BIOS6643. L12 GEE and Marginal Models

Fitting population-averaged models using the gee() function and the geeglm() function in geepack. We will need to install.packages("gee") and install.packages("geepack").

The syntax for each is similar. A general call looks like

fit.object <- gee(model formula, id, corstr, family, data)

- corstr is a specification for the working correlation matrix
- family is a specification for the scaled exponential family that would be relevant under independence; the canonical link is the default

Dental data

Example of continuous outcome

```
## Read in the data
dat.den <- read.csv("/Users/juarezce/Documents/OneDrive - The University of Colorado Denver/BIOS6643/BI
head(dat.den,3)
##
     id age distance gender
## 1 1 8
                21.0 Girls
## 2 1 10
                20.0 Girls
## 3 1 12
                21.5 Girls
##dat.den$id <- as.factor(dat.den$id)</pre>
##dat.den$gender <- as.factor(dat.den$gender)
ind.gee.den <- gee(distance ~ age*gender,</pre>
             id = id,
             data = dat.den,
             family = gaussian,
             corstr = "independence"
## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
## running glm to get initial regression estimate
##
       (Intercept)
                                        genderGirls age:genderGirls
                               age
##
        16.3406250
                         0.7843750
                                          1.0321023
                                                         -0.3048295
summary(ind.gee.den)
##
   GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
##
   gee S-function, version 4.13 modified 98/01/27 (1998)
##
## Model:
```

```
Link:
                                 Identity
##
##
    Variance to Mean Relation: Gaussian
                                 Independent
##
    Correlation Structure:
##
## Call:
   gee(formula = distance ~ age * gender, id = id, data = dat.den,
##
       family = gaussian, corstr = "independence")
##
##
##
   Summary of Residuals:
##
          Min
                       1Q
                              Median
                                              30
                                                         Max
   -5.6156250 -1.3218750 -0.1681818
                                       1.3299006
                                                  5.2468750
##
##
  Coefficients:
##
##
                      Estimate Naive S.E.
                                             Naive z Robust S.E.
                                                                     Robust z
##
   (Intercept)
                    16.3406250
                                1.4162242 11.538163
                                                       1.17148092 13.9486906
##
  age
                                0.1261673
                                            6.216945
                                                       0.09834755
                                                                   7.9755416
                     0.7843750
  genderGirls
                     1.0321023
                                2.2187969
                                            0.465163
                                                       1.37778506
                                0.1976661 -1.542143
  age:genderGirls -0.3048295
                                                       0.11686730 -2.6083390
## Estimated Scale Parameter:
                                5.093818
## Number of Iterations:
##
## Working Correlation
##
        [,1] [,2] [,3] [,4]
## [1,]
           1
                 0
                      0
  [2,]
           0
                      0
                           0
##
                 1
           0
                           0
##
   [3,]
                 0
                      1
           0
## [4,]
                      0
                           1
```

Recall the interpretation of the parameter estimates is at the population level. For instance for boys, the $\hat{\beta}_{age} = 0.78$ may be interpreted as the average (expected) increase in distance per year of age increased.

Seizure data using gee()

Epileptic Seizure Study of a randomized trial reported in Thall and Vail (1990).

