Squid Simulation Project

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Abstract

This project explores a simulation of a population of squid with random variation. The purpose of the simulation is to test a catch function that allows us to consistently catch squid yearly in their breeding cycle while not rendering the species extinct. The preferred catch function came to a feedback loop which allowed for these criteria to be met.

I. Introduction

The clearest realisation of the 21st century is our effect on the environment. One of the systems vital for life on earth is biodiversity. The ocean's biodiversity is no exception. Over-fishing, extracting a live resource faster than natural processes can replace the population levels has a devastating effect on the biodiversity of a region which can cause a cascade of effects on that local environment and economy.
We are at a turning point in human history where we need to consider our impact on the world around us and begin to navigate the nuances and struggles that comes with finally accepting that our assumption of endless economic growth is not compatible with life on earth. At the same time, our species relies on our civilisation and all its structures to carry on living. We need the economy to work in order to experience life as we know it.
Our ability to sustainably extract a resource, that is, extract it at a rate such that the population levels are able to bounce back, is crucial.
This project hopes to be able to begin to examine and explore some of these ideas. we consider the following environment:
For a population of squid with biomass B the population has an annual breeding cycle. Once this cycle occurs we catch a portion of the population. From that point forward the population will decline by natural means until the next breeding cycle.
We have two sources of variation in the simulation.
The recruitment of the squid population will have random variation to simulate the natural fluctuation of the breeding cycle.
The second source of variation comes from our observation of the biomass. Every year between the breeding cycle and the catch event, a survey is conducted to measure the biomass levels. Total accuracy in this type of measurement is impossible. A random error is therefore introduced to simulate this.
The objective is to determine a good catch function which will satisfy the following properties,
maximise biomass catch (and therefore profits);
minimise year-on-year catch variation;
and restrain from over fishing so as not to render the species extinct.
A catch function that can successfully achieve these requirements to a good degree will be able to yield some insights into approaches we might take. This simulation is intended to compete with other implementations of the project with their own unique catch functions.

II. Simulation Outline

i. Analytical Solutions

If the biomass of the squid present at the start of year t is B_t, then the dynamics of the squid population is given by:

$$B_{t+1} = [B_t + R(B_t) - C_t]e^{-M}$$

Where
$R(B_t) = \frac{\alpha B_t}{\beta + B_t}$ is the recruitment in year t, and
C_t is the catch taken in year t, and
M is the instantaneous rate of natural mortality.
We find:

i.1 Carrying Capacity of Population

let $C_t = 0$

$$B_{t+1} = [B_t + \frac{\alpha B_t}{\beta + B_t}]e^{-M}$$

The point of stability in the update equation occurs when $B_{t+1} = B_t$. Therefore

$$B_t = [B_t + \frac{\alpha B_t}{\beta + B_t}]e^{-M}$$

Solving for B_t gives us:

$$B_t = \frac{\alpha}{e^M - 1} - \beta$$

We can define this expression as the carrying capacity for the population of squid:

$$K := \frac{\alpha}{e^M - 1} - \beta$$

i.2 Finding β:

We are given that if the recruitment is $R(K)$, then the gradient of the slope is:

$$R(0.2K) = hR(K)$$

$$R(0.2K) = \frac{\alpha(0.2K)}{\beta + 0.2K} = hR(K)$$

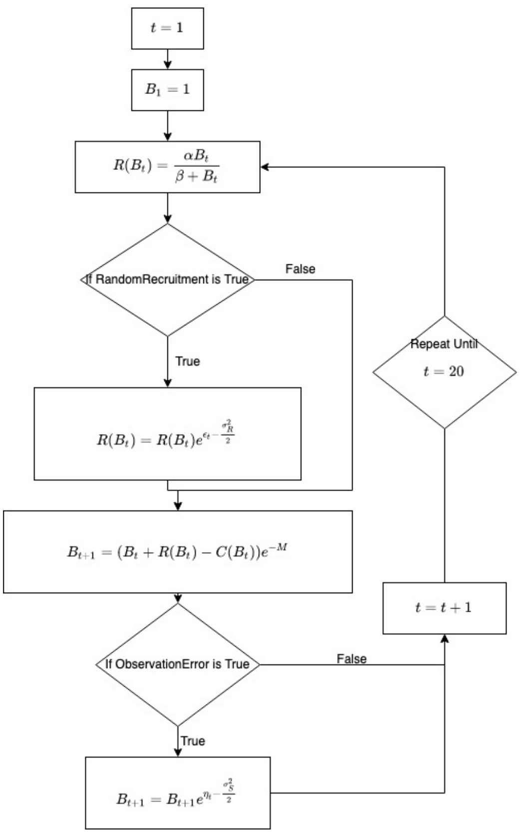
Solving this for β will give:

$$\beta = \frac{0.2K(1-h)}{h - 0.2}$$

ii. Structure of the simulation:

