Quiz II

Discrete Structures
IIIT Hyderabad, Monsoon 2022

October 20, 2023

Consider the following system of congruent-recurrences:

$$\begin{array}{lll} a_n \equiv a_{n-1} + 3n^2 & \text{with} \quad a_0 \equiv 4 & \pmod 5 \\ a_n \equiv 6a_{n-2} - a_{n-1} & \text{with} \quad a_0 \equiv -1, a_1 \equiv 8 & \pmod{11} \\ a_n \equiv 4a_{n-1} - 3a_{n-2} - 2 & \text{with} \quad a_0 \equiv 2, a_1 \equiv 5 & \pmod 7 \end{array}$$

Answer the following:

1. What is a_0 mod 385? 5 marks
2. What is a_2 mod 385? 5 marks
3. What is (a_{100} mod 5) ? 15 marks
4. What is (a_{150} mod 35) ? 20 marks
5. What is (a_{200} mod 385) ? 25 marks
6. With the same initial/boundary conditions, how many values between 0 and 384 can (a_3 mod 385) take, if:
   - all the three congruences are satisfied? 3 marks
   - *none of the three* congrunences are satsfied? 12 marks
   - exactly *one of the three* congrunences are satsfied? 15 marks