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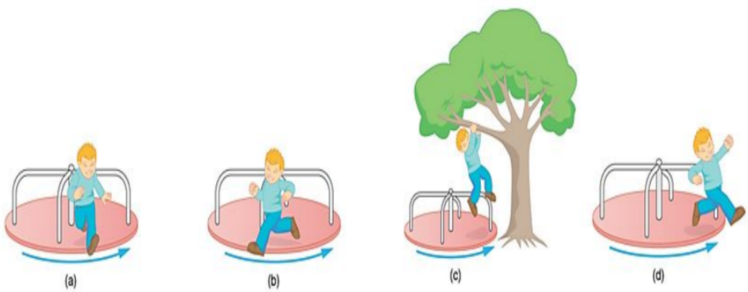
General Physics 1

Quarter 2 - Module 1

Rotational Equilibrium and Rotational Dynamics



https://courses.lumenlearning.com/physics/chapter/10-5-angular-momentum-and-its-conservation/

Department of Education • Republic of the Philippines

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General Physics

Quarter 2 - Module 1
Rotational Equilibrium and Rotational Dynamics

Department of Education ● Republic of the Philippines

Table of Contents

Module 1
Rotational Equilibrium and Rotational Dynamics

What This Module is About

This module demonstrates your understanding on the concepts of Rotational Motion. Rotational motion can be most easily approached by considering how it is analogous to linear of translational motion.

The first part of this module will discuss the kinematics of rotational motion as described in its angular position, speed and acceleration. The second part will discuss its dynamics and explain the forces that causes objects to rotate. In this part, you will understand the Physics of simple events observed daily such as the motion of opening doors, the concept behind see-saw; the motion of ice-skaters and many more.

Specifically, this module will discuss two (2) lessons:

- **Lesson 1 – Rotational Kinematics**
- **Lesson 2 – Rotational Dynamics**

What I Need to Know

At the end of this module, you should be able to:

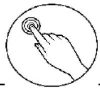
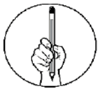
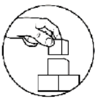
1. Calculate the moment of inertia about a given axis of single-object and multiple object systems **STEM_GP12REDIIa-1**
2. Calculate magnitude and direction of torque using the definition of torque as a cross product **STEM_GP12REDIIa-3**
3. Describe rotational quantities using vectors **STEM_GP12REDIIa-4**
4. Determine whether a system is in static equilibrium or not **STEM_GP12REDIIa-5**
5. Apply the rotational kinematic relations for systems with constant angular accelerations **STEM_GP12REDIIa-6**
6. Solve static equilibrium problems in contexts such as, but not limited to, seesaws, mobiles, cable-hinge-strut system, leaning ladders, and weighing a heavy suitcase using a small bathroom scale **STEM_GP12REDIIa-8**
7. Determine angular momentum of different systems **STEM_GP12REDIIa-9**
8. Apply the torque-angular momentum relation **STEM_GP12REDIIa-1**

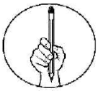
How to Learn from this Module

To achieve the objectives cited above, you are to do the following:

- Take your time reading the lessons carefully.
- Follow the directions and/or instructions in the activities and exercises diligently.
- Answer all the given tests and exercises.

Icons of this Module

	What I Need to Know	This part contains learning objectives that are set for you to learn as you go along the module.
	What I know	This is an assessment as to your level of knowledge to the subject matter at hand, meant specifically to gauge prior related knowledge
	What's In	This part connects previous lesson with that of the current one.
	What's New	An introduction of the new lesson through various activities, before it will be presented to you
	What is It	These are discussions of the activities as a way to deepen your discovery and under-standing of the concept.
	What's More	These are follow-up activities that are in-tended for you to practice further in order to master the competencies.
	What I Have Learned	Activities designed to process what you have learned from the lesson
	What I can do	These are tasks that are designed to show-case your skills and knowledge gained, and applied into real-life concerns and situations.



What I Know

Multiple Choice. Answer the question that follows. Choose the best answer from the given choices.

1. Which of the following statements show the properties of angular displacement and linear displacement?
   A. θ, for points on a rotating object depends on their distance from the axis.
   B. θ, for points on a rotating object does not depends on their distance from the axis.
   C. The displacement, for point on a rotating object, depends on their distance from the axis of rotation.
   D. The displacement, for point on a rotating object, does not depends on their distance from the axis of rotation
2. What is the linear speed of a child on a merry-go-round of radius 3.0 m that has an angular velocity of 4.0 rd/s?
   A.) 0.75 m/s   B.) 10 m/s   C.) 12 m/s   D.) 13.0 m/s
3. What is the angular velocity of an object traveling in a circle of radius 0.75m with a linear speed of 3.5 m/s?
   A.) 4.3 rd/s   B.) 4.8 rd/s   C.) 4.9 rd/s   D.) 4.7 rd/s
4. What is the angular acceleration of a ball that starts at rest and increases its angular velocity uniformly to 5 rd/s?
   A.) 8.0 rd/s^2 B.) 2 rd/s^2 C.) 0.5 rd/s^2 D. 3 rd/s^2
5. What is the angular velocity of a ball that starts at rest and rolls for 5 seconds with a constant angular acceleration of 20 rd/s^2?
   A.) 4 rd/s   B.) 10 rd/s   C.) 100 rd/s   D.) 7 rd/s
6. If no external torque acts on a body, its angular velocity remains conserved.
   A) True   B) False
7. The easiest way to open a heavy door is by applying the force
   A) Near the hinges   B) In the middle of the door
   C) At the edge of the door far from the hinges   D) At the top of the door
8. Does a bridge anchored resting on two pillars have any torque?
   A) No, it isn't moving   B) Yes, but it is at equilibrium
   C) Yes, but it will soon break because of the torque   D) No, Bridges can't have torque
9. When an object is experiencing a net torque
   A) it is in dynamic equilibrium.   B) it is in static equilibrium.
   C) it is rotating.   D) it is translating.
10. A rusty bolt is hard to get turned. What could be done to help get the bolt turned?
    A) use a long-arm lever   B) decrease the force
    C) apply the force at a 30 degree angle   D) use a short-arm lever

Lesson
1

ROTATIONAL KINEMATICS

What’s In

Rotational motion is all around us from molecules to galaxies. The earth rotates about its axis. Wheels, gears, propellers, motors, the drive shaft in a car, a CD in its player, a pirouetting ice skater, all rotate. Our study of rotation is between linear motion and rotational motion. In this lesson, we consider rotation about an axis that is fixed in space, or one that is moving parallel to itself as in a rolling ball.

What I Need to Know

Have you ever watched a Ferris wheel as it turns? How do you feel? Did you ever wonder how it moves? Will you still ride it if it doesn’t turn? This is why rotational motion is a very important motion. It is important to know how this motion affects the movement of a certain body.

As you go along this lesson, you will be able to:

- Define kinematic rotational variables such as angular position, angular velocity, and angular acceleration
- Derive rotational kinematic equations, and
- Solve for the angular position, angular velocity, and angular acceleration of a rotating body.

What's New

Activity 1.1 Am I Important?

List some types of rotating objects and how are they important to society. Write your answer in tabular form below.

