube Plots…)

Response Optimizer: Target at 40cm

Minitab>Stat>DOE>Factorial>Response Optimizer



Response Optimizer: Target at 40cm

Response Optimization: Reaction Time

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Reaction Time	Target	20	40	100	1	1

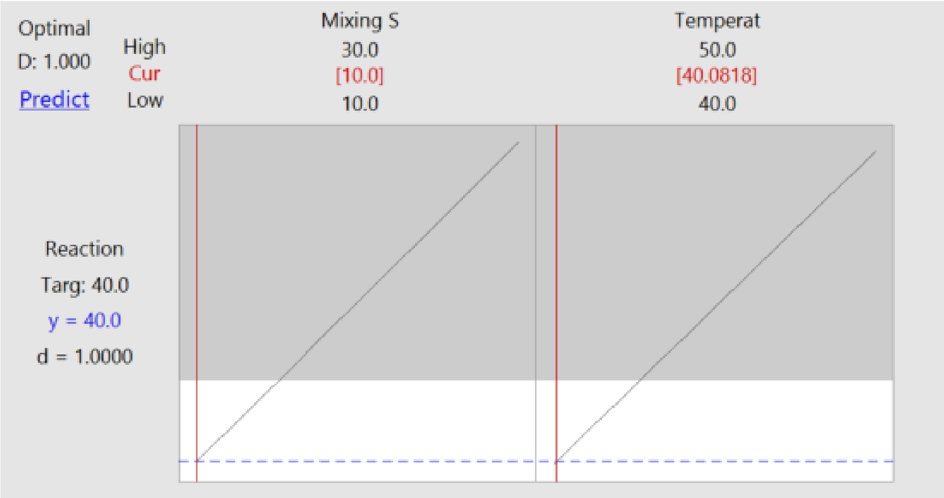
Solution

Solution	Mixing Speed	Temperature	Reaction Time Fit	Composite Desirability
1	10	40.0818	40	1

Multiple Response Prediction

Variable	Setting
Mixing Speed	10
Temperature	40.0818

Response	Fit	SE Fit	95% CI	95% PI
Reaction Time	40.00	4.53	(27.43, 52.57)	(18.11, 61.89)



In order to hit a target at 40 reaction time, one optimized setting is:

- Mixing speed 10
- Temperature 40.0818

Now, you try on the Catapult!