Project - 1

Name – Gopal Gupta **Roll no.- 20CH30008**

The Van-der-Pol (VDP) equation describes a non-linear oscillator given by: $y''(t) - \mu\left(1 - y^2(t)\right)y'(t) + y(t) = 0 \quad for\ \mu > 0$

a)

To use ODE45, one must first convert the above second order ODE to two ODEys, each of which is first order.

Letting $y_1 = y(t)\ and\ y_2 = y'(t)$

$$y_1 = y(t) \rightarrow\ y_1' = y'(t) \rightarrow y_1' = y_2$$

$$y_2 = y'(t) \rightarrow\ y_2' = y''(t) = \mu\left(1 - y^2(t)\right)y'(t) - y(t)$$

$$substituting\ y'(t) = y_1' = y_2\ and\ \ y(t) = y_1$$

$$y_2' = \mu(1 - y_1^2)y_2 - y_1$$

Therefore, the y vector:

$$y = \begin{bmatrix} y_2 \\ \mu(1 - y_1^2)y_2 - y_1 \end{bmatrix}$$

c)

Comparing the speed with which the simulations run with ODE45 and ODE15s we get following runtime of 0.7475s and 0.7024s for mu=100. This not always true that ODE15s is always better than ODE45 because ODE15s is only better when differential equation is stiff.

d)

As the µ value increases from 0 to 100 we the differential equation shift from non-stiffness to stiffness and for less value of µ we see that the oscillator as it converges from initial condition towards a steady state of periodic oscillation.

Code

```
t  = [-10,10];   % time scale
mu  = 1;
y0 = 1;
v0 = 0;
values = @(t,y) [y(2) ; mu*(1-y(1)^2)*y(2)-y(1)];
[t,y] = ode45(values, t, [y0,v0]);

subplot(2,1,1);
plot(t,y(:,1),'r-o',t,y(:,2),'b-*');
xlabel('t'); ylabel('f(t)');
legend('y(t)','v(t)');
title(sprintf('Van Der Pol, position vs time, u=%3.2f',mu));

subplot(2,1,2);
plot(y(:,1),y(:,2),y0,v0,'*r','MarkerSize',10);
xlabel('y(t)'); ylabel('v(t)');
title(sprintf('Phase portait showing limit cycle. y(0)=%3.2f,
v(0)=%3.2f',y0,v0));
axis equal
```