Types of Rotating Objects	Importance to Society

What Is It

Rotational Kinematics

Kinematics is the description of motion. It is concerned with the description of motion without regard to force or mass. But what exactly is rotational kinematics? From the word, you can describe that it's all about any object that can rotate or spin. It's different from linear motion when object simply moves forward. The *kinematics of rotational motion* describes the relationships among rotation angle (θ), angular velocity (ω), angular acceleration (α) , and time (t). You will find that translational kinematic quantities, such as displacement, velocity, and acceleration have direct analogs in rotational motion.

Axis of Rotation

In activity 1.1, you have listed some types of rotating objects and their importance to society right? Everything that you have listed are all rotating about a line somewhere within the object called the axis of rotation. We are also going to assume that all these objects are rigid bodies, that is, they keep their shape and are not deformed in any way by their motion. Look at Figure 1 below. It shows the wheel and axle of a bike. Is the axle (axis of rotation) part of the wheel (rigid body)? The answer is NO. If you were to spin the wheel around its center, the axis of rotation (axle) would be pointing perpendicular to the motion of the wheel.

2

Figure 1.1 Wheel and Axle
Source: https://www.pinterest.ph/pin/764134261749726038/
https://bit.ly/3mmkceW

Angular Displacement

The symbol generally used for angular displacement is θ pronounced "teta" or "theta." θ is the angle swept by the radius of a circle that points to a rotating object. Look at the circle below and assume its rotating about its middle so the axis of rotation is pointing out of the page. Start with a piece of the circle at point A. as the circle rotates counterclockwise, the piece of the circle reaches point B. The point traveled a distance of s along the circumference, and swept out an angle θ. We can also say that the angle θ "substends' an arc length of s. Note that the points A and B are always at the same distance, r, from the axis of rotation.

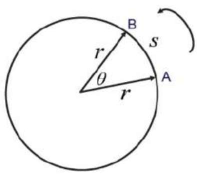


Figure 1.2 The Angle of Displacement
Source: file:///C:/Users/USER/Downloads/rotational_motion_-_day_1___2%20(3).pdf

We will now define the angle of rotation (θ) as the ratio of the arc length (s) to the radius (r) of the circle. We call this angle of rotation (θ) the ***angular displacement***. We denote angular displacement as Θ (theta). In symbol,

$$\theta = \frac{s}{r}$$

Where : θ is the angle of rotation,
S is the arc length, and
r is the radius.

Angular displacement is unitless since it is the ratio of two distances but, we will say that the angular displacement is measured in *radians*. We know degrees, and we know that when a point on a circle rotates and comes back to the same point, it has performed one revolution; let us say from point A, and rotate until we come back to point A.

Refer to Figure 2 again, what distance (s) was covered? How many degrees were swept by this full rotation? The point moved around the entire circumference, so it traveled 2πr while an angle of 360^0 was swept through. Using the angular displacement definition:

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \ radians$$

$$360^0 = 2\pi \ radians$$

$$1 \ radian = \frac{360^0}{2\pi} = 57.3^0$$

When an object makes one complete revolution, it sweeps out an angle of 360^0 or 2π radians. One radian is the angle at which the arc has the same length as the radius r.

1 radian = 57.3^0

The radian is frequently abbreviated as rad.

Sample Problems

1.) An object travels around a circle10.0 full turns in 2.5 seconds. Calculate (a) the
angular displacement, θ in radians.

Given:
# of turns/complete rotations = 10 turns
Time = 2.5 seconds

Find: Angular displacement (θ) in radians

*Note that 1 complete rotation = 360^0 = 2π radians = 6.28 rd

Solution:
Θ = 10.0 turns (6.28 rd / turn) = 62.8 radians.

2.) A girl goes around a circular track that has a diameter of 12 m. If she runs around the entire track for a distance of 100 m, what is her angular displacement?

Given:

Diameter of the curved path = 12m ;

*Note that diameter = 2r therefore,

r= d/2 so,

r= 12m/2= 6m

Linear displacement, s = 100 m.

Find: Angular displacement θ

Solution:

Θ = s/r → θ = 100m / 6 m = 16.67 radians

Angular displacement can now be related to linear displacement. Working on Kinematics problems with linear displacement was tackled in your previous lessons. What other quantities played a key role in linear displacement?

Angular Velocity

In linear motion, velocity (v) is defined as the rate of change of the object's position with respect to a frame of reference and time that is, $v = \Delta x/\Delta t$

while acceleration (a) is the rate of change of velocity. In symbol, we have:

$$a = \Delta v/\Delta t \quad ; \quad a = (v_2 - v_1)/\Delta t$$

In rotational motion, *angular velocity* (ω) is defined as the change in angular displacement (θ) per unit of time (t). In symbol,

$$\omega = \Delta\theta/\Delta t$$

The symbol ω is pronounced "omega" is used to denote angular velocity.

We usually describe the angular velocity as revolution per second (rev/sec, rps), or radian per second. See Figure 1.3. You will often have to convert this number, since it is usually given as a frequency (revolutions per time frame).

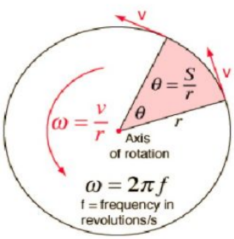


Figure 1.3 Angular Velocity

Source: http://230nsc1.phy-astr.gsu.edu/hbase/rotq.html

From linear velocity conversion, we have:

ω = v/r,

where:

ω is the angular velocity (rad/s),
v is the tangential velocity (m/s), and
r is the radius in circular path (meters).

Starting from angular velocity, let's substitute the linear displacement for the angular displacement we have:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta\left(\frac{s}{r}\right)}{\Delta t}$$
$$\omega = \frac{1}{r}\frac{\Delta s}{\Delta t} = \frac{1}{r}v$$
$$v = r\omega$$

As you can see, the tangential velocity (v) is directly proportional to the product of the angular velocity and the radius of the moving object. This confirms your feeling when riding a merry-go-round. Thus, the farther you are from the center, the faster you feel you are moving.

Sample Problems

1.) If an object travels around a circle with an angular displacement of 70.8 radians in 3.0 seconds, what is its average angular velocity ω in (rd/s)?

Answer

Given: Δθ = 70.8 rd ; Δt = 3 s
Find: ω = ?

Solution: ω = Δθ / Δt = 70.8 rd / 3.0 s = <u>23.6 rd/s.</u>

2.) A bicycle wheel with a radius of 0.28 m starts from rest and accelerates at a rate of 3.5 rad/s^2 for 8 s. What is its final angular velocity?

Answer

Given: r = 0.28 m; α = 3.5 rd/s^2 t = 8 s
Find: ω =?

Solution: From the equation α = ω / t , we can have

ω = αt = 3.5 rd/s^2 (8s) = <u>28 rd/s</u>

Angular Acceleration

If the angular velocity of the rotating object increases or decreases with time, we say that the object experiences an angular acceleration, α. The angular acceleration of a rotating object is the rate at which the angular velocity changes with respect to time. It is the change in the angular velocity, divided by the change in time. The average angular acceleration is the change in the angular velocity, divided by the change in time. The angular acceleration is a vector that points in a direction along the rotation axis. The magnitude of the angular acceleration is given by the formula below. The unit of angular acceleration is radians/s^2.

$$angular\ acceleration = \frac{change\ in\ angular\ velocity}{}$$

$$= \frac{(final\ angular\ velocity) - (initial\ angular\ velocity)}{(final\ time) - (initial\ time)}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

In symbol,

Where:

α = angular acceleration, (radians/s2)
Δω = change in angular velocity (radians/s)
Δt = change in time (s)
ω_1 = initial angular velocity (radians/s)
ω_2= final angular velocity (radians/s)
t_1 = initial time (s)
t_2= final time (s).

