Grover on Quantum Cryptanalysis

Gopal Ramesh Dahale (11840520)
Supervised by:
Dr. Dhiman Saha

Quantum Symmetric-Key Cryptanalysis (CS614)

Department of EECS
Indian Institute of Technology Bhilai

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Outline

Introduction

- Grover's search algorithm recovers key in $O(\sqrt{N})$ calls to quantum oracle where N is the key search space.
- Implementation of quantum circuit of block cipher.
- We study implementation of hardware and software efficient ciphers.
- We study SAES, SIMON 2n/mn, PRESENT, and AES-128.
- Investigation of cost of Grover's Attack with depth constraint as proposed by NIST[1]
- AES-128 under depth constraint.

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SAES

Nibble oriented with block and key size of 16 bits.

$$\underbrace{b_0\,b_1\,b_2\,b_3}_{S_0}\,\underbrace{b_4\,b_5\,b_6\,b_7}_{S_1}\,\underbrace{b_8\,b_9\,b_{10}\,b_{11}}_{S_2}\,\underbrace{b_{12}\,b_{13}\,b_{14}\,b_{15}}_{S_3} = \begin{bmatrix} S_0 & S_2 \\ S_1 & S_3 \end{bmatrix} = State$$

SAES Encryption

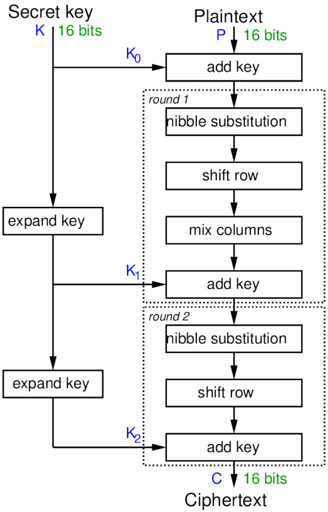

Figure: SAES encryption[2]

[2] Steven Gordon. *Cryptography Study Notes (Chapter 9)*. 2022. URL: https://sandilands.info/crypto/.

Sub Nibbles, Shift Rows and Mix Column

1. Compute the multiplicative inverse x i.e. $y = x^{-1}$ in $GF(2^4)$.
2. The result of the sbox is computed using the follows operation:

$$\begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

3. The Shift Rows operation is the same as AES.

$$\begin{bmatrix} S_0 & S_2 \\ S_1 & S_3 \end{bmatrix} \longrightarrow \begin{bmatrix} S_0 & S_2 \\ S_3 & S_1 \end{bmatrix}$$

4. SAES mix column.

$$\begin{bmatrix} S_0' \\ S_1' \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \end{bmatrix}$$

5. The elements of the matrix are in $\mathbb{F}_{2^4}[x]/(x^2+1)$.

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Key Expansion

- The master key (16 bit) can be thought as 2 bytes $B_0 B_1$.
- 3 Round keys.
- First round key (16 bit) can be thought as 2 bytes $B_2 B_3$.
- Second round key (16 bit) can be thought as 2 bytes $B_4 B_5$.

