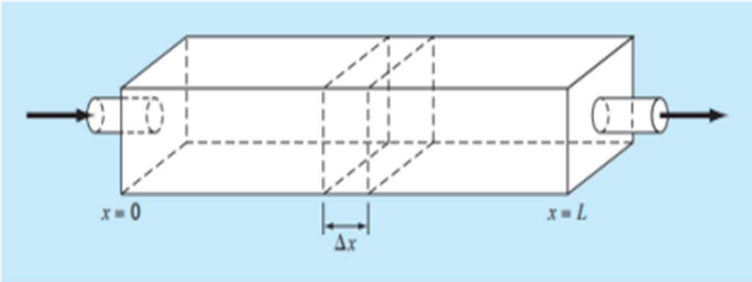
Problem Statement –

Chemical engineers extensively use idealized reactors in their design work. The following figure shows an elongated reactor with a single entry and exit point which can be characterized as a distributed parameter system.



Assuming that the chemical being modeled has a first order decay and the system is vertically and laterally well-mixed, a mass-balance can be performed on a finite segment of length Δx, as shown below

$$\frac{V\Delta c}{\Delta t} = Qc(x) - Q\left[c(x) + \frac{\partial c(x)}{\partial x}\Delta x\right] - DA_c\frac{\partial c(x)}{\partial x} + DA_c\left[\frac{\partial c(x)}{\partial x} + \frac{\partial}{\partial x}\frac{\partial c(x)}{\partial x}\Delta x\right] - kVc$$

where, V = volume (m^3), Q= flow rate (m^3/h), c is concentration ($moles/m^3$), D is a dispersion coefficient (m^2/h), A_c is the tank's cross-sectional area (m^2), and k is the first order decay coefficient (h^{-1}). Assuming, $U=Q/A_c$ and allowing Δt and Δx to approach zero, form a parabolic partial differential equation and use the following boundary conditions:

(i) At t=0, the chemical is injected into the reactor's inflow at a constant level of c_{in}

(ii) $Qc_{in} = Qc_0 - DA_c\frac{\partial c_0}{\partial x}$

(iii) $c'(L,t) = 0$

Then, at steady state, take D=2, U=1, $\Delta x = 2.5$, k=0.2, L=10, and c_{in}=100 to form a system of equations with 5 unknowns, i.e., c_0, c_1, c_2, c_3, c_4 and solve for them.

Hint:

Use centered finite differences for the first and second derivatives