The symbol α is pronounced "Alpha". The unit of measure is radian per second squared (rd/s^2).

All points in the object have the same angular acceleration. Every point on a rotating has, at any instant a linear velocity (v) and a linear acceleration (a). Look at the illustration in Figure 1.4 below, we can relate the linear quantities (v and a) to the angular quantities (ω and α). Linear velocity and angular velocity are related since

v = rω

Where; v is the linear velocity,
r is the radius of the object, and
ω is the angular acceleration.

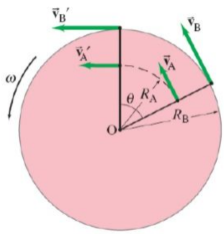


Figure 1.4 Relation of Linear Velocity and Angular Velocity:
Points farther than the axis of rotation will move faster (linear velocity) but the angular velocity for all points is the same.
Source: https://www.slideshare.net/ibidorigin/12-rotational-motion

It is not only the point (we measure) move in that angular velocity. All points in the object rotate with the same angular velocity. Every position in the object move through the same time interval. Conventionally, object moving counterclockwise has a value of positive (+) angular acceleration, while the one moving the clockwise direction has negative (-) value.

Angular acceleration occurs when the angular velocity changes over time. It acts in the direction of rotation in a circular motion (not the same as centripetal acceleration). In this case, we must also introduce tangential acceleration a_t since the tangential velocity is changing. If there is angular acceleration, there will also be tangential acceleration

$$a_{tan} = r\alpha \qquad \text{and} \qquad a_{rad} = \omega^2 r$$

where:

a_t is the tangential (linear) acceleration (m/s)
r is the radius of circular path (meters)
α is the angular acceleration;
ar is the radial (linear) acceleration (m/s);and
ω is the angular acceleration (rd/s²).

Sample Problems

1. A disc in a DVD player starts from rest, and when the user presses "Play", it begins spinning..The disc is spins at 160 radians/s after 4.0 s. What was the average angular acceleration of the disc?

Answer:

Given:

$T_1 = 0$ $T_2= 4.0$ s
$\omega_1 = 0$ $\omega_2 = 160$ rd/s

Find:

Angular acceleration (α) =?

Solution:

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{160\ \text{radians}/s}{4.00\ s} = \frac{160\ \text{radians}/s - 0\ \text{radians}/s}{4.00\ s - 0\ s}$$

$$\alpha = 40.0\ \text{radians}/s^2$$

Between the initial and final times, the average angular acceleration of the disc was 40.0 radians/s².

2.) A car tire is turning at a rate of 5.0 rd / sec as it travels along the road. The driver increases the car's speed, and as a result, each tire's angular speed increases to 8.0 rd /sec in 6.0 sec. Find the angular acceleration of the tire.

Answer

Given: $\omega_1 = 5.0$ rd/s; $\omega_2 = 8.o$ rd/s; $\Delta t = 6.o$ s

Find: α=?

Solution:

α = Δω / Δt = $(\omega_2 - \omega_1)/\Delta t$ = (8.0 rd/s - 5.0 rd/s) / 6.0s = 0.50 rd/s².

2. As a car starts accelerating (from rest) along a straight road at a rate of 2.4 m/s², each of its tires gains an angular acceleration of 6.86 rd/s². Calculate (a) the radius of its tires, (b) the angular speed of **every** particle of the tires after 3.0s, and (c) the angle **every** particle of its tires travels during the 3.0-second period.

Answer

Given: $\omega_1 = 0$ (from rest) $a_t = 2.4\ m/s^2$ $\alpha = 6.86\ rd/s^2$ $\Delta t = 3.0\ s$

Find: a) r b) ω_2 c) θ

Solution:

a.) From the equation

$a_t = r\alpha$, we get $r = a_t/\alpha = [2.4\ m/s^2] / [6.86\ rd/s^2] = 0.35m$

(b) From the equation

$\alpha = (\omega_2 - \omega_{i1})/\Delta t$, we get

$\alpha\ \Delta t = \omega_2 - \omega_1$

$\omega_2 = \omega_1 + \alpha\ \Delta t$

$\omega_2 = 0 + (6.86 rd/s^2)(3.0s) = 21\ rd/s.$

(c) $\theta = (1/2)\alpha\ t^2 + \omega_i\ t = (1/2)(6.86\ rd/s^2)(3.0s)^2 + (0)(3.0s) = 31\ rd.$

Putting these definitions together, you observe a very strong parallel between translational kinematic quantities and rotational kinematic quantities See Table 1.1 below.

Table 1.1a Translational Kinematics

Variable	Translational	Angular
Displacement	Δs	Δθ
Velocity	v	ω
Acceleration	a	α
Time	t	t

Symbols for ang Angular Quantities

Source:https://www.aplusphysics.com/courses/honors/rotation/honors_rot_kinematics.html

It's quite straightforward to translate between translational and angular variables as well when you know the radius (r) of the point of interest on a rotating object.

Variable	Translational	Angular
Displacement	$s = r\theta$	$\theta = \frac{s}{r}$
Velocity	$v = r\omega$	$\omega = \frac{v}{r}$
Acceleration	$a = r\alpha$	$\alpha = \frac{a}{r}$
Time	t	t

Table 1.2 Translational ang Angular Kinematics Quantities (if radius is given)

Source:https://www.aplusphysics.com/courses/honors/rotation/honors_rot_kinematics.html

The rotational kinematic equations (See table below) can be used the same way you used the translational kinematic equations to solve problems. Once you know three of the kinematic variables, you can always use the equations to solve for the other two.

Translational	Rotational

Table 1.3 Translational and Rotational Kinematics Equations
Source:https://www.aplusphysics.com/courses/honors/rotation/honors_rot_kinematics.html

The above Kinematics equations allowed you to explore the relationship between displacement, velocity, and acceleration. You can develop a corresponding set of relationships for angular displacement, angular velocity, and angular acceleration. The equations follow the same form as the translational equations, all you have to do is replace the translational variables with rotational variables.

10

What's More

Activity 1.2 Matched Me Right

h column A with column B according to their meaning. Write the letter of your answer on the space provided before each number.

Column A
(Meaning/Definition)

___ 1. A measure of how angular velocity changes over time.
___ 2. The imaginary or actual axis around which an object may rotate.
___ 3.It is the change in linear velocity divided by time.
___ 4. It is half of the circle's circumference
___ 5. The orientation of a body or figure with respect to a specified reference position as expressed by the amount of rotation necessary to change from one orientation to the other about a specified axis.

___ 6. The rate of rotation around an axis usually expressed in radian or revolutions per second or per minute.
___ 7. A property of matter by which it remains at rest or in uniform motion in the same straight line unless acted upon by some external force.
___ 8. Branch of dynamics that deals with aspects of motion apart from considerations of mass and force.
___ 9. It is the rate of change of the position of an object that is traveling along a straight path.
___ 10. It is an angle whose corresponding arc in a circle is equal

Column B
(Term/s)

A. **Angular position**
B. **Linear velocity**
C. **Axis of rotation**
D. **Tangential Acceleration**
E. **Angular velocity**
F. **Kinematics**
G. **Angular acceleration**
H. Radian
I. Angular displacement
J. **Radius**

What I Have Learned

Activity 1.3 Think Critically

Solve the following in a clean sheet of paper. Show your solution and box your final answer.

