Contents

1 counter Theory

Built: 02 March 2020

Parent Theories: sm

1.1 Datatypes

$ctrcmd$ = load num | count | hold

$ctrOut$ = DISPLAY num

$ctrState$ = COUNT num

1.2 Theorems

[ctr_rules]

$\vdash (\forall ins\ outs.$
  TR (load new) (CFG (load new::ins) (COUNT n) $outs$)
   (CFG ins (COUNT new) (DISPLAY new::$outs$))) $\wedge$
 $(\forall ins\ outs.$
  TR count (CFG (count::ins) (COUNT n) $outs$)
   (CFG ins (COUNT (n - 1)) (DISPLAY (n - 1)::$outs$))) $\wedge$
 $\forall ins\ outs.$
  TR hold (CFG (hold::ins) (COUNT n) $outs$)
   (CFG ins (COUNT n) (DISPLAY n::$outs$))

[ctrcmd_distinct_clauses]

$\vdash$ ($\forall a$. load $a \neq$ count) $\wedge$ ($\forall a$. load $a \neq$ hold) $\wedge$ count $\neq$ hold

[ctrNS_def]

$\vdash$ (ctrNS (COUNT n) (load k) = COUNT k) $\wedge$
 (ctrNS (COUNT n) count = COUNT (n - 1)) $\wedge$
 (ctrNS (COUNT n) hold = COUNT n)

[ctrNS_ind]

$\vdash \forall P.$
  ($\forall n\ k$. P (COUNT n) (load k)) $\wedge$ ($\forall n$. P (COUNT n) count) $\wedge$
  ($\forall n$. P (COUNT n) hold) $\Rightarrow$
  $\forall v\ v_1.\ P\ v\ v_1$

[ctrOut_def]

$\vdash$ (ctrOut (COUNT n) (load k) = DISPLAY k) $\wedge$
 (ctrOut (COUNT n) count = DISPLAY (n - 1)) $\wedge$
 (ctrOut (COUNT n) hold = DISPLAY n)

[ctrOut_ind]

$$\vdash \forall P.\ (\forall n\ k.\ P\ (\texttt{COUNT}\ n)\ (\texttt{load}\ k)) \land (\forall n.\ P\ (\texttt{COUNT}\ n)\ \texttt{count}) \land (\forall n.\ P\ (\texttt{COUNT}\ n)\ \texttt{hold}) \Rightarrow \forall v\ v_1.\ P\ v\ v_1$$

[ctrOut_one_one]

$$\vdash \forall a\ a'.\ (\texttt{DISPLAY}\ a = \texttt{DISPLAY}\ a') \iff (a = a')$$

[ctrState_one_one]

$$\vdash \forall a\ a'.\ (\texttt{COUNT}\ a = \texttt{COUNT}\ a') \iff (a = a')$$

[ctrTR_clauses]

$$\vdash (\forall x\ \mathit{x1s}\ s_1\ \mathit{out1s}\ \mathit{x2s}\ \mathit{out2s}\ s_2.\ \texttt{TR}\ x\ (\texttt{CFG}\ \mathit{x1s}\ s_1\ \mathit{out1s})\ (\texttt{CFG}\ \mathit{x2s}\ s_2\ \mathit{out2s}) \iff \exists \mathit{NS}\ \mathit{Out}\ \mathit{ins}.\ (\mathit{x1s} = x::\mathit{ins}) \land (\mathit{x2s} = \mathit{ins}) \land (s_2 = \mathit{NS}\ s_1\ x) \land (\mathit{out2s} = \mathit{Out}\ s_1\ x::\mathit{out1s})) \land \forall x\ \mathit{x1s}\ s_1\ \mathit{out1s}\ \mathit{x2s}\ \mathit{out2s}.\ \texttt{TR}\ x\ (\texttt{CFG}\ \mathit{x1s}\ s_1\ \mathit{out1s})\ (\texttt{CFG}\ \mathit{x2s}\ (\texttt{ctrNS}\ s_1\ x)\ (\texttt{ctrOut}\ s_1\ x::\mathit{out2s})) \iff \exists \mathit{ins}.\ (\mathit{x1s} = x::\mathit{ins}) \land (\mathit{x2s} = \mathit{ins}) \land (\mathit{out2s} = \mathit{out1s})$$

[ctrTR_rules]

$$\vdash \forall s\ x\ \mathit{ins}\ \mathit{outs}.\ \texttt{TR}\ x\ (\texttt{CFG}\ (x::\mathit{ins})\ s\ \mathit{outs})\ (\texttt{CFG}\ \mathit{ins}\ (\texttt{ctrNS}\ s\ x)\ (\texttt{ctrOut}\ s\ x::\mathit{outs}))$$

