Mid Semester Exam – Solutions

Q1. ***[4 marks for the Equations, 3 marks for process and solving, 3 marks for final condition and answer] OR [7 marks for Truth Table, 3 marks for final condition and answer]***

- Assume Truth = 1, Lying = 0.

- Assume
    - A = Statements made by Abdul
    - B = Statements made by Bishnoi
    - C = Statements made by Carren
    - D = Statements made by David
- Given,
    - $A' = B + C'$
    - $B' = A' + D$
    - $C = A$
    - $D = A'B' + C'$
- The requirement is to find when all these conditions are simultaneously true. Substituting the third statement into first,

$$\Rightarrow A' = B + A'$$

- If $B = 0$,

$$\Rightarrow A' = 0 + A'$$

$$\Rightarrow A' = A'$$

  which is always true.

- If $B = 1$,

$$\Rightarrow A' = 1 + A'$$

$$\Rightarrow A' = 1$$

  which *need not* always be true.

- Hence, $B = 0$. Substituting this into the fourth statement,

$$D = 0 + C'$$

$$\Longrightarrow D = C' = A'$$

- Substituting into the second statement,

$$\Longrightarrow 1 = A' + A'$$

$$\Longrightarrow A' = 1$$

$$\Longrightarrow D = 1, C = 0$$

- Therefore,

$$A = 0, B = 0, C = 0, D = 1$$

- David is telling the truth.

Q2.

- The output is correct only for the highlighted entries truth table, as given in the question.

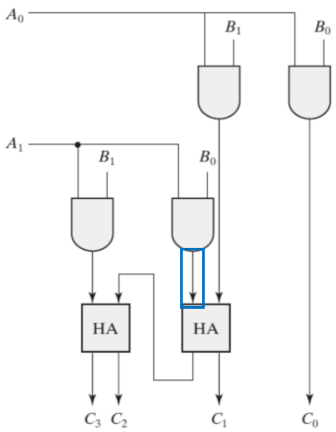
B_1	B_0	A_1	A_0	C_3	C_2	C_1	C_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

[2 Marks for the Truth Table]

- We can clearly see that the product $\boldsymbol{A_1 B_0}$ is 1 in the correct cases *and* it is zero everywhere else.

[7 Marks if the pattern is identified]

- Additionally, it is the output wire of one of the AND gates in the circuit as highlighted below.

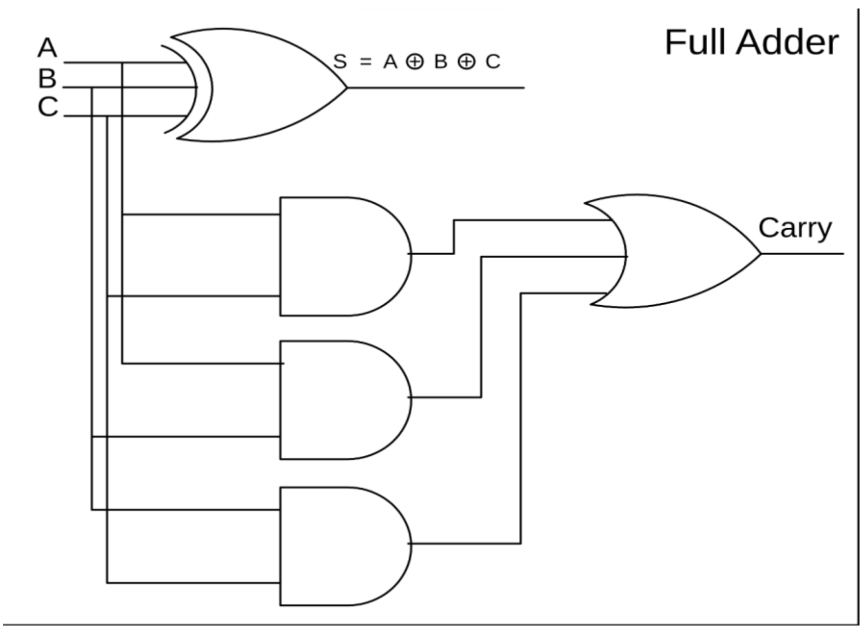


- If that wire is stuck at VDD and everything else in the circuit was operating as it should, we will observe the specified case in the question.
- Hence, the output wire of the AND gate with inputs A_1 and B_0 is problematic, and it is stuck at VDD.

[10 Marks for full solution]

Q3.

Part 1(5 Marks)



$$sum = A \oplus B \oplus C$$

$$carry = AB + BC + AC$$

[2 Marks for expressions]

for $sum = A \oplus B \oplus C$ we need 3 input xor gate which can be realised with 2 xor gates

for sum we need 16*2 Transistors

for carry $AB + BC + AC$ we need 3 - 2 input OR gates and 1 -3 input or gate

for carry we need 6*3 T+ 8*1 T

so for 1 full adder we need 58 transistors

As it is cascaded n bit adder we need 58n transistors

[3 Marks for calculation and final answer]

Part 2

For CLA Adder

$Pi = Ai \oplus Bi$

$Gi = Ai . Bi$

$Si = Pi \oplus Ci$

$C_{i+1} = Gi + PiCi$

[2 Marks for expressions]

