

## BIOS6643. L8 Covariance Structures $R_i$

```
proc import DATAFILE='C:\Users\juarezce\OneDrive - The University of Colorado Denver\BIOS6643\BIOS6643_1'
  replace out=dental dbms=csv; run;

proc print data=dental(obs=2);run;

  title "1. COMMON UNSTRUCTURED";
proc mixed data=dental method=ml;
  class gender id;
  model distance = gender gender*age / noint solution ;
  repeated / type = un subject = id r rcorr;
run;

title "2. COMMON COMPOUND SYMMETRY STRUCTURE";
proc mixed data=dental method=ml;
  class gender id;
  model distance = gender gender*age / noint solution ;
  repeated / type = cs subject = id r rcorr;
run;

  title "3. SEPARATE COMPOUND SYMMETRY BY GENDER";
proc mixed data=dental method=ml;
  class gender id ;
  model distance = gender age*gender / noint solution chisq;
  repeated / type = cs subject=id r=1,4 rcorr=1,4 group=gender;
run;

title "4. COMMON AR(1) STRUCTURE";
proc mixed data=dental method=ml;
  class gender id ;
  model distance = gender age*gender / noint solution chisq;
  repeated / type = ar(1) subject=id r rcorr;
run;

  title "5. COMMON ONE-DEPENDENT STRUCTURE";
proc mixed data=dental method=ml;
  class gender id ;
  model distance = gender age*gender / noint solution chisq;
  repeated / type = toep(2) subject=id r rcorr;
run;

title "6. SPATIAL EXPONENTIAL POWER";
proc mixed data=dental method=ml;
  class gender id;
```

```

model distance = gender gender*age / noint solution chisq;
repeated / type = sp(exp)(age) subject = id r rcorr;
run;

/* */

```

Obs	id	age	distance	gender
1	1	8	21	Girls
2	1	10	20	Girls

# 1. COMMON UNSTRUCTURED

## The Mixed Procedure

### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Unstructured
Subject Effect	id
Estimation Method	ML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

### Class Level Information

Class	Levels	Values
gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3 4 5 6 7 8 9

### Dimensions

Covariance Parameters	10
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

### Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

### Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	419.47721707	0.00000152
2	1	419.47704812	0.00000000

Convergence criteria met.

Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	5.1192	2.4409	3.6105	2.5222
2	2.4409	3.9279	2.7175	3.0624
3	3.6105	2.7175	5.9798	3.8235
4	2.5222	3.0624	3.8235	4.6180

Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.5443	0.6526	0.5188
2	0.5443	1.0000	0.5607	0.7190
3	0.6526	0.5607	1.0000	0.7276
4	0.5188	0.7190	0.7276	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	id	5.1192
UN(2,1)	id	2.4409
UN(2,2)	id	3.9279
UN(3,1)	id	3.6105
UN(3,2)	id	2.7175
UN(3,3)	id	5.9798
UN(4,1)	id	2.5222
UN(4,2)	id	3.0624
UN(4,3)	id	3.8235
UN(4,4)	id	4.6180

Fit Statistics

-2 Log Likelihood	419.5
AIC (Smaller is Better)	447.5
AICC (Smaller is Better)	452.0
BIC (Smaller is Better)	465.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
9	58.76	<.0001

# Solution for Fixed Effects

Effect	gender	Estimate	Standard Error	DF	t Value	Pr >  t
gender	Boys	15.8423	0.9356	25	16.93	<.0001
gender	Girls	17.4254	1.1284	25	15.44	<.0001
age*gender	Boys	0.8268	0.07911	25	10.45	<.0001
age*gender	Girls	0.4764	0.09541	25	4.99	<.0001

## Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
gender	2	25	262.60	<.0001
age*gender	2	25	67.07	<.0001

## 2. COMMON COMPOUND SYMMETRY STRUCTURE

### The Mixed Procedure

#### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Compound Symmetry
Subject Effect	id
Estimation Method	ML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

#### Class Level Information

Class	Levels	Values
gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3 4 5 6 7 8 9

#### Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

#### Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

#### Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	1	428.63905802	0.00000000

Convergence criteria met.

#### Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	4.9052	3.0306	3.0306	3.0306
2	3.0306	4.9052	3.0306	3.0306
3	3.0306	3.0306	4.9052	3.0306
4	3.0306	3.0306	3.0306	4.9052

#### Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.6178	0.6178	0.6178
2	0.6178	1.0000	0.6178	0.6178
3	0.6178	0.6178	1.0000	0.6178
4	0.6178	0.6178	0.6178	1.0000

#### Covariance Parameter Estimates

Cov Parm	Subject	Estimate
CS	id	3.0306
Residual		1.8746

#### Fit Statistics

-2 Log Likelihood	428.6
AIC (Smaller is Better)	440.6
AICC (Smaller is Better)	441.5
BIC (Smaller is Better)	448.4

#### Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	49.60	<.0001

#### Solution for Fixed Effects

Standard

Effect	gender	Estimate	Error	DF	t Value	Pr >  t
gender	Boys	16.3406	0.9631	25	16.97	<.0001
gender	Girls	17.3727	1.1615	25	14.96	<.0001
age*gender	Boys	0.7844	0.07654	79	10.25	<.0001
age*gender	Girls	0.4795	0.09231	79	5.20	<.0001

#### Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
gender	2	25	255.79	<.0001
age*gender	2	79	66.01	<.0001

### 3. SEPARATE COMPOUND SYMMETRY BY GENDER

#### The Mixed Procedure

#### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Compound Symmetry
Subject Effect	id
Group Effect	gender
Estimation Method	ML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

#### Class Level Information

Class	Levels	Values
gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3 4 5 6 7 8 9

#### Dimensions

Covariance Parameters	4
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

#### Number of Observations

Number of Observations Read	108
Number of Observations Used	108

Number of Observations Not Used 0

#### Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	1	408.81297228	0.00000000

Convergence criteria met.

#### Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	4.4704	3.8804	3.8804	3.8804
2	3.8804	4.4704	3.8804	3.8804
3	3.8804	3.8804	4.4704	3.8804
4	3.8804	3.8804	3.8804	4.4704

#### Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.8680	0.8680	0.8680
2	0.8680	1.0000	0.8680	0.8680
3	0.8680	0.8680	1.0000	0.8680
4	0.8680	0.8680	0.8680	1.0000

#### Estimated R Matrix for id 12

Row	Col1	Col2	Col3	Col4
1	5.2041	2.4463	2.4463	2.4463
2	2.4463	5.2041	2.4463	2.4463
3	2.4463	2.4463	5.2041	2.4463
4	2.4463	2.4463	2.4463	5.2041

#### Estimated R Correlation Matrix for id 12

Row	Col1	Col2	Col3	Col4
1	1.0000	0.4701	0.4701	0.4701
2	0.4701	1.0000	0.4701	0.4701
3	0.4701	0.4701	1.0000	0.4701
4	0.4701	0.4701	0.4701	1.0000

#### Covariance Parameter Estimates

Cov Parm	Subject	Group	Estimate
Variance	id	gender Boys	2.7577
CS	id	gender Boys	2.4463
Variance	id	gender Girls	0.5900

CS	id	gender Girls	3.8804
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#### Fit Statistics

-2 Log Likelihood	408.8
AIC (Smaller is Better)	424.8
AICC (Smaller is Better)	426.3
BIC (Smaller is Better)	435.2

#### Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	69.43	<.0001

#### Solution for Fixed Effects

Effect	gender	Estimate	Standard Error	DF	t Value	Pr >  t
gender	Boys	16.3406	1.1130	25	14.68	<.0001
gender	Girls	17.3727	0.8311	25	20.90	<.0001
age*gender	Boys	0.7844	0.09283	79	8.45	<.0001
age*gender	Girls	0.4795	0.05179	79	9.26	<.0001

#### Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
gender	2	25	652.53	326.26	<.0001	<.0001
age*gender	2	79	157.14	78.57	<.0001	<.0001

#### 4. COMMON AR(1) STRUCTURE

##### The Mixed Procedure

##### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Autoregressive
Subject Effect	id
Estimation Method	ML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

##### Class Level Information

Class	Levels	Values
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gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3 4 5 6 7 8 9

#### Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

#### Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

#### Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	440.68100623	0.00000000

Convergence criteria met.

#### Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	4.8910	2.9696	1.8030	1.0947
2	2.9696	4.8910	2.9696	1.8030
3	1.8030	2.9696	4.8910	2.9696
4	1.0947	1.8030	2.9696	4.8910

#### Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.6071	0.3686	0.2238
2	0.6071	1.0000	0.6071	0.3686
3	0.3686	0.6071	1.0000	0.6071
4	0.2238	0.3686	0.6071	1.0000

#### Covariance Parameter Estimates

Cov Parm	Subject	Estimate
AR(1)	id	0.6071
Residual		4.8910

#### Fit Statistics

-2 Log Likelihood	440.7
AIC (Smaller is Better)	452.7
AICC (Smaller is Better)	453.5
BIC (Smaller is Better)	460.5

#### Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	37.56	<.0001

#### Solution for Fixed Effects

Effect	gender	Estimate	Standard Error	DF	t Value	Pr >  t
gender	Boys	16.5920	1.3299	25	12.48	<.0001
gender	Girls	17.3217	1.6040	25	10.80	<.0001
age*gender	Boys	0.7696	0.1147	79	6.71	<.0001
age*gender	Girls	0.4837	0.1384	79	3.50	0.0008

#### Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
gender	2	25	272.27	136.14	<.0001	<.0001
age*gender	2	79	57.23	28.61	<.0001	<.0001

### 5. COMMON ONE-DEPENDENT STRUCTURE

#### The Mixed Procedure

#### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Toeplitz
Subject Effect	id
Estimation Method	ML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

#### Class Level Information

Class	Levels	Values
gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3

4 5 6 7 8 9

# Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

# Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

# Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	589.03603775	0.16283093
2	1	545.67380444	0.15138564
3	1	510.19059372	0.12467398
4	1	484.30189351	0.08645876
5	1	468.14463315	0.04649605
6	1	460.20520640	0.01592441
7	1	457.72394860	0.00214984
8	1	457.42200558	0.00004120
9	1	457.41660393	0.00000002
10	1	457.41660197	0.00000000

Convergence criteria met.

# Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	4.5294	1.6120		
2	1.6120	4.5294	1.6120	
3		1.6120	4.5294	1.6120
4			1.6120	4.5294

# Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.3559		
2	0.3559	1.0000	0.3559	
3		0.3559	1.0000	0.3559
4			0.3559	1.0000

# Covariance Parameter Estimates

Cov Parm	Subject	Estimate
TOEP(2)	id	1.6120
Residual		4.5294

#### Fit Statistics

-2 Log Likelihood	457.4
AIC (Smaller is Better)	469.4
AICC (Smaller is Better)	470.2
BIC (Smaller is Better)	477.2

#### Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	20.83	<.0001

#### Solution for Fixed Effects

Effect	gender	Estimate	Standard Error	DF	t Value	Pr >  t
gender	Boys	16.6208	1.4167	25	11.73	<.0001
gender	Girls	17.3035	1.7086	25	10.13	<.0001
age*gender	Boys	0.7629	0.1253	79	6.09	<.0001
age*gender	Girls	0.4856	0.1512	79	3.21	0.0019

#### Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
gender	2	25	240.22	120.11	<.0001	<.0001
age*gender	2	79	47.36	23.68	<.0001	<.0001

## 6. SPATIAL EXPONENTIAL POWER

### The Mixed Procedure

#### Model Information

Data Set	WORK.DENTAL
Dependent Variable	distance
Covariance Structure	Spatial Exponential
Subject Effect	id
Estimation Method	ML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

#### Class Level Information

Class	Levels	Values
gender	2	Boys Girls
id	27	1 10 11 12 13 14 15 16 17 18 19 2 20 21 22 23 24 25 26 27 3 4 5 6 7 8 9

#### Dimensions

Covariance Parameters	2
Columns in X	4
Columns in Z	0
Subjects	27
Max Obs per Subject	4

#### Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

#### Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	440.79105774	0.00101293
2	1	440.68470622	0.00003002
3	1	440.68100862	0.00000002
4	1	440.68100608	0.00000000

Convergence criteria met.

#### Estimated R Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	4.8908	2.9693	1.8027	1.0944
2	2.9693	4.8908	2.9693	1.8027
3	1.8027	2.9693	4.8908	2.9693
4	1.0944	1.8027	2.9693	4.8908

#### Estimated R Correlation Matrix for id 1

Row	Col1	Col2	Col3	Col4
1	1.0000	0.6071	0.3686	0.2238
2	0.6071	1.0000	0.6071	0.3686
3	0.3686	0.6071	1.0000	0.6071
4	0.2238	0.3686	0.6071	1.0000

#### Covariance Parameter Estimates

Cov Parm	Subject	Estimate
SP(EXP)	id	4.0077
Residual		4.8908

#### Fit Statistics

-2 Log Likelihood	440.7
AIC (Smaller is Better)	452.7
AICC (Smaller is Better)	453.5
BIC (Smaller is Better)	460.5

#### Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	37.56	<.0001

#### Solution for Fixed Effects

Effect	gender	Estimate	Standard Error	DF	t Value	Pr >  t
gender	Boys	16.5920	1.3299	25	12.48	<.0001
gender	Girls	17.3217	1.6040	25	10.80	<.0001
age*gender	Boys	0.7696	0.1147	79	6.71	<.0001
age*gender	Girls	0.4837	0.1384	79	3.50	0.0008

#### Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
gender	2	25	272.27	136.14	<.0001	<.0001
age*gender	2	79	57.23	28.61	<.0001	<.0001