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A TEST OF THE GALACTIC ORIGIN OF GAMMA-RAY BURSTS

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ABSTRACT

The distribution of old neutron stars in the Galaxy was calculated by integrating numerically $\sim$90,000 orbits in the Galactic gravitational potential for up to 10^{10} yr. Neutron stars were assumed to be born in a thin disk with the velocity distribution observed for radio pulsars. The calculated volume density within 0.5 kpc of the Sun may be approximated as $n_{NS} \approx 0.0014$ (pc^{-3}) $(R/8\ \mathrm{kpc})^{(-3+4z/1\ \mathrm{kpc})}\ [1 + (z/0.2\ \mathrm{kpc})^2]^{-1}$, where R is a distance from the Galactic center, z is a distance from the Galactic plane, and assuming that the total number of neutron stars in the Galaxy is 10^9, or equivalently, that there were 1.4×10^6 kpc^{-2} neutron stars born near the Sun over the lifetime of the Galaxy.

The dipole and quadrupole moments, as well as the average value of V/V_{max} were calculated for the expected distribution of gamma-ray bursts, assuming that they originate on neutron stars, and assuming various radial depths and detection thresholds of the surveys. For all intrinsic luminosity functions that were considered, the quadrupole moment, i.e., the concentration of sources to the Galactic plane, should be the easiest to detect. If all the nearest neutron stars are detected as bursters than this concentration should be detectable at a 3 σ level when $\sim$4000 independent events have their positions measured with an accuracy of a few degrees. If only some neutron stars produce gamma-ray bursts during the observational period then a larger volume will be sampled with fewer bursts, and the quadrupole moment in their distribution will become measurable with fewer than 4000 events. Even fewer events will be needed if there is a deficiency in the number of weak bursts, as indicated by the balloon experiment of Meegan *et al.* The GRANAT and GRO missions should discover enough bursts in $\sim$1 yr of their operation to provide evidence for or against the association of gamma-ray bursts with the Galactic disk neutron stars.

Models that require accretion from a cold circumstellar disk or from a close companion are only marginally consistent with the apparently isotropic distribution of the observed bursts. Models that require accretion of interstellar matter are ruled out as they should produce a very strong dipole anisotropy. Models that propose the Galactic halo origin of gamma-ray bursts are briefly discussed.

If the GRANAT and GRO missions confirm the flattening of number-intensity relation for weak bursts, and if the distribution of weak bursts is isotropic, then the bursters are most likely at cosmological distances.

Subject headings: galaxies: The Galaxy — galaxies: stellar content — gamma rays: bursts — stars: neutron — stars: stellar statistics

I. INTRODUCTION

The distribution of gamma-ray bursts has been a subject of many studies, recently reviewed by Hurley (1986). The angular distribution of bursts in the sky is consistent with isotropy (Hurley 1986; Atteia *et al.* 1987; Epstein and Hurley 1988; Hartmann and Epstein 1989). The distribution of intensities of strong bursts as observed with space probes (the number-intensity relation) is consistent with a uniform distribution of bursters in space (Mazets 1986; Paczyński and Long 1988, Schmidt, Higdon, and Hueter 1988). However, a very sensitive balloon experiment (Meegan, Fishman, and Wilson 1985) detected only one weak gamma-ray burst, while 43 were expected if the number of bursts varied with the burst intensities according to a power law with an exponent -1.5.

The Burst and Transient Source Experiment (BATSE) on the *Gamma-Ray Observatory* (Fishman *et al.* 1984) should be at least as sensitive as the balloon experiment, and data from GRO should provide truly unambiguous evidence for the flattening of the number-intensity relation at low burst intensities; i.e., it should detect fewer weak bursts than expected from the extrapolation of the -1.5 power law. As GRO will locate bursts with an error of a few degrees, it will be possible to discriminate between isotropic and anisotropic distribution of weak events. The GRANAT mission (R. Sunyayev, private communication) may provide similar data even earlier. If there is a concentration of sources in the direction of the Galactic plane or to the Galactic center, then a characteristic distance to the sources will be established. However, if the scarcity of weak bursts is accompanied by an isotropic sky distribution, then the Galactic models will be difficult to justify. The aim of this paper is to make this statement more quantitative. In particular, we would like to know whether it is possible for the Galactic sources to display flattening of the number-intensity relation without displaying simultaneously a strong dipole or quadrupole anisotropy.

In the next section we shall calculate the distribution of old neutron stars in the disk of our Galaxy. In § III we shall calculate the dipole moment in the angular distribution of neutron stars as a convenient measure of their concentration toward the Galactic center, the quadrupole moment in their angular distribution as a convenient measure of their concentration toward the Galactic plane, and the average value of V/V_{max} (see Schmidt, Higdon, and Hueter 1988) as a convenient measure of the radial distribution of neutron stars. We have a relation $V/V_{max} = (d/d_{max})^3$, where d is the distance to a neutron star, and d_{max} is the maximum distance at which the neutron star could be detected. These three dimensionless parameters describing the apparent distribution of neutron stars will be calculated as a function of radial depth of the observations, d_{max}. If the distribution of sources is isotropic and homogeneous then the dipole and quadrupole moments vanish, and $\langle V/V_{max}\rangle = 0.5$. This is always the case if d_{max} is very small. Finally, § IV contains a discussion of the results.

Even though the log N − log n_{max} (see Paczyński and Long 1988) relation for gamma-ray bursts contains similar information as the distribution of V/V_{max} (see Schmidt, Higdon, and Hueter 1988), the latter one is easier to work with, and for this reason it is used in this study.

Recently, Hartmann, Epstein, and Woosley (1989*a*, *b*) calculated a model of the distribution of neutron stars in the Galaxy which is very similar to the one presented in this paper. A comparison between their results and those presented in this paper is given in § IV.

II. DISTRIBUTION OF OLD NEUTRON STARS IN THE GALACTIC DISK

We assume that neutron stars are born in the young Galactic disk, with the birth rate per unit volume varying exponentially with distance from the Galactic center, $\sim \exp(-R/R_{exp})$, and varying exponentially with distance from the Galactic plane, $\sim \exp(-|z|/z_{exp})$. We adopted $R_{exp} = 4.5$ kpc, and $z_{exp} = 75$ pc, following van der Kruit (1987). The disk thickness is not important, as long as it is small. We assumed that the thin disk capable of producing neutron stars extends from $R_{min} = 0$ out to $R_{max} = 20$ kpc. Therefore, the probability distribution for the positions of young pulsars is given as

$$p_z(z)dz = e^{-z/z_{exp}} \frac{dz}{z_{exp}}, \tag{1}$$

where z stands for the absolute value of a distance from the Galactic plane, and

$$p_R(R)dR = a_R e^{-R/R_{exp}} \frac{R}{R_{exp}^2} dR, \tag{2}$$

$$a_R \equiv [1 - e^{-R_{max}/R_{exp}}(1 + R_{max}/R_{exp})]^{-1} = 1.0683,$$

where R is a distance from the Galactic center.

We assume that neutron stars have the same distribution of initial velocities as radio pulsars. This is usually described as Maxwellian, with an excess at low and large velocities. We have to be more quantitative, and we adopted the following distribution of absolute values of initial velocities:

$$p(u)du = \frac{4}{\pi}\frac{du}{(1+u^2)^2}, \quad u = \frac{v}{v^*}, \quad v^* = 270 \text{ km s}^{-1}, \tag{3}$$

where $p(u)du$ is the probability that the dimensionless velocity is between u and $u + du$, and the initial velocity is given as $v = uv^*$. The best fit to the observed distribution is for $v^* = 270$ km s^{-1}.