Key Expansion for SAES(K)

```
keys = [B0, B1, B2, B3, B4, B5]
keys[0] = K[0 ... 8]
keys[1] = K[8 ... 16]
for i = 2 to 5
    if i%2 == 0
        keys[i] = keys[i − 2] ⊕ RCON(i/2)⊕
        keys[i] = keys[i] ⊕ Sbox(RotNib(keys[i − 1]))
    else
        keys[i] = keys[i − 2] ⊕ keys[i − 1]
return B0B1, B2B3, B4B5
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- Round Constant is defined as

$$RCON(i) = (x^{i+2} || 0000)$$

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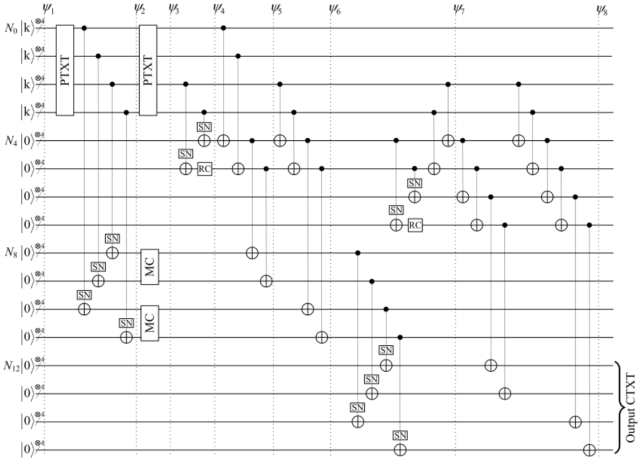

Figure: QSAES18[3]

[3] Mishal Almazrooie et al. "Quantum Grover Attack on the Simplified-AES". In: *Proceedings of the 2018 7th International Conference on Software and Computer Applications*. ICSCA 2018. Kuantan, Malaysia: Association for Computing Machinery, 2018, pp. 204–211. ISBN: 9781450354141. DOI: 10.1145/3185089.3185122. URL: https://doi.org/10.1145/3185089.3185122.

Sub Nibbles

Fermat inversion algorithm (square and multiply method) to find multiplicative inverse in $GF(2^4)$

$$x^{-1} = x^{2^4-2} = x^{16-2} = x^{14} = x^2 \times (x^2)^2 \times ((x^2)^2)^2$$

Figure: Multiplier[4]

Input: $q_0 - q_3$ and $q_4 - q_7$. Output: $q_8 - q_{11}$.

[4] Donny Cheung et al. "On the Design and Optimization of a Quantum Polynomial-Time Attack on Elliptic Curve Cryptography". In: *Theory of Quantum Computation, Communication, and Cryptography*. Ed. by Yasuhito Kawano and Michele Mosca. Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 96–104. ISBN: 978-3-540-89304-2.

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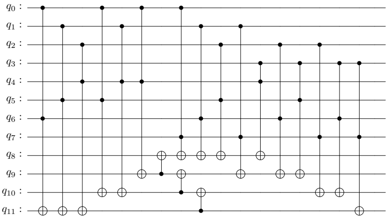

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Squarer circuit obtained using CNOT synthesis algorithm.

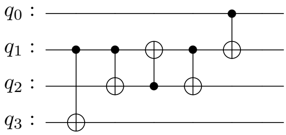

Figure: Squarer

Affine transformation circuit.

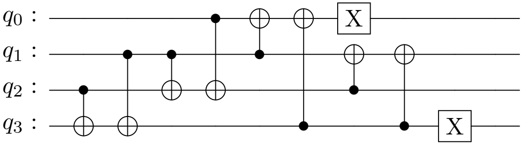

Figure: Affine transformation

Sub Nibbles Complete Circuit

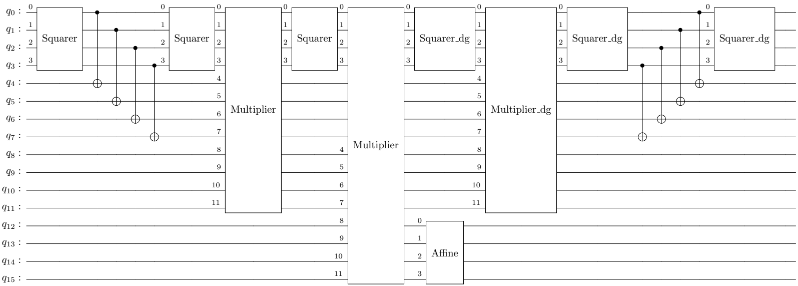

Figure: Sbox

Mix Column

Obtained using CNOT synthesis algorithm.

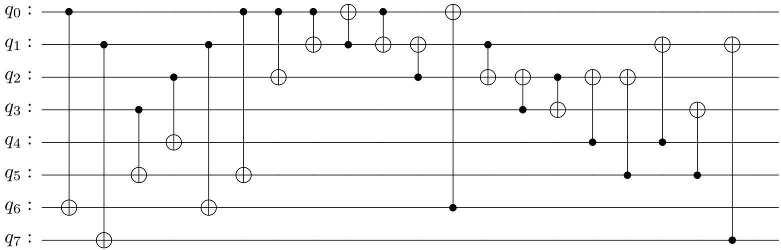

Figure: Mix column

The circuit takes 1 byte (1 column of the state matrix) and outputs the corresponding matrix multiplication in $GF(2^4)$.

Grover's Attack

Boolean function for Grover's oracle:

$$f(k) = \begin{cases} 1 & SAES(k,p) = c \\ 0 & else \end{cases}$$

Figure: Grover's Attack on SAES

Number of iterations:

$$t = \frac{\pi}{4}\sqrt{\frac{2^k}{s}} \tag{2}$$

$s = 2, k = 16$

$$t = \frac{\pi}{4}\sqrt{\frac{2^{16}}{2}} = 142$$

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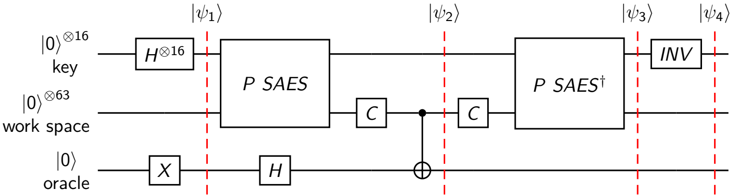

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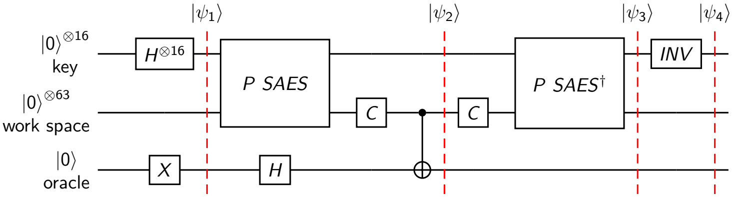

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Grover's Attack (r = 2)

They propose a modified version of Grover's Attack to find the unique key which is shown below.

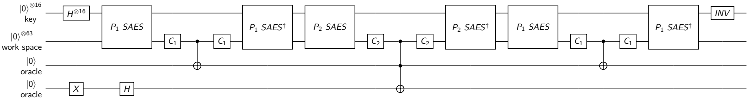

Figure: Grover's Attack to find unique key with r = 2

The corresponding boolean function is described as follows:

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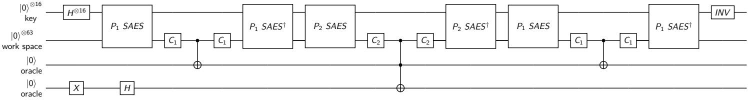

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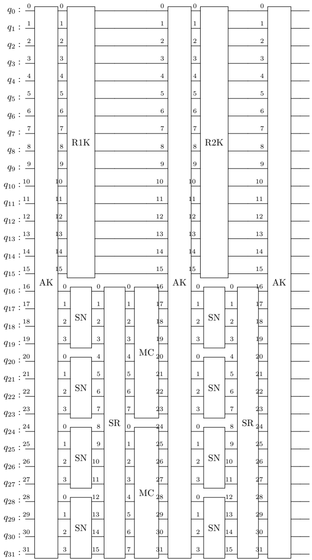
QSAES21[5]



- Only uses 32 qubits and no ancilla qubits.
- The top 16 qubits are used for storing the master key and for the process of key expansion.
- The bottom 16 qubits are used for storing plaintext, round operations, and outputting ciphertext.

