

**Module 3 | Lesson 2**

# **THE INERTIAL MEASUREMENT UNIT**

# The Inertial Measurement

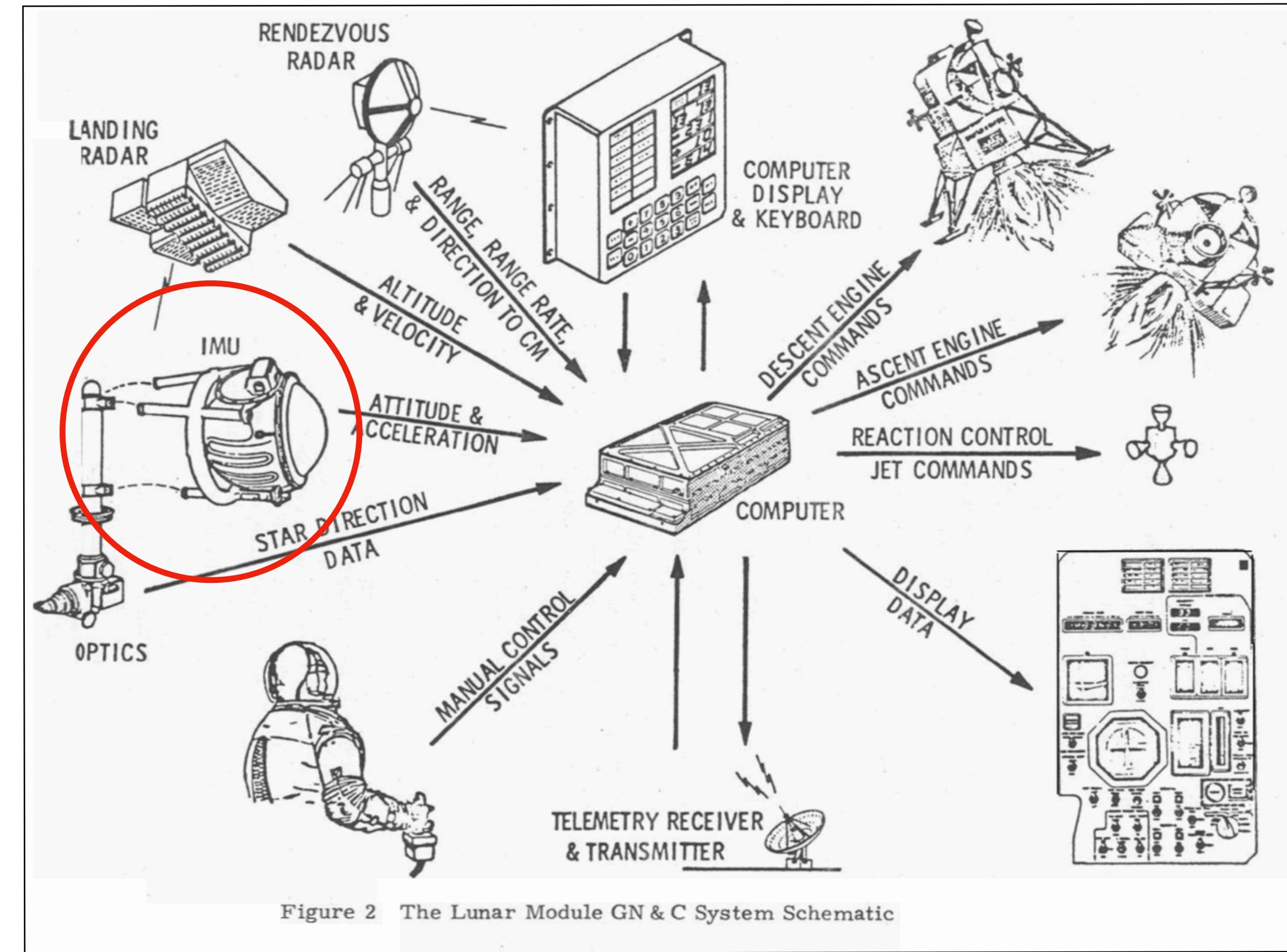
By the end of this video, you will be able to...

