

Sex Ratio: The Secret of Nature

Summary

In nature, most species exhibit a 1:1 sex ratio, but sea lamprey is an exception. Its sex ratio varies with larval growth, influenced by food availability. We create a mathematical model to capture this unique sex ratio dynamic.

To simulate sea lamprey population dynamics, we create the **Sea Lamprey Population Model**. This model combines intrinsic and external factors by applying **Fisher's Principle** to represent natural selection factor and calculating external sex differentiation rates based on food availability-influenced growth rates. Given the **divergent ecological niches** occupied by sea lampreys at different stages of their lifecycle — acting as prey in the juvenile stage and as parasites in adulthood — we employ an age-structured **Leslie Matrix**. Then we introduce the **difference equation** of **Lotka-Volterra Model** to calculate the survival rate in Leslie Matrix.

To assess how the ability to alter sex ratios affects larger ecosystems, we simulated scenarios in **native habitats** and with sea lampreys as invaders in the **Great Lakes**. In native habitats, changing sex ratio capacity modestly increased the population of predators and parasitized fish numbers by **9.78%**. For the Great Lakes, we **exclude predators** and modify our original model to reflect the substantial impact on hosts. Results demonstrate that sex ratio adjustments of sea lamprey **accelerate** host declines and sustain high lamprey populations by reducing the impact of diminished food sources. This indicates that sex ratio adjustment significantly magnifies sea lamprey's invasive effects.

To evaluate the benefits and drawbacks to the sea lamprey population, we conduct simulations in environments with different stability. With abundant resources and stable circumstances, **56%** male ratio aligns with the data provided whereas a fixed sex ratio mirrors Fisher's theoretical value. This reveals that adjustment sex ratio increases sea lamprey flexibility beyond genetic constraints. However, in rapidly changing conditions, variable sex ratios cause larger population fluctuations, potentially **leading to extinction** under severe conditions.

To assess ecosystem stability, we focus on resilience stability by examining the dynamic of top trophic population in the food chain of sea lamprey. Results demonstrate a **38%** difference in population fluctuation between adjustable and fixed sex ratios, which indicates that sea lamprey's sex ratio changes make ecosystems less resilient to severe disturbances.

To assess the impact on other ecosystems, we expanded the Volterra model to include parasites competing with sea lamprey for hosts by adding **competitive terms**. Results show that sea lampreys with adjustable sex ratios contribute to a roughly **16.67% increase** in competitor numbers while their own populations remain the same. This benefits the stability of the corresponding ecosystems for the competitors.

Keywords: Fisher's Principle; Leslie Matrix; Lotka-Volterra Model; Difference equation; Sex ratio

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1 Introduction

1.1 Background

Species exhibit diverse sex ratios. While a 1:1 ratio is typical in nature, sea lampreys can alter their sex ratio in response to larval growth rate influenced by external circumstances including food availability[1]. Sea lampreys occupy a multifaceted ecological niche. As larvae, they prey on plankton and contribute to the decomposition of organic matter. In their adult phase, they assume a parasitic role, subsisting on the blood of other fish species[2]. In their native habitat, sea lampreys constitute a vital component of the food chain. Additionally, they are regarded as a delicacy in certain countries, contributing to the fishing industry. However, they also act as invasive species and pose a significant threat to local ecosystems, particularly in the Great Lakes[3]. Thus, analyzing the impact of sea lampreys' unique ability to adjust sex ratios based on environmental conditions is essential for understanding their interactions with the ecosystem.

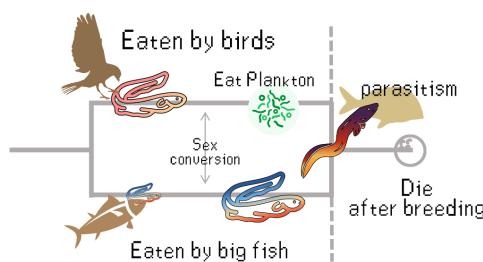


Figure 1: Life of sea lamprey



Figure 2: Photograph of sea lamprey

1.2 Restatement of the Problem

- Assess the impact of sea lampreys' adaptive sex ratio variation on ecosystems based on resource availability.
- Analyze how sex ratio alteration of sea lamprey affects the larger ecosystem's dynamic.
- Model the benefits and drawbacks of sex ratio variation strategy on sea lampreys' population.
- Discuss the impact that sex ratio adjustment has on ecosystem stability.
- Explore the potential benefits offered to other species by sea lampreys' sex ratio adjustment strategy.

1.3 Our Work

Our work is shown in figure 3.

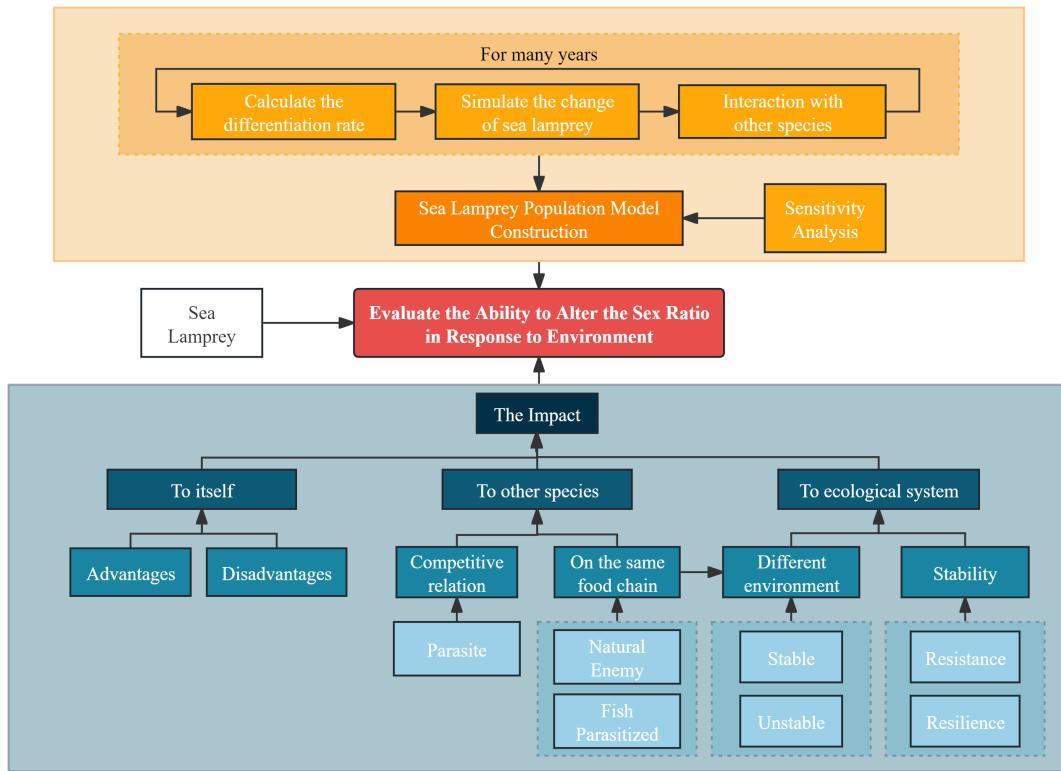
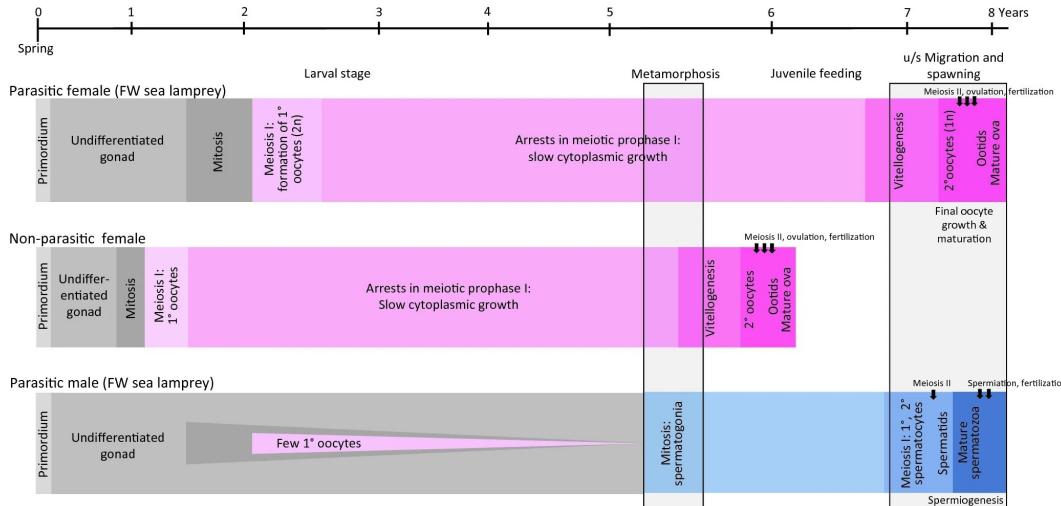


Figure 3: Models and Problems

2 Assumptions

Figure 4: "Timing of key events during lamprey gonadal development in a representative parasitic lamprey and in non-parasitic lampreys" (Partial), from Fig. 1.6 of *The Lamprey Gonad*[4].

Age Group (unit: year)								
	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8
Sex differentiation	Sexually undifferentiated		Immature female					
	Sexually undifferentiated (considered as male)			Immature male		Adult Male		
Feeding	Feeding on plankton				Parasitism			

Figure 5: Simplified timing of key events of a sea lamprey's life

Because sea lampreys are parasitic lampreys, and their parasitism occurs after metamorphosis[2], we produced a simplified timeline (see Figure 5) based on the parasitic portion of the lamprey's life timeline (see Figure 4) provided in *The Lamprey Gonad*[4]. Based on this simplified version, we then made assumptions 1 – 3; to simplify the problem and concentrate on dominant aspects of real-life situations, we also make assumptions 4 – 5, each of which is properly justified.

