

**Contents**

<b>1</b>	<b>counter Theory</b>	<b>3</b>
1.1	Datatypes . . . . .	3
1.2	Theorems . . . . .	3
<b>2</b>	<b>sm Theory</b>	<b>4</b>
2.1	Datatypes . . . . .	4
2.2	Definitions . . . . .	5
2.3	Theorems . . . . .	5



# 1 counter Theory

**Built:** 02 March 2020

**Parent Theories:** sm

## 1.1 Datatypes

*ctrCmd* = load num | count | hold

*ctrOut* = DISPLAY num

*ctrState* = COUNT num

## 1.2 Theorems

[ctr\_rules]

$\vdash (\forall ins\ outs.$   
 $\quad \text{TR } (\text{load } new) (\text{CFG } (\text{load } new::ins) (\text{COUNT } n) outs)$   
 $\quad (\text{CFG } ins (\text{COUNT } new) (\text{DISPLAY } new::outs))) \wedge$   
 $(\forall ins\ outs.$   
 $\quad \text{TR count } (\text{CFG } (\text{count}::ins) (\text{COUNT } n) outs)$   
 $\quad (\text{CFG } ins (\text{COUNT } (n - 1)) (\text{DISPLAY } (n - 1)::outs))) \wedge$   
 $\forall ins\ outs.$   
 $\quad \text{TR hold } (\text{CFG } (\text{hold}::ins) (\text{COUNT } n) outs)$   
 $\quad (\text{CFG } ins (\text{COUNT } n) (\text{DISPLAY } n::outs))$

[ctrCmd\_distinct\_clauses]

$\vdash (\forall a. \text{load } a \neq \text{count}) \wedge (\forall a. \text{load } a \neq \text{hold}) \wedge \text{count} \neq \text{hold}$

[ctrNS\_def]

$\vdash (\text{ctrNS } (\text{COUNT } n) (\text{load } k) = \text{COUNT } k) \wedge$   
 $(\text{ctrNS } (\text{COUNT } n) \text{count} = \text{COUNT } (n - 1)) \wedge$   
 $(\text{ctrNS } (\text{COUNT } n) \text{hold} = \text{COUNT } n)$

[ctrNS\_ind]

$\vdash \forall P.$   
 $(\forall n\ k. P (\text{COUNT } n) (\text{load } k)) \wedge (\forall n. P (\text{COUNT } n) \text{count}) \wedge$   
 $(\forall n. P (\text{COUNT } n) \text{hold}) \Rightarrow$   
 $\forall v\ v_1. P\ v\ v_1$

[ctrOut\_def]

$\vdash (\text{ctrOut } (\text{COUNT } n) (\text{load } k) = \text{DISPLAY } k) \wedge$   
 $(\text{ctrOut } (\text{COUNT } n) \text{count} = \text{DISPLAY } (n - 1)) \wedge$   
 $(\text{ctrOut } (\text{COUNT } n) \text{hold} = \text{DISPLAY } n)$

[ctrOut\_ind]

$$\begin{aligned} &\vdash \forall P. \\ &\quad (\forall n \ k. \ P \ (\text{COUNT } n) \ (\text{load } k)) \wedge (\forall n. \ P \ (\text{COUNT } n) \ \text{count}) \wedge \\ &\quad (\forall n. \ P \ (\text{COUNT } n) \ \text{hold}) \Rightarrow \\ &\quad \forall v \ v_1. \ P \ v \ v_1 \end{aligned}$$

[ctrOut\_one\_one]

$$\vdash \forall a \ a'. \ (\text{DISPLAY } a = \text{DISPLAY } a') \iff (a = a')$$

[ctrState\_one\_one]

$$\vdash \forall a \ a'. \ (\text{COUNT } a = \text{COUNT } a') \iff (a = a')$$

[ctrTR\_clauses]

$$\begin{aligned} &\vdash (\forall x \ x1s \ s_1 \ out1s \ x2s \ out2s \ s_2. \\ &\quad \text{TR } x \ (\text{CFG } x1s \ s_1 \ out1s) \ (\text{CFG } x2s \ s_2 \ out2s) \iff \\ &\quad \exists NS \ Out \ ins. \\ &\quad \quad (x1s = x::ins) \wedge (x2s = ins) \wedge (s_2 = NS \ s_1 \ x) \wedge \\ &\quad \quad (out2s = Out \ s_1 \ x::out1s)) \wedge \\ &\quad \forall x \ x1s \ s_1 \ out1s \ x2s \ out2s. \\ &\quad \text{TR } x \ (\text{CFG } x1s \ s_1 \ out1s) \\ &\quad \quad (\text{CFG } x2s \ (\text{ctrNS } s_1 \ x) \ (\text{ctrOut } s_1 \ x::out2s)) \iff \\ &\quad \exists ins. \ (x1s = x::ins) \wedge (x2s = ins) \wedge (out2s = out1s) \end{aligned}$$