1.) Mark bought a pizza of a radius of 0.5 m. A fly lands on the pizza and walks around the edge for a distance of 80 cm. Calculate the angular displacement of the fly?

2.) What is the angular velocity of an object traveling in a circle of radius 0.75 m with a linear speed of 3.5 m/s?

3.) What is the angular acceleration of a ball that starts at rest and increases its angular velocity uniformly to 5 rad/s in 10 s?

What I Can Do

Activity 1. 4. Correct Me If I'm Wrong

Explain briefly. Write your answer in a separate paper.

1.) Angular acceleration does not change with radius, but tangential acceleration does.

2.) Differentiate angular acceleration from tangential (or linear) acceleration.

3.) On a rotating carousel or merry-go-round, one child sits on a horse near the outer edge and another child sits on a lion halfway out from the center. Which child has the greater linear velocity? Which child has the greater angular velocity?

Lesson 2

ROTATIONAL DYNAMICS

What I Need to Know

In the previous lesson, you learned about the analogy of translational and rotational motion. With this you were able to derived the basic variables necessary in understanding this type of motion. Also, you learned the kinematics of rotating body without taking into account the factors that causes its motion.

In this lesson you will understand rotational motion further through its dynamics; that is how Torque, the force applied, causes a body to rotate. Also, in this lesson, you will learn the conditions of Static Equilibrium; the Work done by the torque and the Angular Momentum and their analogy to Newton’s Laws of Motion. This lesson will help you explore and understand how simple events encountered and observed daily works. Specifically, you are expected to learn the following:

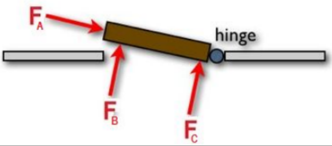
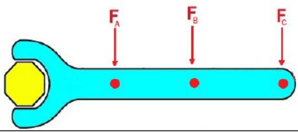
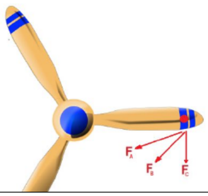
1. Determine whether a system is in static equilibrium or not
2. Determine the conditions of a system under equilibrium and solve static equilibrium problems
3. Define torque and learn how force should be applied in a body to attain maximum torque
4. Determine angular momentum of different systems
5. Apply the torque-angular momentum relation in solving problems

What's New

Investigating Torque! 13

Torque is a physical quantity closely related to rotation. It is the Force's ability to cause an object to rotate.

Perform the following tasks to have a preliminary observation on the relationship between torque to the distance of application and angle of rotation. To do this, apply an estimated constant force to the labelled points to rotate the object and rank the ease of rotation from easiest to hardest.

Situation	Ease of Rotation (Rank the Forces from easiest to hardest)		
	1st	2nd	3rd
A. Opening a Door 			
B. Removing a Bolt using a Wrench 			
C. Rotating A Blade 			

From the results obtained and observed, deduce the relationship of the following:

A. Torque vs the Distance of Application

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B. Torque vs the Angle of Application

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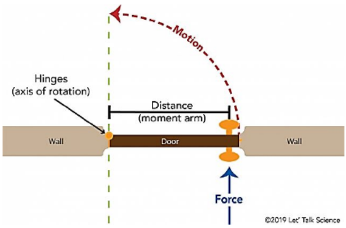
What Is It

A. TORQUE (τ)

Have you ever wondered why doorknobs are situated at the opposite end of the hinges and not near it? And why is it easier to use long-handled wrenches than the short-handled one in removing bolts? How about doing an arm-wrestling with a longer-arm person? What do you think would be your chances of winning?

This lesson will enlighten you on the simple physics behind these things. With the understanding of Torque, you will be able to answer these questions.

To understand this, let us imagine a door. The door has hinges on one side. To successfully open the door, you need to apply a force. This force will cause the door to rotate about its hinges, or its axis of rotation. This rotation creates Torque.



Torque, also called the *Moment of Force,* is the result of the force that can cause an object to rotate about an axis. It is a vector quantity. It is the cross product of the vector Force and the distance from the axis of rotation.

Mathematically, $\vec{\tau} = \vec{r} \; x \; F$

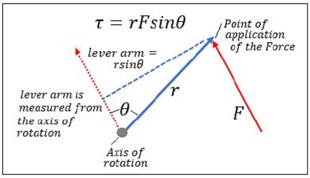
whose magnitude is equal to $\tau = r_{\perp} F$

$\tau = rF \sin\theta$

where $r_{\perp} = r\sin\theta$

And θ is the angle between r and F

S.I. Unit: Nm



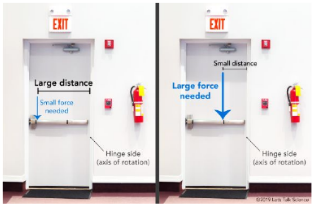
The direction of the torque may either be counterclockwise (CCW) or clockwise (CW). By convention, we take the counterclockwise direction to be positive and clockwise as negative.

From the equation, we see that the effect of the Force on the motion of the rotating body depends on three factors as follows:

1. Magnitude of the Force
2. Lever Arm (Moment Arm) – perpendicular distance of the line of action to the axis of rotation

3. The angle between the Force vector and the lever arm

The torque increases as the force increases, and also as the distance increases. That is why doorknobs are located at the opposite end of the hinges. It is easier to open the door in this case since small force is needed to cause torque to the door.



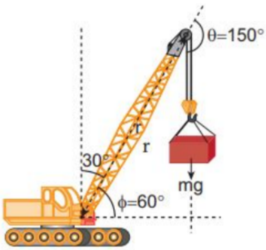
Sample Problems:

1. A Force of $\left(4\hat{i} - 3\hat{j} + 5\hat{k}\right)N$ is applied at a point whose position vector is $\left(7\hat{i} + 4\hat{j} - 2\hat{k}\right)m$. Find the Torque of force about the origin.

Solution:

$$F = \left(4\hat{i} - 3\hat{j} + 5\hat{k}\right)N$$
$$\vec{r} = \left(7\hat{i} + 4\hat{j} - 2\hat{k}\right)m$$
$$\vec{\tau} = \vec{r}\ x\ F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 4 & -2 \\ 4 & -3 & 5 \end{vmatrix} = (20-6)\hat{i} - (35+8)\hat{j} + (-21-16)\hat{k} = \left(14\hat{i} - 43\hat{j} - 37\hat{k}\right)Nm$$

2. A crane has an arm length of 20 m inclined at 30º with the vertical. It carries a container of mass of 2 ton suspended from the top end of the arm. Find the torque produced by the gravitational force on the container about the point where the arm is fixed to the crane. [Given: 1 ton = 1000 kg; neglect the weight of the arm. g = 9.8 m/s^2]

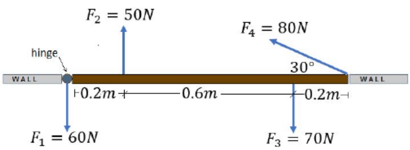


Solution:

$\tau = rF\sin\theta$ where $F = weight = mg$
$= r(mg)\sin\theta$
$= (20m)(2000kg)(9.8m/s^2)(\sin 30)$
$\tau = 1.96x10^5 Nm$

Note: θ is the angle between r and F

3. Consider the door shown in the figure, which is seen from an aerial view. The circle on the left is the hinge (pivot point).
   a. Find the Net Torque acting on the door.
   b. Which way will the door open, up or down?