The observed distribution was taken from Lyne, Anderson, and Salter (1982), who gave the proper motions of 26 pulsars. No allowence was made for any observational selection effects. To make the comparison possible, we have to convert the probability distribution $p(u)du$ as given by equation (3) into the distribution of transverse velocities $p_t(u_t)du_t$, where a transverse velocity is given as $u_t = (u_z^2 + u_y^2)^{1/2}$, while a full velocity is given as $u = (u_x^2 + u_y^2 + u_z^2)^{1/2}$. Notice, that x, y, z is used just to indicate three orthogonal coordinates, with no reference to the Galactic coordinates. The three-dimensional velocity distribution is assumed to be isotropic, i.e., all three velocity components, u_x, u_y, and u_z have the same distributions. After simple algebra we find

$$p_{xyz}(u_x, u_y, u_z)du_x du_y du_z = \frac{1}{\pi^2 u^2 (1+u^2)^2} du_x du_y du_z,$$
$$u^2 = u_x^2 + u_y^2 + u_z^2, \tag{4}$$

so that

$$p(u)du = p_{xyz} 4\pi u^2 du = \frac{4}{\pi}\frac{du}{(1+u^2)^2}, \tag{5}$$

and

$$p_t(u_t)du_t = \left[\int_{-\infty}^{+\infty} 2\pi u_t p_{xyz}(u_x, u_y, u_z)du_z\right]du_t$$
$$= \left[2 - \frac{2u_t}{(1+u_t^2)^{1/2}} - \frac{u_t}{(1+u_t^2)^{3/2}}\right]du_t, \quad u_t^2 = u_x^2 + u_y^2. \tag{6}$$

The probability distributions $p(u)du$ and $p_t(u_t)du_t$, may be integrated to obtain

$$P(u) \equiv \int_0^u p(u)du = \frac{2}{\pi}\left[\frac{u}{1+u^2} + \text{atan}(u)\right], \tag{7}$$

$$P_t(u_t) \equiv \int_0^{u_t} p_t(u_t)du_t$$
$$= 1 + 2u_t - 2(1+u_t^2)^{1/2} + (1+u_t^2)^{-1/2}. \tag{8}$$

Now we are in a position to make a comparison between our distribution function and the observations. We ordered the 26 pulsars observed by Lyne, Anderson, and Salter (1982) according to their transverse velocities, and we got a sequence of 26 values: $v_{t,i}$, $i = 1, 2, \ldots, 25, 26$, with $v_{t,1} = 12$ km s^{-1}, and $v_{t,26} = 365$ km s^{-1}. Using equation (8) we calculated the expected number of pulsars, $N_{cal,i}$, with transverse velocity smaller than or equal to $v_{t,i}$, and compared it with the observed number, $N_{obs,i} = i$, for various assumed values of v^* (see, eq. [3]). The best agreement, as shown in Figure 1, was obtained for $v^* = 270$ km s^{-1}. The analytical formula (3) has an excess of very high velocities, as $N_{cal} = 24.3$ for $N_{obs} = 26$. For this reason the formula gives somewhat too high rms transverse velocity: $\langle v_{t,cal}^2\rangle^{1/2} = 220$ km s^{-1}, as compared to $\langle v_{t,obs}^2\rangle^{1/2} = 170$ km s^{-1}. The corresponding three-dimensional rms velocities are $\langle v_{cal}^2\rangle^{1/2} = v^* = 270$ km s^{-1}, as compared to $\langle v_{obs}^2\rangle^{1/2} = 210$ km s^{-1}.

The random velocity v of a newly born neutron star has to be added to the rotational velocity of the Galactic disk. The rotation curve for our Galaxy was adapted after Burton and Gordon (1987), following an adjustment by Binney and Tremaine (1987, Figs. 2–17, p. 87), so that our distance to the Galactic center is taken as $R_0 = 8$ kpc, and the circular velocity at that distance is $v_c(R_0) = 220$ km s^{-1}.

Given the initial velocity, a neutron star orbit has to be integrated in a Galactic gravitational potential in order to find the density distribution of old neutron stars in the Galaxy. We used a potential proposed by Miyamoto and Nagai (1975). This is made of two components of the form:

$$\Phi_i(R, z) = \frac{GM_i}{\{R^2 + [a_i + (z^2 + b_i^2)^{1/2}]^2\}^{1/2}}, \quad R^2 = x^2 + y^2, \tag{9}$$

where $i = 1$ corresponds to the spheroid, and $i = 2$ corre-

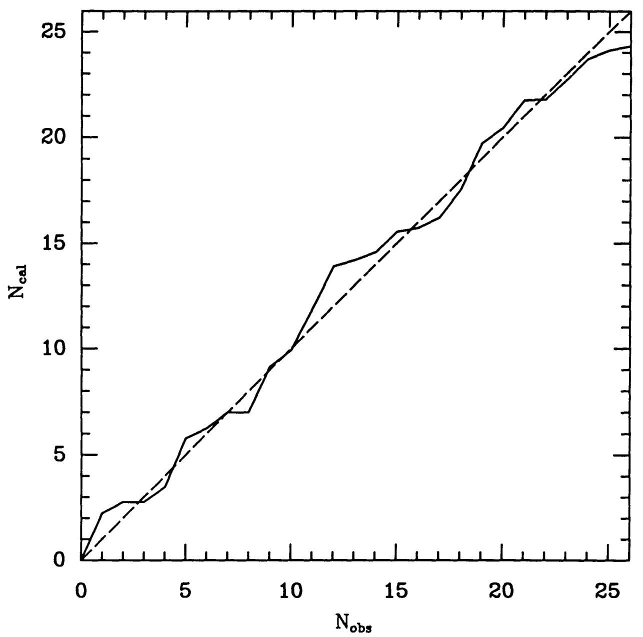

FIG. 1.—A comparison between the observed (Lyne, Anderson, and Salter 1982) and calculated (eqs. [6] and [8]) transverse velocity distributions for radio pulsars is shown with a solid line, for a characteristic velocity constant $v^* = 270$ km s^{-1} (dashed lines shown for reference). N_{obs} and N_{cal} give a number of pulsars observed, and expected, with a transverse velocity below some value. Small velocities are at the lower left corner, high velocities are at the upper right corner.

sponds to the disk. The third component, the halo, has density distribution given as

$$\rho_h = \frac{\rho_c}{1 + (r/r_c)^2}, \qquad r^2 = x^2 + y^2 + z^2, \tag{10}$$

where ρ_c is the central density of the halo component, and r_c is the halo core radius. All three components must satisfy Poisson's equation

$$\nabla^2\Phi_1 = -4\pi G\rho_1, \qquad \nabla^2\Phi_2 = -4\pi G\rho_2, \qquad \nabla^2\Phi_h = -4\pi G\rho_h, \tag{11}$$

where ρ_1 and ρ_2 are the densities of the spheroid and the disk component, respectively. The halo potential corresponding to the density distribution (10) is given as

$$\Phi_h = -\frac{GM_c}{r_c}\left[\frac{1}{2}\ln\left(1 + \frac{r^2}{r_c^2}\right) + \frac{r_c}{r}\,\mathrm{atan}\left(\frac{r}{r_c}\right)\right], \qquad M_c \equiv 4\pi\rho_c r_c^3. \tag{12}$$

The total density, and the total gravitational potential of the model are

$$\rho = \rho_1 + \rho_2 + \rho_h, \qquad \Phi = \Phi_1 + \Phi_2 + \Phi_h. \tag{13}$$

The parameters in the gravitational potential were adjusted so as to provide a good agreement with the Galactic rotation curve as given by Burton and Gordon (1978) and Binney and Tremaine (1987), and to reproduce as closely as possible the local volume density near the Sun: $\rho_0 = 0.18\ M_\odot$ pc^{-3}, and the column density from $z = -700$ pc to $z = +700$ pc: Σ(700 pc) $= 75\ M_\odot$ pc^{-2}, as given by Binney and Termaine (1987, p. 201). The following choice of parameters:

$$a_1 = 0, \quad b_1 = 0.277\ \mathrm{kpc}, \quad M_1 = 1.12 \times 10^{10}\ M_\odot, \tag{16}$$

$$a_2 = 3.7\ \mathrm{kpc}, \quad b_2 = 0.20\ \mathrm{kpc}, \quad M_2 = 8.07 \times 10^{10}\ M_\odot, \tag{17}$$

$$r_c = 6.0\ \mathrm{kpc}, \quad M_c = 5.0 \times 10^{10}\ M_\odot, \tag{18}$$

gave $\rho_0 = 0.186\ M_\odot$ pc^{-3}, Σ(700 pc) $= 80\ M_\odot$ pc^{-2}, $v_c(R_0) = 220$ km s^{-1}, with $R_0 = 8$ kpc, and the Galactic rotation curve as shown in Figure 2.

