Proper Key

given Relations

① $a_n \equiv a_{n-1} + 3n^2$ with $a_0 \equiv 4$ (mod 5)

② $a_n \equiv 6a_{n-2} - a_{n-1}$ with $a_0 \equiv -1, a_1 \equiv 8$ (mod 11)

③ $a_n \equiv 4a_{n-1} - 3a_{n-2} - 2$ with $a_0 \equiv 2, a_1 \equiv 5$ (mod 7)

Pre - Work

① Eqn

$a_1 \equiv (4+3) \bmod 5$

$a_1 \equiv 2 \pmod 5$

$a_2 \equiv (a_1 + 3 \cdot 4) \bmod 5$

$a_2 = 4 \bmod 5$

② Eqn

$a_0 \equiv -1, a_1 \equiv 8$

$a_2 \equiv (6a_0 - a_1) \bmod 11$

$= (-6 - 8) \bmod 11$

$\equiv (22 - 14) \bmod 11$

$a_2 = 8 \bmod 11$

③ Eqn

$a_2 \equiv 4 \times 5 - 3 \times 2 - 2$

$\equiv 20 - 6 - 2$

$\equiv 12 \pmod 7$

$a_2 \equiv 5 \pmod 7$

① What is a_0 mod 385

$a_0 \equiv 219 \pmod{385}$ Use CRT For this

② What is a_2 mod 385

$a_2 \equiv 19 \bmod 385$

③ $a_{100} \bmod 5$

$a_n \equiv \cancel{a_{n-1}} + 3n^2 \pmod 5$

$\cancel{a_{n-1}} \equiv \cancel{a_{n-2}} + 3(n-1)^2 \pmod 5$

$\vdots$

$\cancel{a_1} \equiv a_0 + 3 \pmod 5$

$a_n \equiv a_0 + 3(\Sigma n^2) \pmod 5$

$a_{100} \equiv \left[4 + \cancel{3} \times \frac{(\overset{50}{\cancel{100}})(\overset{0}{\cancel{101}})(201)}{\cancel{6}_2}\right] \bmod 5$

$a_{100} \equiv 4 \bmod 5$

5th Solving the Recurrence Equation

2nd

$a_n = (-3)^n c_1 + c_2 2^n$

$a_0 = c_1 + c_2 = 10$

$a_1 = -3c_1 + c_2 2 = 8$

$a_n = \left[(-3)^n \, 12/5 + 2^n \, 38/5\right] \bmod 11$

$3c_1 + 3c_2 = 30$

$-\ 3c_1 + 2c_2 = 8$

$5c_2 = 38$

$c_2 = 38/5$

$2c_1 + 2c_2 = 20$

$-3c_1 + 2c_2 = 8$

$5c_1 = 12$

$c_1 = 12/5$

3rd

$a[n] = c_2 3^n + c_1 + n + 1/2$

$a_0 = c_2 + c_1 + 1/2 = 2$

$a_1 = 3c_2 + c_1 + 1 + 1/2 = 5$

$a(n) = (3^n + 1 + n) \bmod 7$

We have the following

$$a_n \equiv [4 + 3\Sigma n^2] \bmod 5$$

$$a_n = [(-3)^n \, 12/5 + 2^n \, 38/5] \bmod 11$$

$$a(n) \equiv (3^n + 1 + n) \bmod 7$$

7
14
21
28

Question 4

$$a_{150} \equiv (4 + 3 \; 1\cancel{5}0 \times) \bmod 5$$

$$a_{150} \equiv (3^{150} + 1 + 150) \bmod 7$$

$$\equiv ((3^4)^{37} \times 3^2) \bmod 7 + (151) \bmod 7$$

$$2 + 4$$

$$a_{150} \equiv 6 \bmod 7 \qquad a_{150} \equiv 4 \bmod 5$$

$$a_{150} \equiv 34 \pmod{35} \quad \text{CRT}$$

Question 5

$$a_{200} \equiv \left(4 + \frac{3 \times 200 \times \cancel{2}01 \times 401}{6}\right) \bmod 5$$

$$\equiv 4 \bmod 5$$

$$a_n \equiv \left[\frac{(-3)^{200} \cdot 12}{5} + \frac{2^{200} \cdot 38}{5} \right] \bmod 11$$

$$\equiv \left[\frac{(3)^{200} \cdot 12}{5} + \frac{2^{200} \cdot 38}{5} \right] \bmod 11$$

$$a_{200} \equiv 10 \bmod 11$$

$$a_{200} \equiv (8^{200} + 201) \bmod 7$$

$$\equiv (1 + 201) \bmod 7$$

$$\equiv 202 \bmod 7$$

$$\equiv 6 \bmod 7$$

$$a_{200} \equiv 384 \pmod{385}$$