- **Assumption 1:** Sea lampreys act as prey for the first 0 – 5 years and as parasites during years 5 – 8.

Justification: We acknowledge that sea lampreys have a complex role in ecosystems without a clear transition in their function. Research indicates that during their larval stage, lampreys primarily consume plankton[5]. Because the food resources are abundant, their growth is mainly limited by predators. As adults, they face few predators and shift to predation.

- **Assumption 2:** Sea lampreys undergo sex differentiation only at ages 2 – 3.

Justification: We recognize that the problem of sex differentiation in sea lampreys is complex, such that sexual transitions are possible until metamorphosis. According to Figure 5, sea lampreys that do not differentiate into females at age 2-3 will eventually differentiate into males automatically. In order to model this more effectively, we made this assumption.

- **Assumption 3:** Adult sea lampreys die after breeding. We could exclude seniors from the population analysis.

Justification: Research on the life cycle of sea lampreys indicates they die shortly after spawning[5]. This assumption is realistic and simplifies the model. It highlights the impact of evolutionary strategies in sex ratio changes on the survival of sea lamprey populations[AI: 9].

- **Assumption 4:** The sex ratio of the sea lampreys is determined by both intrinsic genetic factors and extrinsic environmental conditions.

Justification: Based on R.A. Fisher's theory[6][AI: 6, 7, 8], our model assumes a natural selection-driven 1:1 sex ratio when the cost of raising sons and daughters is equal. Our model evaluates the costs of rearing both genders under natural selection and includes extrinsic factors like food availability and larval growth rates[1]. This method, considering multiple factors, more accurately reflects real-life conditions compared to single-factor models.

- **Assumption 5:** The birth rate of sea lampreys is influenced solely by the number of adult male and female fish, with the female fish having a greater impact.

Justification: In practice, birth rates could be affected by temperature and environmental resources, but these factors are outside our study's scope and are disregarded. Generally, males produce far more sperm than females do eggs, making the number of males a smaller factor. For sea lampreys, changes in sex ratios mean that the possibility of low male proportions. Thus, sperm availability could fall below egg numbers and we need to take male into account.

- **Assumption 6:** The survival rate of female is lower than male.

Justification: The exact mechanism by which sea lampreys alter sex ratios in response to their environment is controversial. Some literature states "[f]emales could experience higher mortality under growth-limiting conditions because of higher energetic demands during gonadal development." [1] This explanation is practical for modeling, so we adapt it.

3 Notations

We use the notations in Table 1.

Table 1: Notations used generally

Notations	Meanings
$\mathbf{x}^{(t)}$	distribution vector of sea lamprey population at year t
$x_i^{(t)} \quad i = 0, 1$	population of sea lampreys at age i to $(i + 1)$ at year t
$x_{if}^{(t)} \text{ or } x_{im}^{(t)} \quad i = 2, 3, \dots, 7$	population of male/female sea lampreys at age i to $(i + 1)$ at year t
$X^{(t)}$	total population of sea lampreys at year t
\mathbf{L}	Leslie matrix, indicating the relationship of the population between two years
$S_i^{(t)}, S_{if}^{(t)} \text{ or } S_{im}^{(t)}$	survival rate of sexually undifferentiated/female/male sea lampreys at age i to $(i + 1)$ at time t
b	birth factor
v_r	larvae's relative growth rate ($v_r \in (0, 1)$)
$A^{(t)}$	population of natural enemy at year t
A_{max}	maximum population of natural enemy
$Y^{(t)}$	population of fish parasitized by the sea lampreys at year t
Y_{max}	maximum population of fish parasitized
ρ_A	relative death rate of natural enemy
k_A	coefficient of sea lamprey's ability to feed natural enemy
ρ_Y	relative growth rate of fish parasitized
k_Y	coefficient of sea lamprey's ability to limit the growth of fish parasitized
R	abundance of environmental resources

r	intrinsic sex differentiation rate into males
λ	coefficient of the effect of intrinsic sex differentiation rate on the overall sex differentiation rate
f	sex differentiation rate into females
h	evaluation function for the degree of impact on two variables
$c_j \quad j = 1, 2, 3, 4$	coefficients describing relationship of sea lamprey's survival rate and related creatures

4 Sea Lamprey Population Model

4.1 Model Overview

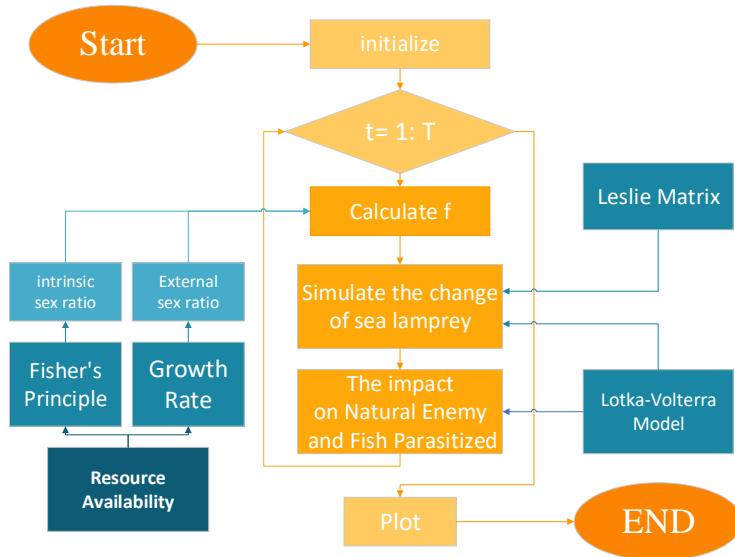


Figure 6: Simulation process of our model

To track long-term ecological changes, we have developed a model that annually calculates the sex transformation rate, sea lamprey population, and the populations of other species within the ecosystem.

The model considers both intrinsic and extrinsic factors for the sex differentiation rate. We use Fisher's Principle to model the intrinsic rate, shaped by natural selection and evolution, which generally stays stable in the short term. The growth rate is used to influence the extrinsic rate.

When it comes to the lamprey population, we acknowledge that the sea lamprey occupies different ecological niches at different life stages. Therefore, we use the Leslie matrix to calculate the population across different age groups. The model accounts for lampreys' dietary transition from consuming plankton to parasitically feeding on host blood, shifting from prey in their first five years to parasites in the following three years. Concurrently, we transform the Lotka-Volterra model's differential equations into difference equations. We use this to correlate the sea lamprey population with that of other relevant species in the ecological setting.

For estimating the population of other species, the Volterra model is similarly utilized to gauge the impact of lampreys on these species within the ecosystem.

This comprehensive approach allows us to understand inter-species relationships and the overall health and evolution of the ecosystem over time.

4.2 Modeling Sex Differentiation Rate

To study the impact of sex ratio alteration, it's essential first to understand why the sex ratio of most creatures is 1:1, which can be explained by the R.A. Fisher's principle[6][AI: 6, 7, 8]. Sea lampreys adapt their sex ratios to environmental shifts, yet the final ratio is determined by both intrinsic and external factors. We utilize a refined version of Fisher's theory to capture these intrinsic effects. We will going to prove this, using the notations in Table 2.

Table 2: Notations used in Fisher model

Notations	Meanings
H	maternal resource to be divided among sons and daughters for each parent
q	proportion of resources given to sons
C_m	the cost of one son, in terms of H
C_f	the cost of daughter, in terms of H
S_m	survivorship of a son to adulthood
S_f	survivorship of a daughter to adulthood
r	the primary ratio of male at birth

In case that child is given appropriate number of resources, each parent will have related survival rates of sons and daughters as follows:

$$m = \frac{HqS_m}{C_m} \quad (1a)$$

$$f = \frac{H(1-q)S_f}{C_f} \quad (1b)$$

From the perspective of inheritance and evolution, every individual wants to maximize their genes passed to their offsprings. Because daughter and sons both contain half the genes from each of their parents, we conclude that parents want to maximize the survival rates of offsprings, thus ensuring the greatest possible transmission of genes to the next generation.

$$mf = \frac{H^2 S_m S_f}{C_m C_f} q(1-q) \quad (2)$$

If the S_m, S_f, C_m, C_f do not interact with q , then Equation 2 reaches its maximum value at $q = \frac{1}{2}$, which means that distributing resources evenly between sons and daughters let parents gain optimal gene propagation. Thus, even distribution resources is selected as a significant evolutionary strategy.

Based on this, we could conclude that in order to let the resources evenly distributed, the following equations should be met:

$$rC_m = (1 - r)C_f \quad (3)$$

We now deduce the relationship between primary sex ratio at birth and the cost of female and male.

$$\frac{r}{1 - r} = \frac{C_f}{C_m} \quad (4)$$

In words this is

$$\frac{\text{Number of sons}}{\text{number of daughters}} = \frac{\text{cost of one daughter}}{\text{cost of one son}}$$

However, in Fisher's initial theory, he did not explain the exact meaning of the cost of sons and daughters. In our model we interpreted it to be composed of two parts:

1. Growth Cost G : the energy consumed by the sons and daughters to grow up. We can quantify this parameter by measuring the weight and length of female and male sea lampreys. Since for one sea lamprey to grow one unit of length or weight, the energy consumed is similar.
2. Propagation Cost P : the energy used by parents to propagate sons and daughters. In the application of our model, we could determine this parameter by collecting data related to spawning consumption.

Let R be the abundance of environmental resources. R is a parameter between 0 and 1, which may vary in response to different aspects, including food availability. Therefore

$$C_m = (1 - R)G_m + RP_m \quad (5a)$$

$$C_f = (1 - R)G_f + RP_f \quad (5b)$$

When the food resources are abundant, which means the R is larger, the sea lampreys will lay more eggs. Thus, the proportion of energy used on propagation is larger than the energy used on the growth, which means that the difference between propagation cost is a more dominant factor. On the contrary, when the environment condition is sever, the resource R is limited, and the larger proportion of food found is used for growth instead of reproduction.