[5] Kyung-Bae Jang et al. "Grover on Simplified AES". In: *2021 IEEE International Conference on Consumer Electronics-Asia (ICCE-Asia)*. 2021, pp. 1–4. DOI: 10.1109/ICCE-Asia53811.2021.9642017.

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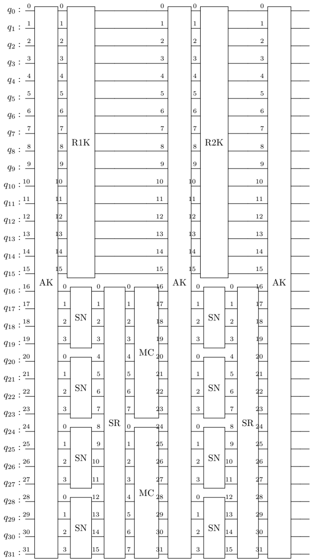

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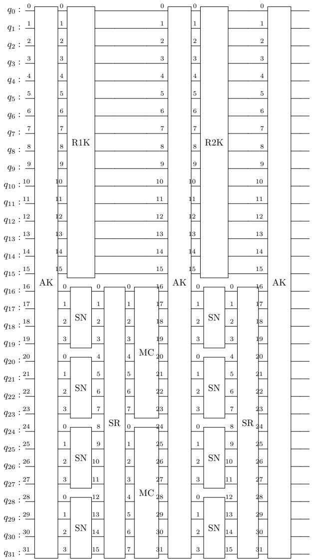

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Sub Nibbles

- Unlike [Alm+18][6] which uses 16 qubits (4 input, 4 output, and 8 ancillae) for Sbox computation, [Jan+21a][7] uses only 4 qubits using LIGHTER-R tool [Das+19][8].
- Output is permuted so we need SWAP gates (Not measured in quantum resources).

Figure: Sbox

[6] Mishal Almazrooie et al. "Quantum Grover Attack on the Simplified-AES". In: *Proceedings of the 2018 7th International Conference on Software and Computer Applications*. ICSCA 2018. Kuantan, Malaysia: Association for Computing Machinery, 2018, pp. 204–211. ISBN: 9781450354141. DOI: 10.1145/3185089.3185122. URL: https://doi.org/10.1145/3185089.3185122.

[7] Kyung-Bae Jang et al. "Grover on Simplified AES". In: *2021 IEEE International Conference on Consumer Electronics-Asia (ICCE-Asia)*. 2021, pp. 1–4. DOI: 10.1109/ICCE-Asia53811.2021.9642017.

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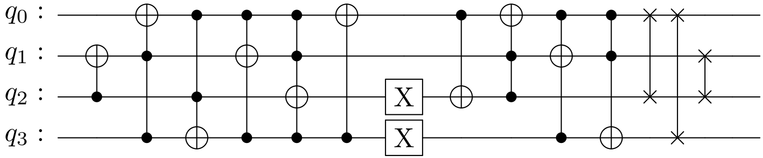
Sub Nibbles

- Unlike [Alm+18][6] which uses 16 qubits (4 input, 4 output, and 8 ancillae) for Sbox computation, [Jan+21a][7] uses only 4 qubits using LIGHTER-R tool [Das+19][8].
- Output is permuted so we need SWAP gates (Not measured in quantum resources).



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Shift Rows and Mix Column

- Shift rows operation using swap gates only whereas [Alm+18][9] used extra qubits for that with additional CNOT gates.
- Compared to [Alm+18], the authors use less number of qubits but require SWAP gates to get the correct result.

Figure: Mix column

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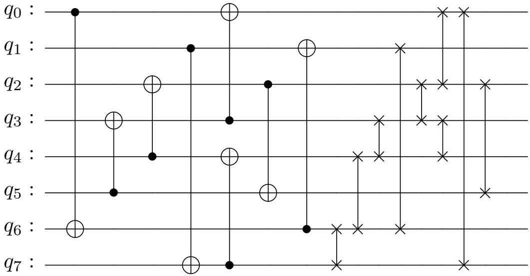

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Key Exapansion

Round keys are generated on the fly.

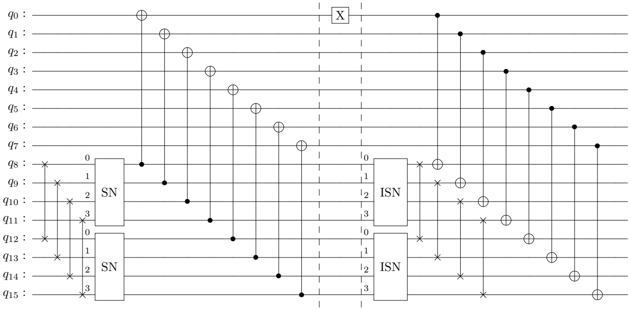

Figure: Round key 1

- Swap and substitution of B_1 then xor it with B_0.
- Add Round constant (10000000).
- Now the first 8 qubits hold B_2. To get B_3, we need to xor B_1 and B_2 but B_1 is lost.
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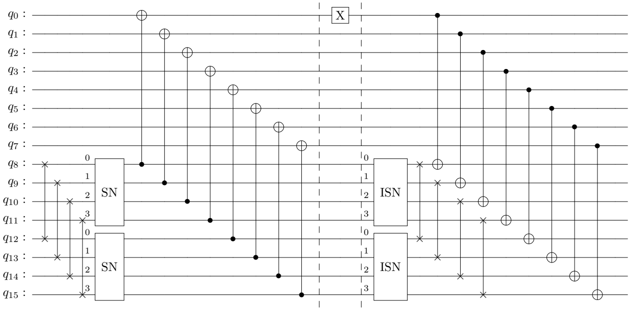

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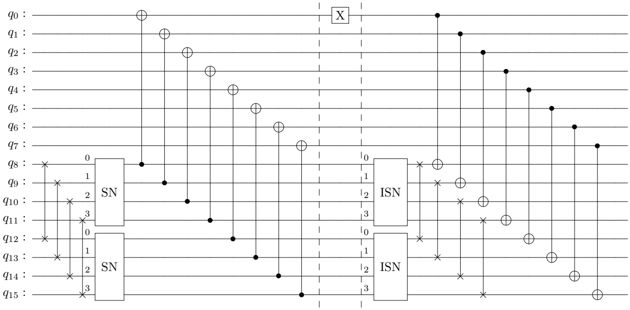

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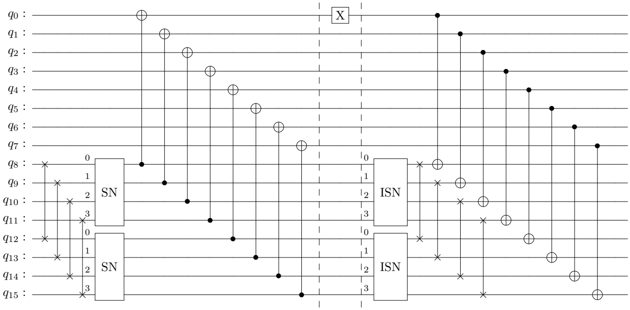

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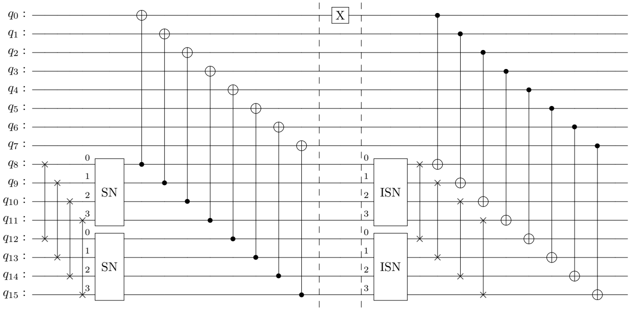

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- Expt1: verified the encryption process on IBMQ QASM Simulator [Qua21][10].
- Expt2 : Superpostion of all keys on IBM's Statevector simulator for 4000 shots.

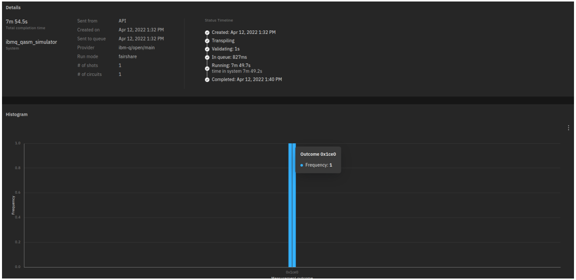

Figure: Output of encryption of plaintext 0110 1111 0110 1011 with key 1010 0111 0011 1011

Output is 0x1*ce*0 in hexadecimal, which in binary is 0001 1100 1110 0000 which is the reverse of the ciphertext.

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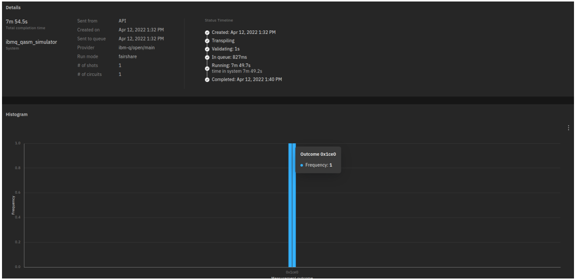

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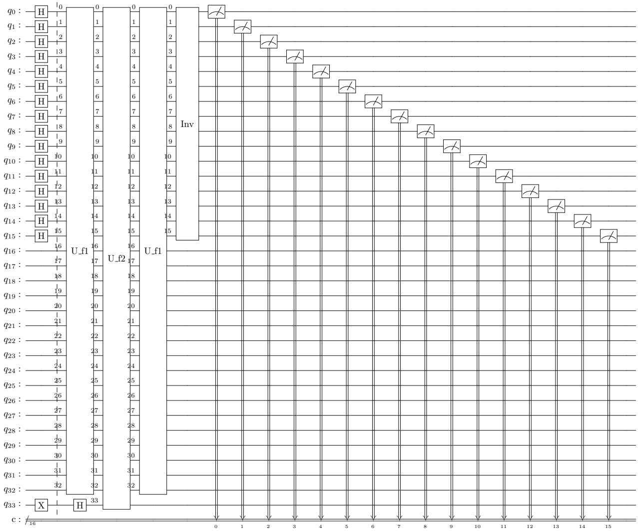

Figure: Grover's Attack for unique key with r = 2 (1 iteration)