- 59 subjects with epilepsy suffering from simple or partial seizures were assigned at random to receive either the anti-epileptic drug progabide or a placebo in addition to a standard chemotherapy regimen all were taking.
- Because each individual might be prone to different rates of experiencing seizures, the investigators first tried to get a sense of this by recording the number of seizures suffered by each subject over the 8-week period prior to the start of administration of the assigned treatment.
 - It is common in such studies to record such baseline measurements, so that the effect of treatment for each subject can be measured relative to how that subject behaved prior to treatment.
- Following initiation of treatment, the number of seizures for each subject was counted for each of 4 consecutive 2-week periods.
- The age of each subject at the start of the study was also recorded, as it was suspected that subject age might be associated with the effect of the treatment.

```
# Read in the data
dat.sz <- read.table("/Users/juarezce/Documents/OneDrive - The University of Colorado Denver/BIOS6643/B
colnames(dat.sz) <- c("subj", "seize", "visit", "trt", "base", "age")</pre>
## trt=0 corresponds to placebo
head(dat.sz,3)
     subj seize visit trt base age
## 1 104
                        0
             11
                    0
                            11
                                31
## 2 104
              5
                    1
                            11 31
## 3 104
              3
                    2
                            11 31
                        0
# Create other covariates
dat.sz$o <- 8*(dat.sz$visit==0)+2*(dat.sz$visit>0)
dat.sz$logo <- log(dat.sz$o)</pre>
dat.sz$vm0 <- as.numeric(dat.sz$visit>0)
# Basic models -- the unstructured fit using gee() does not converge
# even with maxiter set to be much larger than the default
##un.gee <- gee(seize ~ trt + offset(logo), id=subj, family=poisson,
                corstr="unstructured", data=dat.sz, maxiter=100)
cs.gee <- gee(seize ~ trt + offset(logo),id=subj,family=poisson,</pre>
              corstr="exchangeable", data=dat.sz)
## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
## running glm to get initial regression estimate
## (Intercept)
                       trt
## 1.40454203 -0.02685314
summary(cs.gee)
##
##
   GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
   gee S-function, version 4.13 modified 98/01/27 (1998)
##
## Model:
                               Logarithm
## Variance to Mean Relation: Poisson
## Correlation Structure:
                               Exchangeable
##
## Call:
## gee(formula = seize ~ trt + offset(logo), id = subj, data = dat.sz,
       family = poisson, corstr = "exchangeable")
##
## Summary of Residuals:
##
           Min
                       1Q
                                Median
                                                 3Q
                                                            Max
##
   -3.9392609 -0.9392609
                             2.3491032 10.7049211 147.0607391
##
##
## Coefficients:
##
                 Estimate Naive S.E.
                                       Naive z Robust S.E. Robust z
## (Intercept) 1.29497285 0.1392018 9.3028451
                                                 0.1718363 7.5360855
```

```
##
## Estimated Scale Parameter: 20.73326
## Number of Iterations: 3
## Working Correlation
            Γ.17
                     [,2]
                               [.3]
                                        [.4]
## [1,] 1.0000000 0.7692888 0.7692888 0.7692888 0.7692888
## [2,] 0.7692888 1.0000000 0.7692888 0.7692888 0.7692888
## [3,] 0.7692888 0.7692888 1.0000000 0.7692888 0.7692888
## [4,] 0.7692888 0.7692888 0.7692888 1.0000000 0.7692888
## [5,] 0.7692888 0.7692888 0.7692888 1.0000000
## AR1
ar1.gee <- gee(seize ~ trt + offset(logo),id=subj,family=poisson,
            corstr="AR-M",Mv=1,data=dat.sz)
## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
## running glm to get initial regression estimate
## (Intercept)
## 1.40454203 -0.02685314
summary(ar1.gee)
##
   GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
   gee S-function, version 4.13 modified 98/01/27 (1998)
##
## Model:
## Link:
                             Logarithm
## Variance to Mean Relation: Poisson
## Correlation Structure:
                            AR-M , M = 1
##
## Call:
  gee(formula = seize ~ trt + offset(logo), id = subj, data = dat.sz,
      family = poisson, corstr = "AR-M", Mv = 1)
##
##
## Summary of Residuals:
                      1Q
                             Median
                                                       Max
##
  -3.7578756 -0.7578756
                          2.4656263 10.8538753 147.2421244
##
##
## Coefficients:
               Estimate Naive S.E. Naive z Robust S.E. Robust z
## (Intercept) 1.26253613 0.1420223 8.889702 0.1750352 7.2130401
             ## Estimated Scale Parameter: 21.68155
## Number of Iterations: 3
##
## Working Correlation
            [,1]
                     [,2]
                               [,3]
                                        [,4]
## [1,] 1.0000000 0.8098380 0.6558376 0.5311222 0.4301229
## [2,] 0.8098380 1.0000000 0.8098380 0.6558376 0.5311222
## [3,] 0.6558376 0.8098380 1.0000000 0.8098380 0.6558376
```

```
## [4,] 0.5311222 0.6558376 0.8098380 1.0000000 0.8098380
## [5,] 0.4301229 0.5311222 0.6558376 0.8098380 1.0000000
```

