Supplementary Material
Strategic interactions between liquefied natural gas and domestic gas markets: A bilevel model

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Description: This supplementary information file provides the preliminary model and analysis for the simplest version of the model described in the paper.

1. Preliminary model and analysis

In this section, we consider a bilevel model where an LNG operator and NG producer interact with each other in a sequential manner. The LNG operator is represented as the leader who anticipates the response of the NG producer when making his own decisions. The NG producer is the follower who responds optimally to the LNG operator's upper-level choices. In this simple, preliminary model, we assume that the LNG operator has an operational export facility and the NG producer has unlimited production capacity. The NG producer can sell natural gas to two markets: the domestic spot market and the LNG operator, each of which is assigned its own demand curve. The LNG operator can liquefy the gas it procures from the NG producer and sell LNG to a single overseas market, represented by a demand curve. The LNG operator's decisions are the amount of gas to procure from the NG producer, the quantity of LNG to export, and the LNG market price. The NG producer's decisions are the amounts of gas to produce, transmit, and sell to both the spot market and LNG operator (with corresponding prices).

1.1. Notation

Parameters

$C^{\text{prod}}(q)$	NG production unit cost (\$/Mcf[1])
$C^{\text{pipe}}(x)$	NG pipeline installation unit cost (\$/Mcf)

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[1] Mcf = Thousand cubic feet

$C^{\text{ship}}(f)$	NG shipment unit cost (\$/Mcf)
$C^{\text{liq}}(\nu)$	LNG liquefaction unit cost (\$/Mcf)
$C^{\text{LNGship}}(l)$	LNG shipment unit cost (\$/Mcf)
$D^{\text{spot}}(p^{\text{spot}})$	Domestic spot market demand curve (Mcf)
$D^{\text{LNG}}(p^{\text{LNG}})$	LNG market demand curve (Mcf)
$D^{\nu}(p^{\nu})$	LNG facility gas procurement demand curve (Mcf)
Leader's Decision Variables	
ν	Natural gas procured from producer (by LNG operator) (Mcf)
l	LNG exported to overseas market (Mcf)
p^{LNG}	Market price of LNG (\$/Mcf)
Follower's Decision Variables	
f	Natural gas shipped to spot market (Mcf)
x	NG pipeline capacity installed (Mcf)
q	Quantity of NG produced (Mcf)
p^{spot}	Spot market price of natural gas (\$/Mcf)
p^{ν}	NG procurement price for LNG operator (\$/Mcf)

1.2. Mathematical Model

$$\max_{\substack{p^{\text{LNG}},\nu,\\ l\in\mathbb{R}_+}} \; p^{\text{LNG}} l - p^{\nu}\nu - C^{\text{liq}}(\nu) - C^{\text{LNGship}}(l) \tag{1a}$$

subject to

$$l \leq \nu \tag{1b}$$

$$l = D^{\text{LNG}}(p^{\text{LNG}}) \tag{1c}$$

$$p^{\nu} \in \arg\max_{\substack{p^{\text{spot}},p^{\nu},\\ f,q,x\in\mathbb{R}_+}} \left\{ p^{\text{spot}} f + p^{\nu}\nu - C^{\text{prod}}(q) - C^{\text{pipe}}(x) - C^{\text{ship}}(f) \right. \tag{1d}$$