[ctrTR\_rules]

$$\begin{aligned} &\vdash \forall s \ x \ ins \ outs. \\ &\quad \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (\text{ctrNS } s \ x) \ (\text{ctrOut } s \ x::outs)) \end{aligned}$$

[ctrTrans\_Equiv\_TR]

$$\begin{aligned} &\vdash \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (\text{ctrNS } s \ x) \ (\text{ctrOut } s \ x::outs)) \iff \\ &\quad \text{Trans } x \ s \ (\text{ctrNS } s \ x) \end{aligned}$$

## 2 sm Theory

**Built:** 02 March 2020

**Parent Theories:** indexedLists, patternMatches

### 2.1 Datatypes

*configuration* = CFG ('input list) 'state ('output list)

## 2.2 Definitions

[TR\_def]

$$\begin{aligned}
&\vdash \text{TR} = \\
&\quad (\lambda a_0 a_1 a_2. \\
&\quad \quad \forall TR'. \\
&\quad \quad (\forall a_0 a_1 a_2. \\
&\quad \quad \quad (\exists NS \text{ Out } s \text{ ins } \text{ outs}. \\
&\quad \quad \quad \quad (a_1 = \text{CFG } (a_0 :: \text{ins}) \text{ } s \text{ } \text{outs}) \wedge \\
&\quad \quad \quad \quad (a_2 = \text{CFG } \text{ins } (NS \text{ } s \text{ } a_0) (\text{Out } s \text{ } a_0 :: \text{outs}))) \Rightarrow \\
&\quad \quad \quad TR' a_0 a_1 a_2) \Rightarrow \\
&\quad \quad TR' a_0 a_1 a_2)
\end{aligned}$$

[Trans\_def]

$$\begin{aligned}
&\vdash \text{Trans} = \\
&\quad (\lambda a_0 a_1 a_2. \\
&\quad \quad \forall Trans'. \\
&\quad \quad (\forall a_0 a_1 a_2. (\exists NS. a_2 = NS a_1 a_0) \Rightarrow Trans' a_0 a_1 a_2) \Rightarrow \\
&\quad \quad Trans' a_0 a_1 a_2)
\end{aligned}$$

## 2.3 Theorems

[configuration\_one\_one]

$$\begin{aligned}
&\vdash \forall a_0 a_1 a_2 a'_0 a'_1 a'_2. \\
&\quad (\text{CFG } a_0 a_1 a_2 = \text{CFG } a'_0 a'_1 a'_2) \iff \\
&\quad (a_0 = a'_0) \wedge (a_1 = a'_1) \wedge (a_2 = a'_2)
\end{aligned}$$

[TR\_cases]

$$\begin{aligned}
&\vdash \forall a_0 a_1 a_2. \\
&\quad \text{TR } a_0 a_1 a_2 \iff \\
&\quad \exists NS \text{ Out } s \text{ ins } \text{ outs}. \\
&\quad \quad (a_1 = \text{CFG } (a_0 :: \text{ins}) \text{ } s \text{ } \text{outs}) \wedge \\
&\quad \quad (a_2 = \text{CFG } \text{ins } (NS \text{ } s \text{ } a_0) (\text{Out } s \text{ } a_0 :: \text{outs}))
\end{aligned}$$

[TR\_clauses]

$$\begin{aligned}
&\vdash (\forall x \text{ x1s } s_1 \text{ out1s } x2s \text{ out2s } s_2. \\
&\quad \text{TR } x (\text{CFG } x1s \text{ } s_1 \text{ } \text{out1s}) (\text{CFG } x2s \text{ } s_2 \text{ } \text{out2s}) \iff \\
&\quad \exists NS \text{ Out } \text{ins}. \\
&\quad \quad (x1s = x :: \text{ins}) \wedge (x2s = \text{ins}) \wedge (s_2 = NS \text{ } s_1 \text{ } x) \wedge \\
&\quad \quad (\text{out2s} = \text{Out } s_1 \text{ } x :: \text{out1s})) \wedge \\
&\quad \forall NS \text{ Out } x \text{ x1s } s_1 \text{ out1s } x2s \text{ out2s}. \\
&\quad \quad \text{TR } x (\text{CFG } x1s \text{ } s_1 \text{ } \text{out1s}) \\
&\quad \quad (\text{CFG } x2s \text{ } (NS \text{ } s_1 \text{ } x) (\text{Out } s_1 \text{ } x :: \text{out2s})) \iff \\
&\quad \quad \exists \text{ins}. (x1s = x :: \text{ins}) \wedge (x2s = \text{ins}) \wedge (\text{out2s} = \text{out1s})
\end{aligned}$$

