EC2.101 – Digital Systems and Microcontrollers

Practice Sheet 1 (Lec 1 – Lec 8) – Solutions

Q1. Verify whether the inhibition and implication operations follow commutative and associative properties.

Ans.

- Inhibition is $x\ but\ not\ y$. Clearly, the commutative statement, $y\ but\ not\ x$ is not the same. Writing the Boolean expressions, we can see that $xy' \neq yx'$. So, inhibition does not obey the commutative property.
- Implication is $if\ x\ then\ y$. Once again, the commutative statement, $if\ y\ then\ x$ is not the same. Writing the truth table as shown below, we can verify the same, i.e., implication does not obey the commutative property.

$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{x \supset y = x' + y}$	$\boldsymbol{x \subset y = x + y'}$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

- Expanding inhibition to three variables, it becomes $(x\ but\ not\ y)\ but\ not\ z$. The Boolean expression for the same is $(xy')z'$. To verify whether associative property holds, we need to change the order of computation, i.e., $x\ but\ not\ (y\ but\ not\ z)$. The Boolean expression then becomes $x(yz')' = x(y' + z) = xy' + xz$. Clearly, changing the order changes the output. Hence, inhibition does not obey the associative property as well.
- Similarly, expanding implication to three variables, it becomes $(if\ x\ then\ y)\ then\ z$. Writing the Boolean expression, $(x' + y)' + z = xy' + z$. Changing the order of computation to $if\ x\ then\ (\cancel{if}\ y\ then\ z)$. The Boolean expression is $x' + (y' + z)' = x' + yz'$. Clearly, changing the order changes the output. Hence, implication does not obey the associative property as well.

Q2. Express the following functions in sum-of-minterms and product-of-maxterms form.

Ans.

a. $a'b'c' + acd + ab'd' + b'cd$

Let the expression be F. Since there are four variables, we need to convert each literal to have all four variables as well. We can do this by multiplying with the sum of the missing variable and its complement. That is,

$$a'b'c = a'b'c(d + d')$$

$$acd = acd(b + b')$$

$$ab'd' = ab'd'(c + c')$$

$$b'cd = b'cd(a + a')$$

Adding them up, we get (observe that the stricken literals appear twice)

$$F = a'b'cd' + \boldsymbol{a'b'cd} + abcd + \boldsymbol{ab'cd} + ab'c'd' + ab'cd' + \cancel{ab'cd} + \cancel{a'b'cd}$$

$$F = \sum(2, 3, 8, 10, 11, 15)$$

This is the sum-of-minterms form.

To get the product-of-maxterms form, we just obtain the numbers that are not part of the minterms form. That is, if the value of F in the full truth table is 1 at the entries given by the sum-of-minterms representation, it must be 0 everywhere else, by definition. Therefore,

$$F = \prod(0, 1, 4, 5, 6, 7, 9, 12, 13, 14)$$

b. $(b + c'd')(a + bc')$

Let the expression be F. Computing the product-of-maxterms representation first, we expand the given expressions using the OR over AND distributive property.

$$F = (b + c'd')(a + bc')$$

$$= (b + c')(b + d')(a + b)(a + c')$$

Now, each literal should once again be transformed to have all four variables. We can do this as follows.

$$b + c' = (b + c') + aa'$$

Once again, we use the OR over AND distributive property as above.

$$(b + c') + aa' = (b + c' + a)(b + c' + a')$$

Similarly adding dd' and expanding using the distributive property,

$$b + c' = (a + b + c' + d)(a + b' + c' + d')(a' + b + c' + d)(a' + b + c' + d')$$

$$\Rightarrow b + c' = \prod(2, 7, 11, 14)$$

Transforming the remaining literals in a similar manner,

$$b + d' = (a + b + c + d')(a' + b + c + d')(a + b + c' + d')(a' + b + c' + d')$$

$$\Rightarrow b + d' = \prod(1, 3, 9, 11)$$

$$a + b = (a + b + c + d)(a + b + c' + d)(a + b + c + d')(a + b + c' + d')$$

$$\Rightarrow a + b = \prod(0, 1, 2, 3)$$

$$a + c' = (a + b + c' + d)(a + b' + c' + d)(a + b + c' + d')(a + b' + c' + d')$$

$$\Rightarrow b + d' = \prod(2, 3, 6, 7)$$

Therefore,

$$F = \prod(0, 1, 2, 3, 6, 7, 9, 11, 14)$$

$$F = \sum(4, 5, 8, 10, 12, 13, 15)$$

c. $xy'z + x'y'z + w'xy + wx'y + wxy$

Expanding,

$$F = wxy'z + w'xy'z + wx'y'z + w'x'y'z + w'xyz + w'xyz' + wx'yz + wx'yz' + wxyz$$
$$+wxyz'$$

$$F = \sum(1, 5, 6, 7, 9, 10, 11, 13, 14, 15)$$

$$F = \prod(0, 2, 3, 4, 8, 12)$$

d. F', where $F = \sum(0, 1, 3, 8, 9, 13, 15)$

Clearly from the truth table,

$$F' = \sum(2, 4, 5, 6, 7, 10, 11, 12)$$

$$F' = \prod(0, 1, 3, 8, 9, 13, 15)$$

Q3. Find the duals of the following expressions.