We used a Monte Carlo method to choose the initial position of a young pulsar within the Galaxy following the probability distributions (1) and (2). The absolute value of the logarithm of initial velocity was adopted as 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, or 2.8, in units of km s^{-1}. The direction of the initial velocity vector was chosen at random, assuming isotropic distribution, and the local circular velocity of our Galactic model was added to it. Now, we had the initial position, given as R and z, and the initial velocity vector, given as v_R, v_ϕ, and v_z, and the orbit calculations could be initiated. Notice that because of the assumed cylindrical symmetry of the Galactic potential, and the Galactic disk, the distribution of old neutron stars should also be cylindrically symmetric, and therefore two space coordinates are sufficient: R and z.

There are two integrals of motion, angular momentum j_z, and energy E:

$$j_z = Rv_\phi, \quad E = \tfrac{1}{2}(v_R{}^2 + v_z{}^2 + v_\phi{}^2) - 4.30 \times 10^4 \times (\Phi_1 + \Phi_2 + \Phi_h) \quad \mathrm{km^2\ s^{-2}}, \tag{19}$$

where all the velocities are in units of km s^{-1}, all gravitational potentials are in units of $10^{10}\ GM_\odot$ kpc^{-1}, and the conversion factor is 4.30×10^4 km^2 s^{-2} $= 10^{10}\ GM_\odot$ kpc^{-1}. The total energy E is in units of km^2 s^{-2}.

For the purpose of numerical integrations the angular momentum constant was used to reduce the number of equa-

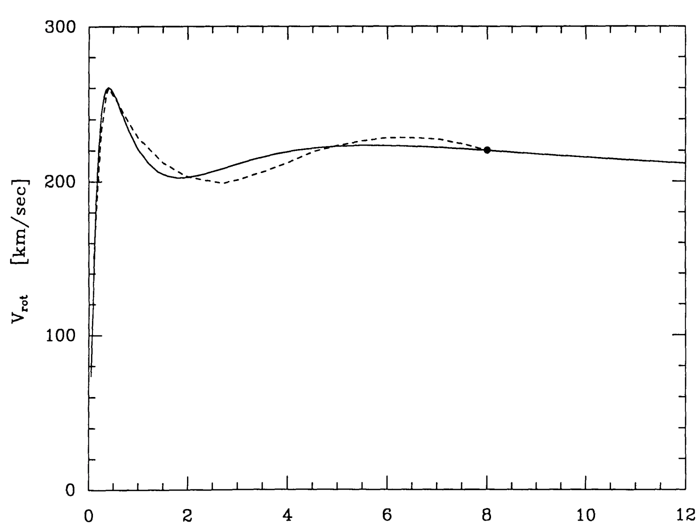

FIG. 2.—The dashed line shows the rotational velocity curve in our Galaxy as observed, according to Burton and Gordon (1978) and Binney and Tremaine (1987). The solar position is at 8 kpc and 220 km s^{-1}. The solid line is the rotation curve calculated with the model adopted in this paper.

tions to four:

$$\frac{dR}{dt} = v_R, \qquad \frac{dz}{dt} = v_z, \tag{19a}$$

$$\frac{dv_R}{dt} = \left(\frac{\partial \Phi}{\partial R}\right)_z + \frac{j_z^2}{R^3}, \qquad \frac{dv_z}{dt} = \left(\frac{\partial \Phi}{\partial z}\right)_R. \tag{19b}$$

These equations were integrated with a fourth-order Runge-Kutta method, and the energy integral was used to control the accuracy of integrations so as to keep it better than 1 part in 10^6.

Almost 90,000 orbits were integrated for up to 10^{10} yr. For every orbit the coordinates (R, z) were calculated at intervals of 10^6 yr, and a density point was added to the corresponding bin in the (R, z) table, for a total of $\sim 2 \times 10^8$ density points. There was a total of 1974 bins, ranging from $\log R = -1$ out to $\log R = 3$, with a step of 0.1, and from $\log z = -1.5$ out to $\log z = 3$, with a step of 0.1, with R and z expressed in kiloparsecs. The orbits of neutron stars with large initial velocities were integrated for 10^{10} yr, i.e., approximately for the age of the Galaxy. However, neutron stars which had low initial velocities never traveled very far in the (R, z) coordinates. Therefore, it was sufficient to integrate their orbits for 10^9 yr to get a good representation of a steady state distribution of their space density. In all cases the resulting density distribution is that of a very old population of disk neutron stars.

The distribution of density obtained from the orbit integrations was stored separately for the eight different absolute values of the initial velocity: $10^{1.4}$, $10^{1.6}$, and so on, up to $10^{2.8}$, all expressed in km s^{-1}. These density distributions were all combined according to the initial velocity distribution adopted in this project, as given with equations (3) and (7). The probabilities calculated for the eight initial velocity bins were: 0.1478, 0.0833, 0.1234, 0.1671, 0.1878, 0.1550, 0.0868, and 0.0488, corresponding to the velocity ranges from 0 to $10^{1.5}$, from $10^{1.5}$ to $10^{1.7}$, and so on, up to velocities larger than $10^{2.7}$ km s^{-1}.

The space density of old neutron stars per 1 pc^3 is shown in Figures 3*a* and 3*b* with density contours in the $R - z$ plane, assuming that the total number of neutron stars in the whole Galaxy is 10^9. Approximately that many neutron stars are required to produce all the heavy elements in the galactic disk (Arnett, Schramm, and Truran 1989). The local density expected near the Sun ($R = 8$ kpc, $z = 0$), is $n_{NS} = 0.0014$ pc^{-3}, falling to 50% at $z_{1/2} = 193$ pc. The radial logarithmic derivative of the number density is $\partial \log (n_{NS})/\partial \log R = -2.84$ at $R = 8$ kpc, and $z = 0$. The density of old neutron stars within 0.5 kpc of the Sun may be described with an accuracy of 5% with a formula:

$$n_{NS} \approx 0.0014 \text{ pc}^{-3} \left(\frac{R}{8 \text{ kpc}}\right)^{(-3+4z/1 \text{ kpc})} \times \left[1 + \left(\frac{z}{0.2 \text{ kpc}}\right)^2\right]^{-1}, \tag{20}$$

assuming that the total number of neutron stars in the whole Galaxy is $N_{tot} = 10^9$. This corresponds to 1.4×10^6 kpc^{-2} neutron stars born near the Sun ($R = 8$ kpc) over the lifetime of the Galaxy.

Lyne, Manchester, and Taylor (1985) estimated the half-density half-thickness of the old radio pulsar distribution to be $z_{1/2} = 260$ pc. They assumed a Maxwellian velocity distribution in the z-direction, and used a local analysis, i.e., the pulsars born at R = 8 kpc remained there forever, and an equilibrium distribution was adopted in the z-direction. Their result is in a reasonable agreement with our value of $z_{1/2} = 193$ pc, based on a very different analysis. Lyne *et al.* estimated that the number of pulsars born near the Sun over the lifetime of the Galaxy was between 0.1 and 0.4×10^6 kpc^{-2}, and the number of all pulsars born in the whole Galaxy was between 1 and 4×10^8, assuming that the birth rate was constant over the age of 1.2×10^{10} yr. As the birth rate was almost certainly higher when the Galaxy was young, the value adopted in this paper: $N_{tot} = 10^9$ is reasonable.

