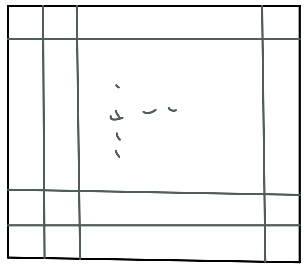
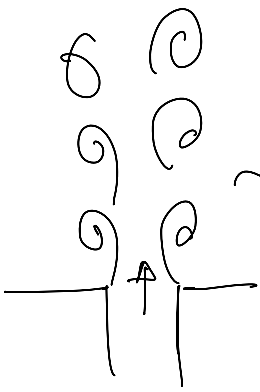
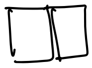


n_2

n_1

$N = n_1 \times n_2$ pixels $\rightsquigarrow 256^N$





256^2

$\rightsquigarrow$

$$\frac{du}{dt} = f(u) \qquad u\big|_{10^8 - 10^{12}}$$

$$\frac{dy}{dt} = g(y) \qquad y\big|_{10^2}$$

Linear Independence :

compression

$v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \qquad \alpha = -3$

dim = 1

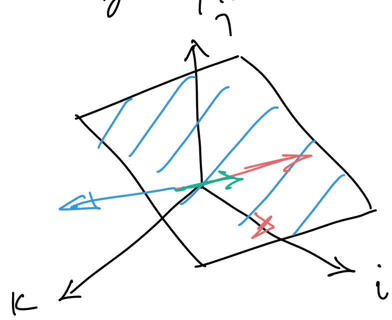
- $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -3 \\ -9 \\ 6 \end{bmatrix} \rightsquigarrow v_2 = -3v_1$, $v_1 = -\frac{1}{3}v_2$ linearly dependent

dim=2

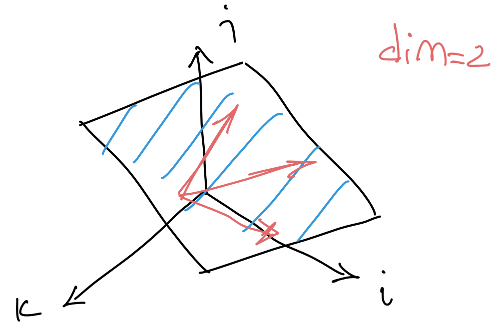
- $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -3 \\ -9 \\ 7 \end{bmatrix} \rightsquigarrow v_2 \neq \alpha v_1$ linearly independent

dim=1

- $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -3 \\ -9 \\ 6 \end{bmatrix} \qquad v_3 = \begin{bmatrix} -2 \\ -6 \\ 4 \end{bmatrix}$



- $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$



$v_3 = v_1 + v_2$

linearly dependent

Dimension: The minimum number of vectors needed to represent the space.

- $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 4 \\ 0.001 \end{bmatrix}$

$v_3 \simeq v_1 + v_2$

Inner Product:

$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$ $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$ $\rightsquigarrow \langle u, v \rangle = u^T v$ Eucleadian I.P.

$= [u_1 \ u_2 \ \cdots \ u_N] \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = u_1 v_1 + u_2 v_2 + \cdots + u_N v_N$

Weighted Inner Product

$\langle u, v \rangle = u^T W v$

$\underset{N \times N}{W} = \begin{pmatrix} \omega_1 & & & \\ & \omega_2 & & \\ & & \ddots & \\ & & & \omega_N \end{pmatrix}$ $\omega_i > 0$

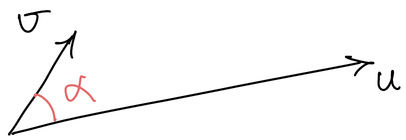
$\langle u, v \rangle = \sum_{i=1}^{N} \omega_i u_i v_i$

- $\langle u, v \rangle = \langle v, u \rangle$
- $\langle u, u \rangle \geq 0$
- $\langle u, u \rangle = 0 \iff u = 0$
- $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$

Length

$$\|u\| = \langle u, u \rangle^{1/2} \quad \overset{E.I.P}{\rightsquigarrow} \quad \|u\| = \left(u_1^2 + u_2^2 + \cdots + u_N^2\right)^{1/2}$$

Angle



$$\cos\alpha = \frac{u^T v}{\|u\| \|v\|}$$

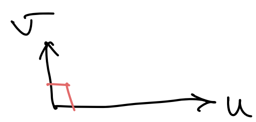
$$\cos\alpha = \frac{\langle u, v \rangle}{\|u\| \|v\|}$$

Orthogonality

u & v are orthogonal : $\langle u, v \rangle = 0$

$$\Updownarrow$$

$$\cos\alpha = 0 \implies \alpha = 90$$

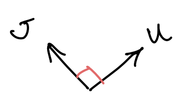


Orthonormality

$\langle u, v \rangle$ are orthonormal : $\langle u, v \rangle = 0$

$\|u\| = 1$

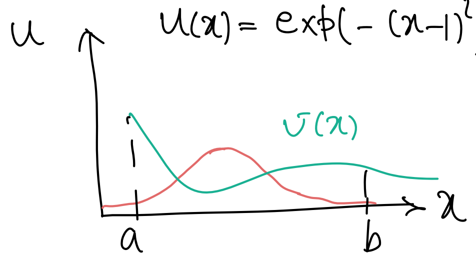
$\|v\| = 1$



A Finite-Dimensional Space

An Infinite-Dimensional Space

$$u(x) = \exp(-(x-1)^2)$$



$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}_{N\times 1}$$

$$u(x) = \begin{bmatrix} \\ u \\ \\ \end{bmatrix}_{\infty \times 1}$$

$$\langle u, v \rangle = u^T v$$

$$\langle u(x), v(x) \rangle = \int_a^b u(x)\, v(x)\, dx$$

$$\|u(x)\| = \langle u(x), u(x) \rangle^{1/2} = \left(\int_a^b u^2(x)\, dx \right)^{1/2}$$

$$\langle \sin(x), \cos(x) \rangle = \int_{-\pi}^{\pi} \sin(x) \cos(x)\, dx = 0$$

$$A|_{n\times m}, \quad B|_{n\times m}$$

$$\langle A, B \rangle = \sum_{j=1}^{m} \sum_{i=1}^{n} A_{ij} B_{ij}$$

$$A = \begin{bmatrix} a_1 & | & a_2 & | & \cdots & | & a_m \end{bmatrix} \xrightarrow{\text{vectorize}} A(:) = \begin{bmatrix} a_1 \\ \hline a_2 \\ \hline \vdots \\ \hline a_m \end{bmatrix}_{nm\times 1}$$

$$\|A\|_F = \langle A, A \rangle^{1/2}$$

Frobenius