Ans.

a. $xy' + x'y$

The dual of $(xy' + x'y)$ is $(x + y') \cdot (x' + y)$.

Now, $(xy' + x'y)' = (xy')' \cdot (x'y)'$

$$= (x' + y) \cdot (x + y')$$

$$= (x + y') \cdot (x' + y)$$

Hence, the complement and dual of the given expression (XOR gate) are the same.

b. $xy' + (y + z)(x + z')$

The dual of the given expression is,

$$(x + y')(yz + xz') = xyz + xz' + xy'z' = xyz + xz' = x(yz + z') = x(y + z')$$

Q4. What is the sum of all 2^n minterms possible with n variables?

Ans.

- Let $n = 1$. The number of minterms possible are 2, and their sum is,

$$S_1 = a' + a = 1.$$

- If $n = 2$, the number of minterms possible are 4, and their sum is,

$$S_2 = a'b' + a'b + ab' + ab = a'(b' + b) + a(b' + b) = (a' + a)(b' + b) = 1.$$

- If $n = 3$, the number of minterms possible are 8, and their sum is,

$$S_3 = a'b'c' + a'b'c + a'bc' + a'bc + ab'c' + ab'c + abc' + abc$$

$$= a'b'(c' + c) + a'b(c' + c) + ab'(c' + c) + ab(c' + c)$$

$$= S_2(c' + c) = 1$$

- Clearly, the pattern suggests that for every new variable, the new sum is just a product of the previous sum and a tautology of the new variable, which is always equal to 1.
- Hence, the sum of all minterms possible with n variables is 1.

Q5. Consider the equation $L + M = N$ where $L = (312)_8$ and $N = (451)_8$. Find the value of M and represent the answer in hexadecimal.

Ans.

$$L = (312)_8 = (202)_{10}$$

$$N = (451)_8 = (297)_{10}$$

$$M = N - L = (95)_{10} = (5F)_{16}$$

Q6. Convert the following numbers to decimal:

Ans.

a. $(101.01)_2$

$$(101.01)_2 = 1 * (2^2) + 0 * (2^1) + 1 * (2^0) + 0 * (2^{-1}) + 1 * (2^{-2})$$
$$= 4 + 1 + 0.25 = (5.25)_{10}$$

b. $(7.12172)_8$

$$(7.12172)_8 = (7 * 8^0) + (1 * 8^{-1}) + \ (2 * 8^{-2}) + \ (1 * 8^{-3}) + \ (7 * 8^{-4}) + \ (2 * 8^{-5})$$
$$= 7 + \frac{1}{8} + \frac{2}{8^2} + \frac{1}{8^3} + \frac{7}{8^4} + \frac{2}{8^5}$$
$$= 7 + \frac{4096 + 1024 + 64 + 56 + 2}{8^5} = \left(7\frac{5242}{8^5}\right)_{10}$$

Q7. Add the BCD numbers 1001 and 0100. Represent the answer in BCD as well.

Ans.

$$\begin{array}{c} 1001 \\ \underline{0100} \\ 1101 \end{array}$$

- However, the answer is greater than 9, hence we need to add 6 to it and represent the carry (if any) as a separate BCD number as follows.

$$\begin{array}{c} 01101 \\ \underline{00110} \\ \mathbf{1}0011 \end{array}$$

- Indeed, there is a carry. So, the final answer is $0001\ 0011$.

Q8. Subtract 010001 from 001001 using 2's complement.

Ans.

- The 2's complement of the subtrahend 010001 is 101111. Adding to the minuend,

$$\begin{array}{c} 001001 \\ \underline{101111} \\ 111000 \end{array}$$

- Since the MSB is a 1, the magnitude of the answer can be found by taking 2's complement again, which turns out to be 001000, or $(8)_{10}$. Therefore, the answer is $(-8)_{10}$.

Q9. If X is the number of distinct integers that can be represented in 16-bit 2's complement notation, and Y is the number of distinct integers that can be represented in 16-bit sign-magnitude notation, what is $X - Y$?

Ans.

- Using n bits in binary, the range of distinct integers that can be represented using 2's complement representation is $[-2^{n-1}, 2^{n-1} - 1]$. That is, the number of distinct integers is $(2 \times 2^{n-1}) - 1 = 2^n - 1 = X$.
- The range of distinct integers that can be represented using sign-magnitude is $[-(2^{n-1} - 1), 2^{n-1} - 1]$. That is, the number of distinct integers is $2 \times (2^{n-1} - 1) = 2^n - 2 = Y$.
- Therefore,

$$X - Y = 1$$

- The reason for this difference is that sign-magnitude representation has two representations for zero, $+0$ and -0. 2's complement has only one representation for zero.

Q10. $(210)_3 = (\quad)_{16}$

Ans.

$$(210)_3 = 2 * 3^2 + 1 * 3^1 + 0 * 3^0 = (21)_{10}$$

$$(21)_{10} = 1 * 16^1 + 5 * 16^0 = (15)_{16}$$