

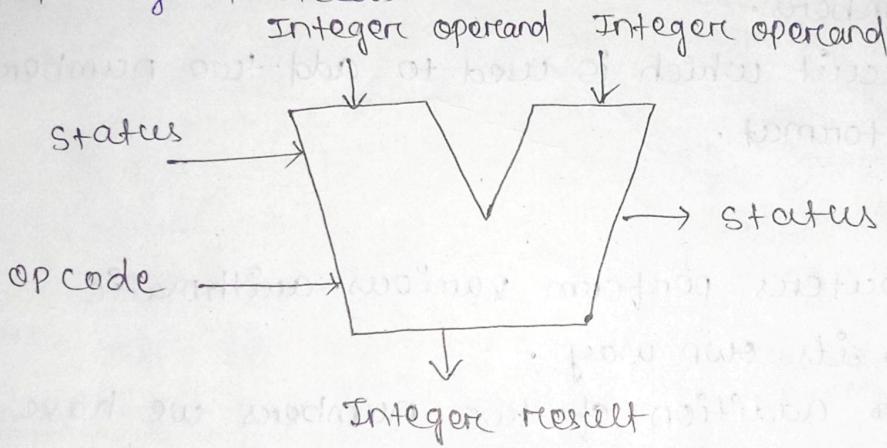
UNIT-2

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Addition or Subtraction Logic Unit: (Arithmetic Logic Unit - ALU)

- Computer's processing unit contains an arithmetic and logic unit (ALU) which is capable of performing logical operations (e.g. AND, OR, NOT) etc. In addition to the arithmetic operations e.g. addition, subtraction etc.
- The control unit supplies the data required by the ALU from memory or from input devices and directs ALU to perform a specific operation based on the ~~in~~ the instructions fetched from the memory.
- ALU can be said as the calculator of the computer.
- It is a major component of the CPU.
- It performs all operations related to arithmetic and logical needs.



(Diagram of Arithmetic logic unit)

- When the operations become more complex then the ALU will also become more and more expensive & also requires more and more space in the CPU.
- It generates more heat.
- Hence it is constructed with powerful and fast architecture.

- It consumes more energy.
- It is costly as per the complexity nature.
- Operations performed by ALU.
 - i) Logical Operations: AND, OR, XOR, NAND, NOR
 - ii) Bit Shifting operations :- At the time of multiplication and division operations to be performed the bit shifting requirements occur either to right shift or left shift by a certain number of place.
 - iii) Arithmetic Operations; Arithmetic operations like addition, subtraction, multiplication etc. are performed.

Binary Adder :-

- Adder is a kind of calculator used to add two binary numbers.
- It is a circuit which is used to add two numbers in binary format.

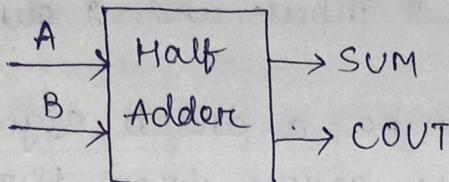
Half Adder -

- Digital computers perform various arithmetic operations in its own way.
- Normally for addition of two numbers we have 4 possible elementary operations.

	SUM	CARRY
0 + 0	0	0
0 + 1	1	0
1 + 0	1	0
1 + 1	0	1

The logic circuit which performs addition of two binary digits and produces a sum and a carry output is called as a half adder.

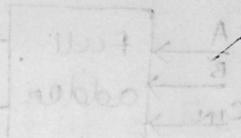
A half adder inputs two bits only A and B and produces sum value SUM and carry value COUT.



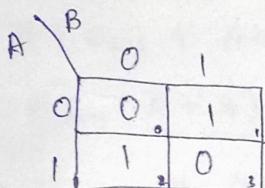
(Functional diagram for half adder)

Truth Table

A	B	SUM	COUT
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



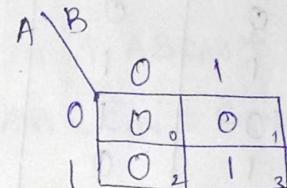
K-map for SUM



$$01 + 10$$

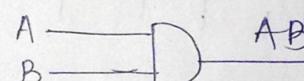
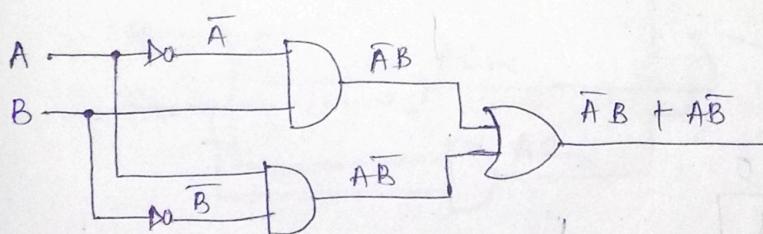
$$\text{SUM} = \overline{AB} + AB$$

K-map for COUT



$$\text{COUT} = 11 = AB$$

Circuit Diagram



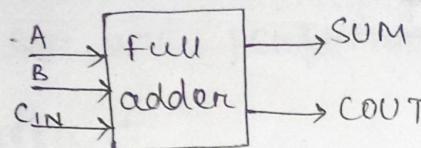
$$010 + 111 + 001 + 100$$

$$101 + 011 + 110 + 111 = 1001$$

Full Adder -

- A full adder is a combinational circuit that takes a carry received from previous bit position.
- It performs addition of 3 bits two significant bit and a previous carry.
- Full adder consists of 3 inputs and 2 outputs (SUM and COUT)
- 2 input variables can be a and b significant bits and C_{IN} ~~is~~ represents carry from the previous lower significant bit position.

