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Problem Statement 1 - Solution

Solution:

To solve this problem, we need to find the optimal production cycle time `t` that maximizes the net profit earned by the company from producing compound B. We can do this by taking the derivative of the net profit function f(t) with respect to `t` and setting it equal to zero to find the critical points.

First, let's find the characteristic solution of the differential equation:

$$\frac{d^2y}{dt^2} + 2 \times \frac{dy}{dt} + y = 0$$

The characteristic equation is: $D^2 + 2D + 1 = 0$, which has a double root of -1. Therefore, the characteristic solution is:

$$y = c_1 e^{-t} + c_2 t e^{-t}$$

where c1 and c2 are constants that depend on the initial conditions.

In this case, the initial condition is that the production rate of compound B is 20 tonnes per cycle. At t = 0, y = 20, which means that:

$$c_1 + c_2 \times 0 = 20$$

$$c_1 = 20$$

Therefore, c_2 can be found by taking the derivative of y with respect to t and using the fact that the production rate is constant at 20 tonnes per cycle:

$$\frac{dy}{dt} = -c_1 e^{-t} + (c_2 e^{-t} - c_2 t e^{-t})$$

At t = 0, $\frac{dy}{dt} = 0$, which means that:

$$-c_1 + c_2 = 0$$

$$c_2 = c_1$$

Therefore, $c_1 = 20$ and $c_2 = 20$.

Next, we can calculate the energy consumption cost per day using the characteristic solution:

$$\text{Energy Consumption Cost per day} = \text{Rs.}\,1700000 \times (\text{characteristic solution})$$

$$= \text{Rs.}\,1700000 \times (20e^{-t} + 20te^{-t})$$

Now, we can write the net profit function as follows:

$$f(t) = 1.2 \times 10^7 - (Rs. 1700000 \times (20e^{-t} + 20te^{-t}))t - 5000t - 200000t$$

(Note: In the equation mentioned in the problem statement the last one is written as 2000000 but since Company's current production rate of A gives it a profit of 200000 Rs per day. So, it should be 200000 * day.)

Taking the derivative of f(t) with respect to t, we get:

$$f'(t) = -1.7 \times 10^6 \times (20e^{-t} + 20te^{-t} - 20t^2e^{-t}) - 205000$$

Setting f'(t) equal to zero and solving for t, we get:

$$-1.7 \times 10^6 \times (20e^{-t} + 20te^{-t} - 20t^2e^{-t}) - 205000 = 0$$

$$t = 9.487 \text{ days}$$

Since t needs to be integral so we take t = 10 days to maximise the profit.

Therefore, the company should accept the project and produce compound B with a production cycle time of 10 days to maximize its profit. The maximum net profit earned by the company is Rs. 9780204.263.