We begin year $t = 1$ with biomass $B_1 = K = 1$. Which represents 1 Million tons of squid biomass. We then find the recruitment and add the random effects if desired. We then catch a portion of this population with catch function C. From this we can find the next year's biomass value, add the observation error if desired. and repeat all the previous steps till $t = 20$.

Figure 1: *Basic 20 year Simulation Structure*



This is the basic structure of the simulation environment. This process will be referred to as the 'Squid Cycle' as in a 20 year cycle of squid recruitment and extraction. Note minor details of the simulation environment have been omitted from the diagram.

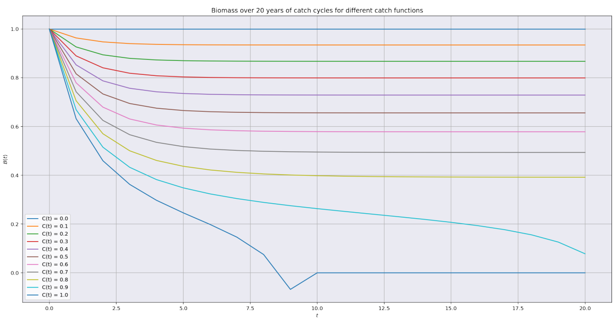
This gives us a functional environment in which to enter different catch functions for a different output of 20 biomass measurements. The random variation of recruitment, as well as the random error of observation have been made toggle-able in the simulation.

iii. Evaluation of Environment

iii.1 Basic Plots of Simulation Environment

With the ability to toggle the random effects in the simulation, we can perform a sanity check on the set up:

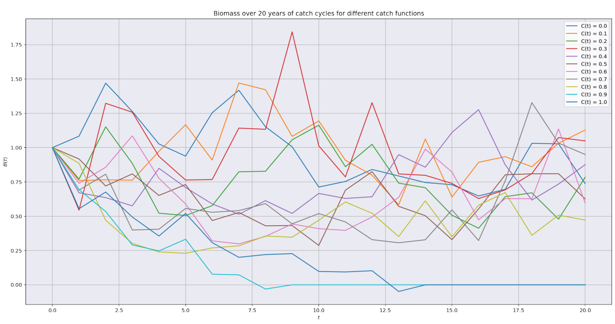
Figure 2: *Simulations with no random effects.*



This follows expected behaviour of the dynamics of the population. As the catch function increases in proportion to the original carrying capacity K, The system settles down to a lower carrying capacity.

Looking at the same range of catch functions with random effects in place shows us:

Figure 3: *Simulations with random effects.*



Which is a useful graph as it shows that although the random effects have a large effect on the end biomass, the catch function still plays the largest role, and the biggest risk to extinction. This is the enviroment we want to be able to begin preparing a catch function.

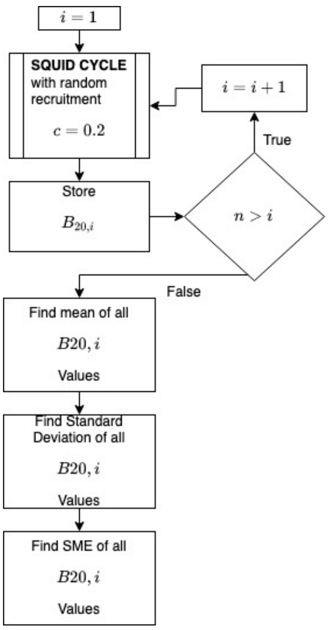
iii.2 Ensuring Reproducible Results

Although in principle we have a working simulation environment, we must be able to establish coherent results in spite of the present sources of random variation. No two simulations will return the same results which creates difficulty in understanding if our environment is behaving appropriately.
We also need to be able to establish consistent results with the other simulations(competitors, work this into the writing style.).
On order to do this the competing simulation teams agreed to test along the catch function: $C = 0.2$.
i.e 20% of the carrying capacity $K = 1$ or 1 million tons of squid biomass.
We run on the order of n 10^3 to n 10^6 simulations with this same catch function and then compare by:

- Creating a histogram of all B_{20} results;
- Calculate the mean, $\bar{B_{20}}$;
- standard deviation, σ_{B20} ;and
- standard mean error ($SME = \frac{\sigma_{B20}}{\sqrt{n}}$).