We deduce that the genetical influence on the intrinsic sex ratio r satisfies Equation 4. Therefore we have

$$\frac{r}{1 - r} = \frac{(1 - R)G_f + RP_f}{(1 - R)G_m + RP_m} \quad (6)$$

In a stable environment with consistent larval growth rates, sea lampreys maintain an equilibrium sex differentiation rate (distinct from sex ratio) at ages 2 ` 3. However, sex differentiation at ages 2 ` 3 is also affected by larval growth velocity, which is primarily determined by environmental resources, R . Consequently, the overall sex differentiation rate into female f satisfies

$$f = \lambda(1 - r) + (1 - \lambda)v_r \in (0, 1) \quad (7)$$

where λ is the coefficient of the effect of intrinsic sex differentiation rate on the overall sex differentiation rate, and

$$v_r \propto R \quad (8)$$

4.3 Age-structured Population Matrix

Based on Patrick H. Leslie's framework for discrete, age-structured population dynamics[7], we have refined this model to specifically analyze the age and sex population distribution of sea lampreys. Our selection of this model is motivated by two key factors: firstly, the survival rates of male and female sea lampreys vary significantly across different age groups; secondly, literature indicates that sea lampreys do not present distinct sexes at birth, making the Leslie matrix an effective tool for incorporating sex differentiation rates into our analysis.

We group sea lampreys by age ($0 - 1, 1 - 2, \dots, 7 - 8$) and sex, with one group per year of sexually undifferentiated ones in the first two years ($0 - 2$) and two groups per year in the later six years ($2 - 8$), giving a total of 14 groups. To describe the population of sea lampreys in each group, we have a 14-dimensional vector, as shown below.

$$\mathbf{x}^{(t)} = (x_0^{(t)}, x_1^{(t)}, x_{2f}^{(t)}, x_{2m}^{(t)}, x_{3f}^{(t)}, x_{3m}^{(t)}, \dots, x_{7f}^{(t)}, x_{7m}^{(t)}) \quad (9)$$

And to describe the year-by-year change of the population of sea lampreys, we have the modified Leslie matrix

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 & bh(x_{7f}^{(t)}, x_{7m}^{(t)}) & b(1 - h(x_{7f}^{(t)}, x_{7m}^{(t)})) \\ S_0^{(t)} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & S_1^{(t)}f & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & S_1^{(t)}(1-f) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{2f}^{(t)} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{2m}^{(t)} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & S_{6f}^{(t)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & S_{6m}^{(t)} & 0 & 0 \end{bmatrix} \quad (10)$$

where b is the birth factor, $S_i^{(t)}$ is proportion of population that survive from i th age group to the $(i + 1)$ th age group. which is intrinsically linked to predator-prey dynamics. It is worth noting that $S_i^{(t)}$ varies from different sexual and age group. To ascertain $S_i^{(t)}$, we consider their hosts as prey and incorporate the Lotka-Volterra model with Leslie matrix. The utilization of Lotka-Volterra model and the definition of $S_i^{(t)}$ is to be discussed in Section 4.4. Variable f is the sex differentiation rate into females at age $2 - 3$, which is discussed in Section 4.2.

$$f = \lambda(1 - r) + (1 - \lambda)v_r$$

And h is an evaluation function for the degree of impact on two variables. The definition of h is shown below.

$$h(m, n) = \frac{1 + \tanh(\alpha \ln \frac{\beta n}{m})}{2} \in (0, 1) \quad \alpha > 0, \beta > 1 \quad (11)$$

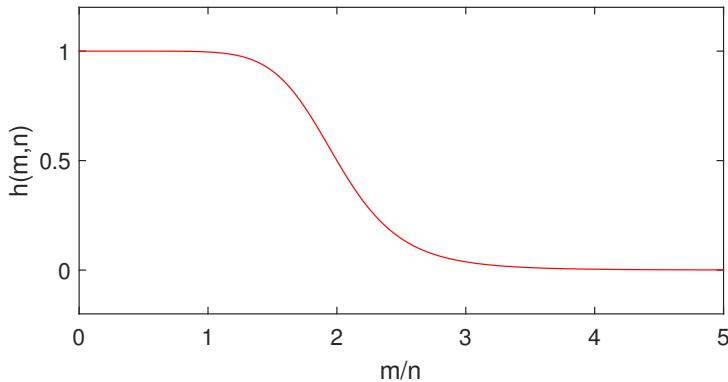


Figure 7: Example $h(m, n)$ at $\alpha = 4$ and $\beta = 2$

In the function, the larger $\frac{m}{n}$ is, the smaller $h(m, n)$ is. The larger α is, the more sensitive the evaluation function is. The larger β is, the more m is the dominant factor. In Formula 10, we substitute m with $x_{7f}^{(t)}$ (adult female) and n with $x_{7m}^{(t)}$ (adult male). Thus, h becomes a coefficient describing the dominance of adult females over fertility. The higher the female-to-male ratio, the more pronounced the limiting effect of the population of males and the less dominant adult females are, i.e., the smaller h is; and vice versa. β being greater than 1 makes adult females more dominant.

The relationship between sea lamprey populations between the two years can be succinctly written as Equation 12.

$$\mathbf{x}^{(t+1)} = \mathbf{L}\mathbf{x}^{(t)} \quad (12)$$

4.4 Simulating the Interaction with Predators and Hosts

Research indicates that once larval sea lampreys have metamorphosed, they become parasitic juveniles and feed on the blood of host fish[5]. We assume metamorphosis occurs in year five. In their early years, sea lampreys feed on abundant plankton. Predators, such as the European catfish[8], primarily limit their survival. As adults, sea lampreys become parasites with few natural predators. Their survival then depends on host population availability. We use two Lotka-Volterra Models to analyze their life stages: one for the prey-predator relationship, with sea lampreys as prey and another for the host-parasite relationship, with sea lampreys as parasites.

To make Lotka-Volterra Model matches with our Leslie matrix, we utilize the difference equation instead of differential equation. Then we have

$$A^{(t+1)} - A^{(t)} = -\rho_A A^{(t)} + \frac{k_A A^{(t)} \sum_{i=0}^4 x_i^{(t)}}{A_{max}} - \frac{\rho_A (A^{(t)})^2}{A_{max}} \quad (13)$$

$$Y^{(t+1)} - Y^{(t)} = \rho_Y Y^{(t)} - \frac{k_Y Y^{(t)} \sum_{i=5}^7 x_i^{(t)}}{Y_{max}} - \frac{\rho_Y (Y^{(t)})^2}{Y_{max}} \quad (14)$$

where $A^{(t)}$ denotes the predator population of juvenile sea lampreys at time t , the coefficient ρ_A represents the predator's mortality rate without prey, and k_A reflects how sea lamprey presence

reduces predator's death by providing a food source. A larger sea lamprey population increases predator growth and decreases mortality. A_{max} signifies the environment's carrying capacity for predators, and we deduce $A_{max} \propto R$. This approach adds a logistic term to the Volterra model, promoting system stability by considering self-limiting effects. Similarly, $Y^{(t)}$ presents the population of sea lampreys' hosts in their adulthood at time t , ρ_Y is the natural growth rate of the hosts, k_Y denotes the reduction in growth rate due to parasitism by sea lampreys, and $Y_{max} \propto R$.

Based on this, we could determine $S_i^{(t)}$ as follows:

$$S_i^{(t)} = (1 - c_1(A^{(t)}/A_{max}))(1 - X^{(t)}/X_{max})^{1/8}, \quad i = 0, 1 \quad (15a)$$

$$S_{if}^{(t)} = \begin{cases} (1 - c_2(A^{(t)}/A_{max}))(1 - X^{(t)}/X_{max})^{1/8} & , i = 2, 3, 4 \\ (1 + c_3(Y^{(t)}/Y_{max} - 1))(1 - X^{(t)}/X_{max})^{1/8} & , i = 5, 6, 7 \end{cases} \quad (15b)$$

$$S_{im}^{(t)} = \begin{cases} (1 - c_1(A^{(t)}/A_{max}))(1 - X^{(t)}/X_{max})^{1/8} & , i = 2, 3, 4 \\ (1 + c_4(Y^{(t)}/Y_{max} - 1))(1 - X^{(t)}/X_{max})^{1/8} & , i = 5, 6, 7 \end{cases} \quad (15c)$$

$$c_1, c_2, c_3, c_4 \in (0, 1), \quad c_1 < c_2, \quad c_3 > c_4$$

Where c_1 and c_2 describes relationship of sea lamprey's survival rate and its natural enemies during larval phase, whereas c_3 and c_4 describes relationship of sea lamprey's survival rate and its host after metamorphosis. The conditions $c_1 < c_2$ and $c_3 > c_4$ assure that survival rate of female is less than male. $X_{max} \propto R$, considering the relationship between abundance of environmental resources and the maximum population. Factor $(1 - X^{(t)}/X_{max})^{1/8}$ describes the effect of maximum environmental carrying capacity.

5 Application of Sea Lamprey Population Model

5.1 Impact of Sex Ratio Alteration on Large Ecosystem

5.1.1 Simulation Under Native Environment

Ecosystems consist of two components: biotic and abiotic factors. To assess the impact of sea lampreys on the ecosystem, it's crucial to consider both. Biotically, sea lampreys interact with their predators during juvenile stages and with their hosts in adulthood.

In a stable ecosystem with constant resource levels where R is a constant, we conducted simulation of sea lamprey population dynamics. We simulate community evolution over four hundred years with annual time steps.

To evaluate the impact of adaptive sex ratio variation, we adjust λ in Equation 7. If λ is close to 1, the intrinsic sex ratio is dominant factors and the growth rate have negligible influence on sex differentiation rate. We regard this case as that the sea lampreys cannot alter their sex ratio based on growth rate. On contrary, we regard it as sea lampreys can adjust their sex ratio when lambda approaches 0.