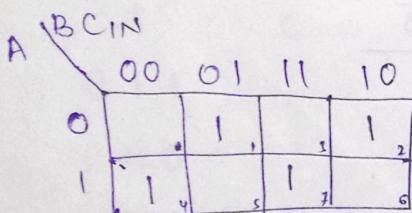
functional Diagram



Truth Table :-

A	B	CIN	SUM	COUT
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

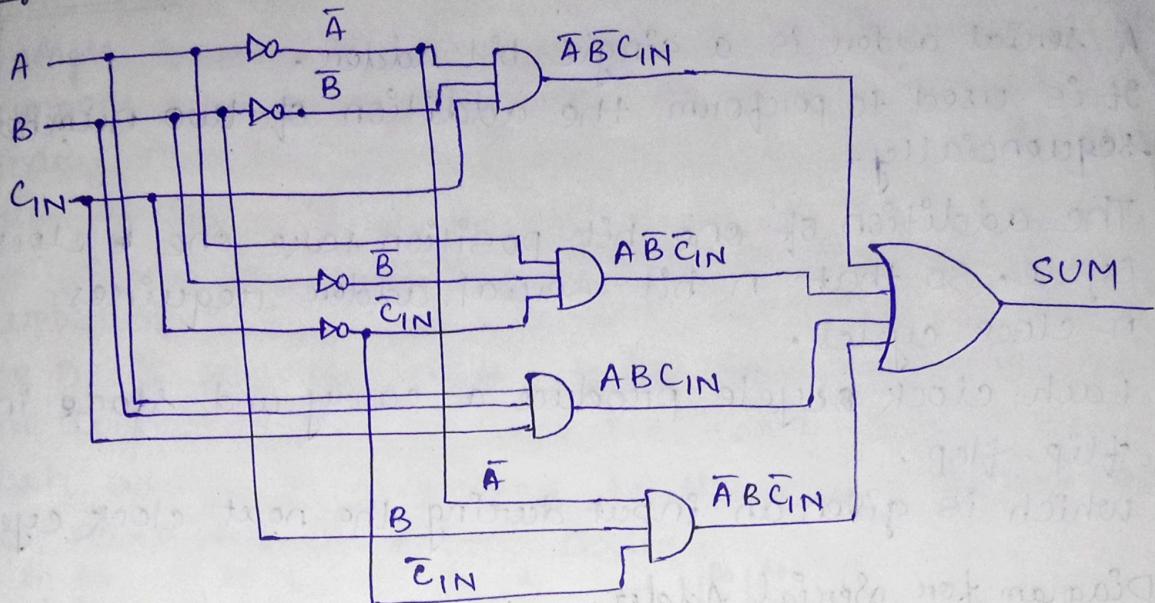
K-map for SUM



$$SUM = \overline{A}\overline{B}C_{IN} + A\overline{B}\overline{C}_{IN} + ABC_{IN} + \overline{A}BC_{IN}$$

$$SUM = \overline{A}\overline{B}C_{IN} + A\overline{B}\overline{C}_{IN} + ABC_{IN} + \overline{A}BC_{IN}$$

Logic gate:



K-map for C_{OUT}:

A \ B	C _{IN}	00	01	11	10
0	.	1	1	1	0
1	1	1	1	0	1

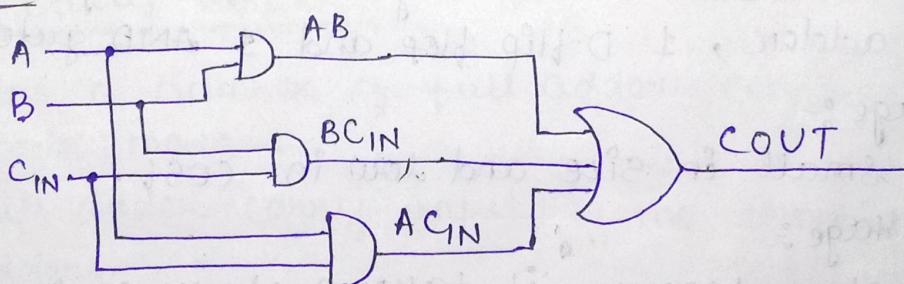
$$\text{Pair}_1: 011 + 111 = \bar{A}B\bar{C}_{IN} + A\bar{B}\bar{C}_{IN}$$

$$\text{Pair}_2: 101 + 111 = A\bar{B}C_{IN} + \bar{A}BC_{IN}$$

$$\text{Pair}_3: 111 + 110 = ABC_{IN} + \bar{A}\bar{B}\bar{C}_{IN}$$

$$\begin{aligned}
 & \bar{A}B\bar{C}_{IN} + A\bar{B}\bar{C}_{IN} + A\bar{B}C_{IN} + \bar{A}BC_{IN} + ABC_{IN} + \bar{A}\bar{B}\bar{C}_{IN} \\
 &= BC_{IN}(\bar{A} + A) + AC_{IN}(\bar{B} + B) + AB(\bar{C}_{IN} + C_{IN}) \\
 &= BC_{IN} + AC_{IN} + AB \\
 &= AB + BC_{IN} + AC_{IN}
 \end{aligned}$$

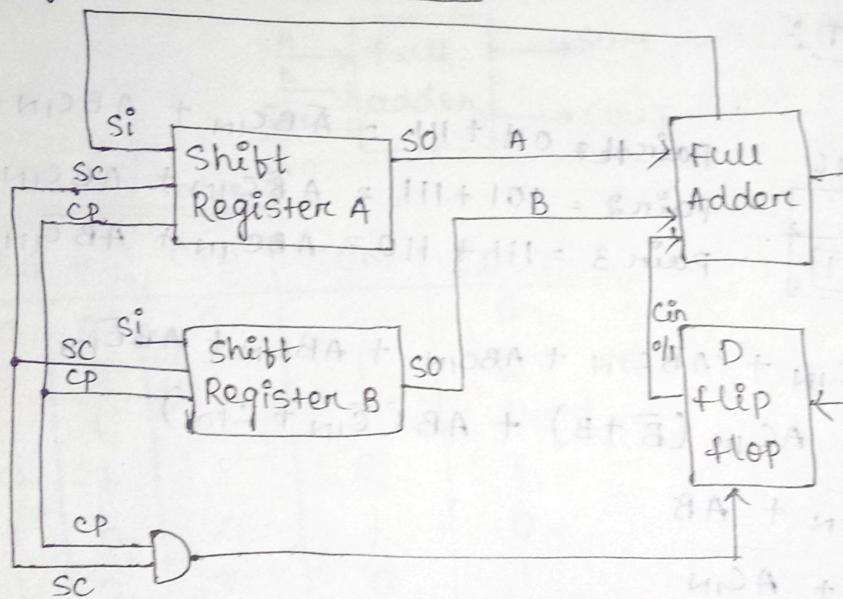
Logic gate:



Serial Adder :-

- A serial adder is a single bit adder.
- It is used to perform the addition of two numbers sequentially.
- The addition of one bit position take one clock cycle. So that n bit serial adder requires n clock cycles.
- Each clock cycle produce a carry and store it in a flip-flop.
- Which is given an input during the next clock cycle.

Diagram for Serial Adder



SI = shift input
SC = shift control
CP = clock pulse
SO = shift output

Carry

- The serial adder uses a right shift register, 1 full adder, 1 D-Flip-flop and 1 AND gate.

