

Real Analysis
Mid-Sem 2023
Time - 1.5 hours
Full marks 50

- 1.a) Prove that for each $n \geq 2$, $(n+1)! > 2^n$.
b) Prove that for all $n \in \mathbb{N}$, $(3 + \sqrt{5})^n + (3 - \sqrt{5})^n$ is an even integer. (4+6)
- 2.a) Prove that the set of natural numbers is not bounded from above.
b) Prove that there is a unique positive real number x , such that $x^2 = 2$ (5+5)
- 3.a) Prove that the union and intersection of finite number of open sets in \mathbb{R} are open sets themselves.**
b) Show that the set \mathbb{N} has no limit points. (10+5)
- 4.a) Prove that $\lim_{n \rightarrow \infty} \frac{S_n}{t_n} = \frac{s}{t}$, given $\lim_{n \rightarrow \infty} S_n = s$ and $\lim_{n \rightarrow \infty} t_n = t$ with $t_n \neq 0 \forall n \in \mathbb{N}$ and $t \neq 0$.
b) Show whether the following sequence (x_n) with $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$ is convergent or not.
c) Given $x \geq 1$, show that $\lim_{n \rightarrow \infty} (2x^{1/n} - 1)^n = x^2$ (5+5+5)