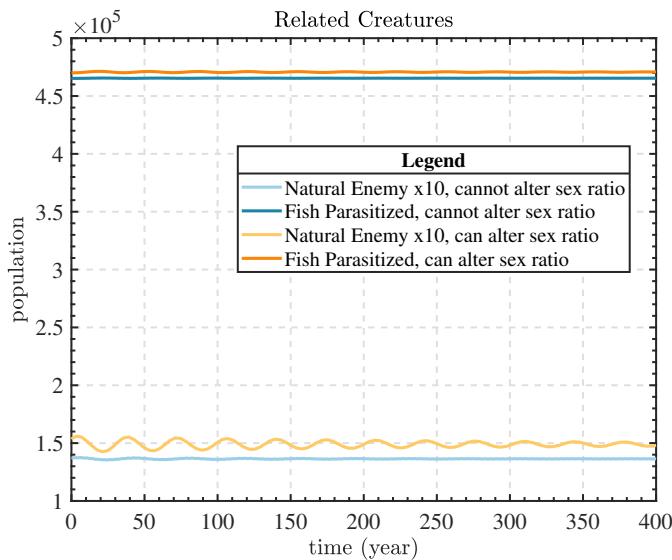


Figure 8: Related creatures in four hundred years

Figure 8 suggests that variations in the sex ratio leads to a marginal increase in the stable populations of both predators and hosts. This finding implies that the sex ratio variability of the sea lamprey confers benefits on other species via food chain interactions. An increase in the stable populations of predators and parasitized species enhances biomass and productivity within the ecosystem's food chain. This enhancement allows the ecosystem to better buffer from external disturbances, thereby maintaining ecological balance more effectively.

On the other hand, the relatively marginal difference in the numbers of predators and parasitized species suggests that the impact of sea lampreys is limited, approximately 9.78%. This result aligns with reality since parasites typically do not directly kill their hosts, and predators have multiple food sources. Thus, changes in a single food source do not significantly affect their numbers. The primary factor determining the equilibrium numbers of predators and parasitized species should be the environment, with the interaction between sea lampreys and these species being a secondary factor.

As shown in Figure 9, the population fluctuations of sea lampreys are less stable if they can alter sex ratio in response to external environment. In stable populations, lampreys with variable sex ratios exhibit fluctuation amplitudes of 1.09, surpassing the 1.01 in those with fixed ratios. In this case, sex ratio alteration is detrimental to the ecosystem, as larger fluctuations in a species' population can lead to instability within the food chain. The oscillations of sea lampreys will possibly amplify through their interactions beyond predator-prey dynamics, such as competitive relationships. This functional will potentially lead to ecosystem instability.

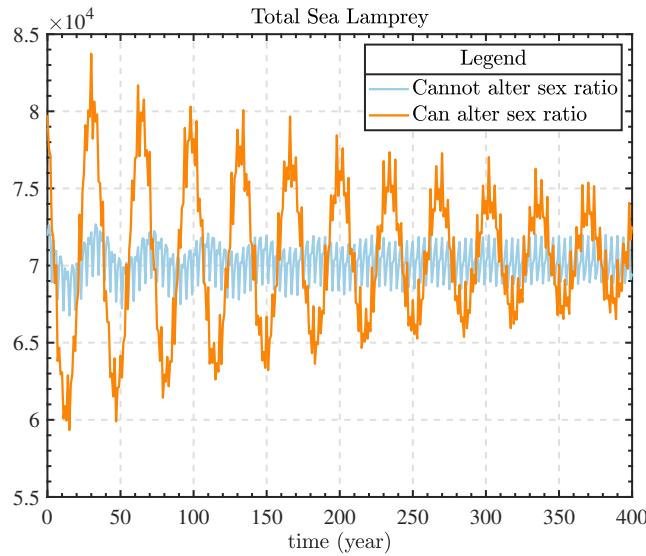


Figure 9: Total population of sea lampreys in four hundred years

5.1.2 Simulation as Invasive Species in Great Lakes

In the Great Lakes, sea lampreys are considered a significant pest. In these freshwater environments, they did not coevolve with local fish species, making the native fish highly vulnerable to lamprey predation. The sea lamprey invasion drastically reduced Great Lakes fishery catches from 15 million to 300 thousand pounds, severely harming the regional economy and ecosystem[3].

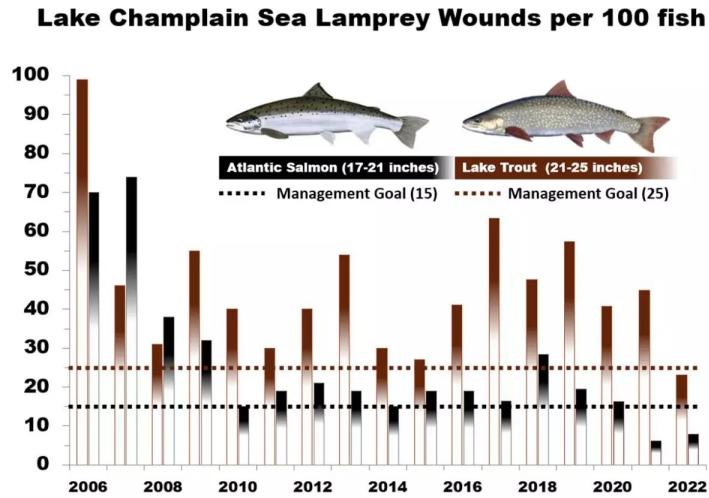


Figure 10: Sea lampreys' impact on the Great Lakes

To model the scenario of sea lampreys as invasive species, we adjust our model slightly. We omit the first Lotka-Volterra equation (Equation 13), which simulates the interaction between lampreys and their predators, due to the lack of natural predators in the Great Lakes. Additionally, we elevate the coefficient k_Y which denotes the sea lampreys' predation ability to limit the growth of

parasitized fish. We also shorten the simulation to one hundred years to capture the swift impact of invasive species. We obtain the following charts.

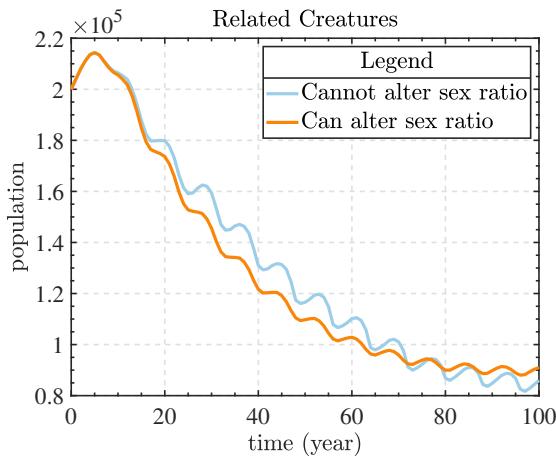


Figure 11: Population change of parasitized fish in the Great Lakes

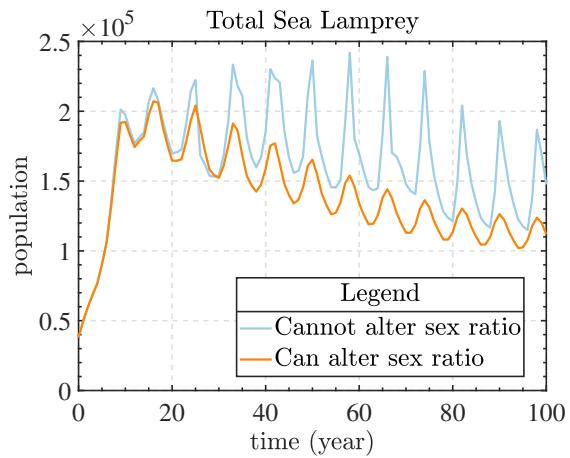


Figure 12: Total population of sea lampreys in the Great Lakes

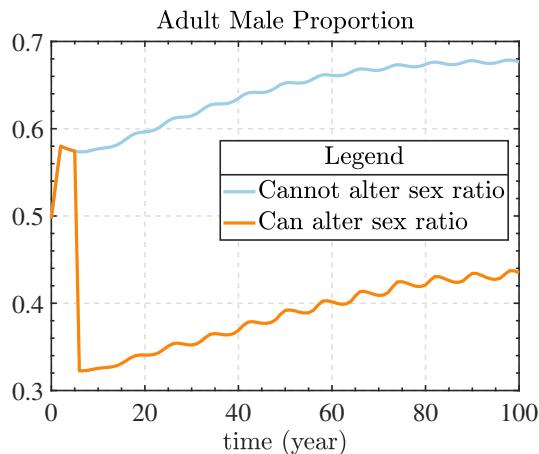


Figure 13: Male proportion of adult sea lampreys in the Great Lakes

Figure 11 illustrates a rapid decrease in parasitized fish populations, aligning with typical biological invasion patterns. Notably, the decline also accelerates at a greater magnitude, indicating that sea lampreys adapt more swiftly to environmental shifts, like resource abundance or predator reduction. This adaptability enhances their invasive impact, detrimentally affecting the ecosystem.

In Figure 12, sea lampreys exhibit an exponential growth phase within the first ten years. It is indicative of natural expansion in predator-free environments with abundant resources. This growth is later constrained by the reduced availability of host fishes. Populations capable of altering their sex ratio surpass those that cannot. By adjusting their sex proportions in response to growth rates, sea lampreys buffer the host scarcity's impact, thus sustaining higher population levels. However, this flexibility leads to increased population volatility.

Figure 13 demonstrates that with the capability of adjusting sex ratio, sea lampreys significantly reduce proportion of male to less than 0.5. This strategy increases the female proportion and enhance the offsprings production under optimal environment condition, thereby amplifying their invasive impact.

