

Lab 4 – Intro to Assembly
Group 8 – Abuke, Gaerlan, Sy, Taruc

(Mishka + Tobey + Ethan) 1.

```
sall $2, %eax
```

- (shifts value in %eax left by 2 bits) $\rightarrow 2^2 = 4$
- shifting is much faster and simpler over multiplication for powers of two
- sall (shift all) shift entire section by a constant (2) number of bits

(Tobey + Mishka + Ethan) 2.

```
movl -4(%rbp), %edx    edx=x
movl %edx, %eax        eax=x
sall $2, %eax          eax = x << 2 = x * 4
addl %edx, %eax        eax = 4x + x = 5x
sall $3, %eax          eax = 5x << 3 = 5x * 8 = 40x
addl %edx, %eax        eax = 40x + x = 41x
```

- the code that's generated is
 - shift %eax by 2 bits ($x * (2^2) = 4x$) and store it back in %eax
 - add the input x (%edx) to %eax ($4x + x = 5x$)
 - shift %eax by 3 bits ($5x * (2^3) = 40x$)
 - add the input x (%edx) to %eax ($40x + x = 41x$)
 - $41x$ was decomposed to $((2^2x + x)(2^3) + x)$

(Tobey + Mishka + Ethan) 3.

```
movl -4(%rbp), %edx    edx = x
movl %edx, %eax        eax = x
sall $6, %eax          eax = x << 6 = x * 64 = 64x
subl %edx, %eax        eax = 64x - x = 63x
```

- the code that's generated is
 - shift %eax by 6 bits ($x * (2^6) = x * 64 = 64x$) and store it back in %eax
 - subtract the input x (%edx) from %eax ($64x - x = 63x$)
 - Essentially, $63x = (2^6)x - x$

(Tobey + Mishka + Alinus + Ethan) 4.

```
movl -4(%rbp), %edx    edx = x
movl %edx, %eax        eax = x
sall $2, %eax          eax = x << 2 = x * 4 = 4x
addl %edx, %eax        eax = 4x + x = 5x
negl %eax              eax = 0 - 5x = -5x
```

- the code that's generated is essentially $2 * 5$ but negated by two's complement
 - shift %eax by 2 bits ($x * (2^2) = 4x$) and store it back in %eax
 - add the input x (%edx) to %eax ($x + 4x = 5x$)
 - negates long ([does two's complement](#)) so the result is $2 * -5$

(Tobey + Mishka + Alinus) 5.

```
imull    $61, %eax, %eax    eax = eax * 61
```

- Compiler just uses imull (integer multiply) directly instead of shifts and adds

The compiler [optimizes](#) using the instruction count; apparently, they use [heuristics](#) which determine if the cost of shifting/adding is lower than the cost of using imull directly.

Looking at the algorithm, gcc tries to factor a number into factors of $x * 2^n \pm 1$, and then factors that number recursively until they reach 1 or a number that is handled by a special case. The compiler also has multiple different cases that attempt to account for as many numbers as possible. This is done through the synth_mult function which is under the expmed.cc of the [gcc compiler](#).

```
2807
2808     /* t = 1 can be done in zero cost. */
2809     if (t == 1)
2810     {
2811         alg_out->ops = 1;
2812         alg_out->cost.cost = 0;
2813         alg_out->cost.latency = 0;
2814         alg_out->op[0] = alg_m;
2815         return;
2816     }

/* t = 0 sometimes has a cost. If it does and it exceeds our limit,
fail now. */
if (t == 0)
{
    if (MULT_COST_LESS (cost_limit, zero_cost (speed)))
        return;
    else
    {
        alg_out->ops = 1;
        alg_out->cost.cost = zero_cost (speed);
        alg_out->cost.latency = zero_cost (speed);
        alg_out->op[0] = alg_zero;
        return;
    }
}

2977
2978     /* If T was -1, then W will be zero after the loop. This is another
2979     case where T ends with ...111. Handling this with (T + 1) and
2980     subtract 1 produces slightly better code and results in algorithm
2981     selection much faster than treating it like the ...0111 case
2982     below. */
2983     if (w == 0
2984         || (w > 2
2985             /* Reject the case where t is 3.
2986             Thus we prefer addition in that case. */
2987             && t != 3))
2988     {
2989         /* T ends with ...111. Multiply by (T + 1) and subtract T. */
2990
2991         op_cost = add_cost (speed, mode);
2992         new_limit.cost = best_cost.cost - op_cost;
2993         new_limit.latency = best_cost.latency - op_cost;
2994         synth_mult (alg_in, t + 1, &new_limit, mode);
2995
2996         alg_in->cost.cost += op_cost;
2997         alg_in->cost.latency += op_cost;
```

```

3007      /* T ends with ...01 or ...011. Multiply by (T - 1) and add T. */
3008
3009      op_cost = add_cost (speed, mode);
3010      new_limit.cost = best_cost.cost - op_cost;
3011      new_limit.latency = best_cost.latency - op_cost;
3012      synth_mult (alg_in, t - 1, &new_limit, mode);
3013
3014      alg_in->cost.cost += op_cost;
3015      alg_in->cost.latency += op_cost;
3016      if (CHEAPER_MULT_COST (&alg_in->cost, &best_cost))
3017      {
3018          best_cost = alg_in->cost;
3019          std::swap (alg_in, best_alg);
3020          best_alg->log[best_alg->ops] = 0;
3021          best_alg->op[best_alg->ops] = alg_add_t_m2;
3022      }
3023  }