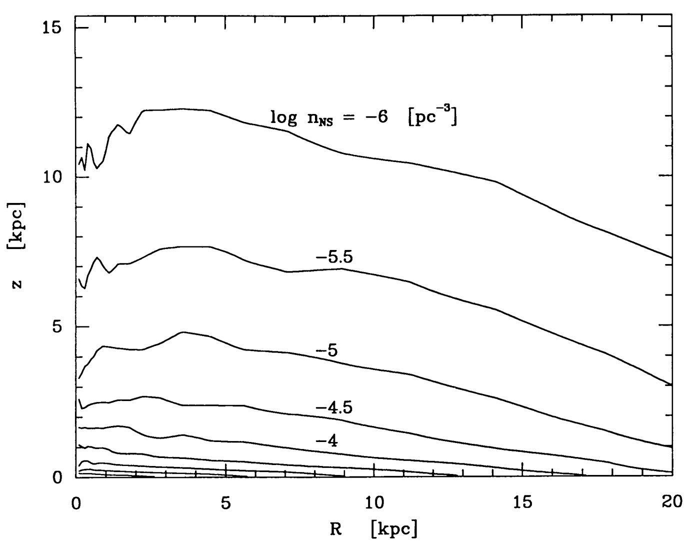

FIG. 3*a*

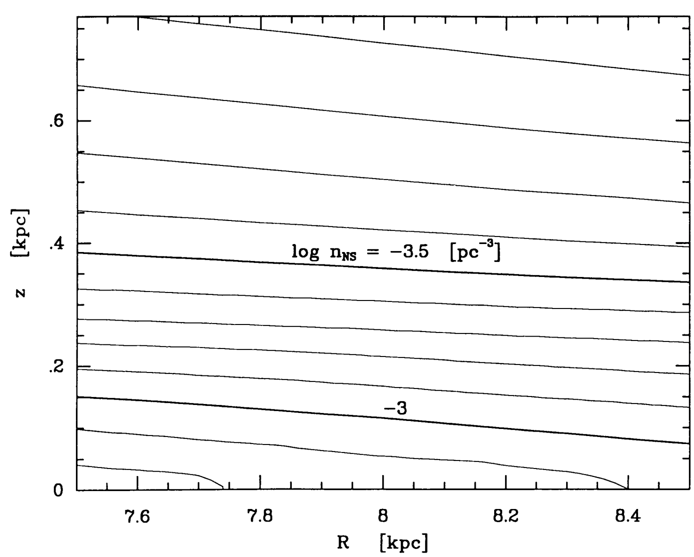

FIG. 3*b*

FIG. 3.—(*a*) The number density of old neutron stars per cubic parsec n_{NS}, is shown with density contours in the $R - z$ plane, where R is a distance from the Galactic center, and z is a distance from the Galactic plane. The total number of neutron stars in the Galaxy was adopted as 10^9. The lines are labeled with a logarithm of n_{NS}. (*b*) The same as (*a*), but on a smaller distance scale.

III. STATISTICAL RESULTS FOR OLD NEUTRON STARS

Following Hartmann and Epstein (1989) and Schmidt, Higdon, and Hueter (1988) we shall consider three parameters characterizing the distribution of neutron stars in the Galaxy: a dipole moment $\langle \cos \Theta \rangle$, which measures the degree of concentration of objects in the sky toward the Galactic center, a quadrupole moment $\langle \sin^2 b - \frac{1}{3} \rangle$, which measures the degree of concentration of objects in the sky toward the Galactic plane and $\langle V/V_{max} \rangle$, which measures the distribution of objects in the radial direction. The angle between the object and the Galactic center, is denoted by Θ, b is the Galactic latitude of the object, $V/V_{max} = (d/d_{max})^3$, where d is the distance to the object, and d_{max} is the maximum distance to which the object can be observed. All three parameters can be calculated as a function of d_{max}.

The density distribution of old neutron stars, as calculated in the previous section, was analyzed. The average values of $\cos \Theta$, $\sin^2 b$, and V/V_{max} were calculated for a large range of d_{max}, and the results are shown with three solid lines in Figure 4. Also shown, with three pairs of dashed lines, are the standard deviations for all three average values, assuming that 100 objects were detected. If the number of objects N is larger, then the standard deviations scale as $N^{-1/2}$. It is clear that when $d_{max} \ll 193$ pc, i.e., when the range of observations is much less then the half-density scale height in the distribution of neutron stars, the distribution is almost isotropic and homogeneous, and we have $\langle \cos \Theta \rangle = 0$, $\langle \sin^2 b \rangle = \frac{1}{3}$, and $\langle V/V_{max} \rangle = \frac{1}{2}$. When the range of observations increases a concentration of sources toward the Galactic plane becomes noticeable as a reduction in the value of $\langle \sin^2 b \rangle$. At still larger d_{max} a concentration toward the Galactic center becomes apparent: $\langle \cos \Theta \rangle$ increases, and reaches a maximum when the Galactic center becomes visible at $d_{max} \approx 10^4$ pc. If all sources were exactly in the Galactic plane then we would have $\langle \sin^2 b \rangle = 0$. If all sources were at the Galactic center we would have $\langle \cos \Theta \rangle = 1$. However, neutron stars are only moderately concentrated, as it is apparent in Figure 4. It is also apparent that when d_{max} increases beyond 10^4 pc then the number density of far away neutron stars decreases sharply, and therefore $\langle V/V_{max} \rangle$ decreases as well.

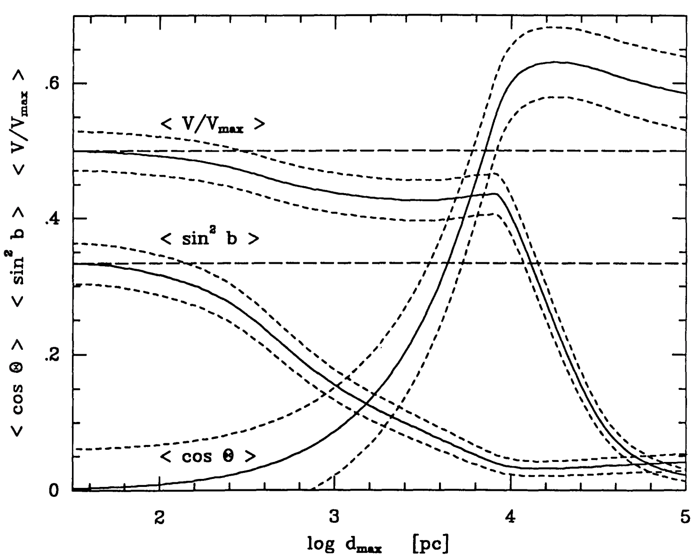

FIG. 4.—The variation of the dipole moment, $\langle \cos \Theta \rangle$, the quadrupole moment $\langle \sin^2 b \rangle - \frac{1}{3}$, and the average ratio $\langle V/V_{max} \rangle = \langle (d/d_{max})^3 \rangle$, in the distribution of old neutron stars is shown as a function of distance d_{max} with three solid lines, and the three pairs of dashed lines show the standard deviation for a random sample of $N = 100$ objects. Θ is the angle between an object and the Galactic center, b is the Galactic latitude, d is a linear distance from the observer located at the solar position, and d_{max} is the range of observations. These results are for a distance limited sample. It corresponds to a flux-limited sample if all sources are standard candles. The initial velocities of newly born neutron stars are assumed to have the same distribution as radio pulsars studied by Lyne, Anderson, and Salter (1982). It is assumed that all neutron stars were born in the exponential disk, with $R_{exp} = 4.5$ kpc, and $z_{exp} = 75$ pc.