5.2 How Sex Ratio Alteration Influence Sea Lamprey Population

We conducted simulations in two distinct environments: a stable environment with a constant resource R value, and an unstable environment with a time-varying resource R value. This analysis evaluates the effect of sex ratio alteration strategies on sea lamprey populations within various circumstances.

5.2.1 Simulation Under Stable Environment

As with section 5.1.1, we simulated the population dynamics and sex ratios of sea lampreys over four hundred years in a stable condition. We obtain Figures 14 and 15.

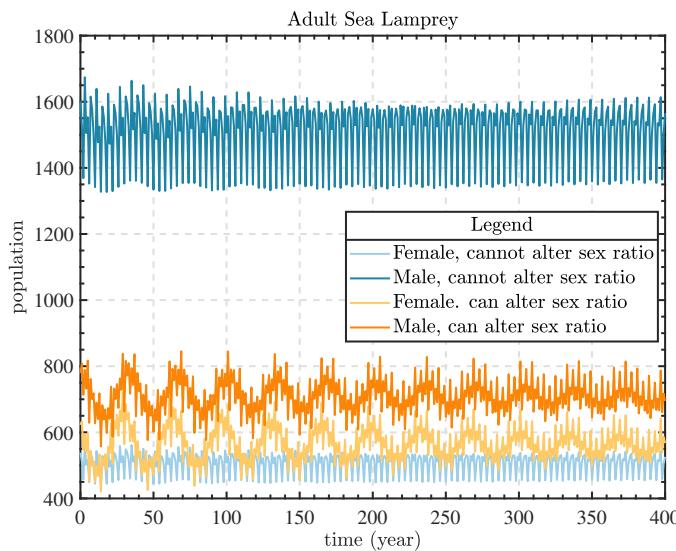


Figure 14: Population of adult sea lampreys under stable condition over time

We simulate under conditions with an environmental resource coefficient $R = 0.8$, indicating abundant resources and high food availability. This setup leads to a higher larval growth rate and male differentiation rate. The observed sex ratio at 56% closely aligns with our model's result represented by the blue line in Figure 15, validating our model's accuracy. Without the capacity for sex ratio alteration, the population's sex ratio gravitates towards a genetically determined equilibrium. This (74.9%) is close to the Fisher model's outcome of 78.54%, which is calculated as $rS_m/(rS_m + (1 - r)S_f)$, where S_m is overall survival rate of male and S_f of female. We conclude that adjustment of the sex ratio in response to growth rates benefits sea lampreys. This strategy offers sea lampreys a flexible and rapid response to environmental changes, without being entirely constrained by genetic factors. Such adaptability is vital for sea lampreys' survival and reproduction in moderately fluctuating environments.

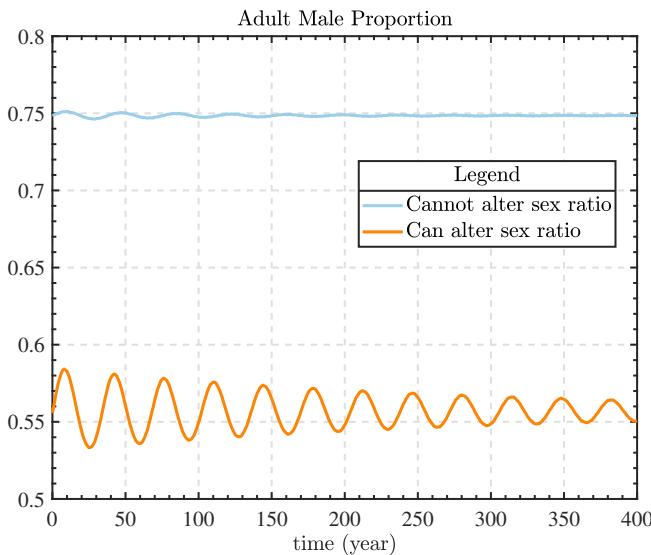


Figure 15: Male proportion of adult sea lampreys under stable condition over time

However, the ability to adjust the sex ratio results in longer stabilization times and greater fluctuation magnitudes for sea lamprey populations, with notably low numbers at fluctuation troughs. This vulnerability increases the risk of population extinction under additional adverse effects, which is harmful to the population of sea lampreys.

5.2.2 Simulation Under Varied Circumstances

To model the situation where the environmental resource coefficient R varies with time t , we defined $R = 0.8$ over the time interval $[0, 10]$ and at $t = 10$, the R suddenly decreases to 0.2 and after another 10 years, at time $t = 20$, the R gradually increase back to 0.8. This fluctuation implies the sudden change of environment caused by factors like human activities and environment change.

To simulate scenarios where the environmental resource coefficient R fluctuates over time, we set $R = 0.8$ for the interval $[0, 10]$. At $t = 10$, R abruptly drops to 0.2, reflecting a sudden environmental shift possibly due to human activities or ecological changes. After a decade, at $t = 20$, R progressively returns to 0.8, modeling the gradual recovery of the environment. This approach captures the dynamic impact of external factors on resource availability.

Figure 16 clearly illustrates the disadvantages associated with the capacity to alter sex ratios in response to larval growth rate and R . This capacity increases sensitivity of sea lamprey's population to environmental changes. It leads to a significant male population predominance over females. Such an imbalance severely restricts reproductive capacity, precipitating a steep decline in population levels. Within a single lifecycle (8 years), populations can plummet to near-extinction levels before any environmental improvement. The sex ratio alteration hinders the recovery of population under rapidly change environments and potentially leads to extinction.

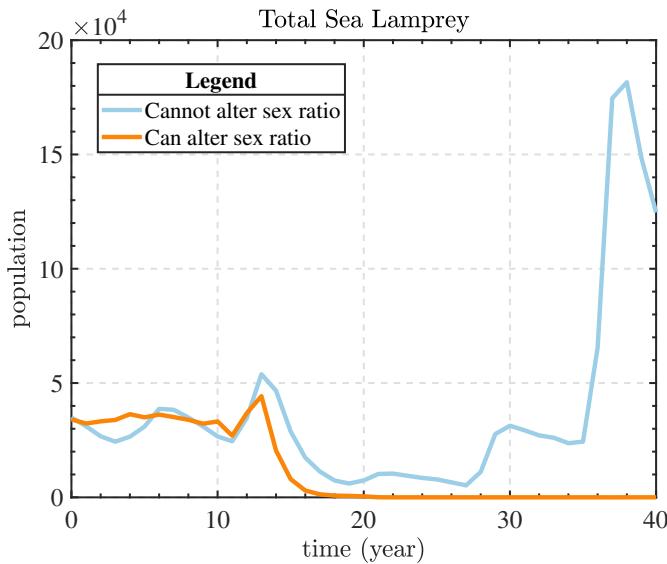


Figure 16: Total population of sea lampreys under varied circumstances over time

5.3 The Impact on Ecosystem Stability

The stability of ecosystems is measured by two characteristics: resilience stability and resistance stability. Resilience stability refers to the ability of an ecosystem to return to its original state or reach a new stable state after being severely disturbed. Resistance stability is the capacity of an ecosystem to maintain its structure and functions without significant changes under the influence of external disturbances. Since resistance stability is characterized by the absence of significant changes, it is challenging to reflect in simulation results. Therefore, our analysis focuses on resilience stability.

At $t = 50$, we reduced the number of hosts to simulate ecosystem disruption caused by natural disasters, impacting the lower trophic levels (parasitized fishes). This impact is expected to cascade up the food chain of the sea lamprey. To assess ecosystem stability, we analyzed the stability of the highest trophic level (predators of the sea lamprey). Typically, the highest trophic level is the least stable and most susceptible to fluctuations. Therefore, we assume that the variability of the highest trophic level can effectively reflect the overall stability of the ecosystem.

The result clearly shows that when the sex ratio of sea lamprey can be altered, the fluctuation at the highest trophic level is significantly greater than when it cannot be altered, with a difference reaching 38%. This indicates that the ability of the sea lamprey to change its sex ratio is detrimental to the resilience stability of the ecosystem. The food chain corresponding to the sea lamprey requires a longer time to return to its original stable equilibrium.

5.4 The Impact on Other Ecosystems

To assess the ecological impact of sea lamprey sex ratio alteration, our model integrates parasite competitors that share large fish hosts with sea lampreys. This analysis clarifies the ecological benefits of adaptive sex ratio variation.

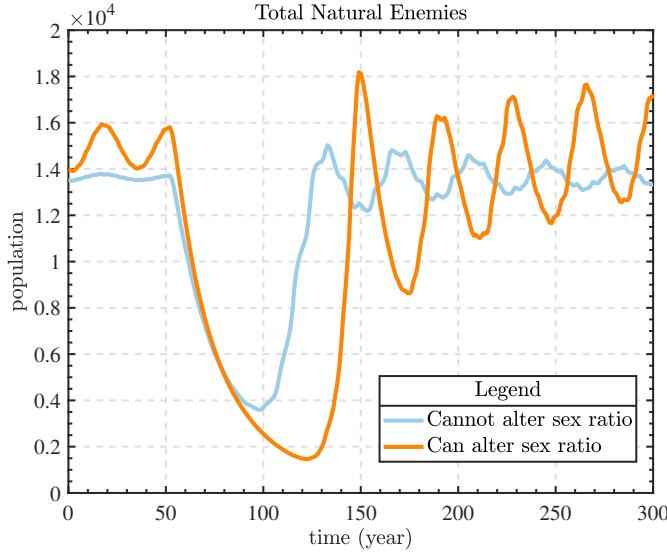


Figure 17: Population of total natural enemy (i.e., the highest trophic level) over time

We define the competitors as other parasites feeding on the same fish as sea lampreys. We simulate the impact of competitors by adding the formula describing the competitors in our Lotka-Volterra Model, and adjusting the formulas of related creatures, i.e. sea lampreys and parasitized fish.