- Describe the individual components (accelerometers and gyroscopes) of an inertial measurement unit and their basic operating principles.
- Define the measurement models used for accelerometers and gyroscopes.

# The Inertial Measurement Unit



*iPhone X IMU*



*Apollo Lunar Module Guidance Navigation & Control*

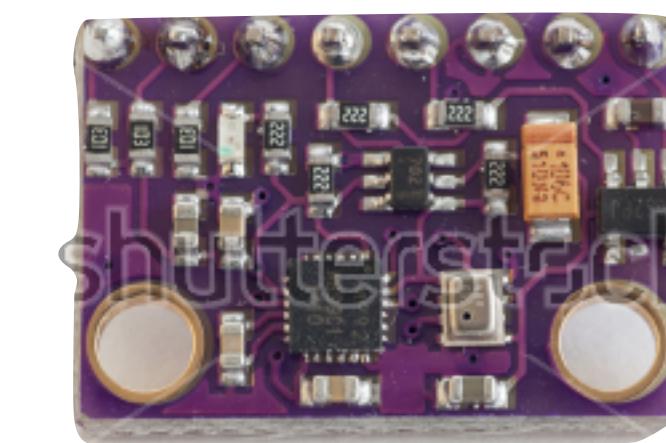
# The Inertial Measurement Unit

- An IMU is typically composed of
  - **gyroscopes** (measure angular rotation rates about three separate axes)
  - **accelerometers** (measure accelerations along three orthogonal axes)
- IMUs come in many form factors; cost varies from ~\$10 to ~\$100K+