Solution:

a. $\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$

$$\tau_1 = rF_1\sin\theta = (0)(60N)\sin 90 = 0$$

$$\tau_2 = rF_2 sin\theta = (0.20m)(50N)\,sin90 = 10Nm$$
$$\tau_3 = rF_3 sin\theta = (0.2m + 0.6m)(70N)\,sin90 = 56Nm$$
$$\tau_4 = rF_4 sin\theta = (0.2m + 0.6m + 0.2m)(80N)\,sin30 = 40Nm$$

Before adding the torque, determine their corresponding direction according to the rotation of the door.

τ_1 has no rotation since the torque is zero
τ_2 and τ_4 : pulling the door upward would make it rotate in the CCW direction (+)
τ_3 : pulling the door downward would make it rotate in the CW direction (-)

$$\tau_{net} = 0 + 10Nm + (-56Nm) + 40Nm = -6Nm$$

b. Since the result of the net torque is negative, this means that the door will rotate in the clockwise or downward direction.

B. STATIC EQUILIBRIUM

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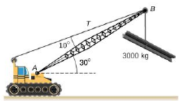
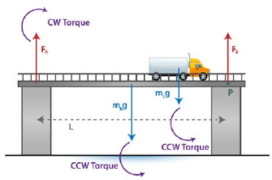
Static equilibrium occurs when an object is at rest – neither rotating nor translating. It is analogous to Newton's 1st Law of motion for rotational system. An object which is not rotating remains not rotating unless acted on by an external torque. Similarly, an object rotating at constant angular velocity remains rotating unless acted on by an external torque.

For an object to maintain in static equilibrium, the following conditions must be met:

1. The net force acting on the object must be zero: $\sum F = 0$
2. The net torque acting on the object must be zero: $\sum \tau = 0$



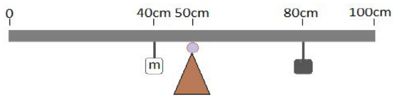


Applications of Static Equilibrium is constantly seen and observed around us. A common example of balanced torques is two children on a see-saw. If the fulcrum is in the center of the see-saw, the two children must have equal mass for it to be balanced. If the fulcrum is not in the center, their masses must vary to create equal torques.

This topic will also help you understand important applications in the field of engineering such as building bridges, the Physics behind crane towers and many more.

Sample Problems:

1. A 0.15kg meterstick is supported at the 50cm mark. A mass of 0.5kg is attached at the 80cm mark.
   a. How much mass should be attached to the 40cm mark to keep the meterstick horizontal?



b. Determine the supporting force from the fulcrum on the meterstick.

Solution:

a. From the 2nd condition of Equilibrium:

$$\sum \tau = 0 \quad \rightarrow \quad \tau_1 + \tau_2 = 0$$

Where τ_1 is the torque caused by the force exerted by mass m

τ_2 is the torque caused by the force exerted by the 0.5kg mass

Hanging mass m would cause the stick to rotate in the CCW direction, thus τ_1 is (+)

Hanging the 0.5kg-mass would cause the stick to rotate in the CW direction, thus τ_2 is (-)

where $F = weight = mg$

$$\left[(0.10m)(m)(9.8m/s^2)sin90\right] - \left[(0.30m)\,(0.5kg)(9.8m/s^2)sin90\right] = 0$$

$$(0.98m^2/s^2)m - 1.47Nm = 0 \qquad \rightarrow \qquad m = 1.47Nm/0.98m^2/s^2) = 1.5kg$$

$$m = 1.5kg$$

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b. From the 1st condition of Equilibrium:

$$\sum F = 0 = \vec{F_1} + \vec{F_2} + \vec{F_m} + \vec{F_f} = 0$$

Where $\vec{F_1} = w = m_1 g$ is the downward force due to mass $m_1 = 1.5kg$

$\vec{F_2} = w = m_2 g$ is the downward force due to mass $m_2 = 0.5kg$

$\vec{F_m} = w = mg$ is the downward force due to the mass of the meterstick

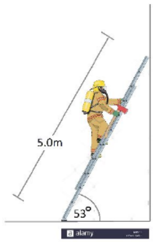
$\vec{F_f} =$ is the upward force exerted by the fulcrum to support the weight of the meterstick and masses

$$-\vec{F_1} - \vec{F_2} - \vec{F_m} + \vec{F_f} = 0 \quad \rightarrow \vec{F_f} = \vec{F_1} + \vec{F_2} + \vec{F_m} = m_1 g + m_2 g + mg$$

$$\vec{F_f} = (1.5kg)(9.8m/s^2) + (0.5kg)(9.8m/s^2) + (0.15kg)(9.8m/s^2)$$

$$\vec{F_f} = 21.07N$$

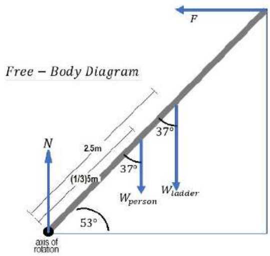
2. A firefighter who weighs 800N climbs a uniform ladder and stops one-third of the way up the ladder. The ladder is 5m long and weighs 180N. It rests againsts a vertical wall making an angle 53° with the ground. Find the normal and the frictional forces on the ladder at its base.



Solution:

We first contruct the Free-Body Diagram to identity the forces present.

Since the Normal Force is located at the y-axis, we get the net force along this axis.



$$\sum F_y = 0$$

$$N - W_p - W_l = 0$$

$$N = W_p + W_l = 800N + 180N$$

$$N = 980N$$

To solve the friction Force, we use get the net force along the x-axis.

$$\sum F_x = 0$$

$$F - f = 0 \quad \rightarrow \quad F = f$$

To solve for F, we use the 2nd condition of equilibrium. $\sum \tau = 0$

$$\tau_N - \tau_{Wp} - \tau_{Wl} + \tau_F = 0$$

$$r_N N \sin\theta - r_{wp} W_p \sin\theta - r_{wl} W_l \sin\theta + r_f F \sin\theta = 0$$

$$F = \frac{-r_N N \sin\theta + r_{wp} W_p \sin\theta + r_{wl} W_l \sin\theta}{r_f \sin\theta}$$

$$F = \frac{(0)(980N \sin\theta) + (\frac{1}{3})(5m)(800N)(\sin 37°) + (2.5m)(180N)(\sin 37°)}{(5m)\sin 53°}$$

$$F = \frac{0 + 802.42Nm + 270.82Nm}{3.99m}$$

$$F = 268.98N = f$$

C. WORK DONE BY A TORQUE

We have seen how Newton's Laws of motion is similar to rotational motion. Newton's Laws may be stated in terms of rotational motion.

1st Law of Rotational Motion:

A body in motion at constant angular velocity will continue in motion at the same angular velocity, unless acted upon by some unbalanced external torque.