Similar to the previous description of natural enemies and parasitized fish, let $B^{(t)}$ denote population of the parasite competitor at time t , B_{max} denote its maximum population, ρ_B denote relative death rate of the competitor, and k_B denote coefficient of parasitized fish's ability to feed the competitor. Additionally, let γ_1 be competition coefficient of the competitor against sea lamprey and γ_2 be competition coefficient of the opposite direction. Thus, we have Equations 16 as adjustments of Equations 15:

$$S_{if}^{(t)} = \left(1 + c_3 \left(\frac{Y^{(t)}}{Y_{max}} - 1 \right) - \gamma_1 \frac{B^{(t)}}{B_{max}} \right) \left(1 - \frac{X^{(t)}}{X_{max}} \right)^{1/8}, i = 5, 6, 7 \quad (16a)$$

$$S_{im}^{(t)} = \left(1 + c_4 \left(\frac{Y^{(t)}}{Y_{max}} - 1 \right) - \gamma_1 \frac{B^{(t)}}{B_{max}} \right) \left(1 - \frac{X^{(t)}}{X_{max}} \right)^{1/8}, i = 5, 6, 7 \quad (16b)$$

and the new equation for the competitor:

$$B^{(t+1)} - B^{(t)} = -\rho_B B^{(t)} + \frac{k_B B^{(t)} \sum_{i=5}^7 x_i^{(t)}}{Y_{max}} - \frac{\rho_B (B^{(t)})^2}{B_{max}} - \frac{\gamma_2 B^{(t)} \sum_{i=5}^7 x_i^{(t)}}{X_{max}} \quad (17)$$

and adjusted equation for fish parasitized as shown below, since its population is affected by the dynamics of both sea lampreys' and their competitors' populations:

$$Y^{(t+1)} - Y^{(t)} = \rho_Y Y^{(t)} - \frac{k_{Y1} Y^{(t)} \sum_{i=5}^7 x_i^{(t)}}{Y_{max}} - \frac{k_{Y2} Y^{(t)} B}{Y_{max}} - \frac{\rho_Y (Y^{(t)})^2}{Y_{max}} \quad (18)$$

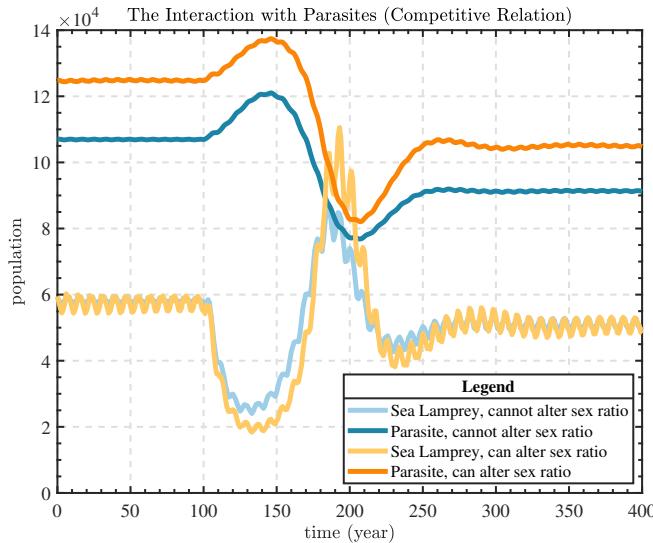


Figure 18: Population of sea lampreys and its competitor in four hundred years

where k_{Y1} denotes sea lamprey's influence while k_{Y2} represents the competitors' impact.

In our simulation, we reduced environmental resource abundance from $R = 0.8$ to $R = 0.7$ at time $t = 100$. As shown in Figure 18, the results indicate that, with the ability to alter sex ratios, the stable number of competitors increased under the same population of sea lamprey. Specifically, there was a increase of 19.05% at $R = 0.8$ and a increase of 16.67% at $R = 0.7$ compared to scenarios without sex ratio alteration. This increase benefits ecosystems associated with the competitors, suggesting more efficient use of environmental resources and higher energy transfer efficiency. It enhances the ability to support other organisms in the corresponding food chain, especially the predators of the competitors. Thus, the sex ratio alteration capability of sea lamprey contributes to the stability of the food chains in other ecosystems.

Although the number of competitors fluctuates more in orange curve when environmental resources change, the trough of fluctuation remains above the blue curve depicted the situation without alteration. Additionally, the time to reach stability is the same in both scenarios.

6 Sensitivity Analysis

To analyze the sensitivity of our model, we varied c_3 , which represents the magnitude of female parasitic capabilities. A larger c_3 indicates weaker parasitic ability and poorer utilization of parasitic capacity. Since this value is difficult to quantify, we estimated its value during the model simulation process. However, this parameter affects the sex ratio and has neccessity for an examination of its sensitivity.

In modeling process, we set parameter $c_3 = 0.6$, now we slightly modify it around 0.6 to observe changes in the sex ratio. Furthermore, at $t = 50$, we altered environmental resources to examine the differences in fluctuations.

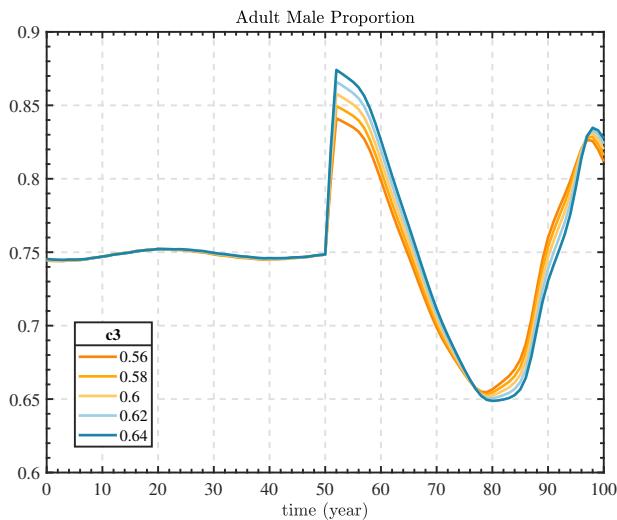


Figure 19: Result of sensitivity analysis focusing on the coefficient c_3 and the change on adult male proportion.

From the result shown in Figure 19, variations in c_3 within a certain range have a minor impact on the stable sex ratio. For fluctuations, when the value of c_3 changes by 3.3%, the magnitude of fluctuations differs by 0.95% around our setting $c_3 = 0.6$. This indicates that estimating this value within a reasonable range is still reliable for our model.

7 Strengths and Weaknesses

7.1 Strengths

- Our method integrates intrinsic genetic and external environmental influences on sea lamprey sex ratios, offering a detailed and comprehensive view for forecasting their long-term dynamics.
- We analyze interspecies interactions through food chains to indirectly reflect ecosystem impacts, offering a strategy for studying the elusive metric of "impact."
- We analyze the impacts of sea lamprey sex ratio strategies in diverse settings like native habitats, as Great Lakes invaders, in stable circumstance and changing environments, which provides insights for targeted population management.

7.2 Weaknesses

- Integrating the Lotka-Volterra Model with Leslie Matrix results in nonlinear difference equations. With characteristics like bifurcation and chaos, it is distinct from differential equations. The non-convergence nature of equilibrium points can cause model simulations to be unstable and unpredictable.

- Our approach of utilizing an age-structured Leslie Matrix results in newborn numbers being set solely by the oldest age class and population fluctuations of 7-year cycles. We extended the simulation period to reduce this effect.
- It is difficult to quantify the costs of producing son and daughter in our modified Fisher's model, potentially causing discrepancies between the calculated intrinsic sex ratio and reality and impacting simulation accuracy.

8 Conclusion

Advantages of sex ratio alteration:

- The lamprey's variable sex ratio capability boosts the stable predator and host numbers. Consequently, it increases the biomass and stability of sea lamprey's ecosystem.
- Adapative sex ratio alteration overcomes genetic limits and enables swift adaptation to environmental shifts, enhancing survival in stable environments and resistance to minor, short-term disturbances.
- The lamprey's variable sex ratio allows for an increase in the stable numbers of competing parasites within a reasonable range, without altering its own population, thereby enhancing the stability of the competitor's ecosystem.

Disadvantages of sex ratio alteration:

- Variable sex ratios in populations cause large fluctuations, with low troughs increasing the risk of extinction under sever environmental deteriorations. [AI: 3]
- As an invasive species, lamprey capable of altering sex ratio accelerates the decline of host fish populations and maintain higher levels of their own population even when food resources sharply decrease, significantly impacting the invaded ecosystems.
- With adaptable sex ratios, the recovery time increases and resilience stability of ecosystems reducenc, under severely deteriorated conditions.

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Appendices

Appendix A MATLAB program

Here is the simulation program we use.

```

%% ... Initialize (omitted)

%% ... Calculate and record initial values to be plotted (omitted)

for t = 1:T
    %% Adjust Resource if needed (omitted)

    % Calculate survival rate S(i)
    for i = 1:2
        S(i) = 1 - c1 * (A / Amax); % linear processed
    end
    for i = 3:2:7
        S(i) = 1 - c2 * (A / Amax);
        S(i+1) = 1 - c1 * (A / Amax);
    end
    for i = 9:2:11 % (change in section 5.4)
        S(i) = (1 + c3 * (Y / Ymax - 1));
        S(i+1) = (1 + c4 * (Y / Ymax - 1));
    end
    S = (1 - X / Xmax)^(1 / 8) * S;

    %% .. Update v_r (omitted)

    h = (1 + tanh(Alpha * log(Beta * x(14) / x(13)))) / 2;
    f = lambda * (1 - r) + (1 - lambda) * v_r; % Sex Differantiate Rate

    %% ... Update Matrix L in terms of S, h, and f (omitted)

    x = L * x; % Calculate population next year

    % Volterra Model with logistic terms (change in section 5.4)
    A = A - rhoA * A + kA * A * X1 / Amax - rhoA * A * A / Amax;
    Y = Y + rhoY * Y - kY * Y * X2 / Ymax - rhoY * Y * Y / Ymax;

    % Update X1 = sum(x0 .. x4m), X2 = sum(x5f .. x7m), and X
    X1 = sum(x(1:8)); X2 = sum(x(9:14));
    X = X1 + X2;

    %% ... Record data to be plotted (omitted)

end

%% ... Plot (omitted)

```

Report on Use of AI

1. In our COMAP's mathematical modeling competition, we utilized LLMs, particularly *ChatGPT-4*, to support our model development. Our use of GPT included:
 1. Researching information through plugins like WebPilot and Scholar AI;
 2. Translation of English literature into Chinese and converting our articles written in Chinese to English
 3. Language polishing to enhance the readability of our articles.