Advantage:-

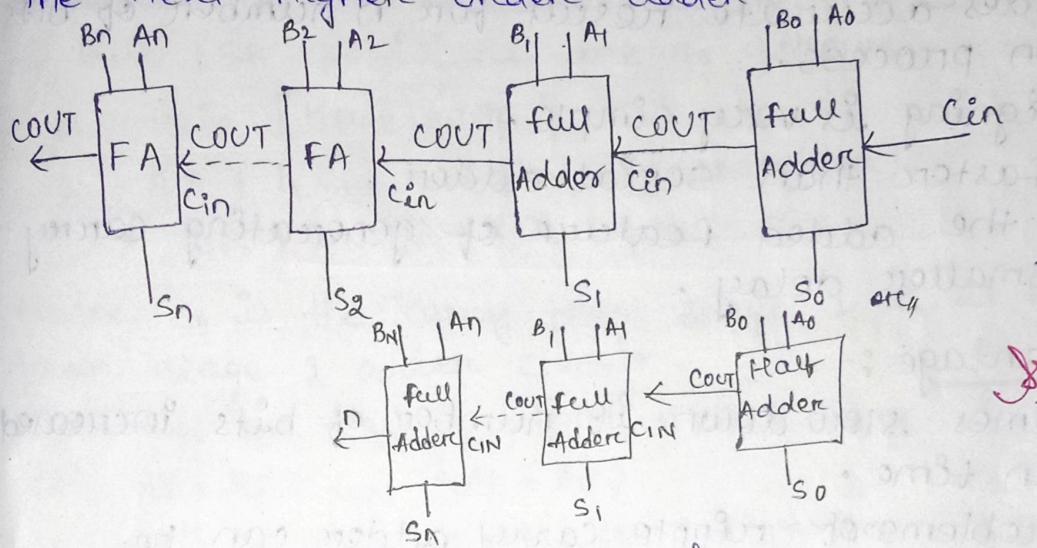
It is small in size and low in cost..

Disadvantage:-

It is slow because it takes n clock cycles for completion of addition of n bit numbers. Because it takes n-clock cycle.

Parallel Adder :-

- A single full adder is capable of adding two 1 bit numbers and an input carry. In order to add binary numbers with more than 1 bit additional full adders must be used.
- A n bit parallel adder can be constructed using number of full adder circuits connected in parallel.
- The n bit parallel adder using full adder circuits are connected in cascade i.e. carry output of each adder is connected to the carry input to the next higher order adder.



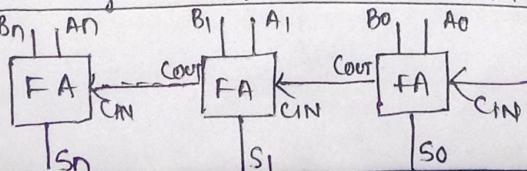
Parallel adder is of 2 types :-

- i) Ripple carry adder
- ii) Carry Look ahead adder

i) Ripple Carry Adder:-

- There are n number of full adders connected into a cascade mode.
- Each full adder carry input to the ~~next~~ next step adders.

Block diagram of n bit ripple carry adder:-



- From the above diagram it can be clear that each stage adder is a combinational circuit in which the current output depends on the current inputs and generates 2 relevant outputs i.e. sum and carry.
- Ripple carry adder is a simple form of parallel adder, where the carry out of each full adder is connected to the carry in of the next full adder.
- Hence the total delay time of the adder is the time it would take for a carry to ripple through all bit pair full adder.

Advantage :-

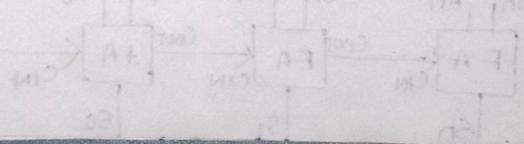
- It provides accurate result for n number of bits addition process.
- Its designing is very simple.
- It is faster than serial adder.
- It has the added feature of generating carry with smaller delay.

Disadvantage :-

- It becomes slow down if number of bits increased at run time.
- The problems of ripple carry adder can be solved using carry look ahead adder with extra circuitry.

Cascading ripple carry adder :-

using cascading ripple carry adder with any bit length, construct n bit adder using two numbers of n bit adders of ripple time and then both the modules can simultaneously perform addition process and allow the carry propagation module to the next module.



Carry Look Ahead Adder :-

- A carry look ahead adder is a high speed adder which allows a special strategy of quick carry generation form.
- At this generation stage without waiting for the carries of the previous stages of the circuit the adder involves early determination of carry input stage without using carry output from the previous stage, it directly determines the carry signals.
- Consider an n-bit adder where the carry output of two LSB positions are as follows -

$$C_0 = A_0 B_0 \quad [\text{Half Adder}]$$

$$\begin{aligned} C_1 &= A_1 B_1 + B_1 C_{\text{IN}} + A_1 C_{\text{IN}} \quad [\text{Full Adder}] \\ &= A_1 B_1 + C_{\text{IN}} (A_1 + B_1) \end{aligned}$$

Hence C_0 is the carry from stage 1 c_1 is the carry from stage 2 adder circuit.

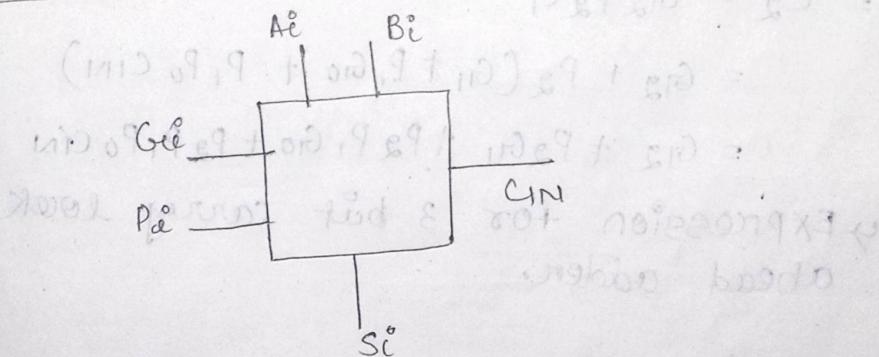
Similarly the carry output from stage i will be

$$C_i = A_i + B_i + C_{\text{IN}} (A_i + B_i)$$

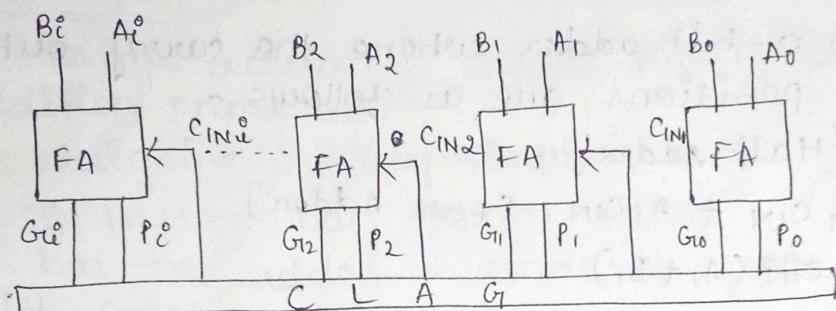
C_{i-1} will be the carry output from stage $i-1$. Hence the role of generators G_i and propagators P_i can be identified in the following manner.

$$C_0 = G_0 + P_0 C_{\text{IN}}^{i-1}$$

The symbolic diagram :-



- When expanding is ~~stage~~ statement for all the carries it is confirmed that all the carries in parallel adder can be determined directly from the input data with the given pattern without actually waiting for the carry of the previous bit position through the parallel adder.
- It means that every stage can perform independently of other stage.
- This procedure increases the speed of addition.
- But it has several gates with multiple inputs.