2nd Law of Rotational Motion:

When an unbalanced external torque acts on a body with moment of inertia, I, it gives that body an angular acceleration α, which is directly proportional to the torque and inversely proportional to the moment of Inertia.

3rd Law of Rotational Motion:

If body A and body B have the same axis of rotation, and if body A exerts a torque on body B, then body B exerts an equal but opposite torque on body A.

We can derive the equation of Torque in terms of the angular acceleration , α ,from Newton's 2nd Law of Motion:

$\vec{F} = m\vec{a}$ multiplying both sides with r

$r\vec{F} = rm\vec{a}$ where $r\vec{F} = \tau$ and $a = r\alpha$

$\tau = (rm)(r\alpha)$

$\tau = mr^2\alpha$; $I = mr^2$

$\tau = I\alpha$

Rotational Work

To calcula the work done by the torque, we derive it from the translational equation of Work.

$W = Fd$ where $d = s = r\theta$ $(rotational\ motion)$

$W = Fr\theta$; $Fr = \tau$

$W = \tau\theta$

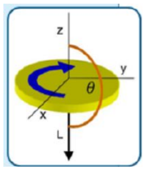
Rotational Kinetic Energy

$$KE = \frac{1}{2}mv^2 \qquad v = r\omega \quad (rotational\ motion)$$
$$KE = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2v^2 \qquad ; \quad mr^2 = I$$
$$KE = \frac{1}{2}I\omega^2$$

For vehicles such as cars and bicycles, the tires exert rotational and translational kinetic energy. Thus, the total kinetic energy is equal to:
$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Angular Momentum

Angular momentum is a quantity that tells us how hard it is to change the rotational motion of a particular spinning body. For a single particle with known momentum. The angular momentum can be calculated using the relationship:



$$L = r\ x\ p$$

Where L is the angular momentum of the object;
r is the distance of the particle from the point of rotation and
p is the linear momentum

$L = r\ x\ (mv) \quad where \quad v = r\omega$
$L = r\ x\ (m)(r\omega) = mr^2\omega \quad where \quad mr^2 = I$

Therefore: $L = I\omega$

The higher the angular momentum of the object, the harder it is to stop. Objects with higher angular momentum have greater orientational stability. That is why in riding a bicycle, if you are going faster, you will not fall ober easily as when you are going slower.

Conservation of Momentum:

"The momentum of a system will not change unless an external torque is applied."

$L_f = L_i \quad (Final\ momentum = Initial\ momentum)$

Kinematic Equations for Linear and Rotational Motion

Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

http://hyperphysics.phy-astr.gsu.edu/hbase/mi.html

Sample Problems:

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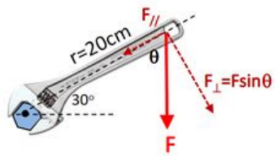
1. Janelle uses a 20cm long wrench to tighten a nut. The wrench handle is tilted 30º above the horizontal and Janelle pulls straight down on the end with a force of 100N. How much torque does Janelle exert on the nut?



Solution:

$$\tau = rF_{\perp} = rF\sin\theta$$
$$\tau = (0.20m)(100N)(\sin 60°) = 17.3Nm$$

2. A flywheel of mass 182kg has a radius of 0.62m (assume the flywheel is a hoop).
   a. What is the torque required to bring the flywheel from rest to a speed of 120rpm in an interval of 30 sec?
   b. How much work is done in this 30-sec period?

Solution

a. $\tau = rF = r(ma) = rm(r\alpha) \quad where\ \alpha = (\Delta\omega/\Delta t)$

$$\tau = mr^2\left(\frac{\Delta\omega}{\Delta t}\right) = mr^2\left(\frac{\omega_f - \omega_i}{\Delta t}\right) = mr^2\left(\frac{\omega_f}{\Delta t}\right) \quad where\ \ \omega_i = 0 \quad (from\ rest)$$
$$\tau = (182kg)(0.62m)\left[\frac{12.57\frac{rad}{s}}{30s}\right] = 29.31Nm$$

Rememeber to be consistent with the units.

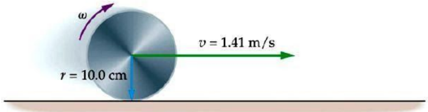
Conversion of the angular velocity ω in rad/sec:

$$\omega_f = \left(120\frac{rev}{min}\right)\left(\frac{2\pi rad}{1\ rev}\right)\left(\frac{1min}{60sec}\right) = 12.57\ rad/sec$$

b. $W = \tau\theta \quad where \quad \theta = \omega_{ave}\Delta t$

$$W = \tau\left(\frac{\omega_i + \omega_f}{2}\right)\Delta t = (29.31Nm)\left(\frac{12.57rad/s)}{2}\right)(30s) = 5,526.4J$$

3. A 1.20kg disk with a radius of 10.0 cm rolls without slipping. The linear speed of the disk is 1.41m/s.
   a. Find the translational KE.
   b. Find the rotational KE.
   c. Find the total kinetic energy.



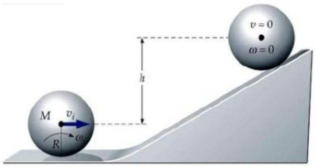
Solution:

a. $K_{trans} = \frac{1}{2}mv^2 = \frac{1}{2}(1.20kg)(1.41m/s)^2 = 1.19J$

b. $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mr^2)(\frac{v}{r})^2 = \frac{1}{4}mv^2 = \frac{1}{4}(1.20kg)(1.41m/s)^2 = 0.596J$

c. $K_{tot} = K_{trans} + K_{rot} = 1.19J + 0.596J = 1.79J$

4. A bowling ball that has an 11cm radius and a 7.2kg mass is rolling without slipping at 2.0m/s on a horizontal ball return. It continues to roll without slipping up a hill to a height *h* before momentarily coming to rest and then rolling back down the hill. Model the bowling ball as uniform sphere and calculate *h.*



Solution:

Since the problem involves the presence of kinetic K, and potential energy U, we use the conservation of mechanical energy to calculate *h*.

$$\Delta E = 0 \quad \rightarrow \quad E_f - E_i = 0 \quad \rightarrow \quad E_f = E_i$$

where $E_i = K_i + U_i$ and $E_f = K_f + U_f$

$$K_f + U_f = K_i + U_i$$

$$mgh_f + \frac{1}{2}mv_f^2 = mgh_i + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega^2 \quad ; \quad v_f = 0 \quad and \quad h_i = 0$$

$$mgh_f + 0 = 0 + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega^2 \quad ; \quad I_{sphere} = \frac{2}{5}mr^2 \quad and\ \omega = v/r$$

$$h_f = \frac{\frac{1}{2}mv_i^2 + \frac{1}{2}(\frac{2}{5}mr^2)(\frac{v}{r})^2}{mg} = \frac{\frac{1}{2}mv_i^2 + \frac{1}{5}mv^2}{mg}$$

$$h_f = \frac{\frac{1}{2}(7.2kg)(\frac{2.0m}{s})^2 + \frac{1}{5}(7.2kg)(\frac{2.0m}{s})^2}{(7.2kg)(\frac{9.8m}{s^2})} = 0.29m$$

5. An ice skater with a moment of inertia of 1.2kg.m^2 initially spins at a rate of 1 revolution every 0.8seconds, when her arms and one leg are extended outward.
   a. Find her angular speed
   b. Find her angular momentum.
   c. When she pulls her arms and legs inward, her moment of inertia changes to 0.9kg.m^2. Find her angular speed.