Figure 4 was prepared assuming that the number density of gamma-ray bursters was proportional to the number density of old neutron stars. The total number of neutron stars located, and in principle observable within a distance d_{max} is N_{obs}. At small d_{max} we have $N_{obs} = 1.4 \times 10^{-3} \times 4\pi d_{max}^3/3$, as the distribution is locally uniform. At very large d_{max} all the Galactic neutron stars are observable, and $N_{obs} = N_{tot} = 10^9$. The dashed lines in Figure 4 indicate the standard deviations in the parameters for the same number of bursts, $N_b = 100$, no matter what d_{max} is. This is equivalent to assuming that a fraction N_b/N_{obs} of all neutron stars generated gamma-ray bursts during the observing period.

Figure 4 was obtained with the initial velocity distribution adopted in the previous section, and that was based on the observational data of Lyne, Anderson, and Salter (1982). It was also assumed that all neutron stars were born within a thin disk with an exponential density distribution, with a radial scale of $R_{exp} = 4.5$ kpc, and a vertical scale of $z_{exp} = 75$ pc. One may wonder what would be the distribution of old neutron stars if they were all born in the same disk, but all had very high velocities at formation. As an extreme case we may take objects with initial velocities $v_{init} = 10^{2.8} = 631$ km s^{-1}, and a uniform distribution of initial directions, and add to them the Galactic rotational velocity. Assuming there are 10^9 such old neutron stars within our Galaxy, born with a uniform rate over the last 10^{10} yr, we find that their local space density near the Sun would be only $n_{NS} = 1.5 \times 10^{-5}$ pc^{-3}, their space density would fall off by 50% at a height of $z_{1/2} = 480$ pc, and the logarithmic radial gradient of their space density would be $\partial \ln (n_{NS})/\partial \ln R = -1.7$ at $R = 8$ kpc and $z = 0$. In spite of very high velocities these neutron stars do show a concentration toward the galactic center and toward the Galactic plane, because their birth rate is concentrated in these directions.

Statistical properties of the distribution of very high velocity neutron stars, i.e., the dipole and quadrupole moments as well as $\langle V/V_{max} \rangle$, are shown in Figure 5. The lines are more "noisy" because Figure 5 is based on the statistics of $\sim 10^4$ orbits only, while Figure 4 was based on $\sim 9 \times 10^4$ orbits.

Even at very high velocities the effect of Galactic rotation is not negligible and makes it easier for stars to reach very large distances from the Galactic center, if their random initial velocities are in the direction of Galactic rotation. As a result, even at very large distances from the Galactic center, like 100 kpc to 1 Mpc, the population of very high velocity stars is concentrated toward the Galactic plane.

We calculated the local density of neutron stars, n_{NS}, and the half-density height above the Galactic plane, $z_{1/2}$, for each initial velocity bin. Assuming that the total number of neutron stars in the whole galaxy is 10^9, and that all of them are in a single velocity bin, we obtained $n_{NS} = 0.0038$, 0.0028, 0.0019, 0.0012, 0.000 65, 0.000 28, 0.000 095, 0.000 015 pc^{-3}, and $z_{1/2} =$ 167, 193, 224, 247, 254, 353, 432, and 483 pc, for $\log v_{init} = 1.4$, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, and 2.8, respectively. With the initial

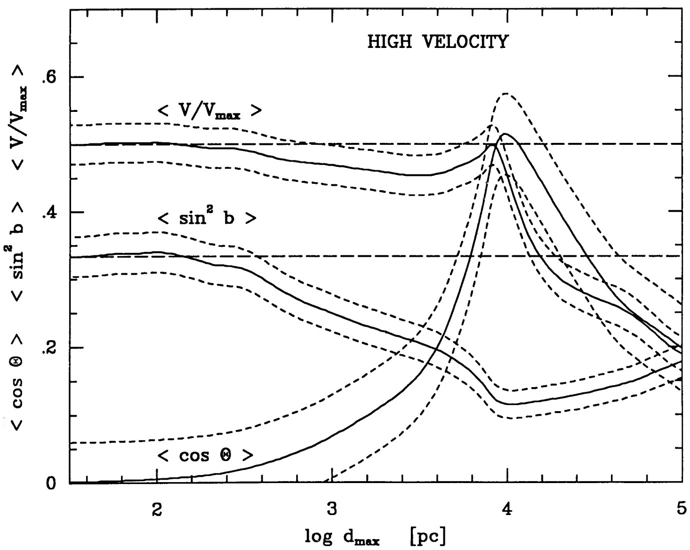

FIG. 5.—The same as Fig. 4, but assuming that all neutron stars were born in the exponential disk, with randomly oriented velocities which had the absolute value of $10^{2.8} = 631$ km s^{-1}.

velocity distribution given with equations (3) and (7) the resulting probabilities for each velocity bin are 0.1478, 0.0833, 0.1234, 0.1671, 0.1878, 0.1550, 0.0868, and 0.0488, respectively. Combining these two sets of numbers we may calculate the contribution to the local density of neutron stars due to the eight velocity bins to be 0.000 56, 0.000 24, 0.000 23, 0.000 20, 0.000 12, 0.000 044, 0.000 008, and 0.000 000 7 pc^{-3}, respectively. Even though many neutron stars have very high initial velocities, those that contribute significantly to the local density had low or moderate initial velocities.

All the results presented in Figures 4 and 5 are valid for a distance-limited sample of bursts. All real surveys are at best "apparent luminosity" limited, which in the case of gamma-ray bursts corresponds to a sharp detection threshold at some photon count rate per time bin, n_0 (see Paczyński and Long 1988). We define n_0 as the photon count rate per time bin which has to be exceeded in order to trigger the gamma-ray burst detector. The count rate is a "photon luminosity" integrated over the duration of a time bin, Δt. It is expressed as a number of photons per time bin, and not as a number of ergs per second. It should be stressed that an instantaneous maximum luminosity is not a measurable quantity. Figures 4 and 5 may be relevant only for sources that are "standard candles" defined so that for a given detection threshold we could detect all bursts out to a distance $d_{\max}$, and none beyond that distance. There is no reason to think that all gamma-ray bursters have the same peak value of a number of photons emitted per time bin Δt. Therefore, it is necessary to look at the effect of a broad intrinsic "luminosity function." As we have no knowledge of that, it was necessary to assume an ad hoc distribution. We shall use the term *absolute luminosity* L, but we shall really mean the maximum intrinsic photon emission rate, related to the observed maximum photon count rate $n_{\max}$ (integrated over a time bin Δt), with a simple relation: $L \equiv 4\pi d^2 n_{\max}$, where d is the distance to the source (see Paczyński and Long 1988). We shall assume that the observed sample is flux limited, i.e., that all sources with the observed photon count rate above some threshold, $n_{\max} > n_0$, are detected. Therefore, the maximum distance at which a source with a given observed count rate can be seen is given as $d_{\max} = (n_{\max}/n_0)^{1/2} d$, and for this particular source we have $V/V_{\max} = (n_0/n_{\max})^{1.5}$.

The following luminosity function was adopted:

$$p(L)dL = \frac{1}{\ln(L_{\max}/L_{\min})} \frac{dL}{L}, \quad \text{for} \quad L_{\min} < L < L_{\max}, \tag{21}$$

and five cases, with $\log L_{\max}/L_{\min} = 0$, 0.5, 1, 1.5, and 2, were considered. The first case corresponded to the sources that were standard candles, while in the last case the sources covered two orders of magnitude with their intrinsic luminosities.

