MA2.101 Linear Algebra
Spring'24

Tutorial 1

Problems:

1. Which of the following sets of real numbers are subfields of the field R?
   - Q, the set of rational numbers.
   - Z, the set of integers.
   - $\{\ldots, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \ldots\}$, the set of integer powers of 2.
   - R, the set of all real numbers.
   - $\{0\}$, the singleton set consisting of the number zero.
   - $\{0, 1\}$
2. Let F be the field of complex numbers. Are the two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of the equations in the other system

   System 1:
   $$\begin{aligned} -x_1 + x_2 + 4x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}$$

   System 2:
   $$\begin{aligned} x_1 + 3x_2 + 8x_3 &= 0 \\ x_2 + 3x_3 &= 0 \\ \frac{1}{2}x_1 + x_2 + \frac{5}{2}x_3 &= 0 \end{aligned}$$

3. Let A be a 2×2 matrix with complex entries. A $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Suppose that A is row-reduced and also that $a+b+c+d = 0$. Prove that there are exactly three such matrices.
4. Let F be a field. Prove that the identity element in F is unique. Additionally, show that for any element a in F, the inverse (if it exists) is unique.
5. Suppose R and R' are 2×3 row-reduced echelon matrices and that the systems $RX = 0$ and $R'X = 0$ have exactly the same solutions. Prove that $R = R'$.