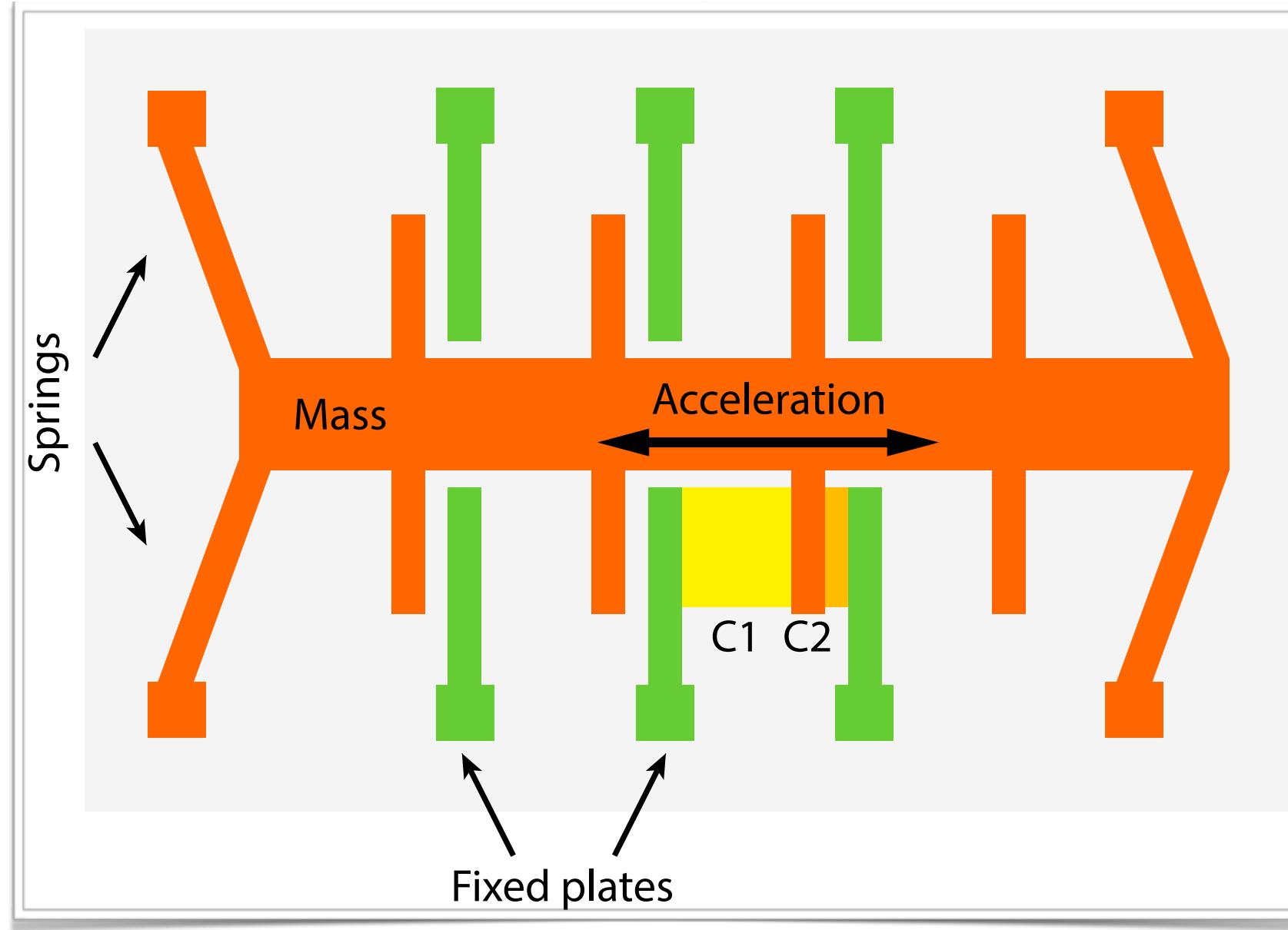


# The Gyroscope

- Gyroscope: γῦρος gûros, “circle” + σκοπέω skopéō, “to look”
  - Historically: a spinning disc that maintains a specific orientation relative to *inertial space*, providing an orientation reference
  - Modern spinning-disk gyroscopes spin at up to 24,000 RPM!
- Microelectromechanical systems (MEMS) are much smaller and cheaper
  - Measure rotational *rates* instead of orientation directly
  - Measurements are **noisy** and **drift** over time



# The Accelerometer



- Accelerometers measure acceleration relative to *free-fall* - this is also called the *proper acceleration* or *specific force*:

$$\mathbf{a}_{\text{meas}} = \mathbf{f} = \frac{\mathbf{F}_{\text{non-gravity}}}{m}$$

Sitting still at your desk, your *proper* acceleration is  $g$  upwards! (think of the 'normal' force holding you up)

In localization, we typically require the acceleration relative to a fixed reference frame

- 'coordinate' acceleration
- computed using fundamental equation for accelerometers in a gravity field:

$$\mathbf{f} + \mathbf{g} = \ddot{\mathbf{r}}_i$$

# The Accelerometer | Examples

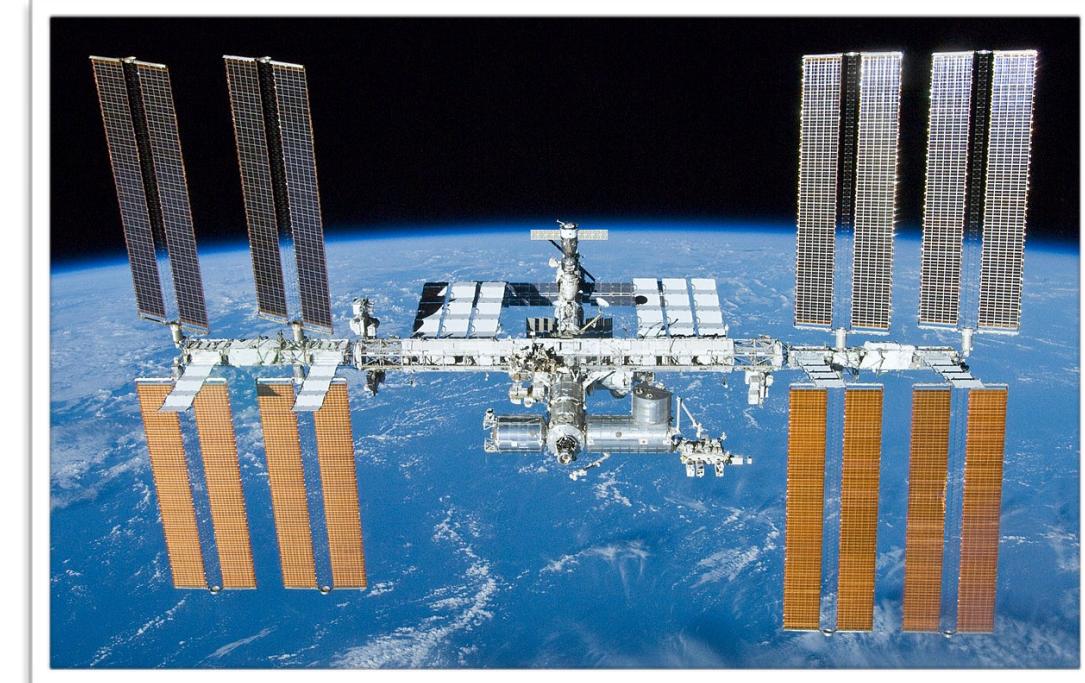
- An accelerometer in a stationary car measures:

$$\mathbf{f} = \ddot{\mathbf{r}}_i - \mathbf{g} \approx \mathbf{0} - \mathbf{g} \approx -\mathbf{g} \quad g \text{ 'up'}$$



- An accelerometer on the International Space Station measures:

$$\mathbf{f} = \ddot{\mathbf{r}}_i - \mathbf{g} \approx \mathbf{g} - \mathbf{g} \approx \mathbf{0} \quad (\text{zero-g!})$$



