Problem 1

Determine which of the following sets are linearly independent. For those sets that are linearly dependent, write one of the vectors as a linear combination of the others.

1. $\left[\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}1\\5\\9\end{pmatrix}\right]$

2. $\left[\begin{pmatrix}1\\2\\0\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\1\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\1\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\0\\3\\1\end{pmatrix}\right]$

Problem 2

Which of the following sets of functions are linearly independent?

1. $\{\sin x, \cos x, x \sin x\}$.
2. $\{e^x, xe^x, x^2 e^x\}$.
3. $\{\sin^2 x, \cos^2 x, \cos 2x\}$.

Problem 3

For $\mathbf{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$, $\mathbf{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$, determine which of the following are inner products for $\mathbb{R}^3$.

1. $\langle \mathbf{x}, \mathbf{y} \rangle = x_1 y_1 + x_3 y_3$,
2. $\langle \mathbf{x}, \mathbf{y} \rangle = x_1 y_1 - x_2 y_2 + x_3 y_3$,
3. $\langle \mathbf{x}, \mathbf{y} \rangle = 2x_1 y_1 + x_2 y_2 + 4x_3 y_3$,
4. $\langle \mathbf{x}, \mathbf{y} \rangle = x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2$.

Problem 4

Show the following basis functions are orthogonal to each other in the interval of $x \in [-\pi, \pi]$. In the following m and n are nonzero integer numbers.

1. Fourier modes of $\{\sin(mx)\}$ and $\{\sin(nx)\}$ when $n \neq m$.
2. Fourier modes of $\{\cos(mx)\}$ and $\{\cos(nx)\}$ when $n \neq m$.
3. Fourier modes of $\{\sin(mx)\}$ and $\{\cos(nx)\}$.