Cost Estimates

	Qubits	X	CX	CCX	Ancilla
Key expansion	32	10	568	192	8
Encryption	32	16	512	384	-
Total	64	26	1080	576	8
Key expansion	16	19	56	48	-
Encryption	16	16	88	48	-
Total	32	35	144	96	-
My Code	32	35	144	96	-

Table: Comparison of cost for QSAES18[Alm+18] and QSAES21[Jan+21a]

Cost was heavily reduced due to optimizations in Sbox, key expansion, and mix columns circuit. Cost of Grover's Attack is :

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Outline

SIMON 2n/mn[11]

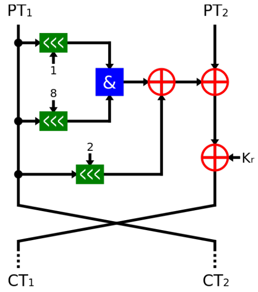

Figure: One round of SIMON [con22a]

Round Function

$$F(x,y) = (y \oplus (S^1(x) \wedge S^8(x)) \oplus S^2(x) \oplus k, x) \quad (3)$$

- $S^j(x)$ denotes left circular shift by j bits.
- PT_1, PT_2 are also referred as L_i, R_i.
- CT_1, CT_2 are also referred as L_{i+1}, R_{i+1}. L_i, R_i are n bit strings as input to the i^{th} round and k is the round key.

[11] Ray Beaulieu et al. *The SIMON and SPECK Families of Lightweight Block Ciphers.* Cryptology ePrint Archive, Report 2013/404.

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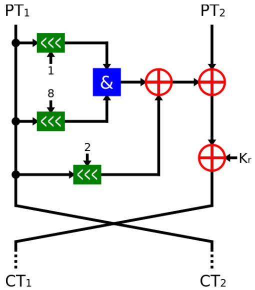

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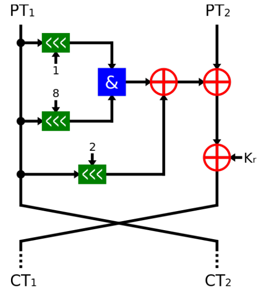

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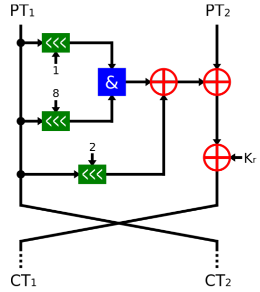

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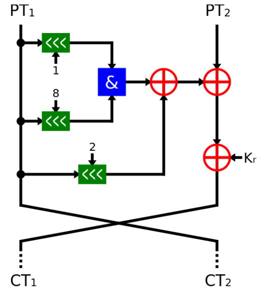

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Key Expansion

- For the first m rounds, the round keys are initialized from the master key.
- For the remaining T-m rounds, use the below function:

$$k_{m+i} = \begin{cases} c_i \oplus k_i \oplus S^{-3}(k_{i+1}) \oplus S^{-4}(k_{i+1}) & m = 2 \\ c_i \oplus k_i \oplus S^{-3}(k_{i+2}) \oplus S^{-4}(k_{i+2}) & m = 3 \\ c_i \oplus k_i \oplus S^{-1}(k_{i+1}) \oplus S^{-3}(k_{i+3}) \oplus S^{-4}(k_{i+3}) & m = 4 \end{cases} \quad (4)$$

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QSIMON[12]

- The path for i^{th} bit from R_0 which can be written as

$$R_2(i) = L_1(i) = R_0(i) \oplus (L_0(i+1)mod(n) \wedge L_0(i+8)mod(n)) \oplus$$
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- Use the qubits of R_0 for storing L_1 and L_0 becomes R_1. This eliminates the need for ancilla qubits.
- For computing $R_2(i)$, we can use the following operations:
  1. $CCX(L_0(i+1)mod(n), L_0(i+8)mod(n), R_0(i))$
  2. $CX(L_0(i+2)mod(n), R_0(i))$
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QSIMON Contd.

Define three functions that together form one round of the QSIMON.

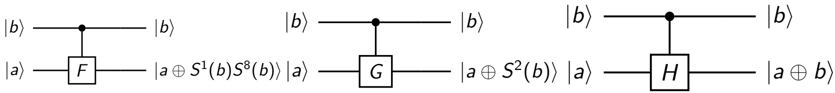

Figure: Subroutines for one round of QSIMON

Figure: 2 Round of QSIMON

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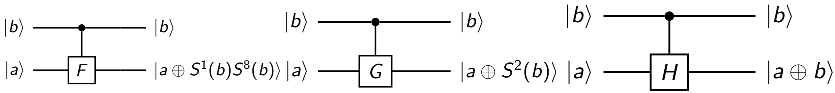

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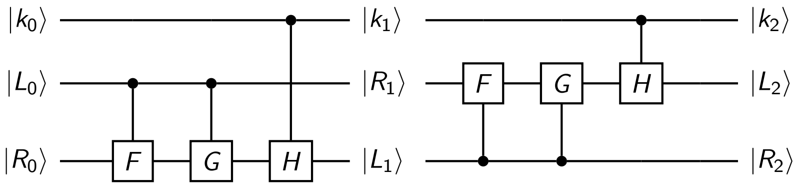

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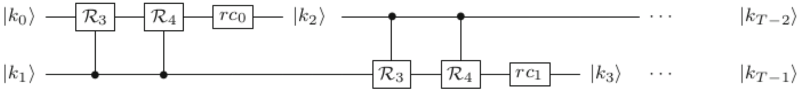
Define a function as

$$R_q(a, b) = (S^{-i}(b) \oplus a, b)$$

This function will be used in the circuit of key expansion.

Figure: Quantum circuit for $R_q(a, b)$

Case for $m = 2$.



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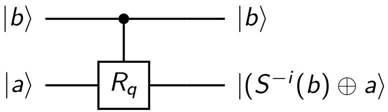

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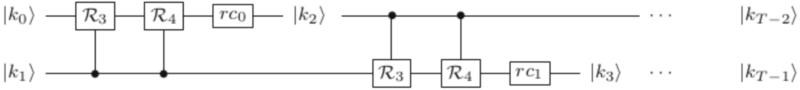

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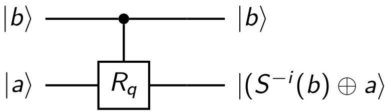

