ME 2061 — Reduced Order Modeling for Engineering
Spring 2024

Course Description

This course will provide an introduction to dimension-reduction techniques and reduced order modeling. The course covers the mathematical foundation of all the techniques discussed and includes a diverse set of applications. In particular, the course covers the utility of dimension reduction in the diagnostics of complex data, deterministic and stochastic reduced order modeling and their applications in optimization and uncertainty quantification.

Topics

The topics covered in this course are:

1. Preliminaries
   - (a) Introduction to dimension reduction and reduced-order modeling.
   - (b) Review of relevant subjects from linear algebra: linear independence, matrix rank, second norm and Frobenius norms, inner product, angle, orthogonal projection, and oblique projection.
   - (c) Finite-dimensional (vectors) versus infinite-dimensional (functions) representations.
   - (d) Bases: piece-wise linear, monomials, spectral polynomials, and Fourier.
   - (e) Interpolation, regression, and projection.
   - (f) Discretization and approximating a partial differential equation with a system of ordinary differential equations.
2. Dimension reduction with singular value decomposition (SVD)
   - (a) SVD and optimality.
   - (b) Low-rank approximation with SVD.
   - (c) Different interpretations of SVD.
   - (d) Functional SVD.
   - (e) Applications.
3. Reduced-order modeling of vector differential equations
   - (a) Proper-orthogonal decomposition (POD).
   - (b) POD-Galerkin reduced order models.
   - (c) Linear time-dependent partial differential equations.
   - (d) Nonlinear time-dependent partial differential equations.
   - (e) Examples: transient conduction, linear advection, and Burgers.

4. Sparse sampling
   (a) Motivation: computational cost issues in nonlinear reduced-order modeling.
   (b) The direct empirical interpolation method (DEIM) and QDEIM.
   (c) CUR low-rank approximation.
   (d) Application: Sensor placement and field reconstruction from sparse measurements.
5. Advanced topics: On-the-fly low-rank approximation with time-dependent bases
   (a) Matrix differential equations (MDEs)
   (b) Low-rank approximation of MDEs with time-dependent bases
   (c) Dynamical low-rank approximation and its variants
   (d) Oblique projection for the low-rank approximation

Evaluation

The grading policy is as follows:

1. Quiz: Plickers multiple-choice questions. Grade percentage: (10%).
2. Homework Assignments. Grade percentage: 60%.
3. Final Project: Includes a computer project, final report and a presentation. Grade percentage: 30%.

All assessments will be submitted electronically on the Canvas site. Late submissions will be penalized on an hourly basis. The late penalties per assignment type are given in the table below.

Assignment type	**Late submission penalty**
Homework	20% per 24 hours
Project	10% per hour

Office Hours

Office hours will conducted via Zoom on Mondays from 3:30 pm- 5:00 pm. The Zoom link for the office hours is: `https://pitt.zoom.us/j/6177518304`. To avoid having multiple students on the same Zoom session, please contact ahead time for an appointment.

Academic Integrity

Include repercussions for academic integrity violations. Students in this course will be expected to comply with the University of Pittsburghs Policy on Academic Integrity and the Swanson School of Engineering Policy. Any student suspected of violating this obligation for any reason during the semester will be required to participate in the procedural process, initiated at the instructor level, as outlined in the University Guidelines on Academic Integrity and the Swanson School procedures. This may include, but is not limited to, the confiscation of the examination of any individual suspected of violating University Policy.

All students are expected to adhere to the standards of professional conduct and academic honesty. Any student engaged in cheating, plagiarism, or other acts of academic dishonesty would be subject to disciplinary action. Any student suspected of violating this obligation for any reason during the semester will be required to participate in the procedural process, initiated at the instructor level, as outlined in the SSOE Academic Integrity Policy found at: `https://www.engineering.pitt.edu/Academic-Integrity-Guidelines/`.

References

For the majority of the topics covered in this course the students may use the following resources. You will not be required to read any of the books in their entirety.

1. S. Brunton and J. N. Kutz, *Data-Driven Science and Engineering*, Cambridge University Press, Second Edition, 2022. This book is freely available at https://www.databookuw.com/.

2. G. Strang, *Linear Algebra and Learning from Data.* Cambridge Press, 2019.

3. P. Holmes, J. L. Lumley, and G. Berkooz, C. W. Rowley *Turbulence, Coherent Structures, Dynamical Systems and Symmetry.*, Second Edition, Cambridge: Cambridge University Press, 2012.

4. J. N. Kutz, S. Brunton, B. W. Brunton, J. L. Proctor, *Dynamic Mode Decomposition*, SIAM, 2016.