Solution:

a. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8sec} = 7.85 rad/sec$

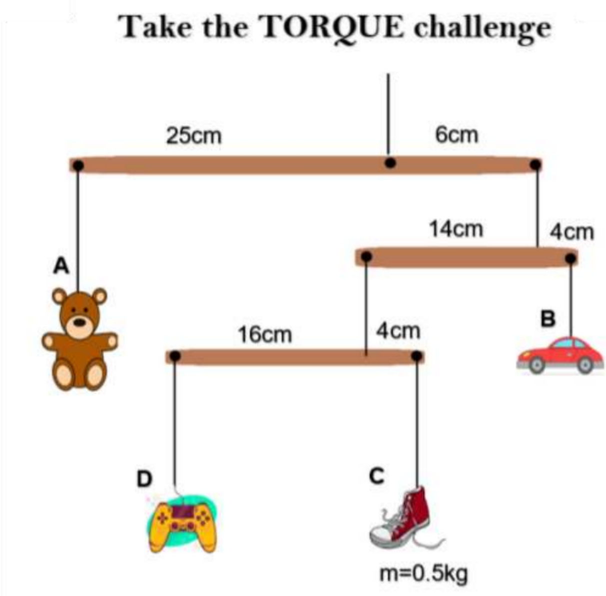
b. $L = I\omega = (1.2kgm^2)\left(\frac{7.85rad}{sec}\right) = 9.42kg.m^2/sec$

c. $L_f = L_i \quad (Conservation\ of\ Angular\ Momentum)$

$I_f\omega_f = I_i\omega_i \quad \rightarrow \omega_f = \frac{I_i\omega_i}{I_f} = \frac{9.42kg.m^2/sec}{0.9kgm^2} = 10.47 rad/sec$

What's More

Direction.Copy the figure in a separate paper and calculate the mass of each item. Show your solutions.

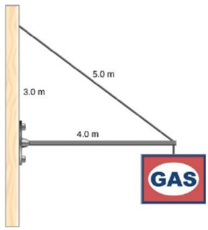


What is the mass of each item?

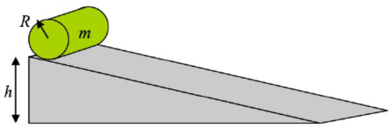
What I Have Learned

Direction. Solve the following problems in a separate paper. Show your solutions systematically and clearly.

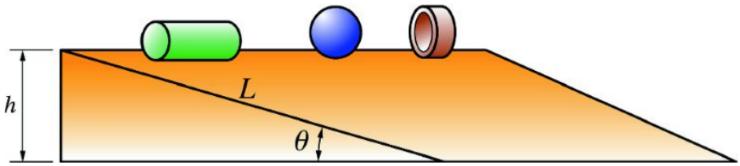
1. A 400.0-N sign hangs from the end of a uniform strut. The strut is 4.0 m long and weighs 600.0 N. The strut is supported by a hinge at the wall and by a cable whose other end is tied to the wall at a point 3.0 m above the left end of the strut. Find the tension in the supporting cable and the force of the hinge on the strut.



2. A cylinder of mass m and radius R has a moment of inertia of $\frac{1}{2}mr^2$. The cylinder is released from rest at a height h on an inclined plane, and rolls down the plane without slipping. What is the velocity of the cylinder when it reaches the bottom of the incline?



3. A uniform solid cylinder, sphere, and hoop roll without slipping from rest at the top of an incline. Find out which object would reach the bottom first.



What I Can Do

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It's undersTORQUEable!

From the lessons learned, list down 3 sports/events that utilizes the concept of Torque and briefly explain how this concept is used.

Sports/Events	Torque Concept
1.	
2.	
3.	

Summary

ROTATIONAL KINEMATICS

Angular Displacement θ is the ratio of the arc length (s) to the radius (r) of the circle. Mathematically, $\theta = s/r$

Angular velocity (ω) is defined as the change in angular displacement (θ) per unit of time (t). In symbol, $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$

The angular acceleration of a rotating object is the rate at which the angular velocity changes with respect to time and is given by the equation $\alpha = \Delta\omega/\Delta t$.

If there is angular acceleration, there will also be tangential acceleration

$a_{tan} = r\alpha$ and $a_{rad} = \omega^2 r$

Analogy between Rotational and Translational Kinematics

Variable	Translational	Angular
Displacement	Δs	$\Delta\theta$
Velocity	v	ω
Acceleration	a	α
Time	t	t

Variable	Translational	Angular
Displacement	$s = r\theta$	$\theta = \frac{s}{r}$
Velocity	$v = r\omega$	$\omega = \frac{v}{r}$
Acceleration	$a = r\alpha$	$\alpha = \frac{a}{r}$
Time	t	t

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

ROTATIONAL DYNAMICS

Torque, also called the *Moment of Force,* is the result of the force that can cause an object to rotate about an axis. It is a vector quantity. It is the cross product of the vector Force and the distance from the axis of rotation. Mathematically, $\tau = rF\sin\theta$

The Torque is dependent on the following factors:

1. Magnitude of the Force
2. Lever Arm (Moment Arm) – perpendicular distance of the line of action to the axis of rotation
3. The angle between the Force vector and the lever arm

Static equilibrium occurs when an object is at rest – neither rotating nor translating.

For an object to maintain in static equilibrium, the following conditions must be met:

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1. The net force acting on the object must be zero: $\sum \vec{F} = 0$
2. The net torque acting on the object must be zero: $\sum \tau = 0$

To calcula the work done by the torque, we derive it from the translational equation of Work and is equal to $W = \tau\theta$

The Rotational Energy is given by: $KE = \frac{1}{2}I\omega^2$

For motion involving rotarional and translational kinetic energy, the total energy is equal to the sum of the two energies. Mathematically, $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Angular momentum is a quantity that tells us how hard it is to change the rotational motion of a particular spinning body. $L = I\omega$ where *I* is the moment of inertia of the object.

Conservation of Momentum:

"The momentum of a system will not change unless an external torque is applied."

$L_f = L_i$ (Final momentum = Initial momentum)

Kinematic Equations for Linear and Rotational Motion

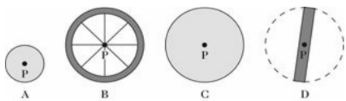
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0t + \frac{1}{2}at^2$	$\theta = \omega_0t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

http://hyperphysics.phy-astr.gsu.edu/hbase/mi.html

Assessment: (Post-Test)

Multiple Choice. Answer the question that follows. Choose the best answer from the given choices.