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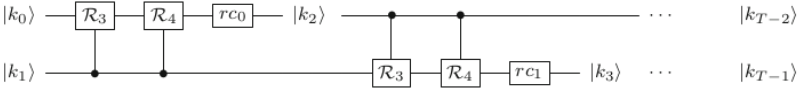


Figure: Key expansion for m = 2 [AMM20]

QSIMON (m = 2)

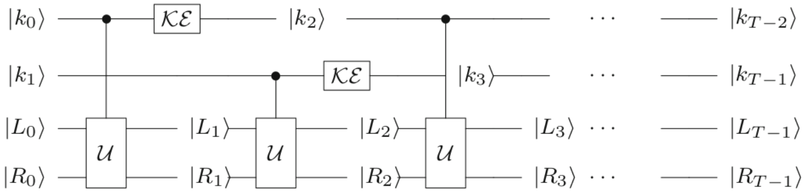


Figure: QSIMON for m = 2 [AMM20]

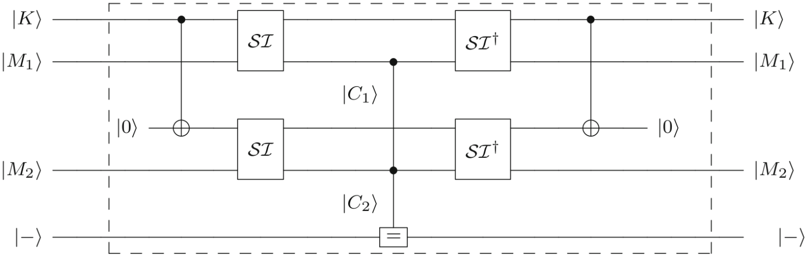
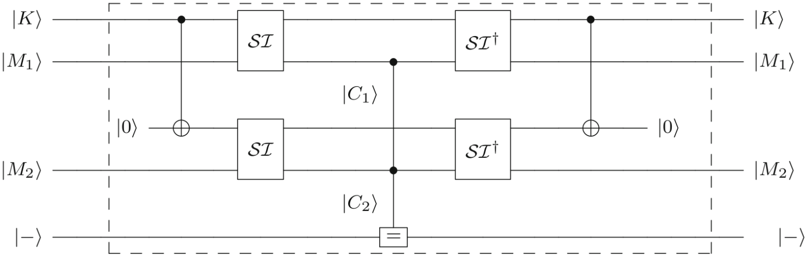


Figure: Grover's Attack on QSIMON [AMM20]

- For finding unique key we need r = 2.
- In general, we require $2nr$ qubits for messasges and mn qubits for master key.
- $O(mn + 2nr)$.

Grover's Attack



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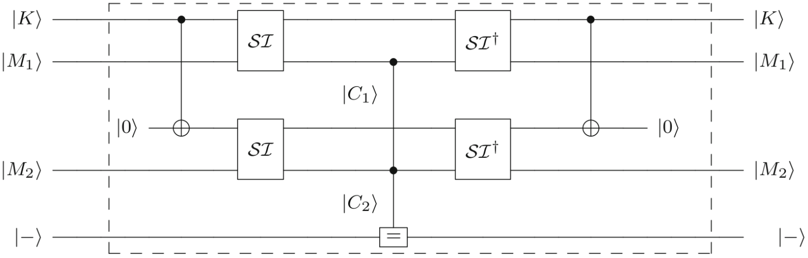

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Outline

PRESENT

- Ultra-lightweight block cipher and has a substitution permutation network.
- Block length of 64 bits and 80 and 128-bit key sizes.

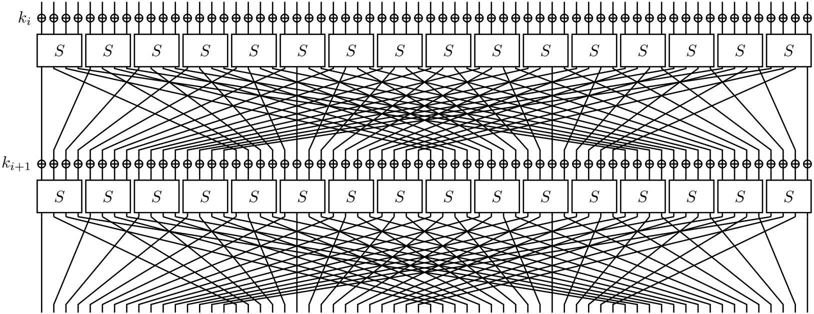

Figure: SP network for PRESENT cipher [Vik07][13]

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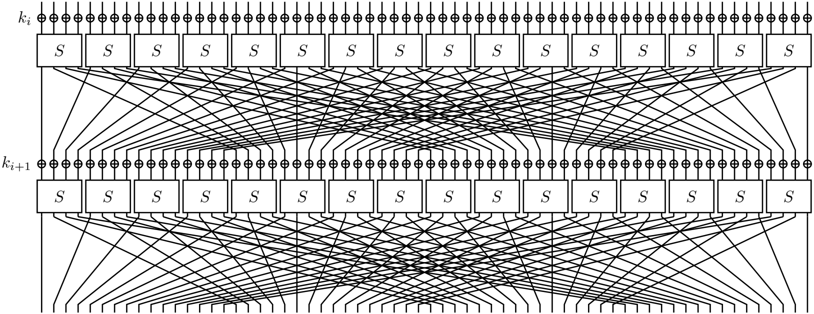

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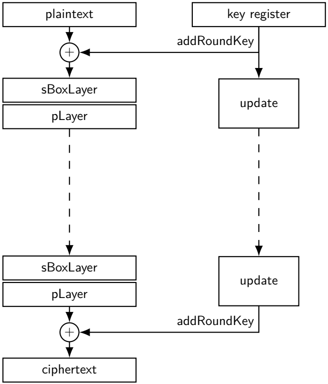
Cipher Design

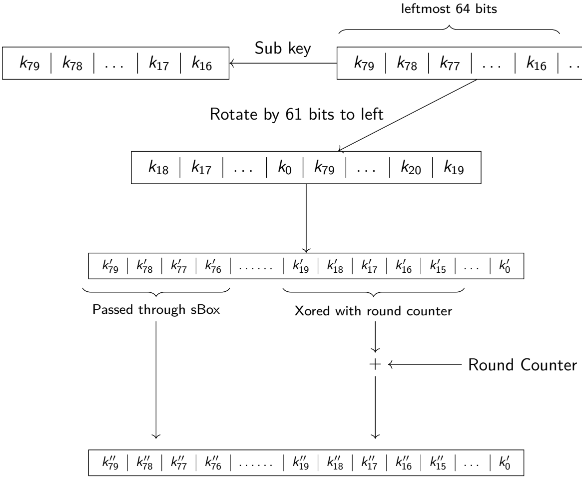
Psuedo-code

generateRoundKeys()
for $i = 1$ **to** 31 **do**
  addRoundKey($\textsc{State}$,K_i)
  sBoxLayer($\textsc{State}$)
  pLayer($\textsc{State}$)
addRoundKey($\textsc{State}$,K_{32})



Key schedule Algorithm

We discuss the 80-bit key schedule algorithm.



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QPRESENT[15]

- Authors used LIGHTER-R tool [Das+19][14] for optimized implementation of Sbox with no ancilla qubits

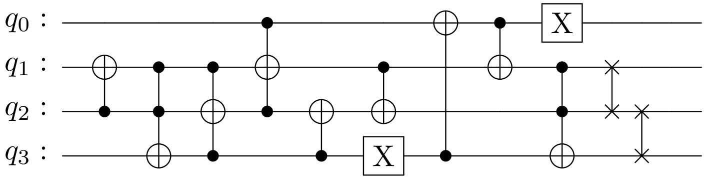

Figure: Sbox for QPRESENT

- Permutation layer can be implemented using only SWAP gates. The quantum cost for the permutation layer of QPRESENT is zero.

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[15] Kyungbae Jang et al. "Efficient Implementation of PRESENT and GIFT on Quantum Computers". In: *Applied Sciences* 11.11 (2021). ISSN: 2076-3417. DOI: 10.3390/app11114776. URL: https://www.mdpi.com/2076-3417/11/11/4776.

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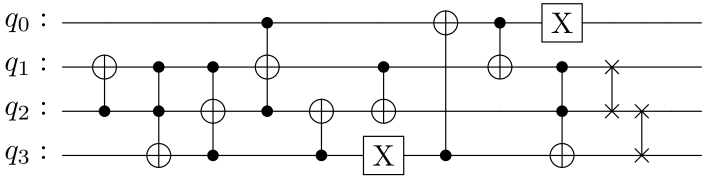

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The input is an 80-bit key and the output is a 64-bit round key.

KEY EXPANSION FOR QPRESENT($K = k_{79}k_{78}..k_0$)

1 $k = \mathrm{k}_{63}\mathrm{k}_{62}..\mathrm{k}_{0\mathrm{s}} = k_{79}k_{78}..k_{16}$
2 $k = k \gg 19$
3 $[k_{79}k_{78}k_{77}k_{76}] = S[k_{79}k_{78}k_{77}k_{76}]$
4 $[k_{19}k_{18}k_{17}k_{16}k_{15}] = X[k_{19}k_{18}k_{17}k_{16}k_{15}]$
5 **return** k

Instead of rotating 61 bits to left, rotate 19 bits to right using SWAP gates.

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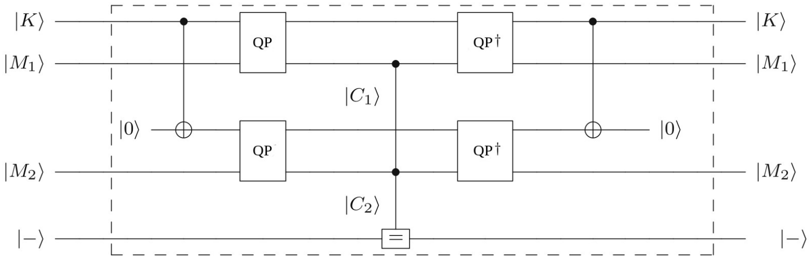

Figure: Grover's Attack on QPRESENT for r = 2

Cipher	Qubits	X	CX	CCX	Ancilla	Depth
QPRESENT 64/80	144	1118	4683	2108	-	311
QSIMON 64/128	192	1216	7396	1408	-	2643

Table: Comparison of cost for QPRESENT 64/80 [Jan+21b] and QSIMON 64/128[AMM20]

Grover's Attack and Cost Estimates

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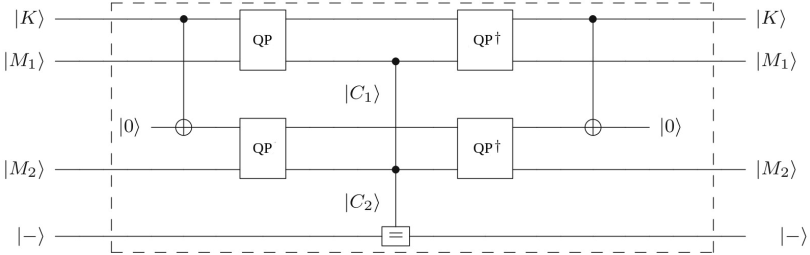

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Outline

Grover on AES[16]

$N = 2^k$ be the key search space. $M \geq 1$ is the number of solutions. The probability of finding one of the M solutions after t iterations is defined as

$$p(t) = sin^2((2t+1)\theta)$$

which after solving for 1 gives $t \approx \frac{\pi}{4\theta} = \frac{\pi}{4}\sqrt{\frac{M}{N}}$.