(Carry Look Ahead Generator)

Equations :-

$$C_0 = G_{10} + P_0 C_{IN}$$

$$\begin{aligned} \text{At stage 1} = C_1 &= G_{11} + P_1 C_0 \\ &= G_{11} + P_1 (G_{10} + P_0 C_{IN}) \\ &= G_{11} + P_1 G_{10} + P_1 P_0 C_{IN} \end{aligned}$$

↳ This is the expression for 2 bit carry look ahead adder.

$$\begin{aligned} \text{At stage 2} = C_2 &= G_{12} P_2 C_1 \\ &= G_{12} + P_2 (G_{11} + P_1 G_{10} + P_1 P_0 C_{IN}) \end{aligned}$$

$$= G_{12} + P_2 G_{11} + P_2 P_1 G_{10} + P_2 P_1 P_0 C_{IN}$$

↳ Expression for 3 bit carry look ahead adder.

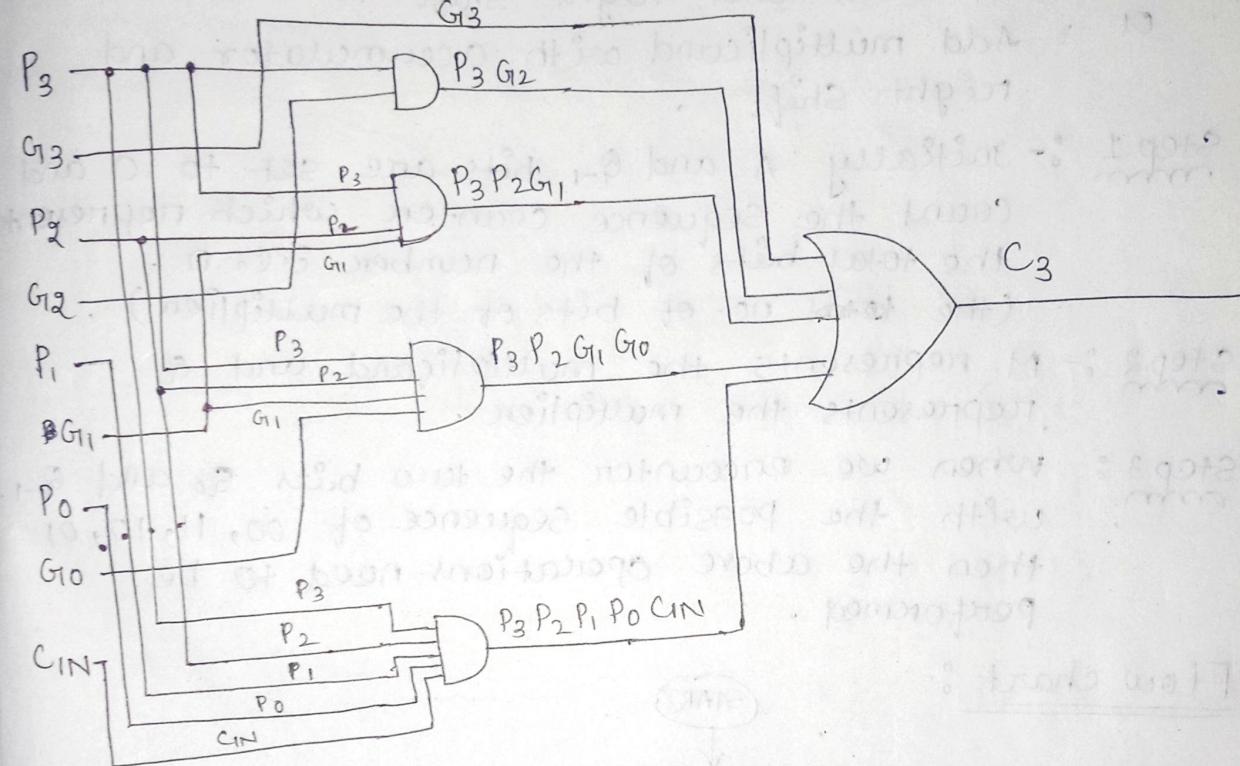
$$\text{At stage } n = G_n + P_n G_{n-1} + P_n P_{n-1} G_{n-2} + \dots + P_n P_{n-1} P_{n-2} C_{IN}$$

[n bit carry look ahead adder]

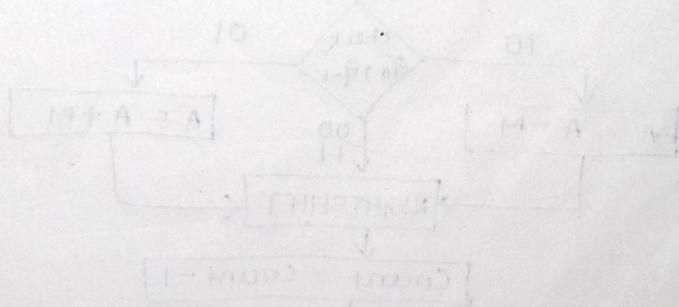
construct the circuit diagram for 4 bit carry look ahead adder :-

the expression for the carry look ahead adder
at stage 3 -

$$C_3 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_{IN}$$



0	0	0	0	= A
1	0	0	0	= B
0	1	0	0	= C
0	0	1	0	= D



Booth's Multiplication :-

Steps involved in Booth's multiplication :-

- Use the LSB (current LSB) and previous LSB to determine the algorithmic operation.
- The possible algorithmic operations can be 00}, where no change only right shift is required . 11,

10 :- then Subtract multiplicand from the accumulator A and right shift .