[ctrTrans_Equiv_TR]

$$\vdash \texttt{TR}\ x\ (\texttt{CFG}\ (x::\mathit{ins})\ s\ \mathit{outs})\ (\texttt{CFG}\ \mathit{ins}\ (\texttt{ctrNS}\ s\ x)\ (\texttt{ctrOut}\ s\ x::\mathit{outs})) \iff \texttt{Trans}\ x\ s\ (\texttt{ctrNS}\ s\ x)$$

2 sm Theory

Built: 02 March 2020

Parent Theories: indexedLists, patternMatches

2.1 Datatypes

configuration = `CFG ('input list) 'state ('output list)`

2.2 Definitions

[TR_def]

$$
\begin{array}{l}
\vdash \texttt{TR} = \\
\quad (\lambda\, a_0\ \ a_1\ \ a_2\,. \\
\qquad \forall\, TR'. \\
\qquad\quad (\forall\, a_0\ \ a_1\ \ a_2\,. \\
\qquad\qquad (\exists\, NS\ \ Out\ \ s\ \ ins\ \ outs\,. \\
\qquad\qquad\quad (a_1 = \texttt{CFG}\ (a_0 :: ins)\ \ s\ \ outs) \wedge \\
\qquad\qquad\quad (a_2 = \texttt{CFG}\ \ ins\ \ (NS\ \ s\ \ a_0)\ \ (Out\ \ s\ \ a_0 :: outs))) \Rightarrow \\
\qquad\qquad TR'\ \ a_0\ \ a_1\ \ a_2) \Rightarrow \\
\qquad\quad TR'\ \ a_0\ \ a_1\ \ a_2)
\end{array}
$$

[Trans_def]

$$
\begin{array}{l}
\vdash \texttt{Trans} = \\
\quad (\lambda\, a_0\ \ a_1\ \ a_2\,. \\
\qquad \forall\, Trans'. \\
\qquad\quad (\forall\, a_0\ \ a_1\ \ a_2\,.\ (\exists\, NS.\ a_2 = NS\ \ a_1\ \ a_0) \Rightarrow Trans'\ \ a_0\ \ a_1\ \ a_2) \Rightarrow \\
\qquad\quad Trans'\ \ a_0\ \ a_1\ \ a_2)
\end{array}
$$

2.3 Theorems

[configuration_one_one]

$$
\begin{array}{l}
\vdash \forall\, a_0\ \ a_1\ \ a_2\ \ a_0'\ \ a_1'\ \ a_2'. \\
\qquad (\texttt{CFG}\ \ a_0\ \ a_1\ \ a_2 = \texttt{CFG}\ \ a_0'\ \ a_1'\ \ a_2') \iff \\
\qquad (a_0 = a_0') \wedge (a_1 = a_1') \wedge (a_2 = a_2')
\end{array}
$$

[TR_cases]

$$
\begin{array}{l}
\vdash \forall\, a_0\ \ a_1\ \ a_2\,. \\
\qquad \texttt{TR}\ \ a_0\ \ a_1\ \ a_2 \iff \\
\qquad \exists\, NS\ \ Out\ \ s\ \ ins\ \ outs\,. \\
\qquad\quad (a_1 = \texttt{CFG}\ (a_0 :: ins)\ \ s\ \ outs) \wedge \\
\qquad\quad (a_2 = \texttt{CFG}\ \ ins\ \ (NS\ \ s\ \ a_0)\ \ (Out\ \ s\ \ a_0 :: outs))
\end{array}
$$

[TR_clauses]

$$
\begin{array}{l}
\vdash (\forall\, x\ \ x1s\ \ s_1\ \ out1s\ \ x2s\ \ out2s\ \ s_2\,. \\
\qquad \texttt{TR}\ \ x\ \ (\texttt{CFG}\ \ x1s\ \ s_1\ \ out1s)\ \ (\texttt{CFG}\ \ x2s\ \ s_2\ \ out2s) \iff \\
\qquad \exists\, NS\ \ Out\ \ ins\,. \\
\qquad\quad (x1s = x :: ins) \wedge (x2s = ins) \wedge (s_2 = NS\ \ s_1\ \ x) \wedge \\
\qquad\quad (out2s = Out\ \ s_1\ \ x :: out1s)) \wedge \\
\quad \forall\, NS\ \ Out\ \ x\ \ x1s\ \ s_1\ \ out1s\ \ x2s\ \ out2s\,. \\
\qquad \texttt{TR}\ \ x\ \ (\texttt{CFG}\ \ x1s\ \ s_1\ \ out1s) \\
\qquad\quad (\texttt{CFG}\ \ x2s\ \ (NS\ \ s_1\ \ x)\ \ (Out\ \ s_1\ \ x :: out2s)) \iff \\
\qquad \exists\, ins.\ (x1s = x :: ins) \wedge (x2s = ins) \wedge (out2s = out1s)
\end{array}
$$