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Key search

Let $E_K(m) = c$ denote the encryption of message $m \in \{0,1\}^n$ by key $K \in \{0,1\}^k$ to ciphertext c. Then the Grover's Oracle is defined as:

$$f(K) = \begin{cases} 1 & E_K(m_i) = c_i \\ 0 & else \end{cases}$$

It is possible that multiple keys other than K lead to the same ciphertext from the given plaintext. Call them spurious keys.

Problem

Find the optimal number r such that the probability of finding a spurious key is minimal.

Let K be the correct key and K' is spurious. Then

$$P_{K \neq K'}(E_K(m) = E_{K'}(m)) = \frac{1}{2^n}$$

for r plaintext-ciphertext pairs, we have:

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Key search Contd.

Y be a binomially distributed random variable that describes the count of spurious keys for given key K and r plaintext-ciphertext pairs.

$$P(Y = y) = \binom{2^k - 1}{y} p^y (1-p)^{2^k - 1 - y}$$

Approximate this to poission distribution with $\lambda = (2^k - 1)p = (2^k - 1)2^{-rn}$

$$P(Y = y) = \frac{e^{-\lambda}\lambda^k}{y!} \approx \frac{e^{-2^{k-rn}} 2^{(k-rn)y}}{y!}$$

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Parallelization of Grover

- Two ways described by [KHJ18][17]. Inner and outer. Multiple instances of the full Grover's algorithm are run on different machines simultaneously for a reduced number of iterations in outer parallelization.
- The search space is divided into multiple disjoint subsets and each machine is assigned one subset, in case of inner parallelization.
- [Zal99][18] found that there is a gain of $\sqrt{S}$ in the number of iterations for S parallel machines. This is inefficient as we gain only $\sqrt{S}$ factor in the depth of the quantum circuit whereas the width has become S times the original. [Jaq+20][19] uses inner parallelization.

[17] Panjin Kim, Daewan Han, and Kyung Jeong. "Time–space complexity of quantum search algorithms in symmetric cryptanalysis: applying to AES and SHA-2". In: *Quantum Information Processing* 17 (Oct. 2018). DOI: 10.1007/s11128-018-2107-3.

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Parallelization of Grover

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- In outer parallelization, the probability that we find the correct key after t iterations is $p_S(t) = 1 - (1 - p(t))^S$.
- In each machine the number of iterations will be $t_S = \frac{\pi}{4\theta\sqrt{S}}$.
- General expression by using the series expansion of $sin(x)$ for larger values of S.

$$p_S(t_S) = 1 - \left(1 - \frac{\pi^2}{4S} + O\left(\frac{1}{S^2}\right)\right)^S, \sum_{y=1}^{\infty} P(Y = y) = 1 - e^{-\frac{2^{k-rn}}{S}}$$

- As S tends to ∞, the above value approaches to $1 - e^{\frac{-\pi^2}{4}} \approx 0.91$.
- This implies by just by increasing the number of parallel machines S, one cannot get a probability near 1 for finding the correct key.
- For inner parallelization, the correct key exists in one of the subsets only, and with t_S iterations, the machine has a near 1 probability of finding it and other machines will not find it.
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Quantum Circuit Design

Use AND gate instead of Toffoli gate. The decomposition of the Toffoli gate is to 7 T gates, 8 Clifford gates with a T-depth 4 and total depth 11 whereas AND gate used 4 T gates, 11 Clifford gates with T-depth 1 and total depth 8. AND gate uses one ancilla qubit which is released after the operation.