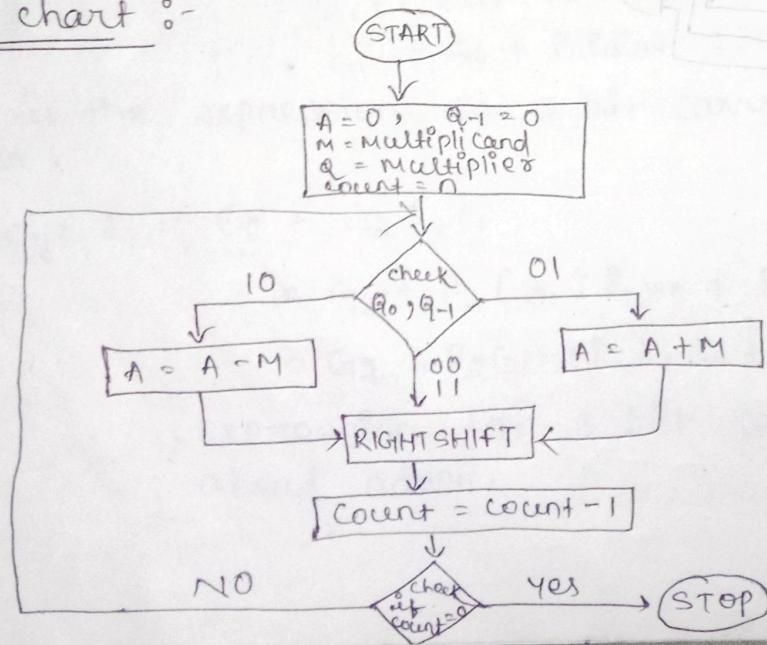
01 :- Add multiplicand with accumulator and right shift .

Step 1 :- Initially A and Q_{-1} bits are set to 0 and count the sequence counter which represents the total bits of the number i.e. n . (the total no. of bits of the multiplier) .

Step 2 :- M represents the multiplicand and Q represents the multiplier .

Step 3 :- when we encounter the two bits Q_0 and Q_{-1} with the possible sequence of 00, 11, 10, 01 then the above operations need to be performed .

Flow chart :-



Rules for MSB :-

- For the positive number multiplicand at MSB of A
Put LSB of Q of the previous cycle.
 - For the negative number multiplicand at MSB of A
Put MSB of A of previous cycle.

1) Multiply -9×6

$$M_2 \rightarrow q = 10111$$

$$q = 6 = 00110$$

$$\text{M} = 10111$$

$$-M = 0 \mid 000$$

01001(-M)

M	A	Q	Q-1	operation	count
10111	000000 G000000	00110 00011	0 0	Initialisation Right shift	5 10
	000000 01001			$A = A - M$	$A = A + (C - M)$
	01001 G00100 000010	00011 10001 01000	0 1 1	Right shift	4 11 01
	00010 10111			$A = A + M$	2
	11001 G11000 11110	01000 10100 01010	1 0 0	Right shift	1 00
	11110	01010			0

$$\underline{\text{check}} := -9 \times 6 = -54$$

$$54 = 000011010$$

$$-54 = 1111001001$$

1111001010

$$\begin{array}{r}
 \cancel{2} \cancel{(5)} \cancel{4} - 0 \\
 \cancel{2} \cancel{(2)} \cancel{7} - 1 \\
 \cancel{2} \cancel{(1)} \cancel{3} - 1 \\
 \cancel{2} \cancel{(6)} - 0 \\
 \cancel{2} \cancel{(3)} - 1 \\
 \cancel{2} \cancel{(1)} - \\
 \hline
 0
 \end{array}$$

④ Multiply -5×4

Multiplicand = $M = -5 = 11011$
Multiplier = $Q = 4 = 00100$

$$M = 11011$$

$$-M = 00100$$

$$\frac{00101}{00101}$$

M	A	Q	Q-1	operations	count
11011	00000	00100	0	Initialisation Right shift	5 00
	00000	00010	0	Right shift	4 00
	00000	00001	0	$A = A - M$ Right shift	3 A $A = A + (-M)$ 01
	00010	00001	0	$A = A + M$	2
	11011	10000	1	Right shift	00
	11110	10000	0	Right shift	1
	11111	01100	0		0

check $-5 \times 4 = -20$

$$-20 = 00000010100$$

$$-20 = 11111010111$$

$$\begin{array}{r} & & 1 \\ & & 1 \\ \hline 1111101100 & = -20 \end{array}$$

$$\begin{array}{r} 2 | 20 - 0 \\ 2 | 10 - 0 \\ 2 | 5 - 0 \\ 2 | 2 - 0 \\ 2 | 1 - 1 \\ \hline 0 \end{array}$$

$$010110000 = 100$$

$$100100111 = 101$$

$$010100111$$

3) Multiply -13×6

$$\text{Multiplicand} = M = -13 = 10011$$

$$\text{Multiplier} = Q = 6 = 00110$$

$$M = 10011$$

$$-M = 01100$$

$$\underline{01101}$$

M	A	Q	Q-1	operation	count
10011	00000	00110	0	initialisation	5 00
	00000	00011	0	Right shift	10
	00000 +01101			$A = A - M$	11
	01101	00011	1	Right shift	4 $A = A + (-M)$
	000110	10001	1	Right shift	3 01
	000011	01000	1	Right shift	2
	000111 +10011			$A = A + M$	$A = A + M$
	10110	01000	0	Right shift	00
	11011	00100	0	Right shift	1
	11101	10010	0	Right shift	0

$$\text{check: } -13 \times 6 = -78$$

$$-78 = 0001001110$$

$$\begin{array}{r} -78 \\ +1110110001 \\ \hline 1110110010 = -78 \end{array}$$

$$\begin{array}{r} 2 \mid 78 - 0 \\ 2 \mid 39 - 1 \\ 2 \mid 19 - 1 \\ 2 \mid 9 - 1 \\ 2 \mid 4 - 0 \\ 2 \mid 2 - 0 \\ 2 \mid 1 - 1 \\ 2 \mid 0 \end{array}$$

$$4) 14 \times -5$$

$$\text{Multiplicand} = M = 14 = 01110$$

$$\text{Multiplier} = Q = -5 = 10101$$

$$-M = 10001$$

$$\begin{array}{r} \\ \\ \hline 10010 \end{array}$$

M	A	Q	Q-1	Operation	Count
01110	00000	11011	0	Initialisation A = A - M	5
A	00000				10
-M	10010			Right Shift	
		11011			
		11001 01101	1		11
		11100 10110	1	R.S	4
A	11100			A = A + M	3
M	01110				A = A + M
discard	101010 10110			R.S	
	00101 01011	0		A = A - M	2
A	00101				10
-M	10010				A + (-M)
		10111 01011		R.S	
		11011 10101	1		11
		11100 11010	1	R.S	1
	11100	11010			0

check :-

$$14 \times -5 = -70$$

$$-70 = 0001000110$$

$$-70 = 1110111001$$

$$+$$

$$\hline 1110111010$$

$$\begin{array}{r} 2 \longdiv{70} \\ -0 \\ \hline 2 \longdiv{35} \\ -1 \\ \hline 2 \longdiv{17} \\ -1 \\ \hline 2 \longdiv{18} \\ -0 \\ \hline 2 \longdiv{4} \\ -0 \\ \hline 2 \longdiv{2} \\ -0 \\ \hline 2 \longdiv{0} \\ -1 \\ \hline \end{array}$$