# **IMU Measurement Models**

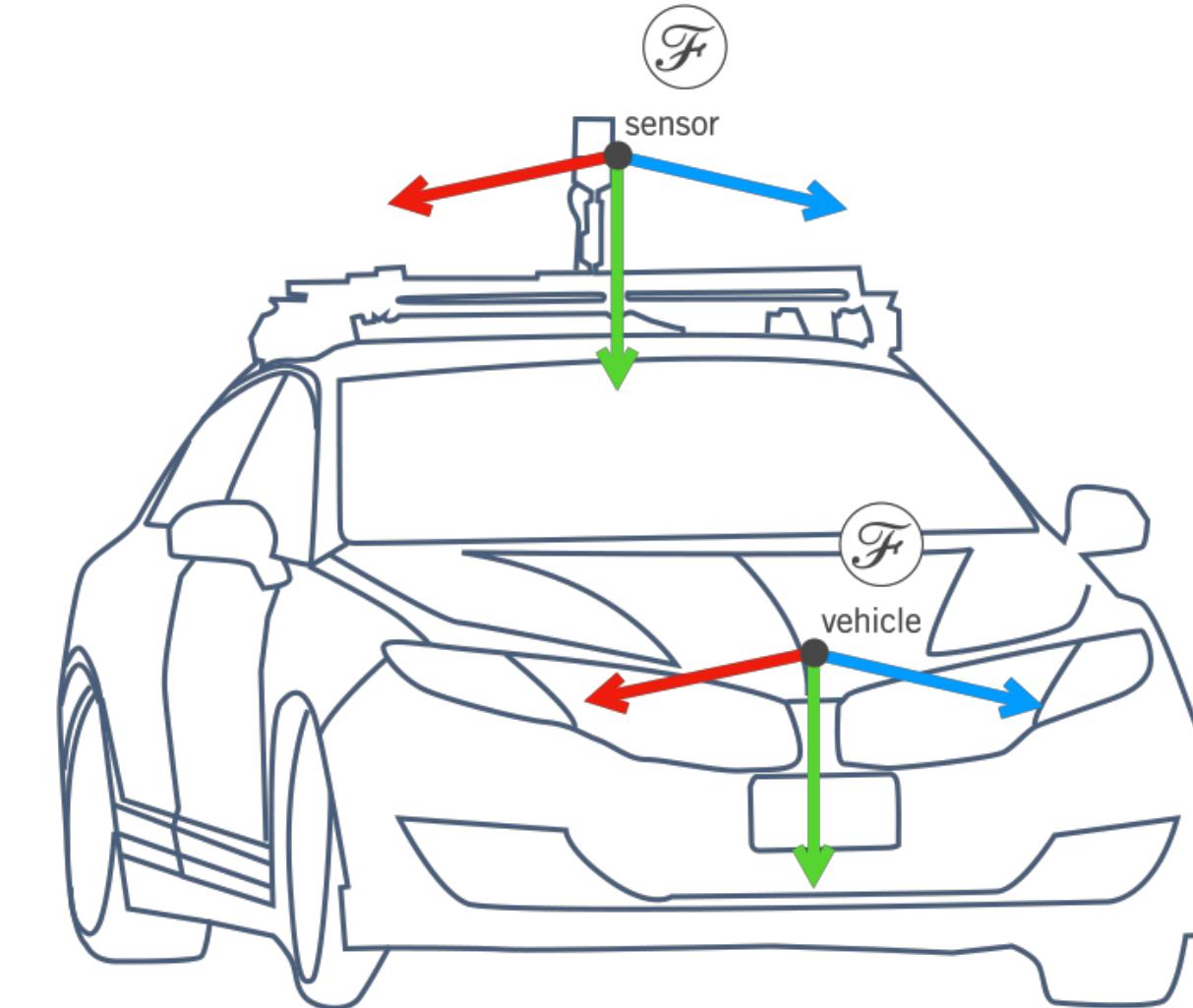