As T.S. Eliot's query, "Where is the wisdom we have lost in knowledge? Where is the knowledge we have lost in information?" This poem emphasizes our aim: to transform information into knowledge, use that knowledge to foster wisdom, solve problems, and foresee the future. In mathematical modeling competitions, which showcase human intellect and creativity, AI serves as a bridge between information and knowledge. This allows us to leverage our wisdom and creativity rather than exhaustively searching for information. Instead of rejecting AI over concerns of objectivity, accuracy, and copyright, we advocate for its judicious use to boost our team's efficiency during the four-day competition.

To ensure the academic integrity and transparency of our team's work, we responsibly wrote an AI Use Report to provide a detailed account of how we utilized artificial intelligence in our work.

- Research

We utilized AI to search relevant information related to sea lampreys, including data and literature. Recognizing the potential for LLMs to produce inaccurate literature, we verified the authenticity and reliability of all data and literature used to construct our model. Rather than directly accepting information from ChatGPT, we preferred to use it to guide us to sources where data and materials could be found, which ensures a thorough validation process.

The following are the inputs and outputs of ChatGPT-4. The use for research purposes is primarily during the first and second days. We only list the main questions and omit follow-up discussions. We promise that the follow-up inquiries will focus on the same topic addressed as the main questions.

2. OpenAI *ChatGPT* (Feb, 2, 2023 version, ChatGPT-4)

Query: Given these 4 questions, what is their difference? We should answer them from what aspects?

- What is the impact on the larger ecological system when the population of lampreys can alter its sex ratio?
- What are the advantages and disadvantages to the population of lampreys?
- What is the impact on the stability of the ecosystem given the changes in the sex ratios of lampreys?

- Can an ecosystem with variable sex ratios in the lamprey population offer advantages to others in the ecosystem, such as parasites

Output: The four questions you've posed about the impact of variable sex ratios in lamprey populations on ecological systems and the ecosystem's stability, as well as the potential advantages or disadvantages, each require a nuanced approach to answer. Here's a breakdown of their differences and the aspects from which we should answer them:

1. What is the impact on the larger ecological system when the population of lampreys can alter its sex ratio?
 - Aspect of Inquiry: This question asks for a broad understanding of how changes in lamprey sex ratios affect the ecological system at large. It encompasses looking into how such alterations might influence biodiversity, predator-prey relationships, competition, and the overall health of the ecosystem.
 - Answer Approach: Investigate ecological theories and empirical studies that detail the role of sex ratios in population dynamics and ecosystem function. This includes examining how changes in sex ratios can lead to shifts in genetic diversity, species interactions, and ecosystem resilience.
2. What are the advantages and disadvantages to the population of lampreys?
 - Aspect of Inquiry: This question focuses on the internal population dynamics of lampreys, considering how varying sex ratios might benefit or harm their population. It involves analyzing reproductive success, survival rates, and potential for population growth or decline.
 - Answer Approach: Review literature on sexual selection, reproductive strategies, and their ecological and evolutionary implications for lampreys. This includes understanding how sex ratio variations can affect mating systems, fecundity, and population viability.
3. What is the impact on the stability of the ecosystem given the changes in the sex ratios of lampreys?
 - Aspect of Inquiry: Similar to the first question but with a specific focus on ecosystem stability. This question seeks to understand how fluctuations in lamprey sex ratios might influence the equilibrium of the ecosystem, potentially affecting its resilience to disturbances and its capacity to maintain biodiversity.
 - Answer Approach: Explore studies on ecosystem stability and resilience, particularly those that model the impact of key species and their traits (like sex ratios) on ecosystem dynamics. This involves assessing how changes in lamprey populations might cascade through trophic levels and affect ecosystem services.
4. Can an ecosystem with variable sex ratios in the lamprey population offer advantages to others in the ecosystem, such as parasites?
 - Aspect of Inquiry: This question shifts the focus to interspecies relationships and the broader ecological community. It asks how lamprey sex ratio variability might confer benefits or disadvantages to other species within the ecosystem, including parasites that interact with lampreys.

- Answer Approach: Investigate ecological and evolutionary interactions between species, focusing on mutualism, parasitism, and commensalism. This includes examining how sex ratio dynamics in a host population (lampreys) can influence parasite transmission, diversity, and evolutionary strategies.

Each question requires a distinct analytical lens, ranging from internal population dynamics and reproductive strategies to broader ecosystem impacts and interspecies interactions. Answering these questions comprehensively would involve synthesizing knowledge from ecological theory, evolutionary biology, and empirical research on lampreys and their ecosystems.

3. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query: For a species, is the ability of changing sex ratio according to resource availability good or bad?

Output: The ability of a species to change its sex ratio according to resource availability is a fascinating aspect of biological adaptation that can have both positive and negative implications, depending on the context and perspective from which it is viewed. This ability is known as sex ratio adjustment or sex allocation and is observed in various species, including some plants, fish, and insects. Here are some considerations on whether it is good or bad:

Positive Aspects (Good)

Enhanced Reproductive Success: Changing the sex ratio in response to environmental conditions can maximize reproductive success. For example, in conditions where one sex has a higher reproductive value due to resource availability, increasing the proportion of that sex can enhance the population's growth or stability.

Population Regulation: This ability can help regulate population size in a way that is sustainable with the available resources, potentially preventing overpopulation and its associated problems.

Adaptive Evolution: The flexibility in sex ratio adjustment can be seen as an evolutionary strategy that allows species to adapt to changing environmental conditions, enhancing their survival and reproductive success over generations.

Resource Optimization: By adjusting the sex ratio, a species can optimize the use of available resources, ensuring that the most reproductively valuable individuals are supported.

Negative Aspects (Bad)

Population Imbalance: In some cases, extreme shifts in sex ratios can lead to population imbalances that may affect the long-term viability of a species, especially if one sex becomes significantly outnumbered.

Genetic Diversity: A skewed sex ratio might reduce genetic diversity if a smaller proportion of the population is contributing genetically to the next generation. This reduction in genetic diversity can make the population more susceptible to diseases and reduce its ability to adapt to environmental changes.

Social Structure Disruption: For species with complex social structures, significant changes in sex ratios can disrupt social dynamics, potentially leading to increased competition, aggression, or other social issues.

Ecological Impact: Changes in the sex ratio of a species can have cascading effects on the ecosystem, potentially disrupting predator-prey relationships, competition, and other ecological interactions.

In summary, the ability to change sex ratios according to resource availability can be advantageous in terms of adaptation and survival. However, it can also pose risks to population stability and genetic diversity. The impact of such changes often depends on the specific ecological context, the degree of sex ratio variation, and the species in question. Would you like to explore scientific literature for more detailed insights on this topic?

4. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query: Where can I find literature related to the sex ratio alteration ability of creatures?

Output: For literature related to the sex ratio alteration ability of creatures, academic databases such as PubMed, Google Scholar, and specific journals in evolutionary biology, genetics, and ecology are valuable resources. These platforms provide peer-reviewed articles, reviews, and studies focusing on the mechanisms, evolutionary implications, and case studies of sex ratio alteration across various species. Searching with keywords like "sex ratio manipulation," "sexual selection," "parental investment," and the names of specific creatures or mechanisms (e.g., "Wolbachia" for bacteria-induced sex ratio alterations) will yield relevant literature.

5. OpenAI *ChatGPT* (Feb, 3, 2024 version, ChatGPT-4)

Query: where can I find the data of lampreys? I need their population of different age and gender, sex ratio, weight

Output: For detailed data on lamprey populations, including aspects like population demographics, distribution, threats, and potentially age and gender-specific data, you can explore several resources:

Freshwater Lamprey Survey Data on GBIF (Global Biodiversity Information Facility) provides access to a dataset that includes information on species distribution, composition, and abundance for river, brook, and sea lampreys in specific river catchments. This could be a valuable resource for understanding the distribution and relative abundance of different lamprey species within Welsh Special Areas of Conservation (SACs). The dataset is a result of surveys conducted using standard methodologies, including electrofishing, to assess the condition and populations of lampreys.

The U.S. Geological Survey (USGS) offers insights into the historical data on adult sea lamprey captures in the Great Lakes, aiming to understand pre-control abundance and evaluate the effectiveness of control measures like lampricide. This information might help in estimating population sizes and understanding the impact of human interventions on lamprey populations over time.

The Pacific Lamprey Conservation Initiative (PLCI) provides assessments every four to five years on Pacific Lamprey, evaluating habitat conditions, population demographics, distribution, and threats across their historical range. These assessments are crucial for conservation efforts and provide a comprehensive overview of the species' status, with the latest updates aiming to fill information gaps in specific regions. This resource is particularly focused on Pacific Lamprey but offers a model for how assessments can inform conservation strategies.

Each of these sources offers different types of data that could be relevant to your research on lampreys, including population sizes, distribution, demographic information, and conservation status. For the most accurate and detailed data, consider reaching out directly to these organizations or accessing their databases and reports through the provided links:

Freshwater Lamprey Survey Data on GBIF

Understanding sea lamprey populations in the Great Lakes - USGS

Pacific Lamprey Conservation Initiative - Assessment

These platforms may require direct contact or further exploration of their databases to find specific data on age, gender, sex ratio, and weight for lampreys.