In this repeated simulation we are only allowing random variation in the recruitment function.
A visual of how the simulation is used:

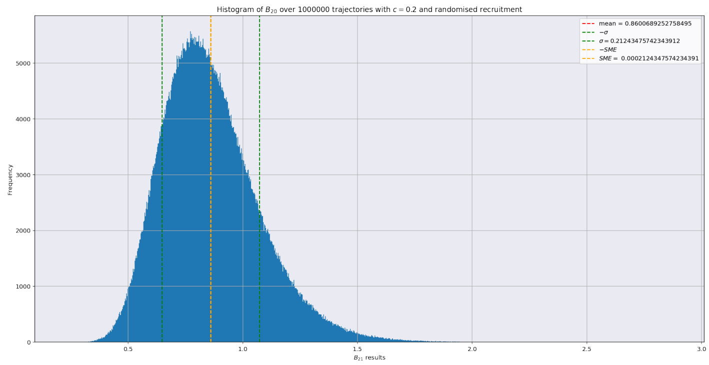
Figure 4: *Multiple Simulation Structure*



These results of this came to:

$$\bar{B_{20}} = 0.8600$$
$$\sigma_{B20} = 0.2124$$
$$SME = 0.0002$$

Figure 5: *B_{20} Histogram*



III. The Catch Function

Now that the environment has been set up and verified, it is now time to move to the main event: Finding a catch function.

i. Considerations

As stated before our recruitment function is blind to our pick of how much squid to catch. Additionally, we have a error variation in the biomass observation.

We want to pick a catch function that is able to adapt to times of increasing population and times of decreasing population appropriately.

ii. The Feedback Loop Model

The catch function that this project will aim to make use of is rather simple. We begin with an initial observation of the biomass B_t and catch a portion of that population with the catch function:

$$C_t = p * B_t$$

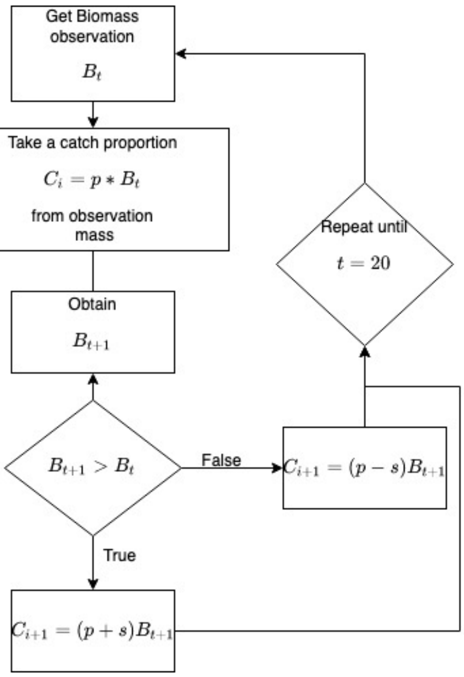
We can then go on to calculate the next year's biomass level. If the next year's biomass level is smaller than the current year, the next catch function C_{t+1} will be adjusted by a parameter s to make:

$$C_{t+1} = (p - s)B_{t+1}$$

However if $B_{t+1} > B_t$, then we know the population has grown and we can afford to take a little more the next year given by:

$$C_{t+1} = (p + s)B_{t+1}$$

Figure 6: *Diagram of feedback loop catch function.*



iii. Model Assessment Criteria

In order to compare our catch functions with the other simulations, the following assessment method and summary statistics were defined: for n 20 year simulations we can find:

- C_{avg}: Median of the average catches for each simulation.
- AAV: For each year's catch we can find the catch variation. We then average all these values for each 20 year simulation and take the median of all these values
- B_{20} lower $5th$ percentile: we take the $B20, i$ values for all n squid cycles. we order these and find the fifth percentile value.
- B_{low}: we take the lowest B value in each simulation and take the median of these values

iv. Optimisation of Parameters

For this simple feedback loop model there are several parameters we may vary in order to try reach the 'best' combination for this simulation. A metric must be defined in order to compare different 'tunings' of the feedback loop. Looking at the summary statistics we can conclude that the 'ideal' catch function will be some combination of parameters such that we achieve:

- Largest C_{avg}
- Smallest AAV
- Largest B_{20} lower $5th$ percentile
- Largest B_{low}

There is clearly a trade off between these statistics. If we have a more sensitive catch function (a larger value of s), we will achieve a larger average catch value. However this will also naturally increase the annual average variation. Additionally, our original proportion of observation that we extract will also have a trade off effect. Large initial proportions will increase C_{avg} but have the risk of upsetting the stability of the environment which will send the other metrics in the opposite direction of what we want as outlined above.