$$5 \times -4$$

$$M = 5 = 00101$$

$$Q = -4 = 11100$$

M	A	Q	Q_{-1}	Operation	Count	
00101	00000	11100	0	Initialisation	5	00
		00000 01110	0	Right shift		00
		00000 00111	0	Right shift	4	10
A	00000			$A - M$	3	$A + (-M)$
-M	11011			Right shift		
	11011	00111				11
	11101	10011	1			
	11110	11001	1	Right shift	2	11
	11111	01100	1	Right shift	1	01011 A
	11111	01100			0	01011 M

check :-

$$5 \times -4 = -20$$

$$20 = 0000010100$$

$$\begin{array}{r} -20 = 1111101011 \\ \hline 1111101100 \end{array}$$

$$\begin{array}{r} 2(20) - 0 \\ 2(10) - 0 \\ 2(5) - 1 \\ 2(2) - 0 \\ 2(1) - 1 \end{array}$$

for. M5

$$6) -13 \times -9$$

$$\begin{aligned}M &= -13 = 10011 \\Q &= -9 = 10111\end{aligned}$$

$$\begin{array}{r} -M = 01100 \\ 01101 \end{array}$$

M	A	Q	Q_{-1}	Operation	Count
10011	00000	10111	0	Initialisation	5
A	00000			$A = A - M$	10
-M	01101			R.S	$A + (-M)$
	01101	10111			
	00010	11011	1	Right shift	4
	00001	01101	1		11
	00001	10110	1	Right shift	3
A	00001			$A = A + M$	01
M	10011				$A + M$
	10100	10110			
	11010	01011	0	R.S	
A	11010			$A = A - M$	10
-M	01101				$A + (-M)$
discard	10011	01011	0	Right shift	
	00010	10101	0		
	00011	10101	0		0

check

$$-13 \times -9 = 117$$

$$117 = 0001110101$$

$$\begin{array}{r} 2 \cancel{117} - 1 \\ 2 \cancel{58} - 0 \\ 2 \cancel{29} - 1 \quad 1 \\ 2 \cancel{14} - 0 \\ 2 \cancel{7} - 1 \\ 2 \cancel{3} - 1 \\ 2 \cancel{1} - 1 \end{array}$$

$$7) -5 \times -4$$

$$M = -5 = 11011 \quad -M = \frac{00100}{00101}$$

$$Q = -4 = 11100$$

M	A	Q	Q_{-1}	operation	count
11011	00000	11100	0	Initialisation	5
	00000	01110	0	Right shift	00
	00000	00111	0	Right shift	4
A	00000			$A = A - M$	3
-M	00101			Right shift	11
	00101	00111	1	Right shift	11
	00010	10011	1	Right shift	11
	00001	01001	1	Right shift	1
	00000	10100	1	Right shift	1
	00000	10100	0		0

$$\text{check} = -5 \times -4 = 20 = 0000010100$$

$$8) -14 \times -5$$

$$M = -14 = 10010$$

$$Q = -5 = 11011$$

$$-M = \frac{01101}{01110}$$

M	A	Q	Q_{-1}	operation	count
10010	00000	11011	0	Initialisation	5
A	00000			$A = A - M$	10
-M	01110			Right shift	
	01110	11011			
	10011	01101	1	R.S	4
	10001	10110	1	R.S	01
A	00011			$A = A + M$	
M	10010			R.S	
	10101	10110			
	11010	11011	0	$A = A + M$	3
A	11010			R.S	
-M	01110			$A = A - M$	10
discard	101000	11011		Right shift	
	00100	01101	1	Right shift	1
	00010	00110	1	Right shift	11
	00010	00110	0		0

$$\text{check: } -14 \times -15 = 70 = 0001000110$$

$\begin{array}{r} 2 \longdiv{70} \\ 2 \longdiv{35} \\ 2 \longdiv{17} \\ 2 \longdiv{8} \\ 2 \longdiv{4} \\ 2 \longdiv{2} \\ 2 \longdiv{0} \end{array}$

$$9) 5 \times 4$$

$$\begin{aligned} M &= 5 = 00101 \\ Q &= 4 = 00100 \end{aligned}$$

$$-M = \frac{11010}{11011}$$

M	A	Q	Q_{-1}	operation	cocent
00101	00000	00100	0	Initialisation	5
	00000	00010	0	Right shift	00
	00000	00001	0	Right shift	4
A	00000			$A = A - M$	3
-M	11011			Right shift	
	11011	00001	1	$A = A + M$	2
A	11101			Right shift	01
M	00101				
discard	100010	10000			
	00001	01000	0	Right shift	1
	00000	10100	0	Right shift	00
	00000	10100			0100

Check

$$5 \times 4 = 20 = 0000010100$$

$$\begin{array}{r} 2 \longdiv{20} \\ 2 \longdiv{10} \\ 2 \longdiv{5} \\ 2 \longdiv{2} \\ 2 \longdiv{0} \end{array}$$

$$0000010100 = 20$$

Restoring Division Algorithm or Booth's Division Algorithm :-

(divisor) $10 \Big| 11110 \Big| 110$ (quotient)

$$\begin{array}{r} 10 \\ \underline{\times} 101 \\ 101 \\ \underline{\times} 101 \\ 00 \\ \underline{\times} 0 \\ 0 \end{array}$$

0 (remainder)

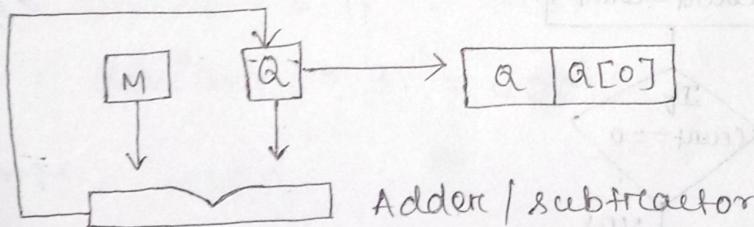
check :- $11110 = 2^4 + 2^3 + 2^2 + 2^1$
 $= 16 + 8 + 4 + 2$
 $= 30$