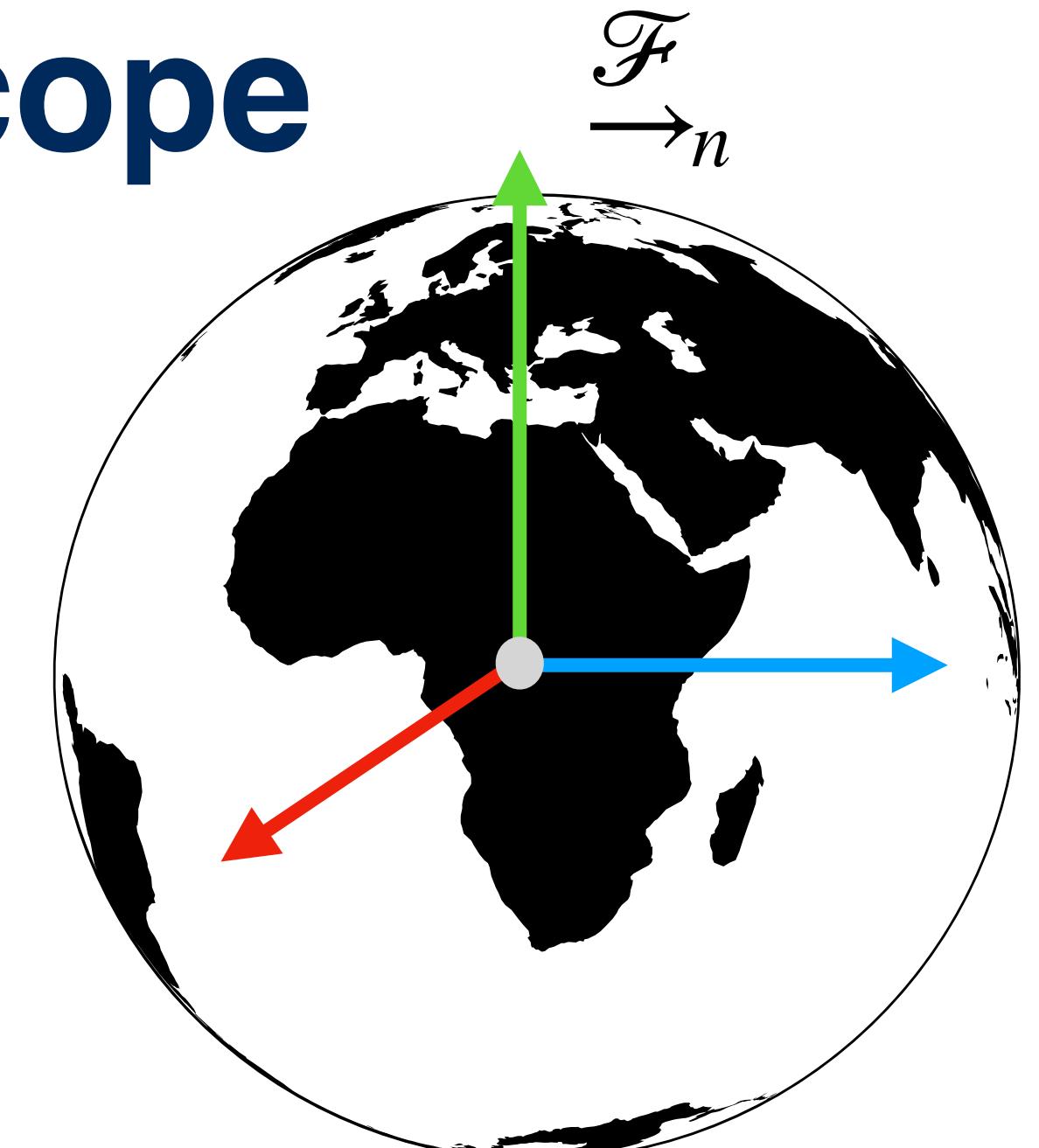
# Measurement Model: Gyroscope