Seizure data using geeglm()

```
## UN using geeglm
un.geeglm <- geeglm(seize ~ trt, id=subj,family=poisson("log"),</pre>
            offset=logo, corstr="unstructured",data=dat.sz)
summary(un.geeglm)
##
## Call:
## geeglm(formula = seize ~ trt, family = poisson("log"), data = dat.sz,
       offset = logo, id = subj, corstr = "unstructured")
##
##
##
   Coefficients:
##
              Estimate Std.err Wald Pr(>|W|)
## (Intercept)
                 -1846
                             0 Inf
                                      <2e-16 ***
                  1852
                             0 Inf
                                      <2e-16 ***
## trt
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation structure = unstructured
## Estimated Scale Parameters:
##
               Estimate
                          Std.err
## (Intercept) 9.954e+17 3.302e+17
##
   Link = identity
##
## Estimated Correlation Parameters:
            Estimate Std.err
##
## alpha.1:2 1.0262 0.14746
## alpha.1:3 0.9145 0.07836
## alpha.1:4 0.9718 0.18927
## alpha.1:5 0.8637 0.09313
## alpha.2:3 0.3005 0.04790
## alpha.2:4 0.3326 0.08001
## alpha.2:5 0.2680 0.04504
## alpha.3:4 0.3202 0.09746
## alpha.3:5 0.2423 0.02407
## alpha.4:5 0.3068 0.10557
## Number of clusters:
                        59 Maximum cluster size: 5
## CS using geeglm
cs.geeglm <- geeglm(seize ~ trt,id=subj,family=poisson("log"),</pre>
            offset=logo, corstr="exchangeable",data=dat.sz)
summary(cs.geeglm)
##
## Call:
## geeglm(formula = seize ~ trt, family = poisson("log"), data = dat.sz,
##
       offset = logo, id = subj, corstr = "exchangeable")
##
```

```
## Coefficients:
##
              Estimate Std.err Wald Pr(>|W|)
## (Intercept) 1.2943 0.1721 56.54 5.5e-14 ***
                0.0766 0.2141 0.13
                                         0.72
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation structure = exchangeable
## Estimated Scale Parameters:
##
##
              Estimate Std.err
                  20.6
## (Intercept)
                          9.81
##
    Link = identity
##
## Estimated Correlation Parameters:
##
        Estimate Std.err
           0.772 0.0953
## alpha
## Number of clusters:
                        59 Maximum cluster size: 5
ar1.geeglm <- geeglm(seize ~ trt,id=subj,family=poisson("log"),</pre>
            offset=logo, corstr="ar1",data=dat.sz)
summary(ar1.geeglm)
##
## geeglm(formula = seize ~ trt, family = poisson("log"), data = dat.sz,
##
      offset = logo, id = subj, corstr = "ar1")
##
##
  Coefficients:
##
              Estimate Std.err Wald Pr(>|W|)
                 1.216  0.195  38.78  4.7e-10 ***
## (Intercept)
                 0.095
                         0.237 0.16
                                         0.69
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation structure = ar1
## Estimated Scale Parameters:
##
##
              Estimate Std.err
## (Intercept)
                  22.2
                          10.9
##
    Link = identity
## Estimated Correlation Parameters:
        Estimate Std.err
## alpha
           0.889 0.0645
## Number of clusters: 59 Maximum cluster size: 5
```

Investigate if there is a different different effect after baseline visit 0

This means include an interaction between the indicator variable for visit>0 and the treatment indicator. Use two different working correlation structures and compare the results.

```
ar2.gee <- gee(seize ~ vm0*trt + offset(logo),id=subj,family=poisson,</pre>
        corstr="AR-M",Mv=1,data=dat.sz)
## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
## running glm to get initial regression estimate
## (Intercept)
                      vmO
                                   trt
                                          vm0:trt
##
        1.3476
                   0.1108
                               0.0265
                                          -0.1037
summary(ar2.gee)
##
   GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
##
   gee S-function, version 4.13 modified 98/01/27 (1998)
##
## Model:
## Link:
                               Logarithm
## Variance to Mean Relation: Poisson
## Correlation Structure:
                              AR-M , M = 1
##
## Call:
## gee(formula = seize ~ vm0 * trt + offset(logo), id = subj, data = dat.sz,
      family = poisson, corstr = "AR-M", Mv = 1)
##
## Summary of Residuals:
     Min
             1Q Median
                            3Q
##
  -4.32 -1.32 2.13 10.45 147.22
##
##
## Coefficients:
              Estimate Naive S.E. Naive z Robust S.E. Robust z
## (Intercept) 1.3120
                            0.143 9.1786
                                             0.162
                                                0.112
## vm0
                0.1513
                            0.169 0.8976
                                                         1.356
                            0.196 0.0961
                                                0.212
                                                         0.089
## trt
                0.0189
                            0.241 -0.5322
## vmO:trt
               -0.1282
                                                0.260 -0.493
## Estimated Scale Parameter: 20.2
## Number of Iterations: 3
##
## Working Correlation
         [,1] [,2] [,3] [,4] [,5]
## [1,] 1.000 0.810 0.657 0.532 0.431
## [2,] 0.810 1.000 0.810 0.657 0.532
## [3,] 0.657 0.810 1.000 0.810 0.657
## [4,] 0.532 0.657 0.810 1.000 0.810
## [5,] 0.431 0.532 0.657 0.810 1.000
```