1. Why are you more stable when riding a bicycle at a faster speed?
   A) You have more mass
   B) The wheels have angular momentum
   C) It's not easier to ride at a faster speed
   D) The bike has more momentum
2. What does the rotational inertia describe?
   A) The average position of mass in an extended object.
   B) How the mass of an object is distributed
   C) How a force can rotate an object.
   D) The tendency of an object to move in a straight line.
3. 2600 rev/min is equivalent to which of the following?
   A) 2600 rad/s B) 43.3 rad/s C) 273 rad/s D) 60 rad/s
4. A 0.12-m-radius grinding wheel takes 5.5 s to speed up from 2.0 rad/s to 11.0 rad/s. What is the wheel's average angular acceleration?
   A) 9.6 rad/s/s B) 4.8 rad/s/s C) 3.1 rad/s/s D) 1.6 rad/s/s
5. Suppose you are rotating in a chair with 2 equal masses held in each outstretched hand and you drop them. What happens to your angular velocity?
   A) Increase B) Decrease C) Stays the same D) Is lost
6. When seen from below, the blades of a ceiling fan are seen to be revolving anticlockwise and their speed is decreasing. Select correct statement about the directions of its angular velocity and angular acceleration.
   A) Angular velocity upwards, angular acceleration downwards
   B) Angular velocity downwards, angular acceleration upwards
   C) Both angular velocity and angular acceleration upwards
   D) Both angular velocity and angular acceleration downwards
7. You exert a force on a friend who is holding a 4.0-m-long rope. Now suppose you exert the same force on your friend, but the friend is holding an 8.0-m-long rope. How will this affect the rotational acceleration?
   A) It will be quartered B) It will be halved C) It will double D) It will quadruple
8. The figure shows scale drawings of four objects, each of the same mass and uniform thickness, with the mass distributed uniformly. Which one has the greatest moment of inertia when rotated about an axis perpendicular to the plane of the drawing at point P?



   A B C D

9. A person sits on a freely spinning lab stool that has no friction in its axle. When this person extends her arms,
   A) her moment of inertia increases and her angular speed decreases.
   B) her moment of inertia decreases and her angular speed increases.
   C) her moment of inertia increases and her angular speed increases.
   D) her moment of inertia increases and her angular speed remains the same.
10. What is the rotational kinetic energy of the cylinder at t = 2 s?
    A)2.0J B)2.5J C)5.0J D) It cannot be determined without knowing the radius
11. A force of magnitude, F is applied to a doorknob and a second force of magnitude, 2F is applied to the same door at the middle of the door. Both forces are perpendicular to the door plane. Which of the following is the correct ratio between the torque of the first force and the torque of the second force? A)1:2 B)1:1 C)2:1 D)4:1
12. A 4.0-kg block travels around a 0.50-m radius circle with an angular velocity of 12 rad/s. Its angular momentum about the center of the circle is:
    A) 12 kg.m2 /s B) 24 kg.m2 /s C) 48 kg.m2 /s D) 6 kg.m2 /s
13. A solid sphere rolling without slipping down an incline has:
    A) rotational kinetic energy. B) constant angular momentum.
    C) zero external torques. D) translational kinetic energy.
14. A seesaw with mass x is perfectly balanced with a fulcrum in the center. If mass x changes uniformly does the net torque change?
    A) Yes, because the Gravitational force increases with more mass, and torque is related to Force
    B) No, because the net torque is still zero despite increases in one of the individual components.
    C) No, because it is balanced that means net torque is zero, a uniform increase in weight keeps it balanced keeping net torque at zero
    D) Yes, any increase in weight will increase torque
15. You apply force to a wrench to tighten a bolt. The wrench breaks in half. Select the best answer for what happens next. Assume the force applied remains constant.
    A) The torque is now half of what it was, because torque and distance are inversely proportional
    B) The torque is now half of what it was, because torque and distance are directly proportional
    C) The torque is now double of what it was, because torque and distance are inversely proportional
    D) The torque is now double of what it was, because torque and distance are directly proportional
16. A construction worker is worried about applying too much torque to a bolt using his wrench. What could he do to reduce his torque on the bolt?
    A) Increase the length of the wrench B) Increase the force on wrench
    C) Reduce the length of the wrench D) Reduce the friction between the bolt and wrench
17. Which has greater linear speed, a horse near the outside rail of a merry-go-round or a horse near the inside rail? A) the inside horse B) the outside horse C) Neither
18. Which force produces the greatest torque?
    A) 20N with a lever arm of 1m B) 15N with a lever arm of 3m
    C) 10 N with a lever arm of 4m D) 8N with a lever arm of 5m
19. A meter stick is balanced. If a 0.5 kg mass is fastened 0.3 m to the left of the fulcrum and a 0.6 kg mass is fastened 0.25 m to the right of the fulcrum, the meterstick will...
    A) rotate counterclockwise (left) B) rotate clockwise (right)
    C) remain in balance D) move up and down
20. Which of these objects is in static equilibrium?

Key to Answers

Pre-Test

- B
- C
- D
- C
- C
- B
- C
- B
- C
- A

Post-Test

- B
- B
- C
- D
- C
- A
- B
- B
- A
- B
- B
- A
- A
- C
- B
- C
- B
- B
- C
- D

Activity 1.2

- G
- C

Activity 1. 4. Correct Me If I'm Wrong

- For a rotating wheel for example that is speeding up, a point on the outside covers more distance(radius) in the same amount of time as a point closer to the center. It has a much larger tangential acceleration than the portion closer to the axis of rotation. However, the angular acceleration of every part of the wheel is the same because the entire object moves as a rigid body through the same angle in the same amount of time. *(Answers vary)*
- Angular acceleration is the change in angular velocity divided by time, while tangential acceleration is the change in linear velocity divided by time

1. 1167 N; 980 N directed upward at $18°18°$ above the horizontal

$$U_g = K_{translational} + K_{rotational}$$
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$I_{cylinder} = \tfrac{1}{2}mR^2,\ and\ \ \omega = \frac{v}{R}$$
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$
$$gh = \tfrac{3}{4}v^2$$
$$v = \sqrt{\tfrac{4}{3}gh}$$

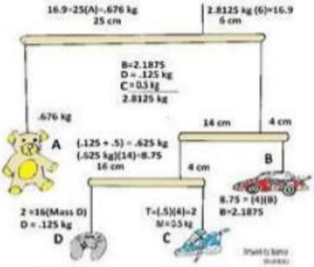
For each object, we have

$$K_i + U_i = K_f + U_f$$
$$0 + Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2$$
$$v_{cm} = \sqrt{\frac{2gh}{1 + I_{cm}/MR^2}}$$

Hence, the speed of the center of mass of any object at the bottom of the incline does not depend on its mass or size; it depends only on its shape. Therefore, all objects of the same shape such as spheres (of any mass or size) have the same speed at the bottom. That is, the smaller the ratio I_{cm}/MR^2 the faster the object moves since less of its energy goes to rotational kinetic energy and more goes to translational kinetic energy The ratio I_{cm}/MR^2 is equal to 0.4, 0.5, and 1 for a sphere, cylinder, and hoop, respectively Therefore, these objects will finish in the order of any sphere, any cylinder, and any hoop.

Name: ____________

Take the Torque Challenge



What is the mass of each item?

A: .676 kg C: .5 kg
B: 2.1875 kg D: .125 kg

Situation	Ease of Rotation (Rank the Forces from easiest to hardest)		
	1st	2nd	3rd
A. Opening a Door	[illegible]	[illegible]	[illegible]
B. Removing a Bolt using a Wrench	[illegible]	[illegible]	[illegible]
C. Rotating A Blade	[illegible]	[illegible]	[illegible]

A. Torque vs the Distance of Application
Torque is Directly Proportional to the Distance to where the Force is applied. As the distance increases, the rotation increases.

B. Torque vs the Angle of Application
Torque is Directly Proportional to the Angle to where the Force is applied. As the angle increases, the rotation increases.

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