$$\boldsymbol{\omega}(t) = \boldsymbol{\omega}_s(t) + \mathbf{b}_{\text{gyro}}(t) + \mathbf{n}_{\text{gyro}}(t)$$

$\boldsymbol{\omega}_s(t)$  angular velocity of the sensor expressed in the sensor frame.

$\mathbf{b}_{\text{gyro}}(t)$  slowly evolving bias

$\mathbf{n}_{\text{gyro}}(t)$  noise term



# Measurement Model: Accelerometer

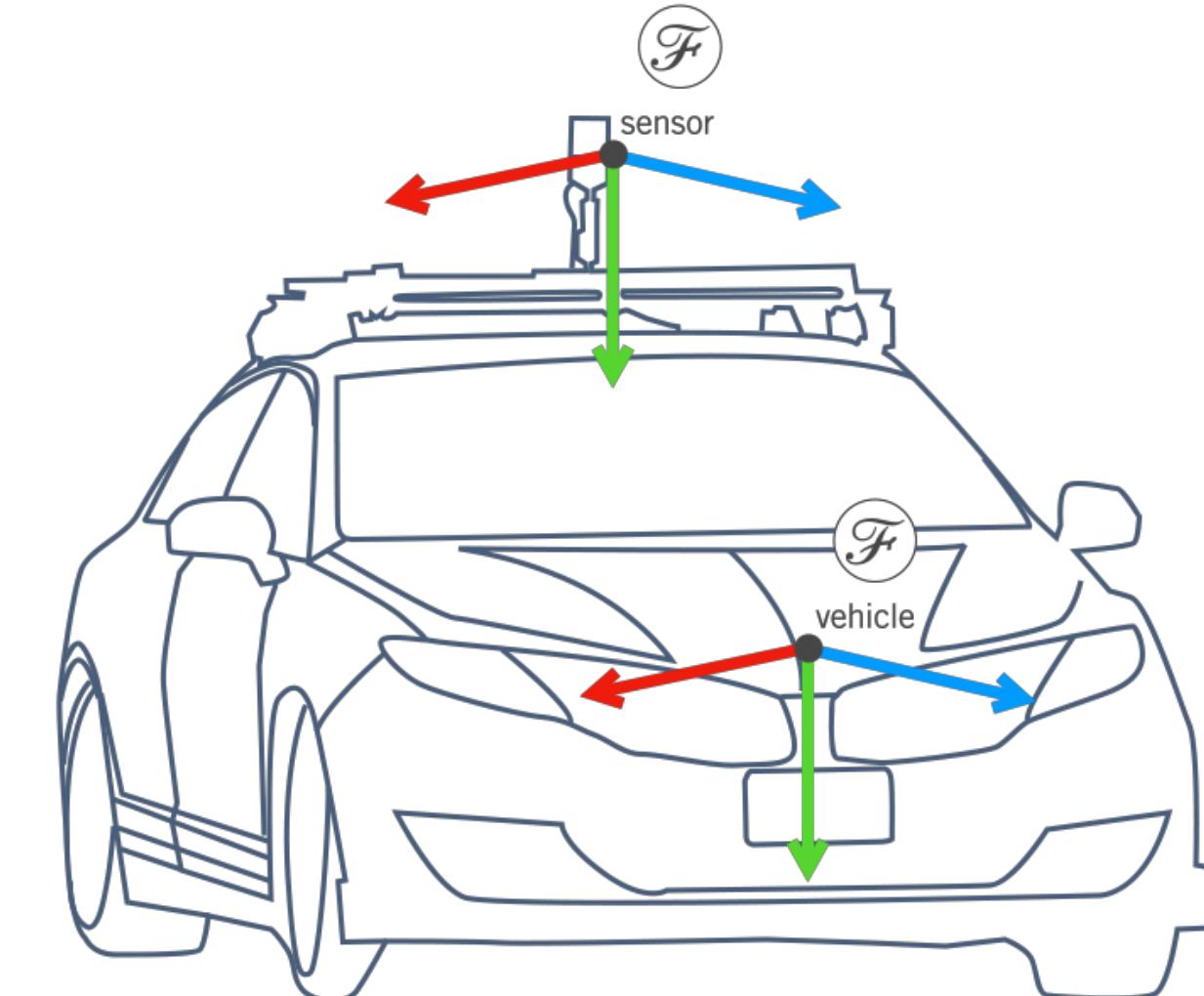
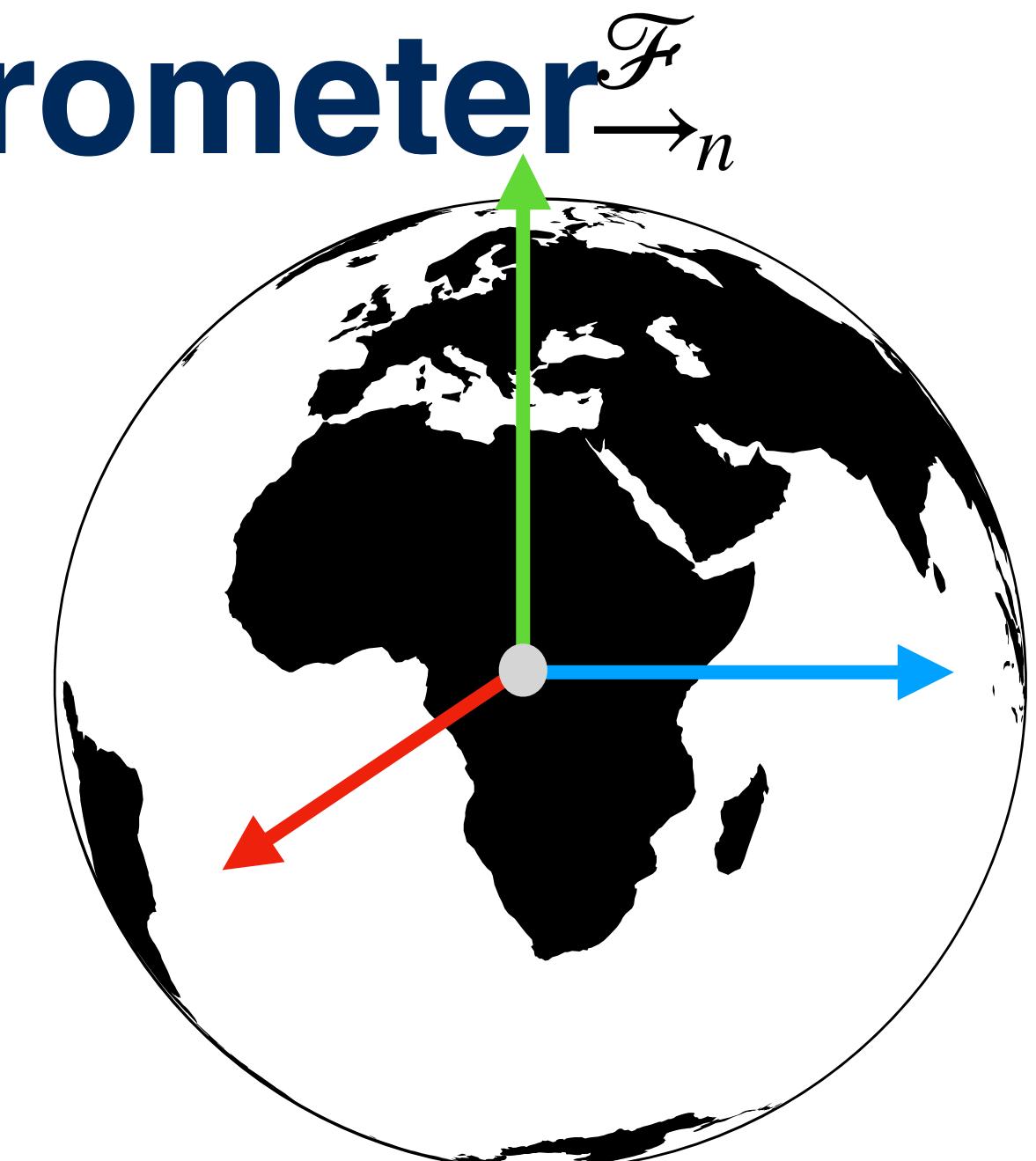
$$\mathbf{a}(t) = \mathbf{C}_{sn}(t)(\dot{\mathbf{r}}_n^{sn}(t) - \mathbf{g}_n) + \mathbf{b}_{accel}(t) + \mathbf{n}_{accel}(t)$$

$\mathbf{C}_{sn}(t)$  orientation of the sensor (computed by integrating the rotational rates from the gyroscope)

$\mathbf{b}_{accel}(t)$  bias term

$\mathbf{n}_{accel}(t)$  noise term

$\mathbf{g}_n$  gravity in the navigation frame



# Inertial Navigation: Important Notes

- When using an IMU for localization, keep in mind:
  1. If we inaccurately keep track of  $\mathbf{C}_{sn}(t)$ , we incorporate components of  $\mathbf{g}_n$  into  $\dot{\mathbf{r}}_n^{sn}(t)$ . This will ultimately lead to terrible estimates of position ( $\mathbf{r}_n^{sn}(t)$ ).
  2. Both measurement models ignore the effect of **Earth's rotation**.
  3. We only consider **strapdown IMUs** - where the individual sensors are rigidly attached to the vehicle and are not gimballed.

# Summary | The Inertial Measurement Unit

- A 6-DOF IMU is composed of three gyroscopes and three accelerometers, mounted orthogonally.
- A strapdown gyroscope measures a rotational rate in the sensor frame.
- A strapdown accelerometer measures a specific force (or acceleration relative to free-fall) in the sensor frame.