

Quizz 2 MA3.101: Linear Algebra Spring 2022

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Answer all questions: (Time - 45 mins) (Full Marks- 30)

1. Let Q be an orthogonal matrix, then show that
 - (i) Q^{-1} is orthogonal.
 - (ii) $\det(Q) = \pm 1$.
 - (iii) If λ is an eigenvalue of Q , then $|\lambda| = 1$.(6)
2. Prove that an orthogonal 2×2 matrix must have the form,
 $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ or $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$ where $\begin{pmatrix} a \\ b \end{pmatrix}$ is a unit vector. (4)
3. Let A be a nilpotent matrix (that is $A^m = O$ for some m). Show that $\lambda = 0$ is the only eigen value of A . (2)
4. Let A be an idempotent matrix (that is $A^2 = A$). Show that $\lambda = 0$ and $\lambda = 1$ are the only eigen value of A . (2)
5. Let v is an eigen vector of A , with corresponding eigen value λ and c is scalar. Show that v is an eigen vector of $A - cI$ with corresponding eigen value $\lambda - c$. (2)
6. Compute the (a) characteristic polynomial, (b) the eigen values, (c) basis for each eigen space, (d) algebraic and geometric multiplicity of each eigen values, for the following matrix,
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ -2 & 1 & 2 & -1 \end{pmatrix}$$
 (4)
7. Apply Gram Schmidt process to find an orthogonal basis for the column spaces of the matrix
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & 0 \\ 1 & 5 & 1 \end{pmatrix}$$
 (4)

8. Suppose that u , v and w are vectors in inner product space such that,
 $\langle u, v \rangle = 1$, $\langle u, w \rangle = 5$, $\langle v, w \rangle = 0$, $\|u\| = 1$, $\|v\| = \sqrt{3}$, $\|w\| = 2$,
then evaluate the expressions,
- (i) $\langle u + w, v - w \rangle$
 - (ii) $\langle 2v - w, 3u + 2w \rangle$
 - (iii) $\|u + v\|$
- (6)