With a luminosity function extending from $L_{\min}$ to $L_{\max}$ all sources out to some distance d_1 can be seen, a decreasing fraction of bright sources can be seen between distances d_1 and d_2, and no source beyond d_2 can be seen. We have $d_2/d_1 = (L_{\max}/L_{\min})^{1/2}$. The total number of observable sources, N_{obs}, becomes a more convenient measure of the radial range of observations, as there is no longer an abrupt cutoff to the observed distribution at some distance $d_{\max}$. The variation of $\langle V/V_{\max}\rangle$, $\langle \sin^2 b\rangle$, and $\langle \cos \Theta\rangle$ with the number of observable sources is shown in Figures 6*a* and 6*b* for $\log L_{\max}/L_{\min} = 0$ and 2, respectively. The first case, shown in Figure 6*a*, corresponds to all sources being "standard candles," and this is just another representation of the case shown in Figure 4. The second case, shown in Figure 6*b*, corresponds to a very broad intrinsic luminosity function. It is somewhat surprising how little difference there is between these two cases. Notice that when the range of observations is very large, we can see the whole Galaxy, and the total number of observable sources becomes 10^9, because that was the number we selected.

The dashed lines in Figures 4, 5, 6*a*, and 6*b*, show a standard deviation for the case when the number of sources actually observed in $N_b = 100$. As the number of observable sources N_{obs} varies from 10^2 up to 10^9, these lines correspond to a small fraction of all observable sources seen as bursters. Suppose now that we see all sources that are observable. In the case of gamma-ray bursts this corresponds to carrying the observations for so long that every burster is detected at least once. If a given burster is seen many times, it should be counted just once. When the number of observable sources increases, the radial range of observations increases as well. Therefore, the departures of $\langle \sin^2 b\rangle$, $\langle V/V_{\max}\rangle$, and $\langle \cos \Theta\rangle$ from their "isotropic" and "homogeneous" values increase, while the standard deviations decrease. The ratio of those departures Δ to the standard deviations σ is shown in Figure 7 as a function of the number of observable sources N_{obs}. The departures from uniform and isotropic distributions are measurable at a 3 σ level when the number of observable sources is a few thousand, and the first measurable departure is in $\langle \sin^2 b\rangle$, i.e., the first property of the distribution of neutron stars that becomes measurable is their concentration toward the galactic disk.

Figure 7 represents a case when the total number of neutron stars in the galaxy is $N_{\text{tot}} = 10^9$, and when they are standard candles, i.e., when $L_{\max}/L_{\min} = 1$. However, we do not know what the correct values of N_{tot} and $L_{\max}/L_{\min}$ are for gamma-ray bursters. Therefore, the number of observable objects N_{obs} that would be required in order to detect a concentration of sources toward the Galactic disk was calculated for a large range of values of N_{tot} and $L_{\max}/L_{\min}$. The results are shown in

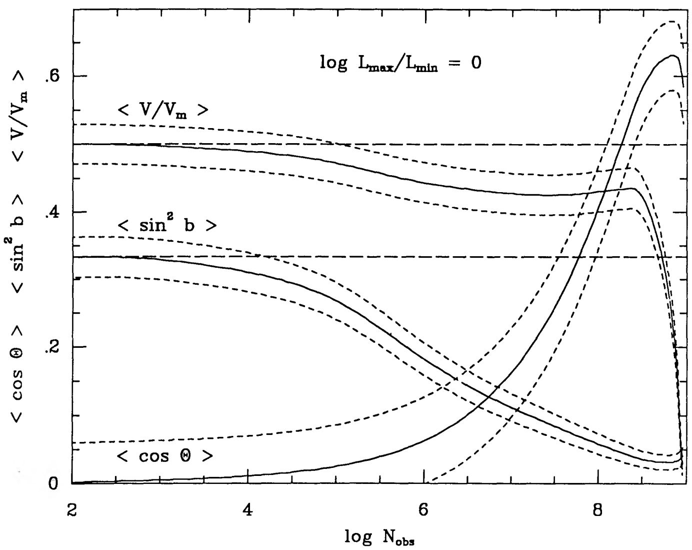

FIG. 6a

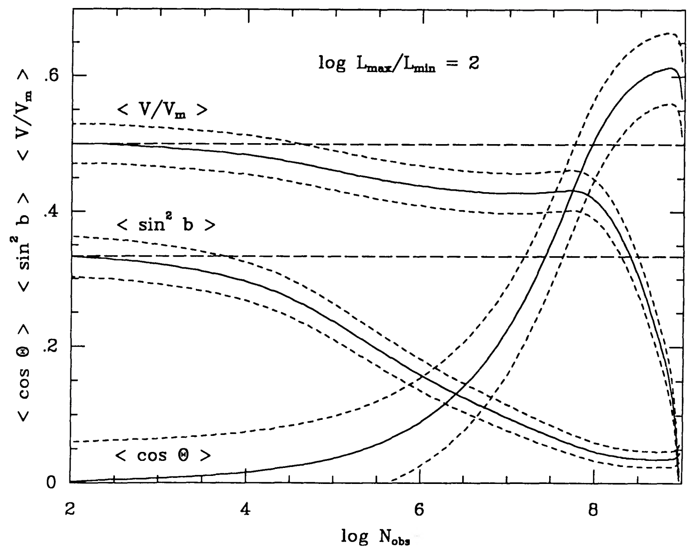

FIG. 6b

FIG. 6.—(*a*) The same as Fig. 4, but with the radial depth of the sample measured with the total number of objects N_{obs} observable out to the limiting distance d_{max}. All sources are assumed to have the same intrinsic luminosity L. (*b*) The same as Fig. 6*a*, but with the sources having a power-law distribution of intrinsic luminosities, with a probability distribution $p(L)dL \sim L^{-1}dL$ for $L_{min} < L < L_{max}$, and $\log L_{max}/L_{min} = 2$.

Figure 8, and may be well approximated with a simple formula

$$\begin{aligned} \log N_{obs,3\sigma} &\approx 3.60 + 0.54(\log N_{tot} - 9) \\ &\quad - 0.12 \log (L_{max}/L_{min}) \\ &\approx 3.60 + 0.54 \log (n_{NS}/1.4 \times 10^{-3}\ \mathrm{pc}^{-3}) \\ &\quad - 0.12 \log (L_{max}/L_{min}) , \end{aligned} \tag{22}$$

where n_{NS} is the number density of old neutron stars near the Sun, i.e., at $R = 8$ kpc, and $z = 0$ (see eq. [20]).

This result has the following interpretation. The detectability of a concentration of sources to the Galactic plane, i.e., the ability to measure a quadrupole moment in their distribution, depends on the distance of the observable sources, and on their number. The farther away the sources are, the more pronounced is their concentration toward the Galactic plane. The more sources are observed the smaller is the statistical error. We do not know intrinsic luminosity of the sources, and therefore we do not know what the radial range of any particular observational sample is. For a given range of a sample the number of observable sources N_{obs}, is directly proportional to the total number of sources N_{tot}, and the error in the estimate of the quadrupole moment is reduced by a factor proportional to $N_{tot}^{1/2}$. However, the detectability of the quadrupole moment

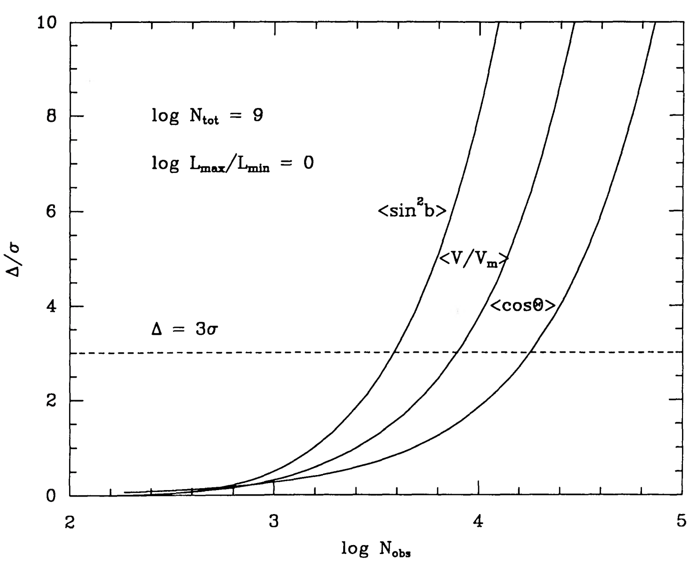

FIG. 7.—The average values of $\Delta(\langle \cos \Theta \rangle) = \langle \cos \Theta \rangle$ (a dipole moment), $\Delta(\langle \sin^2 b \rangle) = \langle \sin^2 b - \frac{1}{3} \rangle$ (a quadrupole moment), and $\Delta(\langle V/V_{max} \rangle) = \langle V/V_{max} - \frac{1}{2} \rangle$, in units of their standard deviations, are shown as a function of the number of observable sources N_{obs}. All sources are assumed to be standard candles. The dashed horizontal line is a 3 σ line. Notice that the quadrupole moment, i.e., the concentration of neutron stars toward the Galactic plane, is the easiest to detect.