subject to

$$f + \nu \leq q \qquad (\alpha) \tag{1e}$$

$$f \leq x \qquad (\beta) \tag{1f}$$

$$f = D^{\text{spot}}(p^{\text{spot}}) \qquad (\gamma) \tag{1g}$$

$$\left. \nu = D^{\nu}(p^{\nu}) \qquad (\tau) \right\} \tag{1h}$$

The upper-level problem (1a) – (1c) is the optimization problem faced by the LNG operator, who maximizes the revenue earned from LNG sales minus the costs of procuring natural gas from the NG producer, liquefying it, and shipping LNG to the overseas market. The lower-level problem (1d) – (1h) belongs to the NG producer, who maximizes revenue earned from sales of natural gas to the spot market and LNG operator, minus the costs of production, pipeline investment, and shipment via the pipeline. The variables $\alpha \in \mathbb{R}_+$, $\beta \in \mathbb{R}_+$, $\gamma \in \mathbb{R}$, and $\tau \in \mathbb{R}$ are the dual variables associated with constraints (1e) – (1h), respectively. Spatially, this simple formulation assumes that gas production and the LNG terminal are located in the same place, whereas the domestic spot market demand is located elsewhere. The NG producer thus needs to build and use a pipeline to deliver gas to the spot market. Note that the spatial configuration could be changed easily by making minor modifications to the model structure. Our model naturally fits within the bilevel framework because the upper-level gas procurement quantity appears in the lower-level problem, while the lower-level price for gas sales to the LNG operator appears in the upper-level problem.

1.3. Analytical insights

The preliminary model allows us to derive analytical insights on optimal gas pricing in a bilevel setting. The lower-level problem is an LP, so we can replace it with its equivalent KKT conditions.

Proposition 1. *At optimality, the constraints $f + \nu \leq q$ and $f \leq x$ of the lower-level problem hold with equality.*

Proof. When the optimal production quantity $q^* = 0$ and the optimal pipeline capacity $x^* = 0$, given the non-negativity of pipeline flow f and gas procurement by the LNG operator ν, we have $f^* = x^*$ and $f^* + \nu^* = q^*$.

Next, if the optimal production quantity $q^* > 0$, then the complementarity conditions yield $\alpha^* = \frac{dC^{\text{prod}}(q)}{dq}$ and $\alpha^* \left(f^* + \nu^* - q^*\right) = 0$. Assuming $\frac{dC^{\text{prod}}(q)}{dq} > 0$, we have $f^* + \nu^* = q^*$ at optimality. Similarly, if the optimal pipeline capacity $x^* > 0$, then the respective complementarity conditions yield $\beta^* = \frac{dC^{\text{pipe}}(x)}{dx}$ and $\beta^* \left(f^* - x^*\right) = 0$. Assuming a non-negative unit pipeline investment cost, we get $f^* = x^*$ at optimality since the objective has a negative term in x with the associated constraint $x \geq f$, so the lowest value x can take on is f^*. $\square$

We see from Proposition 1 that the NG producer will only produce as much gas as he sells to the spot market and LNG operator, and will only invest in as much pipeline capacity as he will actually use to ship gas from the production location to the spot market.

Proposition 2. *Under optimal pricing, the change in the NG producer's revenue from selling the marginal unit of gas to the spot market is equal to the total marginal cost of producing and shipping the gas.*

Proof. When the optimal flow $f^* > 0$, the complementarity condition yields an expression for the optimal spot market price $p^{\text{spot}*}$, which is

$$p^{\text{spot}*} - \gamma^* = \frac{dC^{\text{prod}}(q)}{dq} + \frac{dC^{\text{pipe}}(x)}{dx} + \frac{dC^{\text{ship}}(f)}{df}. \tag{2}$$

Similarly, when $p^{\text{spot}*}, p^{\nu*} > 0$, we have

$$\gamma^* = \frac{-1}{\frac{dD^{\text{spot}}(p^{\text{spot}})}{dp^{\text{spot}}}}\ f^*,$$

$$\tau^* = \frac{-1}{\frac{dD^{\nu}(p^{\nu})}{dp^{\nu}}}\ \nu^*.$$

Observe that $f^* + \nu^*$ represents total sales in both markets, $\frac{dp^{\text{spot}}}{dD^{\text{spot}}(p^{\text{spot}})}$ represents the change in spot price when an additional unit of gas is sold in the spot market, and $\frac{dp^{\nu}}{dD^{\nu}(p^{\nu})}$ represents the change in LNG facility gas procurement price when an additional unit of gas is sold to the LNG operator. Thus, we can interpret γ^* and τ^* as the losses in revenue from existing sales when an additional unit of gas is sold to the spot market and LNG operator, respectively (because selling more gas depresses the price). We can see that Eq. (2) represents the condition equating marginal revenue to marginal cost, where marginal revenue ($p^{\text{spot}*} - \gamma^*$) is the price of the marginal unit sold in the spot market minus the effect of that one unit increase in spot sales on the revenue earned from all previous units of gas sold. $\square$