MA2.101 Linear Algebra
Spring'24

Tutorial 3

1. (a) Let $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 3 & 1 & 2 \\ 1 & 3 & 1 & 2 \end{bmatrix}$. Find a basis for the range $R(A)$ of A that consists of columns of A.

   (b) Find the rank and nullity of the matrix A in part (a).

2. Let V be the vector space of all 2×2 matrices whose entries are real numbers. Let $W = \{A \in V \mid A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$, for any $a, b, c \in \mathbb{R}\}$.

   (a) Show that W is a subspace of V.

   (b) Find a basis of W.

   (c) Find the dimension of W.

3. In each part, V is a vector space and S is a subset of V . Determine whether S is a subspace of V .

   (a) $V = \mathbb{R}^3$

   $$S = \{ \begin{bmatrix} x \\ 12 \\ 3x \end{bmatrix} : x \in \mathbb{R} \}$$

   (b) $V = \mathbb{R}^2$

   $$S = \{ \begin{bmatrix} x \\ y \end{bmatrix} : 2x - 5y = 11 \}$$

   (c) $V = \mathbb{R}^n$
   $S = \{\vec{x} \in \mathbb{R}^n : A\vec{x} = 2\vec{x}\}$,where A is a particular $n \times n$ matrix.

   (d) $V = F(-\infty, \infty)$ denote the space of functions defined on the interval $(-\infty, \infty)$

   $S = \{f : f(x) = a\cos(x) + b\sin(x) + c\}$

4. Let $V = P_\infty$ be the vector space of polynomials. Is the set

   $$S = \{1 + x + x^2, 1 - x, 1 - x^3\}$$

   linearly independent? Prove your claim.