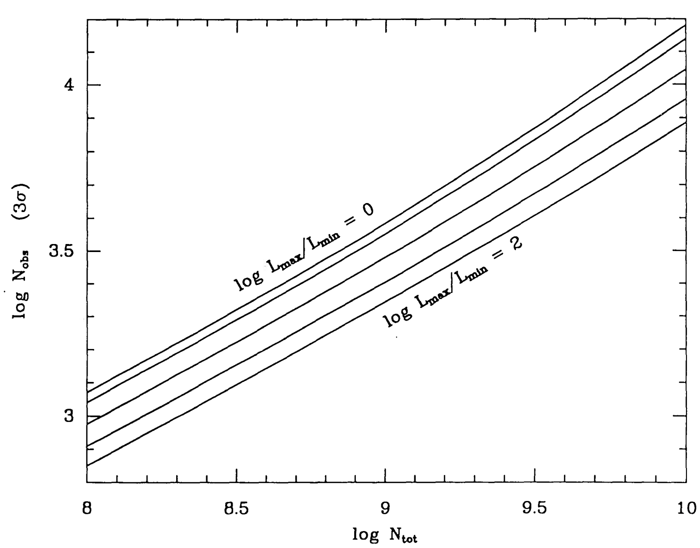

FIG. 8.—The number of observable sources N_{obs} required to detect a quadrupole moment in their distribution at a 3 σ level is shown as a function of the total number of neutron stars in the Galaxy, N_{tot}, for sources with five different luminosity functions. The probability distribution of the intrinsic source luminosity is $p(L)dL \sim L^{-1}\,dL$ for $L_{min} < L < L_{max}$, with $\log (L_{max}/L_{min}) = 0$, 0.5, 1, 1.5, and 2, respectively, for the five luminosity functions.

depends also on the ratio of the geometrical range of the sample to the half-density thickness of the neutron star distribution. Numerically, it turns out that in order to have a 3 σ measurement the required number of observable sources is proportional to $N_{\rm tot}^{0.54}$. If the range of intrinsic luminosities is increased then some very distant luminous sources enter the sample, and its effective depth is increased. For this reason the number of observable sources required to get a 3 σ result is somewhat reduced, $N_{\rm obs} \sim (L_{\rm max}/L_{\rm min})^{-0.12}$.

The results presented in Figure 8 correspond to the case when all observable sources are detected, i.e., when every burster was observed at least once. However, in practice the number of sources that are observed down to some intensity threshold is smaller than the number of sources that would be observable during a very long observing period, because some, perhaps most sources may have a recurrence period much longer than the duration of an experiment. Therefore, if a given gamma-ray burst detection threshold n_0 corresponds to some number of observable sources $N_{\rm obs}$, then the number of bursters N_b that are actually detected during an experiment with a finite duration, say 1 or 10 yr, will be smaller, i.e., $N_b < N_{\rm obs}$, perhaps even $N_b \ll N_{\rm obs}$. Because of this incompleteness of gamma-ray burst surveys the actual number of bursts N_b required for a 3 σ detection of their concentration toward the Galactic plane is smaller than $N_{\rm obs}$ as given with Figure 8 or equation (22).

IV. DISCUSSION

A very striking result of our model of the distribution of old disk neutron stars is that the half-density half-thickness turned out to be as small as 193 pc, while newly born neutron stars, i.e., radio pulsars, are known to have a very large rms velocity, of the order of 200 km s^{-1}. This paradox can be explained in the following way. Neutron stars that are born with large space velocities do not contribute much to the local density of old neutron stars because their velocities are so high. They spend most of their time far out in the halo. As their velocities are high they dominate the value of $\langle v^2 \rangle^{1/2}$ for the pulsars. At the same time there is a large fraction of neutron stars that are born with low velocities. These stay near their birth place for ever, they dominate the local density, and they are responsible for the small value of $z_{1/2}$.

Our calculations assumed that gravitational potential of the Galaxy is axially symmetric, and there was no stochasticity in it. If we started with a "very cold" population it would remain "very cold" for ever. However, it is known that the oldest stars in the galactic disk have velocity dispersion in the z-direction of up to 25 km s^{-1}, much larger than 4 km s^{-1} that is observed for the youngest stars (see Wielen 1977 Table 1, and references therein). According to Bahcall (1984) old K giants have velocity dispersion in the z direction of 20.3 ± 1.5 km s^{-1}, and the half-density half-thickness of their disk distribution, $z_{1/2}$, was estimated to be 190, 220, and 160 pc by Hill (1960), Oort (1960), and Upgren (1962), respectively. We expect that any old population, no matter how "cold" it initially was, must become at least as "warm" as the population of old K giants, and the half-density scale height should be ~ 190 pc. This is about the thickness of our model. Therefore, it may be necessary to introduce some "heating" to our model, and to increase somewhat our $z_{1/2}$. This has not been done in this paper.

Our estimate of $z_{1/2} \approx 193$ pc is consistent with $z_{1/2} \approx 260$ pc as estimated by Lyne, Manchester, and Taylor (1985), as the latter authors assumed Maxwellian distribution of initial velocities, while we allowed for the observed excess of low-velocity radio pulsars. Recently, Hartmann, Epstein, and Woosley (1989*a*, *b*) obtained $z_{1/2} \approx 500$ pc. This is much more than our estimate. The main reason for the difference is the assumption by Hartmann *et al.* that all neutron stars were born with a velocity of 200 km s^{-1}, with very little scatter. We know from our computations that the local density and the value of $z_{1/2}$ is dominanted by the low-velocity objects. Some difference in the scale-heights is due to a difference in the adopted Galactic gravitational potentials.

There is a possibility that gamma-ray bursts are related to very old neutron stars that may populate the Galactic halo (Shklovskii and Mitrofanov 1985). This scenario is not considered here. However, it should be pointed out that there are fewer neutron stars in the Galactic halo than in the disk, as the total mass of heavy elements present in the halo stars is much smaller than in the disk. As far as we know a formation of a neutron star is always associated with an ejection of heavy elements (see Arnett, Schramm, and Truran 1989, and references therein). A possible halo origin of gamma-ray bursts will be discussed in another paper. In the following discussion we shall consider the consequences of an assumption that gamma-ray bursters are old ($\sim 10^{10}$ yr old) neutron stars born in the Galactic disk with the same distribution of initial velocities as that observed for radio pulsars.

Our results were obtained for a resonable, but probably not the best Galactic potential. We do not know at this time how sensitive our results are to the changes of the potential. The observed distribution of pulsar velocities is poorly known at low velocities, which dominate the local density of old neutron stars. For these reasons all the discussion that follows should be considered to be only preliminary.