It would therefore be useful to create some raw score by which we can compare different combinations of parameters in order to select the best catch function.

If we were to simply divide the numbers we want to be high by the number we want to be low this would give us some sense of performance. However as the AAV variation gets low this metric will blow up to infinity, which certainly does not reflect how important the low AAV variation is.

instead the following was preferred:

$$R_{performance} = e^{M_1+M_3+M_4-M_2}$$

This can be usable since all the metrics can be formatted to be a percent (AAV being clear, but also because the biomass numbers are essentially just a percentage of the carrying capacity $K = 1$.) Since we're only working with numbers between 0 and 1 we do not need to worry about infinities.

We can then extend the simulation to run n simulations with parameters *prop* and *sensitivity* which we can vary according to how many overall simulations we want to run. for each of these simulation groups we can run the summary statistics to get the four metrics. From this we can get a score $R_{performance}(p, s) = e^{M_1+M_3+M_4-M_2}$ dependant on parameters p and s.

We now can select the parameters that yield the best overall performance score.

IV. Results

i. Assessment of Feedback loop

Running the parameter optimisation yields a $R_{Performance}$ table which can be drawn as a heat-map for visual simplicity:

Therefore the feedback loop with parameters

$$p = 0.5$$

,

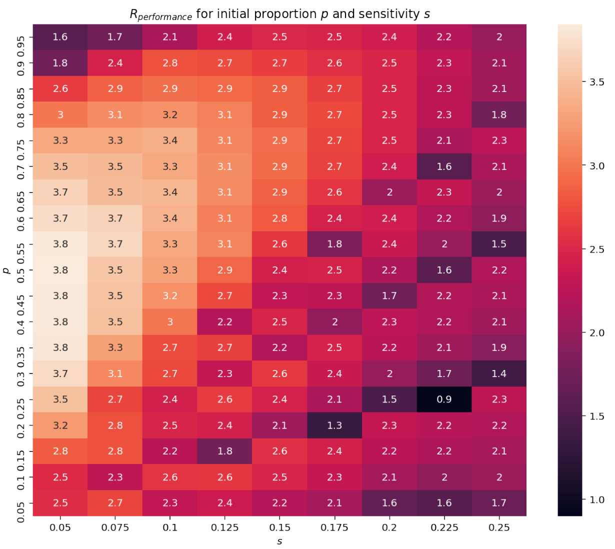
$$s = 0.05$$

was selected as the catch function

which has the summary statistics:

- $C_{avg} = 0.5238$
- $AAV = 0.09629 = 9.63\%$
- $B_{20Lower5thPercentile} = 0.2633$
- $B_{low} = 0.3125$

Figure 7: *$R_{Performance}$ scores for different catch function parameters.*



ii. Additional Measurements

Thinking about the cumulative biomass extracted over the course of the 20 year cycle is an additional metric that can be considered. It can be calculated by taking the same 1000 simulations used to determine the summary statistics. For each simulation, sum all the catch numbers. We can now order this list and take the median value as

$$C_{cumulative} = 11.0$$

This refers to 11 times the value of the original carrying capacity. This is a good overall yield while still maintaining stable population levels.

V. Interpretation of Results

The feedback loop catch function works well at consistently attaining decent catches while limiting the stress on the resource. The simplicity of the function makes it easy to implement. Additionally, the range of parameters in which the function performs comparatively well forms a 'safety net' that allows for more aggressive or conservative parameters to be used. Further models could include the cost of the yearly catch and the average pricing of the resource in order to assess profitability of the catch function.

VI. Conclusions

Simulations are powerful tools. In a few hundred lines of code we can approximate the behaviour of a population. From there we can essentially gain a collective hundreds of thousands of experience.

This project has shown the possibility and benefit of using models to find more sustainable methods of supporting our way of life.