$C1 = G0 + P0Cin$

$C2 = G1 + P1C1 = G1 + P1G0 + P1P0Cin$

$C3 = G2 + P2G1 + P2P1G0 + P2P1P0Cin$

So on

So for computing ci using Gi,Pi

c1 we need 6 transistors for AND ,6 transistors for OR gate =12T

c2=6 for 2 input AND gate, 8 for 3 input AND gate and 8 for 3 input OR gate=22T

$c3 = 6 + 8 + 10 + 10 = 34T$

$Cn = n^2 + 7n + 4$

For computing S we need 2 input XOR gate that is 16 transistors

For computing Pi we need 2 input XOR gate that is 16 transistors

For computing GI we need 2 input AND gate that is 6 transistors

Total count $=\sum C_i + 38 = \sum i^2 + 7i + 4 + 38 = \frac{n(2n+1)(n+1)}{6} + 7\frac{n(n+1)}{2} + 4n + 38n$

$== \frac{1}{3}n^3 + 4n^2 + \frac{11}{3}n + 42n = \frac{1}{3}n^3 + 4n^2 + \frac{137}{3}n$

[3 Marks for calculation and final answer]

Q4.

- It is given in the question that $aabb$ is a perfect square and a decimal number.

$$aabb = 10^3a + 10^2a + 10b + b$$

$$= 10^2a(10+1) + 11b$$

$$= 11(10^2a + b)$$

- For the above quantity to be a perfect square, $(10^2a + b)$ should be divisible by 11 *and* the quotient should be a perfect square, i.e.

$$(10^2a + b) = 11n^2$$

[2 Marks till here] - So if guessing starts here and concludes in right answer, 3 Marks (No extra mark if final answer is wrong).

- Since $a, b \in Z^+$ and $a, b \in [0, 9]$,

$$10a^2 + b = (a0b)_{10}$$

- The divisibility rule of 11 dictates that the quantity $(a + b - 0)$ should be zero or divisible by 11. Since the former is not possible, and $a, b \in [0, 18]$,

$$\boldsymbol{a + b = 11}$$

[4 Marks till here] – If guessing starts here and concludes in right answer, 6 Marks (No extra mark if final answer is wrong).

- Performing the division,

$$\begin{array}{l} 11\,]\; a\;\; 0\;\; b\,[\,x \\ \underline{\quad\;\;\, x\;\; x \qquad\quad} \end{array}$$

- The remainder contains two digits $(a - 1 - x)$, and $(10 - x)$. The remainder should also $\in [0, 10]$, because we are dividing with 11.

$$\Rightarrow a - 1 - x = 1\ \boldsymbol{AND}\ 10 - x = 0$$

$$(OR)$$

$$a - 1 - x = 0, in\ which\ case\ the\ second\ digit \in [0,9])$$

- The first condition above leads to $a = 13$, which is a contradiction.

$$\Rightarrow a = x + 1$$

- Now, continuing the division

$$\begin{array}{l} 11\ ]\ a\ \ 0\ \ b\ [\ xy \\ \quad\ \ \underline{x\ \ x\qquad\qquad\quad} \\ \qquad\ (10 - x)\ \ b \\ \qquad\quad y \qquad\ \ y \end{array}$$

$$\Rightarrow 10 - x = y\ \boldsymbol{AND}\ b = y$$

$$(OR)$$

$$\Rightarrow 9 - x = y\ \boldsymbol{AND}\ b + 10 = y$$

- Expanding the first condition,

$$\Rightarrow b = 10 - x$$

$$\Rightarrow a + b = 10 - x + x + 1 = 11$$

which is true. Therefore,

$$a = x + 1 \Rightarrow x = a - 1$$

$$b = y = 10 - x$$

$$\Rightarrow y = 10 - a + 1 = 11 - a$$

- The final condition is that $(xy)_{10}$ should be a perfect square.

$$(xy)_{10} = 10(a - 1) + 11 - a$$

$$= 9a + 1$$

$$\Rightarrow \boldsymbol{9a + 1 = n^2}$$

[7 Marks till here] – If guessing starts here and concludes in right answer, 8 Marks (No extra mark if final answer is wrong).

Since $a \in [0,9]$ and $a \in Z^+$

$$\Rightarrow n^2 \in [1, 82]$$

$$\Longrightarrow n \in [1, 9] \; and \; n \in Z^+$$

Now

$$a = \frac{n^2 - 1}{9}$$

$$= \left(\frac{n-1}{3}\right)\left(\frac{n+1}{3}\right)$$

For $a \in Z^+$, both $n-1$ and $n+1$ should be divisible by 3. However, $n-1$, n, $n+1$ are consecutive integers. Hence, if $n-1$ is divisible by 3, $n+1$ is not — and vice versa. The only possibility for $a \in Z^+$ is if either $n-1$ or $n+1$ is equal to 9 (Multiple of 9 is also enough, but $n \in [1,9]$).

$$\Rightarrow n = 8$$

$$\Rightarrow \boldsymbol{a} = \mathbf{7}$$

$$\Rightarrow \boldsymbol{b} = \mathbf{4}$$

Therefore, the original decimal number is **7744**.

[10 Marks for full solution]