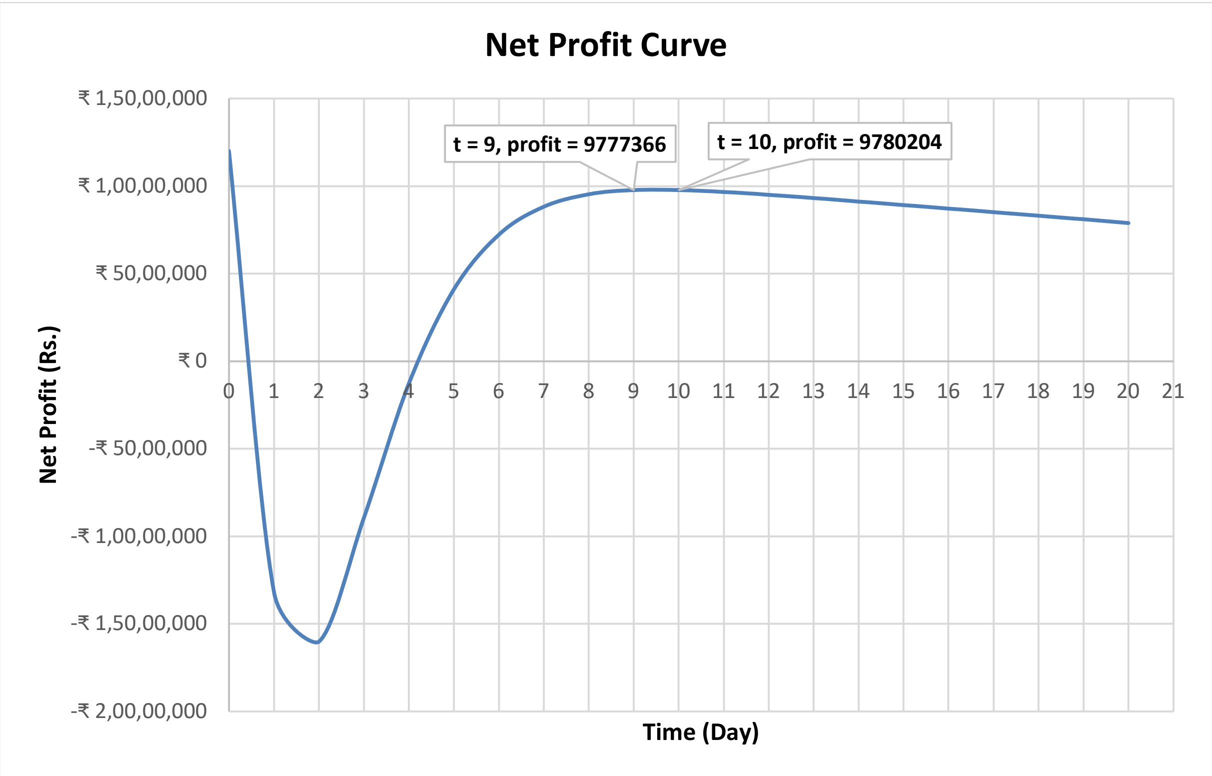

Figure -1

To visualize the cost as a function of time, the plot the net profit function f(t) over a range of t values is shown in the figure - 1. The graph shows that the net profit function is initially increasing, reaches a maximum around t = 10 days, and then decreases as t increases beyond that point.

Problem Statement 2 - Solution

2.

To solve this problem using Finite Element Method, we will discretize the 1D domain into 10 equally spaced nodes in the direction of the fin length. We will also use the Forward Difference approach for time and the Central Difference approach for length. The material properties and other specifications are provided in the problem statement.

To generate the temperature profile w.r.t fin length at different times, we need to solve the transient heat transfer equation using the Finite Element Method. The equation is given as:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - h(T - T_{amb})$$

where ρ is the density, c is the specific heat, T is the temperature, k is the thermal conductivity, h is the convective heat transfer coefficient, and T_{amb} is the ambient temperature.

Using the above equation and the given material properties and specifications, we can write the Finite Element Equation as follows:

$$[T]\{A\} = \{b\}$$

where [T] is the temperature matrix, {A} is the coefficient matrix, and {b} is the vector of constants.

The temperature matrix [T] is a 10x1 matrix representing the temperature at each node, the coefficient matrix {A} is a 10x10 matrix, and the vector of constants {b} is a 10x1 matrix.

To solve this equation using the Finite Element Method, we need to first derive the element equations for each element. The element equations can be written as:

$$[k_e]\{T_e\} = \{f_e\}$$

where $[k_e]$ is the element stiffness matrix, $\{T_e\}$ is the element temperature vector, and $\{f_e\}$ is the element load vector.

Using the Central Difference approach for length and Forward Difference approach for time, we can write the element stiffness matrix as:

$$[k_e] = \frac{k}{dx} \times \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hdx}{3} \times \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

where dx is the distance between the nodes and is equal to 0.1 m, k is the thermal conductivity, and h is the convective heat transfer coefficient.