[TR_complete]

$$\vdash \forall s\ x\ ins\ outs.\ \exists s'\ out.\ \texttt{TR}\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\ (\texttt{CFG}\ ins\ s'\ (out::outs))$$

[TR_deterministic]

$$\begin{aligned}\vdash \forall NS\ Out\ x_1\ ins_1\ s_1\ outs_1\ ins_2\ ins_2'\ outs_2\ outs_2'.\\ &\texttt{TR}\ x_1\ (\texttt{CFG}\ (x_1::ins_1)\ s_1\ outs_1)\\ &\quad(\texttt{CFG}\ ins_2\ (NS\ s_1\ x_1)\ (Out\ s_1\ x_1::outs_2))\ \wedge\\ &\texttt{TR}\ x_1\ (\texttt{CFG}\ (x_1::ins_1)\ s_1\ outs_1)\\ &\quad(\texttt{CFG}\ ins_2'\ (NS\ s_1\ x_1)\ (Out\ s_1\ x_1::outs_2'))\iff\\ &(\texttt{CFG}\ ins_2\ (NS\ s_1\ x_1)\ (Out\ s_1\ x_1::outs_2) =\\ &\ \texttt{CFG}\ ins_2'\ (NS\ s_1\ x_1)\ (Out\ s_1\ x_1::outs_2'))\ \wedge\\ &\texttt{TR}\ x_1\ (\texttt{CFG}\ (x_1::ins_1)\ s_1\ outs_1)\\ &\quad(\texttt{CFG}\ ins_2\ (NS\ s_1\ x_1)\ (Out\ s_1\ x_1::outs_2))\end{aligned}$$

[TR_ind]

$$\begin{aligned}\vdash \forall TR'.\\ &(\forall NS\ Out\ s\ x\ ins\ outs.\\ &\quad TR'\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\\ &\qquad(\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs)))\Rightarrow\\ &\forall a_0\ a_1\ a_2.\ \texttt{TR}\ a_0\ a_1\ a_2 \Rightarrow TR'\ a_0\ a_1\ a_2\end{aligned}$$

[TR_rules]

$$\begin{aligned}\vdash \forall NS\ Out\ s\ x\ ins\ outs.\\ &\texttt{TR}\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\\ &\quad(\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs))\end{aligned}$$

[TR_strongind]

$$\begin{aligned}\vdash \forall TR'.\\ &(\forall NS\ Out\ s\ x\ ins\ outs.\\ &\quad TR'\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\\ &\qquad(\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs)))\Rightarrow\\ &\forall a_0\ a_1\ a_2.\ \texttt{TR}\ a_0\ a_1\ a_2 \Rightarrow TR'\ a_0\ a_1\ a_2\end{aligned}$$

[TR_Trans_lemma]

$$\begin{aligned}\vdash\ &\texttt{TR}\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\\ &\quad(\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs))\Rightarrow\\ &\texttt{Trans}\ x\ s\ (NS\ s\ x)\end{aligned}$$

[Trans_cases]

$$\vdash \forall a_0\ a_1\ a_2.\ \texttt{Trans}\ a_0\ a_1\ a_2 \iff \exists NS.\ a_2 = NS\ a_1\ a_0$$

[Trans_Equiv_TR]

$\vdash$ $\texttt{TR}\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)$
$\quad (\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs)) \iff \texttt{Trans}\ x\ s\ (NS\ s\ x)$

[Trans_ind]

$\vdash \forall\, Trans'.$
$\quad (\forall\, NS\ s\ x.\ Trans'\ x\ s\ (NS\ s\ x)) \Rightarrow$
$\quad \forall\, a_0\ a_1\ a_2.\ \texttt{Trans}\ a_0\ a_1\ a_2 \Rightarrow Trans'\ a_0\ a_1\ a_2$

[Trans_rules]

$\vdash \forall\, NS\ s\ x.\ \texttt{Trans}\ x\ s\ (NS\ s\ x)$

[Trans_strongind]

$\vdash \forall\, Trans'.$
$\quad (\forall\, NS\ s\ x.\ Trans'\ x\ s\ (NS\ s\ x)) \Rightarrow$
$\quad \forall\, a_0\ a_1\ a_2.\ \texttt{Trans}\ a_0\ a_1\ a_2 \Rightarrow Trans'\ a_0\ a_1\ a_2$

[Trans_TR_lemma]

$\vdash \texttt{Trans}\ x\ s\ (NS\ s\ x) \Rightarrow$
$\quad \texttt{TR}\ x\ (\texttt{CFG}\ (x::ins)\ s\ outs)\ (\texttt{CFG}\ ins\ (NS\ s\ x)\ (Out\ s\ x::outs))$

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