```

```

3025      /* We may be able to calculate a * -7, a * -15, a * -31, etc
3026      quickly with a - a * n for some appropriate constant n. */
3027      m = exact_log2 (-orig_t + 1);
3028      if (m ≥ 0 && m < maxm)
3029      {
3030          op_cost = add_cost (speed, mode) + shift_cost (speed, mode, m);
3031          /* If the target has a cheap shift-and-subtract insn use
3032          that in preference to a shift insn followed by a sub insn.
3033          Assume that the shift-and-sub is "atomic" with a latency
3034          equal to it's cost, otherwise assume that on superscalar
3035          hardware the shift may be executed concurrently with the
3036          earlier steps in the algorithm. */
3037          if (shiftsub1_cost (speed, mode, m) ≤ op_cost)
3038          {
3039              op_cost = shiftsub1_cost (speed, mode, m);
3040              op_latency = op_cost;
3041          }
3042          else
3043              op_latency = add_cost (speed, mode);
3044
3045          new_limit.cost = best_cost.cost - op_cost;
3046          new_limit.latency = best_cost.latency - op_latency;
3047          synth_mult (alg_in, (unsigned HOST_WIDE_INT) (-orig_t + 1) >> m,
3048              &new_limit, mode);

```

```

3141  /* Try shift-and-add (load effective address) instructions,
3142  i.e. do a*3, a*5, a*9. */
3143  if ((t & 1) != 0)
3144  {
3145      do_alg_add_t2_m:
3146          q = t - 1;
3147          m = ctz_hwi (q);
3148          if (q && m < maxm)
3149          {
3150              op_cost = shiftadd_cost (speed, mode, m);
3151              new_limit.cost = best_cost.cost - op_cost;
3152              new_limit.latency = best_cost.latency - op_cost;
3153              synth_mult (alg_in, (t - 1) >> m, &new_limit, mode);
3154
3155              alg_in->cost.cost += op_cost;
3156              alg_in->cost.latency += op_cost;
3157              if (CHEAPER_MULT_COST (&alg_in->cost, &best_cost))
3158              {

```

And then finally, the main algorithm, which is too large to show here, focuses on recursing through possible $2^n \pm 1$ scenarios

```

3065  /* Look for factors of t of the form
3066  t = q(2**m +- 1), 2 ≤ m ≤ floor(log2(t - 1)).
3067  If we find such a factor, we can multiply by t using an algorithm that
3068  multiplies by q, shift the result by m and add/subtract it to itself.
3069
3070  We search for large factors first and loop down, even if large factors
3071  are less probable than small; if we find a large factor we will find a
3072  good sequence quickly, and therefore be able to prune (by decreasing
3073  COST_LIMIT) the search. */
3074
3075  do_alg_addsub_factor:
3076  for (m = floor_log2 (t - 1); m ≥ 2; m--)
3077  {
3078      unsigned HOST_WIDE_INT d;
3079
3080      d = (HOST_WIDE_INT_1U << m) + 1;
3081      if (t % d == 0 && t > d && m < maxm
3082      && (!cache_hit || cache_alg == alg_add_factor))
3083      {
3084          op_cost = add_cost (speed, mode) + shift_cost (speed, mode, m);
3085          if (shiftadd_cost (speed, mode, m) ≤ op_cost)
3086              op_cost = shiftadd_cost (speed, mode, m);
3087
3088          op_latency = op_cost;

```

We can try manually if 61 has good factors of 2^n , 2^n-1 , and 2^n+1 . We can get 61 with $31 + 15 + 15$. Which are three shifts and subtracts added together for a total of 9. Other factors are around or take more operations than this. Since imul takes [around 3 cycles](#). It is faster to just use imul.

(Tobey + Alinus + Ethan) 6.

Lots of using the leal function which seems to follow the [format](#) of

leal displacement(base register, offset register, scalar multiplier)

where

*base register + (offset register * scalar multiplier) + displacement*

Which often compresses and performs the operations of these functions without dereferencing the addresses

Leal is being used as an arithmetic instruction rather than for memory access. It allows the compiler to [combine multiple operations like addition and multiplication](#) by constants into a single instruction, instead of using several separate shift and add instructions. leal is a faster way of doing addition + multiplication operations. Leal works best for shift and add operations (see screenshot) $2^n + 1$. So if the synth_mult stumbles upon a $2^n + 1$ number, it can use leal for it instead of multiple shifts and adds.

In addition to that, when using the -O flag, the compiler also removes unnecessary instructions such as extra moves between registers, and redundant computations. Some functions keep the same basic ALU instructions because they are already optimal, while others are rewritten to use leal since it can do the same work in fewer instructions.

The first case is now

```
leal    0(,%rdi,4), %eax
```

Which roughly translates to $0 + (a * 4) + 0 \rightarrow \text{eax} = a * 4$

The second case is now

```
leal    (%rdi,%rdi,4), %eax
leal    (%rdi,%rax,8), %eax
```

Which translates to

$a + (a * 4) + 0 \rightarrow \text{eax} = a * 5$
 $a + (a * 5 * 8) + 0 \rightarrow \text{eax} = a * 41$
(rax is the 64 bit extended eax)

The third case is actually still the same.

The fourth case is now

```
leal    (%rdi,%rdi,4), %eax
negl    %eax
```

Which translates to

$a + (a * 4) + 0 \rightarrow \text{eax} = a * 5$
 $0 - a * 5 \rightarrow \text{eax} = -a * 5$

The fifth case is still using imull

```
imull    $61, %edi, %eax
```