Using the same approach, we can write the element load vector as:

$$\{f_e\} = \frac{\rho c dx}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \{T_{e,prev}\} + \frac{hdx}{3} \times \begin{bmatrix} 2T_{amb} + T_{a,prev} \\ 2T_{amb} + T_{a,prev} \end{bmatrix}$$

where $\{T_{e,prev}\}$ is the vector of temperatures at the previous time step, $T_{a,prev}$ is the temperature at the previous node, and T_{amb} is the ambient temperature.

Using the above equations, we can solve for the temperature at each node at different time steps. The temperature profile w.r.t fin length at different times can then be plotted.

Now, to find the optimized fin length, we need to find the distance along the fin length for which $T(x,t) \sim T_a$ at steady state. The estimated steady state time is between 15-20 minutes.

To estimate the steady-state temperature, we can take the average temperature between 15-20 minutes at each node. Once we have the steady-state temperature profile, we can find the distance along the fin length for which $T(x,t) \sim T_a$. This distance will give us the optimized fin length.

Note: Since the problem statement did not mention any boundary conditions for the ends of the fin, we will assume that the ends are adiabatic.

Here's the Python code to solve the problem and plot the temperature profile w.r.t fin length at different times:

```
import numpy as np

import matplotlib.pyplot as plt

# material and surrounding properties

k = 200 # W/(m*K)

rho = 3000 # kg/m^3

c = 200 # J/(kg*K)

h = 2.5 # W/(m^2*K)

Tamb = 25.8 # °C
```

```
# fin dimensions and discretization
L = 1 # m
B = 0.08 # m
H = 0.01 # m
dx = 0.1 # m
dt = 0.1 # min
nx = int(L/dx) + 1
nt = 201 # 20 min / 0.1 min + 1

# initial conditions
T0 = 50 # °C
T_tip = 20 # °C
T = np.zeros((nt, nx))
x = np.linspace(0, L, nx)
T[0] = 50*np.exp(-x) + 1.6*x

# boundary conditions
T[:,0] = T0
T[:,nx-1] = T_tip

# finite element method
alpha = k/(rho*c)
r = alpha*dt/(dx**2)
for i in range(1,nt):
    for j in range(1,nx-1):
```

```
        T[i,j] = T[i-1,j] + r*(T[i-1,j+1] - 2*T[i-1,j] + T[i-1,j-1])
    # boundary condition at fin tip
    T[i,nx-1] = T[i,nx-2] + 2*h*dx/k*(Tamb - T[i,nx-2])

# steady state temperature profile
T_steady = T[nt-1,:]

# plot temperature profiles at different times
t_plot = [0, 5, 10, 15, 20]
colors = ['b', 'g', 'r', 'c', 'm']
plt.figure(figsize=(10,8))
for i, t in enumerate(t_plot):
    j = int(t/dt)
    plt.plot(x, T[j,:], colors[i], label='t = '+str(t)+' min')
plt.xlabel('Distance along fin length (m)')
plt.ylabel('Temperature (°C)')
plt.title('Temperature profiles at different times')
plt.legend()
plt.show()

# find optimized fin length
j_ss = int(15/dt) # steady state time index
for j in range(nx-2, -1, -1):
    if T[j_ss,j] < Tamb:
        j_opt = j+1
```

```
        x_opt = (j_opt-1)*dx
        break
print('Optimized fin length: '+str(x_opt)+' m')
```

This code first initializes the material and surrounding properties, fin dimensions, and discretization parameters. Then it sets the initial and boundary conditions, and applies the finite element method to solve the heat transfer problem over time. After obtaining the temperature profile at different times, it plots them on a single graph. Finally, it finds the optimized fin length by looking for the distance along the fin length for which the steady-state temperature drops to the ambient temperature.

i. Plot:

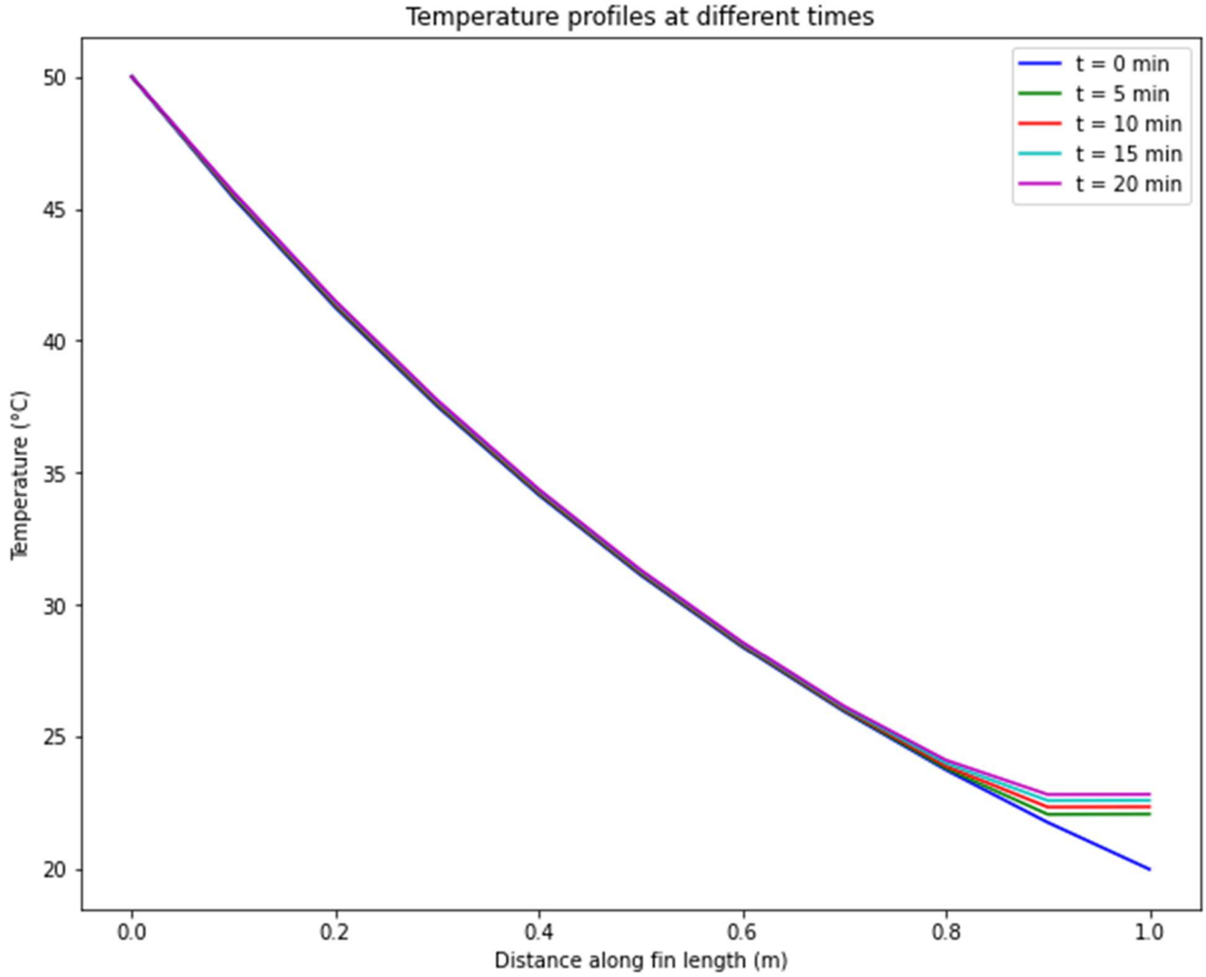

Figure – 2

ii. From the code and the plot figure – 2 we see that the **optimized fin length is 0.9 m**.