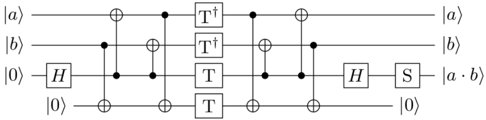
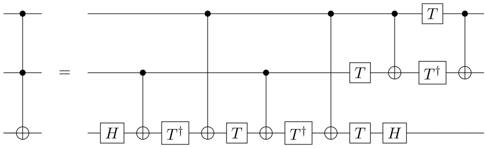


Figure: AND Gate [Jaq+20]



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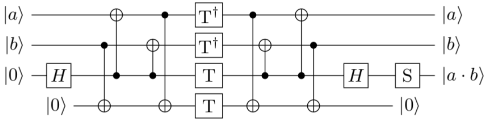

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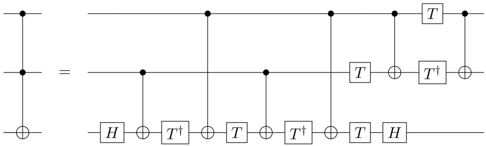

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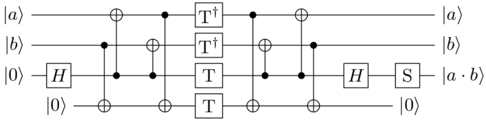

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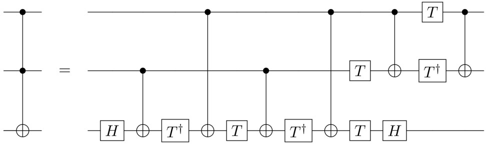

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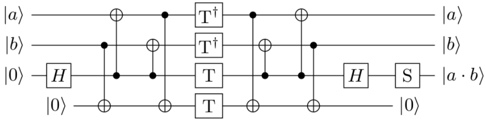

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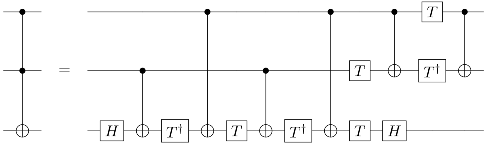

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Depth limit D_{max}. Two types of cost: G-cost for the total number of gates and DW-cost which is the product of the depth and width of the circuit. $N = 2^k$ be the key search space. $S = 2^s$ is the count of parallel machines. Assume that G is the oracle for Grover has G_G gates with G_D depth using G_W qubits. Number of iterations for a probability p i.e. $t_p = \frac{sin^{-1}(\sqrt{p})/\theta - 1}{2} \approx \frac{sin^{-1}(\sqrt{p})}{2}\sqrt{\frac{N}{S}}$. Assume $sin^{-1}(\sqrt{p})/2 = c_p$.

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Depth limit D_{max}. Two types of cost: G-cost for the total number of gates and DW-cost which is the product of the depth and width of the circuit. $N = 2^k$ be the key search space. $S = 2^s$ is the count of parallel machines. Assume that G is the oracle for Grover has G_G gates with G_D depth using G_W qubits. Number of iterations for a probability p i.e. $t_p = \frac{sin^{-1}(\sqrt{p})/\theta - 1}{2} \approx \frac{sin^{-1}(\sqrt{p})}{2}\sqrt{\frac{N}{S}}$. Assume $sin^{-1}(\sqrt{p})/2 = c_p$.

$$D = t_p G_D \approx c_p 2^{\frac{k-s}{2}} G_D \tag{5}$$

The total gate cost over all S machines (G-cost) is:

$$G = t_p G_G S \approx c_p 2^{\frac{k+s}{2}} G_G \tag{6}$$

The total width $W = G_W S$ qubits. Therefore the DW-cost is :

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We can run at most $t_{max} = D_{max}/G_D$ iterations of G. For probablitiy p of finding the correct key, we calculate S i.e. $p = sin^2((2t_{max}+1)\sqrt{\frac{S}{N}})$. This gives:

$$S = \frac{(sin^{-1}(\sqrt{p}))^2 N}{(2\frac{D_{max}}{G_D}+1)^2} \approx c_p^2 2^k \frac{G_D^2}{D_{max}^2} \quad (8)$$

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Outline

QAES[23]

- Authors compared various previously proposed Sbox designs on the G-cost and DW-cost metrics by reconstructing them.
- [BP11] Sbox was effective in terms of G-cost and DW-cost hence they chose it.

Sbox	CNOT	Clifford	T	M	T-depth	full depth	width	DW
[Gra+15][20]	8683	1028	3584	0	217	1692	44	74,448
[BP10][21]	818	264	164	41	35	497	41	20,377
[BP11][22]	654	184	136	34	6	101	137	13,837

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- The authors compared two variants of the mix column.
- In-place which does not require the use of ancilla qubits which saves the width.
- Not in-place and requires ancilla qubits but saves the depth of the circuit.

MC	CNOT	Clifford	T	M	T-depth	full depth	width	DW
In place	1108	0	0	0	0	111	128	14,208
[Max19][24]	1248	0	0	0	0	22	318	6,996

Table: Comparison of cost of mix column variants

The authors chose [Max19] mix column variant due to its low DW cost. The DW cost is mainly affected by the G_D^2 term and therefore it is crucial to minimize the depth of the oracle used, here its mix column.