**[TR\_complete]**

$$\begin{aligned} &\vdash \forall s \ x \ ins \ outs. \\ &\quad \exists s' \ out. \\ &\quad \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \ (\text{CFG } ins \ s' \ (out::outs)) \end{aligned}$$
**[TR\_deterministic]**

$$\begin{aligned} &\vdash \forall NS \ Out \ x_1 \ ins_1 \ s_1 \ outs_1 \ ins_2 \ ins'_2 \ outs_2 \ outs'_2. \\ &\quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\ &\quad \quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2)) \wedge \\ &\quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\ &\quad \quad (\text{CFG } ins'_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs'_2)) \iff \\ &\quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2)) = \\ &\quad \quad \text{CFG } ins'_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs'_2)) \wedge \\ &\quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\ &\quad \quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2)) \end{aligned}$$
**[TR\_ind]**

$$\begin{aligned} &\vdash \forall TR'. \\ &\quad (\forall NS \ Out \ s \ x \ ins \ outs. \\ &\quad \quad TR' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\ &\quad \forall a_0 \ a_1 \ a_2. \text{TR } a_0 \ a_1 \ a_2 \Rightarrow TR' \ a_0 \ a_1 \ a_2 \end{aligned}$$
**[TR\_rules]**

$$\begin{aligned} &\vdash \forall NS \ Out \ s \ x \ ins \ outs. \\ &\quad \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs)) \end{aligned}$$
**[TR\_strongind]**

$$\begin{aligned} &\vdash \forall TR'. \\ &\quad (\forall NS \ Out \ s \ x \ ins \ outs. \\ &\quad \quad TR' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\ &\quad \forall a_0 \ a_1 \ a_2. \text{TR } a_0 \ a_1 \ a_2 \Rightarrow TR' \ a_0 \ a_1 \ a_2 \end{aligned}$$
**[TR\_Trans\_lemma]**

$$\begin{aligned} &\vdash \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\ &\quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs)) \Rightarrow \\ &\quad \text{Trans } x \ s \ (NS \ s \ x) \end{aligned}$$
**[Trans\_cases]**

$$\vdash \forall a_0 \ a_1 \ a_2. \text{Trans } a_0 \ a_1 \ a_2 \iff \exists NS. \ a_2 = NS \ a_1 \ a_0$$

**[Trans\_Equiv\_TR]**

$$\vdash \text{TR } x \text{ (CFG } (x::\text{ins}) \text{ } s \text{ outs)} \\ (\text{CFG ins } (NS \text{ } s \text{ } x) \text{ (Out } s \text{ } x::\text{outs})) \iff \text{Trans } x \text{ } s \text{ (NS } s \text{ } x)$$
**[Trans\_ind]**

$$\vdash \forall \text{Trans}' . \\ (\forall NS \text{ } s \text{ } x . \text{Trans}' \text{ } x \text{ } s \text{ (NS } s \text{ } x)) \Rightarrow \\ \forall a_0 \text{ } a_1 \text{ } a_2 . \text{Trans } a_0 \text{ } a_1 \text{ } a_2 \Rightarrow \text{Trans}' \text{ } a_0 \text{ } a_1 \text{ } a_2$$
**[Trans\_rules]**

$$\vdash \forall NS \text{ } s \text{ } x . \text{Trans } x \text{ } s \text{ (NS } s \text{ } x)$$
**[Trans\_strongind]**

$$\vdash \forall \text{Trans}' . \\ (\forall NS \text{ } s \text{ } x . \text{Trans}' \text{ } x \text{ } s \text{ (NS } s \text{ } x)) \Rightarrow \\ \forall a_0 \text{ } a_1 \text{ } a_2 . \text{Trans } a_0 \text{ } a_1 \text{ } a_2 \Rightarrow \text{Trans}' \text{ } a_0 \text{ } a_1 \text{ } a_2$$
**[Trans\_TR\_lemma]**

$$\vdash \text{Trans } x \text{ } s \text{ (NS } s \text{ } x) \Rightarrow \\ \text{TR } x \text{ (CFG } (x::\text{ins}) \text{ } s \text{ outs)} \text{ (CFG ins } (NS \text{ } s \text{ } x) \text{ (Out } s \text{ } x::\text{outs}))}$$





## Index

### **counter Theory**, 3

- Datatypes, 3
- Theorems, 3
  - ctr\_rules, 3
  - ctrcmd\_distinct\_clauses, 3
  - ctrNS\_def, 3
  - ctrNS\_ind, 3
  - ctrOut\_def, 3
  - ctrOut\_ind, 4
  - ctrOut\_one\_one, 4
  - ctrState\_one\_one, 4
  - ctrTR\_clauses, 4
  - ctrTR\_rules, 4
  - ctrTrans\_Equiv\_TR, 4

### **sm Theory**, 4

- Datatypes, 4
- Definitions, 5
  - TR\_def, 5
  - Trans\_def, 5
- Theorems, 5
  - configuration\_one\_one, 5
  - TR\_cases, 5
  - TR\_clauses, 5
  - TR\_complete, 6
  - TR\_deterministic, 6
  - TR\_ind, 6
  - TR\_rules, 6
  - TR\_strongind, 6
  - TR\_Trans\_lemma, 6
  - Trans\_cases, 6
  - Trans\_Equiv\_TR, 7
  - Trans\_ind, 7
  - Trans\_rules, 7
  - Trans\_strongind, 7
  - Trans\_TR\_lemma, 7