- Model Development

In the modeling process, we initially crafted models independently upon receiving our topic. Through discussion, we decided to integrate the Fisher model, analyzing the sex ratio through genetic and extrinsic factors. To grasp Fisher's theory, we consulted GPT. Aware of the potential risks in relying solely on GPT, we initially used it for a basic understanding of Fisher's theory, then sought further literatures to enrich our comprehension and validate GPT's accuracy.

6. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query1: help me explain Fisher principle that explain the sex ratio of different species

Output: The Fisher principle, formulated by British statistician and geneticist Ronald Fisher in his 1930 book "The Genetical Theory of Natural Selection," offers an explanation for the roughly equal sex ratio observed in many species, including humans. This principle is based on the premise of natural selection and involves a sophisticated argument about parental investment and reproductive success. Here's a simplified explanation of the Fisher principle:

1. **Starting Assumption:** The principle begins with the assumption that if there is any deviation from a 1:1 sex ratio in offspring (for example, more males than females or vice versa), then the rarer sex will have a higher reproductive value. This is because parents producing offspring of the rarer sex will have a greater chance of having grandchildren, given that their offspring (being in short supply) will have a higher mating success.
2. **Natural Selection Adjusts the Ratio:** Given this advantage of producing the rarer sex, natural selection will favor parents genetically disposed to produce offspring of the less common sex. Over time, this selection pressure will push the sex ratio back towards equality.
3. **Equilibrium at 1:1 Ratio:** The equilibrium point, where neither sex has a reproductive advantage, is at a 1:1 ratio of males to females. When this ratio is reached, neither sex is rarer, so there's no selective advantage to producing offspring of one sex over the other. Any deviation from this equilibrium would self-correct over time as per the mechanisms described above.

4. **Underlying Assumptions:** Fisher's principle is based on a few key assumptions. Firstly, it assumes that the cost of producing male and female offspring is roughly the same, which might not be true for all species. Secondly, it presumes no external factors skewing the sex ratio, such as environmental conditions or parental manipulation.
5. **Extensions and Criticisms:** While the Fisher principle provides a robust theoretical foundation for understanding sex ratios, real-world observations sometimes deviate from the predicted 1:1 ratio due to various factors like differential mortality rates between sexes before reproductive age, local resource competition, and local mate competition. These factors can lead to adaptive deviations from the 1:1 ratio in some species.

In essence, the Fisher principle explains how natural selection can influence sex ratios in populations, driving them towards an equilibrium that maximizes reproductive success. It is a fundamental concept in evolutionary biology, demonstrating how genetic and environmental factors interplay to shape the characteristics of populations over generations.

7. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query1: in which literature can we find the math equation for fisher's principle, please offer me the link

Output: It seems that my current environment does not allow me to directly fetch or provide links to specific documents, including scholarly articles from Google Scholar or other academic databases. However, I can guide you on how to find the information you're looking for:

1. **Visit Google Scholar:** Go to Google Scholar. It's a freely accessible web search engine that indexes the full text or metadata of scholarly literature across an array of publishing formats and disciplines.
2. **Search Terms:** Use search terms like "Fisher principle mathematical expression" or "Fisher's sex ratio theory mathematics". These terms are likely to lead you to academic papers discussing the mathematical aspects of the Fisher principle.
3. **Look for Review Articles:** Review articles often provide comprehensive summaries and are more likely to discuss foundational concepts like Fisher's principle in both conceptual and mathematical terms.
4. **Specific Journals:** Consider searching within specific evolutionary biology or theoretical biology journals. Journals such as "Theoretical Population Biology", "Evolution", and "Journal of Theoretical Biology" often publish articles relevant to this topic.
5. **University Libraries:** If you have access to a university library, they often have subscriptions to journals and electronic resources where you can access articles that discuss the Fisher principle in detail.

Remember, accessing some scholarly articles may require a subscription or institutional access. However, you can often find abstracts or pre-publication versions of papers (preprints) for free, which might include the mathematical expressions of the Fisher principle you're interested in.

8. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query1: please explain fisher's principle in math equation and provide me with relevant literature

Output: The mathematical explanation of the Fisher principle usually involves quantifying the relationship between sex ratios and reproductive success. Let's assume we have a population, in which the cost of producing males and females is the same, and we can use a mathematical model to elucidate the Fisher principle.

Basic Model

Let N_m and N_f be the number of males and females in the population, respectively. Suppose the sex ratio of the population is defined by the proportion of males p , where $p = \frac{N_m}{N_m + N_f}$. At equilibrium, the average reproductive success of each sex should be equal, hence the sex ratio tends towards 1:1.

Reproductive Success Rate

Assume each male on average can pair with b_f females, and each female can pair with b_m males. In a population with a balanced sex ratio, ideally, $b_f = b_m$. However, if one sex becomes less in number, then its opportunity for pairing increases, thus increasing its reproductive success rate.

Reproductive Value and Natural Selection

Let V_m and V_f represent the reproductive value of males and females, respectively. If one sex becomes rarer, then its reproductive value relative to the other increases. The Fisher principle suggests that if $V_m > V_f$, natural selection will favor the increase in the proportion of males being produced, and vice versa. This can be expressed as follows:

$$V_m = \frac{1}{p}$$

$$V_f = \frac{1}{1-p}$$

When the sex ratio is 1:1, $V_m = V_f$, indicating no sex has a higher reproductive value over the other.

Limitations of the Mathematical Model

Although Fisher's sex ratio principle seems mathematically straightforward, its practical application must consider many other factors, such as the cost of producing each sex, differences in mortality rates between sexes before reaching reproductive age, mate choice, environmental factors, etc. These factors can all lead to observed sex ratios that deviate from theoretical predictions.

Providing Resources

To delve deeper into the mathematical background and applications of the Fisher principle, here are some recommended readings:

1. "**The Genetical Theory of Natural Selection**" - Ronald Fisher. This is the original source of the Fisher principle, and while the discussion in the book is more theoretical, it's crucial for understanding the formation and application of the principle.
2. "**Principles of Evolutionary Ecology**" - This book discusses various aspects of evolutionary ecology, including the evolutionary theory of sex ratios.
3. **Academic Papers and Review Articles** - Searching for scholarly articles with keywords like "evolution of sex ratios," "Fisher's principle" can yield a wealth of research that delves into the Fisher principle and its application across different species.

These resources can provide a deeper mathematical explanation and theoretical background, helping to better understand the role of the Fisher principle in biological evolution.

- Coding

During the code development process, we did not utilize AI. The Matlab program was entirely written by our team members. Although LLMs possess strong capabilities in code writing, ChatGPT have learned numerous algorithms. We opted to write the code ourselves due to the originality of our model and the uniqueness of the corresponding code.

- Translation and Language Polishing

We extensively utilize LLMs in translation and language polishing. We believe LLMs offer the most benefits with the least risk in these kinds of tasks. The ambiguity of language and the AI's lack of contextual understanding can make machine-translated texts appear rigid and potentially deviate in meaning. To minimize this impact, we endeavor to carefully review and confirm the accuracy of AI-translated sentences before incorporating them into our papers.

Given that we use the same prompt words, we hereby provide the prompt words.

- Prompt Words for Translation

Please translate the following paragraphs into English, ensuring the translation is academic in nature. The language style should be formal, concise, and precise.

- Prompt Words for Polishing

Below is a paragraph for a biological paper. Polish the writing to meet the academic style, improve the spelling, grammar, clarity, concision and overall readability.

We declare that most of the language in this document has been translated or polished by ChatGPT. To avoid making the report overly lengthy due to the use of AI, only two examples are provided. The prompt words mentioned above are omitted.

9. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query1: 海七鳃鳗在产卵后多少时间内即死亡，这个假设是符合实际的并且能够简化模型的研究，突出性别比率改变的进化策略对海七鳃鳗种群生存的影响

Output: The assumption that hagfish die shortly after spawning simplifies research models by focusing on how evolutionary strategies in sex ratio changes affect the survival of hagfish populations. This premise is realistic and aids in streamlining the study.

Query2: 在海七鳃鳗的生命周期中，1-4年它作为被捕食者，5-7年作为寄生者

Output: In the life cycle of the hagfish, it acts as prey for the first 1-4 years and as a parasite during years 5-7.

10. OpenAI *ChatGPT* (Feb 2, 2024 version, ChatGPT-4)

Query1: Background:

Animal of different species have various sex ratio. While most of them exhibit the typical 1:1 sex ratio, some of the like lampreys can change sex ratio based on external circumstances, including food availability. Sea lampreys plays a complex role in ecosystem—their larvae are predator to plankton and decomposer to remains while in their adulthood, they are parasite, feeding on other fishes' blood. At their native habitat, they are important link in the food chain. Moreover, sea lampreys are palatable and considered a delicacy in some country, serving a role in fishing industry. However, they could also be invasive species and threatthe local ecosystem seriously, especially in Great Lake. Thus, analysing the effect of sea lampreys' special ability to alter sex ratio based on environment is crucial for us to gain insights into their interactions with ecosystem.

Output: Revised Paragraph:

Background:

Species across the animal kingdom display a diversity of sex ratios; while the majority adhere to the canonical 1:1 distribution, certain species, such as lampreys, possess the remarkable ability to adjust their sex ratio in response to environmental factors, including the availability of food. Sea lampreys occupy a multifaceted niche within ecosystems: their larvae function as predators of plankton and act as decomposers, whereas adults adopt a parasitic lifestyle, subsisting on the blood of other fish. In their native environments, they constitute a critical component of the food web. Additionally, sea lampreys are regarded as a culinary delicacy in some regions, contributing to the fishing industry. Nevertheless, they can also emerge as invasive species, posing a significant threat to local ecosystems, particularly in the Great Lakes. Therefore, exploring the sea lamprey's unique capacity to modify its sex ratio based on environmental conditions is essential for understanding their impact on ecosystems.