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Generate keys on the fly which do not require ancilla qubits.

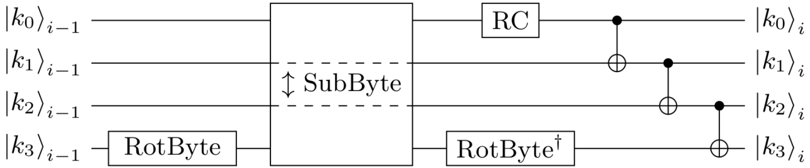

Figure: AES-128 key expansion [Jaq+20]

$|k_j\rangle_i$ represent the j^{th} word (4 bytes) of the i^{th} round key.

QAES-128 circuit

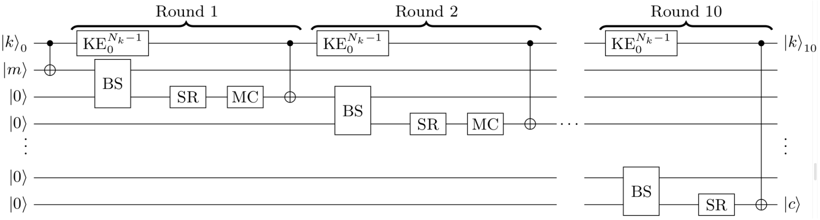

Figure: QAES-128 [Jaq+20]

Each wire represents 4 words (128 qubits). $|k\rangle$ represents the master key and m represents the message.BS is Sbox, SR is shift rows and MC is mix column. Here we have used an in-place version of mix columns.

MC	CNOT	Clifford	T	M	T-depth	full depth	width	DW
QAES-128 In place	2,91,150	83,116	54,400	13,600	120	2,827	1,785	50,46,195
QAES-128 [Max19][25]	2,93,730	83,236	54,400	13,600	120	2,094	2,937	61,50,078

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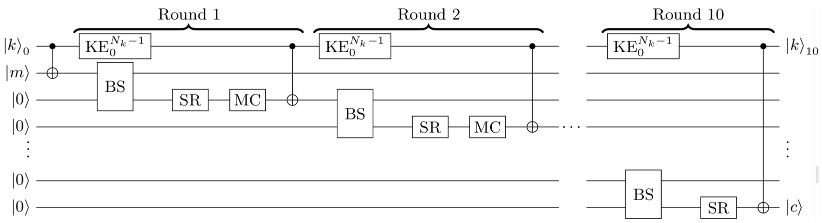

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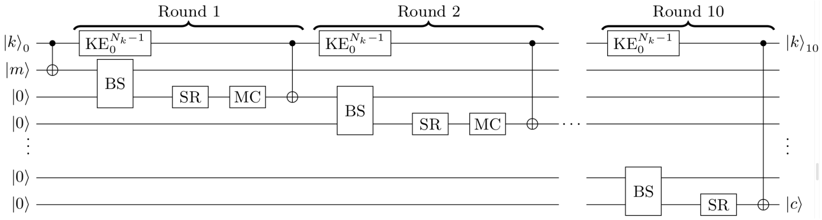

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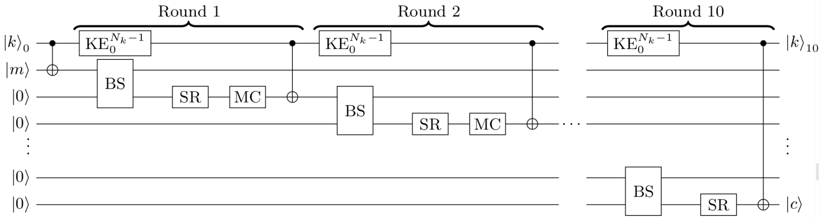

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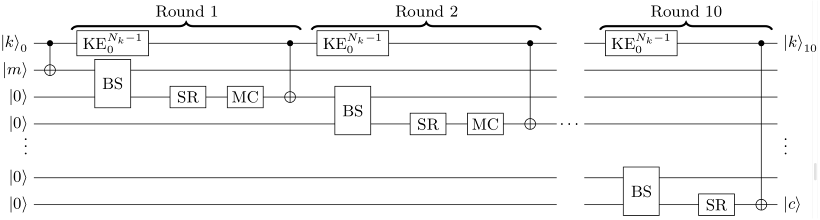

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Grover's Attack

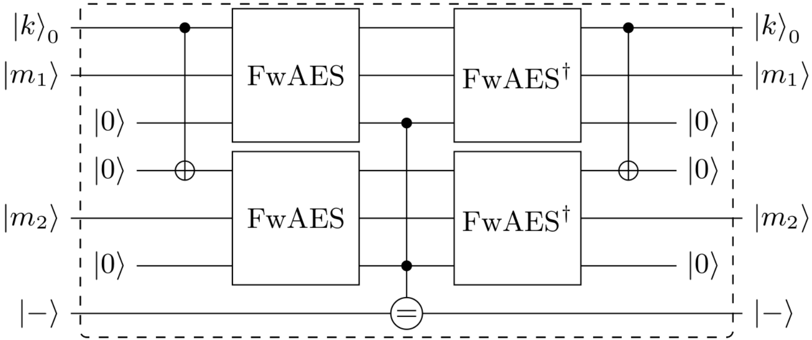

Figure: Grover's Attack on QAES-128 [Jaq+20]

MC	CNOT	Clifford	T	M	T-depth	full depth	width	DW
QAES-128 In place	5,85,051	1,69,184	1,09,820	27,455	121	2,815	3,329	93,71,135
QAES-128 [Max19][26]	5,89,643	1,68,288	1,09,820	27,455	121	2,096	5,633	1,18,06,768

Table: QAES-128 Grover's Oracle cost of both variants of mix columns (r=2)

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Cost Estimates

Let's calculate the cost estimates of Grover's Attack on QAES-128 in place with r = 2 with and without depth constraint.

$$G_G = 5,85,051 + 1,69,184 + 1,09,820 + 27,455 = 8,91,510 \approx 1.7 \times 2^{19}$$
$$G_D = 2,815 \approx 1.37 \times 2^{11}$$
$$G_W = 3,329 \approx 1.62 \times 2^{11}$$

Therefore the cost estimates without depth constraints keeping $S = 1$ is:

$$D \approx 1.37 \times 2^{11} \times 2^{64} = 1.37 \times 2^{75}$$
$$G \approx 1.7 \times 2^{19} \times 2^{64} = 1.7 \times 2^{83}$$
$$DW \approx 1.37 \times 2^{11} \times 1.62 \times 2^{11} \times 2^{64} \approx 1.1 \times 2^{87}$$

Similarly, the cost estimates with depth constraint of 2^{40} are:

$$S \approx 2^{128} \times 1.37^2 \times 2^{22} \times 2^{-80} \approx 1.87 \times 2^{70}$$
$$G \approx 2^{128} \times 1.37 \times 2^{11} \times 1.7 \times 2^{19} \times 2^{-40} \approx 1.16 \times 2^{119}$$
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Therefore the cost estimates without depth constraints keeping $S = 1$ is:

$$D \approx 1.37 \times 2^{11} \times 2^{64} = 1.37 \times 2^{75}$$

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- For $\text{MAXDEPTH} = 2^{40}$, [16] has bounded the count of quantum gates by $2^{170}/2^{40} = 2^{130}$.
- From the above calculation of G-cost we can see that the number of gates required by AES-128 is much less after parallelization (2^{119}).

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- We briefly studied hardware and software-friendly ciphers.
- We designed Quantum circuits for SAES, SIMON, PRESENT and optimized on qubits.
- As a result, the quantum architecture is not a barrier to a quantum adversary carrying out any potential quantum attack.
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