We do not know the intrinsic "luminosities" of gamma-ray bursters, and for that reason we cannot meaningfully define the radial depth of the burst surveys, $d_{\rm max}$. However, it is easy to find how many bursts were detected, N_b. If these were not repeating then they had to come from different neutron stars. Therefore, the number of neutron stars observable with a given instrument, $N_{\rm obs}$, must be larger than N_b. If we could carry out the experiment for an indefinite period of time, we might see the bursts from all neutron stars. We may expect that $N_{\rm obs} = N_b$ in the limit of very long observing time $\Delta t_{\rm obs}$, but under normal conditions we expect $N_{\rm obs} > N_b$, most likely $N_{\rm obs} \gg N_b$. It is most convenient to discuss the results presented in the previous section in terms of the range of observations measured with $N_{\rm obs}$ rather than $d_{\rm max}$. If gamma-ray bursters have a broad intrinsic "luminosity function" then $N_{\rm obs}$ is a very natural quantity to use as a measure of the depth of the observed sample.

Let us consider a gamma-ray burst detector with a constant detection threshold, n_0 counts per time bin. As long as we do not see the bursts repeating we know that the number of detected bursts N_b is only a lower limit to the number of neutron stars observable with this instrument $N_{\rm obs}$. Figures 7 and 8 and equation (22) are expressed in terms of $N_{\rm obs}$, as this is the only quantity that comes out from our model. While the experiment is continued for a longer time, and a larger number of nonrepeating bursters is detected, we are getting a more stringent limit on $N_{\rm obs}$. Let us discuss a few specific examples.

According to Melia (1988) gamma-ray bursts may originate near the surface of isolated neutron stars surrounded by cold, degenerate disks. He estimated their number density near sun to be $\sim 10^{-6}$ pc^{-3}, i.e., ~ 1000 times smaller than the expected density of all old neutron stars (see eq. [20]). A catalog of Atteia *et al.* (1987) contains positions of 54 single error box, nonrepeating sources. We must reach out to a distance of at least $d_{\max} \approx 254$ pc in order to see 54 neutron stars if their local space density is as low as 10^{-6} pc^{-3}, and if they have the space distribution similar to that expected of old disk single neutron stars with $z_{1/2} = 193$ pc. At this depth of the sample we would expect to detect a quadrupole moment, i.e., a concentration of sources toward the Galactic plane at the level of 1.5 σ, which is not very convincing. However, it seems most unlikely that we have seen all objects of this kind, and yet none of them bursted twice. If we make a very modest assumption that only 20% of all Melia's objects have been detected as bursters during a short observational period then we would have to look out to a distance $d_{\max} \approx 500$ pc in order to see 54 of them, and the quadrupole moment in their distribution would be detectable at the 3 σ level. A likely spread of intrinsic burst intensities brings some distant sources into the observed sample and makes the quadrupole moment even more pronounced (see eq. [22]). Hartmann and Epstein (1989) found no dipole or quadrupole moment in the distribution of 54 single error box sources in the Atteia *et al.* catalog and no statistically significant correlation between the positions of various sources; the distribution was fully consistent with that expected for 54 randomly located sources. This makes the scenario proposed by Melia rather unlikely, unless we were so lucky as to see almost all the sources once and none of them twice. A possible way out would be to assume that binary neutron stars have velocity distribution even more strongly biased toward high values than that of the known radio pulsars or that they are much more common than Melia estimated.

The same reasoning may be applied to models that require thermonuclear explosions on accreting neutron stars (see Taam 1987, and Woosley 1987 for references), as the expected number density of neutron stars with close companions is comparable to that proposed by Melia (1988). In their recent review Hurley and Hartmann (1988) point out that nondetection of substantial X-ray flux from the few small error-boxes of gamma-ray bursts implies source distances in excess of ~ 500 pc in the context of the thermonuclear model. At such distances a concentration of sources toward the Galactic plane should be striking. Again, the concentration toward the Galactic plane could be reduced if all binary neutron stars were high-velocity objects.

In general, any model with fewer than 10^6 bursters distributed over the whole Galactic disk in the way expected of old single neutron stars is difficult to reconcile with the apparently isotropic distribution of known bursts.

The distribution of interstellar medium is known to be very nonuniform (Paresce 1984). The density in the direction $l \approx 10° \pm 25°$, and 150 pc away from the Sun is 100 times higher than the density at $l \approx 225° \pm 25°$ and the distance between 0 and 200 pc. The density changes by a factor of 10 within 20 pc of the Sun. Any model that requires accretion from interstellar medium to trigger gamma-ray bursts (e.g., Ruderman and Cheng 1988) would produce highly anisotropic distribution of events, no matter how many neutron stars are used. A strong dipole moment pointing in the direction ($l \approx 10°$, $b \approx 0°$) would be expected, but it is not observed (Hartmann and Epstein 1989; see also Epstein and Hurley 1988).

Models in which gamma-ray bursters are young neutron stars may be difficult to reconcile with the isotropic distribution of the observed bursts, because there are so few of those young stars. However, our density distribution corresponds to old, steady state population and cannot be directly used to analyze the statistics expected in this scenario. This will be a subject of another study.

If every old neutron star can produce gamma-ray bursts more frequently than once every 100 yr or so, then there is no problem with the apparently isotropic distribution (see neutron starquake models: Tsygan 1975; Muslimov and Tsygan 1986; Blaes *et al.* 1989; and references therein). However, the next generation of gamma-ray observatories will have sensitivity so high that much more distant sources will be discovered and a clear concentration toward the Galactic plane should be apparent. A detection of such a concentration will provide a direct measure of a distance scale, a quantity very badly needed for theoretical models. Let us estimate the minimum number of bursts required to detect the quadrupole moment.

If there are $N_{\rm tot} = 10^9$ old neutron stars in the Galactic disk, and if gamma-ray bursters have a modest range of intrinsic "luminosities," $L_{\max}/L_{\min} > 10$, then ~ 4000 nearest neutron stars have to be discovered to measure their concentration to the Galactic plane at a 3 σ level (see eq. [22]). However, if the recurrence time is long then only a fraction of nearby neutron stars will be discovered during a few year observing program. In this case the number of observable objects $N_{\rm obs}$ will be larger than the number of bursts actually detected N_b. The effective depth of the survey depends on $N_{\rm obs}$, and not on N_b. Therefore, a small $N_b/N_{\rm obs}$ implies a large depth of the survey, and the detection of a quadrupole moment is easier. In other words, if the recurrence time of gamma-ray bursts is long then their concentration to the Galactic disk will become apparent with fewer than 4000 events. Therefore, either GRO or GRANAT should have no difficulty in detecting this concentration, provided the bursts are related to the old disk neutron stars.

On the other hand, if the concentration to the Galactic plane is not seen with the GRANAT and GRO instruments then gamma-ray bursts are not related to the old disk neutron stars.

This conclusion will be much stronger if these instruments confirm the deficiency in the number of weak bursts as compared to the number expected from the -1.5 power law. According to the balloon data there is a very strong deficiency (see Meegan, Fishman, and Wilson 1985). Figures 4–6 demonstrate very clearly that any sources related to the Galactic disk should show anisotropy in their angular distribution first, and a reduction in the number density later, i.e., the departure of $\langle \sin^2 b \rangle$ from its "uniform" value is much more pronounced than the corresponding departure of $\langle V/V_{\max} \rangle$ (see Fig. 7). Therefore, the dramatic deficiency in the number of weak bursts claimed by Meegan *et al.* must be accompanied with even more dramatic concentration of weak bursts toward the Galactic plane if the sources are related to the old disk neutron stars. It may be shown that a halo distribution of any known class of stars has a dipole anisotropy pointing toward the Galactic center whenever a $V/V_{\max}$ test points to a departure from a uniform distribution (Paczyński 1990).

If the flattening of the number-intensity relation is confirmed, in other words if $\langle V/V_{\max} \rangle$ is much less than 0.5 for weak bursts to be discovered by BATSE, and if there is no strong dipole or quadrupole anisotropy in their angular distribution, then the most natural conclusion will be that weak bursts come from cosmological distances (see Paczyński 1986 and references therein).

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