$101 = 5$

$110 = 6$

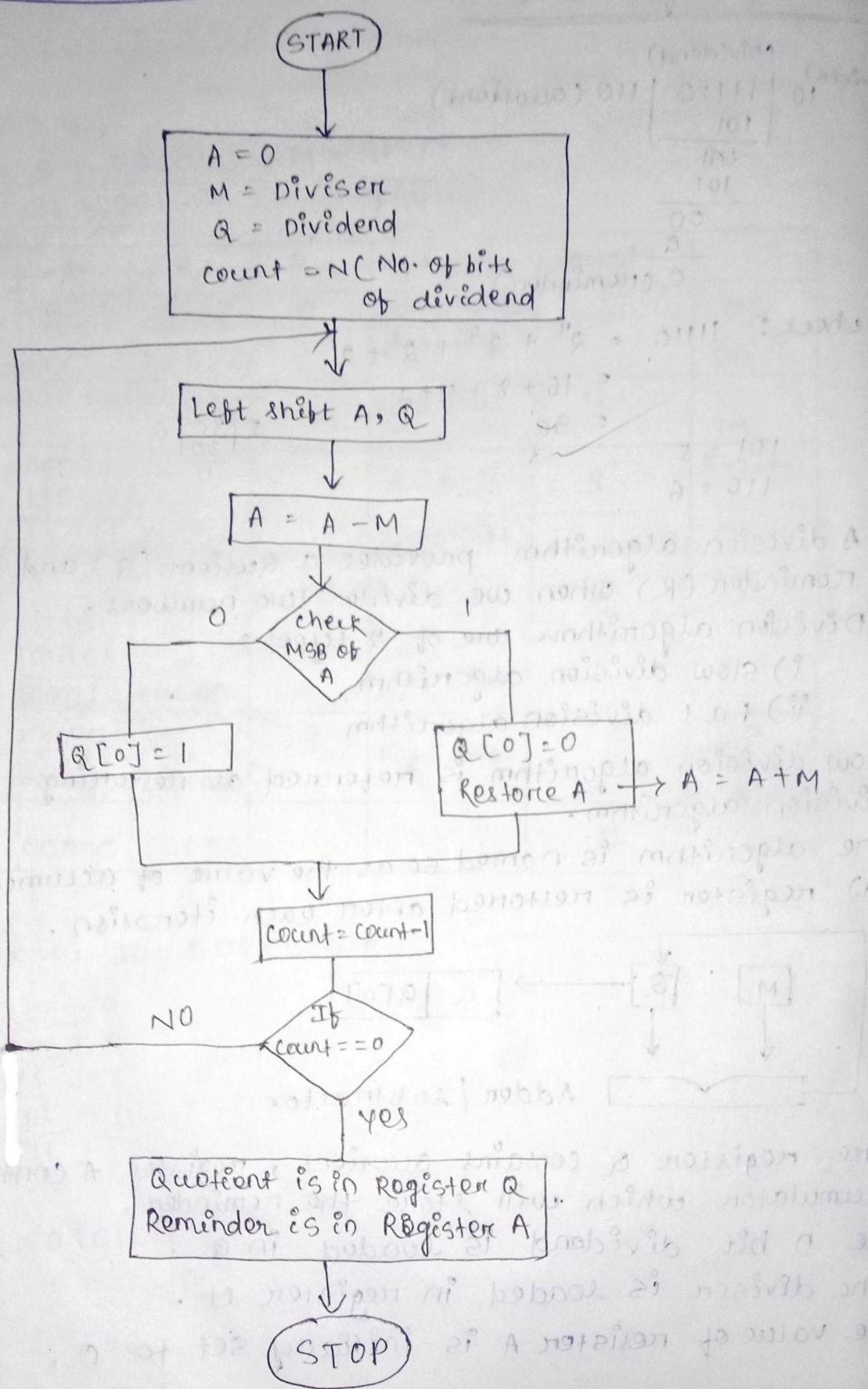
$$5 \Big| 30 \Big| 6$$

- A division algorithm provides a Quotient (Q) and a remainder (R) when we divide two numbers.
- Division algorithms are of 2 type :-
 i) slow division algorithm
 ii) fast division algorithm
- slow division algorithm is referred as restoring division algorithm.
- The algorithm is named so as the value of accumulators (A) register is restored after each iteration.



- Here register Q contains quotient, register A contains Accumulator which will store the remainder.
- The n bit dividend is loaded in Q.
- The divisor is loaded in register M.
- The value of register A is initially set to 0.

Algorithm :-



Steps involved in restoring Division:

- Step 1: The values will be initialized as $A=0$, $M=\text{divisor}$, $Q=\text{dividend}$, $\text{count}=\text{no. of bits available in } Q$.
- Step 2: Registers A and Q will be treated as single unit and shifted left.
- Step 3: Here value of register M will be subtracted from A and restored in A ($A = A - M$)
- Step 4: Now check the most significant Bit (MSB) of Register A, if this bit of A is 0 then the LSB of register Q will be set to 1. If MSB of A is 1 then LSB of Q will be 0 and restore the value of A ($A = A + M$) before subtraction with M.
- Step 5: After that the value of count will be decremented to 1.
- Step 6: If value of count is 0 we will break the loop otherwise go to step 2.
- Step 7: Here we will found the quotient will be available in register Q and remainder will be available in register A.
- Step 8: End or exit

e.g.

Divide 11 by 3.

$$Q = \text{Dividend} = 11 = 01011$$

$$M = \text{divisor} = 3 = 00011$$

$$-M = 11100$$

$$\begin{array}{r} & 1 \\ \hline 1 & 1 & 0 \\ \hline 11101 \end{array}$$

$$\begin{array}{r} 3)11(3, 11 \\ \underline{-2})10 \\ \hline \end{array}$$

MA	A	Q	Operation	Count
000111	000000 111111	010111 101111	Initialisation left shift $A = A - M$	5
A -M	000000 111011	101100	$Q[0] = 0, S_0$ restore A $A = A + M$	
A M	111011 000111			
discard ①	000000 111111	101100		
A -M	000001 111011	011000	left shift $A = A - M$	4
A M	111100 000111	011000	$Q[0] = 0, S_0$ represent A $A = A + M$	
discard ①	000001 111111	011000		
A -M	000100 111011	110000	left shift $A = A - M$	3
A M	111111 000111	110000	$Q[0] = 0, S_0$ represent A $A = A + M$	
discard ①	000100 111111	110000		
A -M	001001 111011	100000	left shift $A = A - M$	2
discard ①	000100 111111	100001	$Q[0] = 1$	
A -M	001001 111011	000011	left shift $A = A - M$	1
discard ①	000100 111111	000011	$Q[0] = 1$	
	000100	000011		0
			remainder coefficient	

check : 3 | 11 | 3 111
 $\underline{9}$
 $\underline{2} \quad 10$

i) Divide 13 by 6

$$Q: \text{Dividend} = 13 = 01101$$

$$M: \text{Divisor} = 6 = 00110$$

$$\begin{array}{r} -M = 01101 \\ \hline 11010 \end{array}$$

M	A	Q	operation	count
00110	00000	01101	Initialisation	
	00000	1101	left shift	5
A	00000		$A = A - M$	
-M	11010		$Q[0] = 0, \text{ so } \cancel{\text{discard}}$	
	11010	11010	restore A	
A	11010		$A = A + M$	
M	00110			
discard ①	00000	11010		
	00001	10110	Left shift	
A	00001		$A = A - M$	4
-M	11010		$Q[0] = 0, \text{ so } \cancel{\text{discard}}$	
	11011	10100	restore A	
A	11011		$A = A + M$	
M	00110			
discard ①	00001	10100		
	00011	01000	Left shift	
A	00011		$A = A - M$	3
-M	11010		$Q[0] = 0, \text{ so } \cancel{\text{discard}}$	
	11101	01000	restore A	
A	11101		$A = A + M$	
M	00110			
discard ①	00011	01000		
	00110	10000	Left shift	
A	00110		$A = A - M$	2
-M	11010		$Q[0] = 1, \text{ so } \cancel{\text{discard}}$	
	00000	10001		
discard ①	00000	10001		
	00001	00011	Left shift	
A	00001		$A = A - M$	1
-M	11010		$Q[0] = 0, \text{ so } \cancel{\text{discard}}$	
	11011	00010	restore A	
A	11011		$A = A + M$	
M	00110			
discard ①	00001	00010		
	00001	00010	(remainder)	0
	00001	00010	(coefficient)	

check:

$$6) 13(2 \quad | \quad 110$$

$$\underline{12}$$

$$110$$

2) Divide 17 by 4 :-

$$Q = \text{dividend} = 17 = 10001$$

$$M = \text{Divisor} = 4 = 00100$$

$$-M = 11011$$

$$11100$$

M	A	Q	operation	Count
00100	00000 ' ' ' ' 0' 0' 0' 1	10001 ' ' ' ' 0' 0' 0' 1	Initialisation Left shift	5
A	00001		$A = A - M$	
-M	11100		$Q[0] = 0, \text{ so } A = A$	
A	11101	00010	restore A	
M	00100		$A = A + M$	
discard ①	00001	00010		
	0' 0' 0' 1' 0	0' 0' 1' 0	Left shift	01011
A	00010		$A = A - M$	0101
-M	11100		$Q[0] = 0, \text{ so } A = A$	10000
A	11110	00100	restore A	01011
+M	00100		$A = A + M$	00001
discard ①	00010	00100		11011
	0' 0' 1' 0' 0	0' 1' 0' 0	Left shift	01011
A	00100		$A = A - M$	0101
-M	11100		$Q[0] = 1, \text{ so } A = A$	11000
discard ①	00000	01001		01011
	0' 0' 0' 0' 0	1' 0' 0' 1'	Left shift	00010
A	00000		$A = A - M$	10111
-M	11100		$Q[0] = 0, \text{ so } A = A$	10111
A	11100	10010	restore A	01100
M	00100		$A = A + M$	00010
discard ①	00000	10010		11000
	0' 0' 0' 0' 0	1' 0' 0' 1' 0	Left shift	00010
A	00001		$A = A - M$	10111
-M	11100		$Q[0] = 0, \text{ so } A = A$	10111
A	11101	100100	restore A	01000
+M	00100		$A = A + M$	11011
discard ①	00001	100100		01000
	0' 0' 0' 0' 1	0' 0' 1' 0' 0		0
	reminder	quotient		

check =

$$4) 17(4 \text{) } 16 \\ \underline{16} \\ 1101$$

3) Divide 12 by 3

$$8 = 12 \div 3 = 01100 \quad M = \frac{11100}{1} \\ M = 3 = 00011$$

M	A	S	operation	count
00011	00000	01100	Initialisation	
	00000	11'00	left shift	5
-M	11101		$A = A - M$	
	11101	11000	$S[0] = 0, \text{ so}$ restore A	
A	11101		$A = A + M$	
M	00011			
discard ①	00000	11000		
	11111			
	00001	1000	left shift	4
A	00001		$A = A - M$	
-M	11101		$S[0] = 0, \text{ so}$ restore A	
	11110	10000	$A = A + M$	
A	11110			
M	00011			
discard ①	00001	10000		
	11111			
	00011	0000	Left shift	3
A	00011		$A = A - M$	
-M	11101		$S[0] = 1$	
discard ①	00000	0000		
	11111			
	00000	0001	Left shift	2
A	00000		$A = A - M$	
-M	11101		$S[0] = 0, \text{ so}$ restore A	
	11101	00010	$A = A + M$	
A	11101			
M	00011			
discard ①	00000	00010		
	11111			
	00000	0010	left shift	1
A	00000		$A = A + M$	
-M	11101		$S[0] = 0, \text{ so}$ restore A	
	11101	00100	$A = A + M$	
A	11101			
M	00011			
discard ①	00000	00100		
	11111			
	00000	00100	content	0

Floating Point Representation:-

- In digital systems floating point numbers containing integer part and the fractional part. Which reserved a certain number of bits in the name Mantissa Part and the exponent part.
- Hence the floating point representation consists of two parts,
 - 1st part represents signed fixed point Mantissa &
 - 2nd part designates the position of the decimal or binary digits called as exponent.

The format for this is $M \times r^e \rightarrow \text{exponent}$

Mantissa radix

- Consider the floating point numbers A and B expressed as $A = M_a \times r^{e_a}$
 $B = M_b \times r^{e_b}$

sign	mantissa	Exponent
1 bit	8 bit	23 bits

where M_a and M_b are Mantissa, r is the radix & $e_a - e_b$ are exponents values of A and B respectively.

> Before performing any arithmetic operation between A and B the operands must be ~~restricted~~ normalised, so that both A and B have same exponent values. This is done by shifting radix point of the operands.

$$\text{e.g. } 2 = 2 \times 10^0$$

$$300 = 3 \times 10^2$$

$$4321.768 = 4.321768 \times 10^3$$

$$6720000000 = 6.72 \times 10^9$$

$$67.2 = 6.72 \times 10^1$$

$$0.2 = 2 \times 10^{-1}$$

$$0.25 = 2.5 \times 10^{-1}$$

$$0.0025 = 2.5 \times 10^{-3}$$

Floating Point Addition Algorithm :-

Consider the addition of A and B floating point numbers.

Step-1 :- Check whether A or B is 0. If so then the result is the other operand.

Step-2 :- Calculate difference between the exponents.

$$\begin{aligned} \text{Suppose } A &= M_a \times 10^{e_a} \\ B &= M_b \times 10^{e_b} \end{aligned}$$

$$\text{difference } = e_d = e_b - e_a / e_b - e_a$$

Step-3 :- Select the number A or B who has smaller exponent, then right shift the mantissa of the selected number by number of time = difference e_d .

Step-4 :- Perform addition of two mantissa i.e. $M_a + M_b$

Step-5 :- Represent the result as $\text{sum} = (M_a + M_b) \times 10^{e_d}$

Step-6 :- Normalise the result. This is done by shifting left the Mantissa till the MSB is a non-zero number. For each shift, the exponent of