Applied Mathematical Modelling 36 (2012) 4944–4951

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# A multi-periodic optimization formulation for bike planning and bike utilization

Hamidreza Sayarshad [a,*], Sepideh Tavassoli [b], Fang Zhao [c]

[a] Department of Industrial Engineering, Mazandaran University of Science and Technology, 16388-95114, Iran
[b] Department of EMBA, Azad University, Science and Research Branch, Tehran 14716, Iran
[c] Department of Civil and Environmental Engineering, Florida International University, EAS 3677, USA

ARTICLE INFO

*Article history:*
Received 26 August 2011
Received in revised form 11 December 2011
Accepted 15 December 2011
Available online 24 December 2011

*Keywords:*
Bike-sharing system design
Cost analysis
Optimization

ABSTRACT

The number of policy initiatives to promote the use of bike, or the combined use of bicycle and public transport for one trip, has grown considerably over the past decade as part of the search for more sustainable transport solutions. This paper presents an optimization formulation to design a bike-sharing system for travel inside small communities, or as a means to extend public transport for access and egress trips. The mathematical model attempts to optimize a bike-sharing system by determining the minimum required bike fleet size that minimizes simultaneously unmet demand, unutilized bikes, and the need to transport empty bikes between rental stations to meet demand. The proposed approach is applied to an example problem and is shown to be successful, ultimately providing a new managerial tool for planning and analyzing bike utilization more effectively.



## 1. Introduction

Non-motorized transport modes are often considered as vital elements of sustainable transport systems. Their emissions of pollutants and noise, and the accident risks they pose for other road users are very low. The use of non-motorized transport modes also promotes a healthy lifestyle and helps to reduce obesity. For these reasons, bicycles are a desirable transportation mode, especially for short to medium range travel in an urban area. Compared to private automobiles, bike-and-ride offers a number of environmental and social benefits [1]. These include reduction in energy use, air and noise pollution, and congestion levels on specific corridors and access routes to public transport stops.

The number of policy initiatives to promote the use of bike-and-ride or the use of the bicycle for entire trips has seen a substantial increase over the past decade in many industrialized countries as part of the search for more sustainable transport solutions (e.g., [2–5]). The use of the bicycle in access trips (at the home-end of a trip) and/or egress trips (at the activity-end of a trip) can substantial reduce the door-to-door travel time of public transport trips and expand public transport service areas, thus helping to reduce automobile use. Bicycles are substantially faster than walking and more flexible than public transport due to its "continuous" and "door-to-door" character, eliminating waiting and scheduling costs, suggesting that the use of the bicycle in access and/or egress trips can help closing the 'travel time gap' between car and public transport [6].

In recent years, many vehicle sharing systems, such as bicycle and electric car rental programs, have been implemented throughout the world. In these systems, customers arrive to rental stations, utilize bicycles for some amount of time, and then return the bicycle to the same or a different bike station. These systems are often quite large – for example [7], a bicycle rental program in Paris, maintains over 2000 bicycles across approximately 1500 locations. The large scale and dynamic

* Corresponding author.
*E-mail address:* hamsayar@yahoo.com (H. Sayarshad).

0307-904X/$ - see front matter 
doi:10.1016/j.apm.2011.12.032

nature of these systems make optimal fleet management critical to the success of the program. One of the fundamental problems facing these programs is determining the proper number of bicycles to maintain in the fleet. Although fleet size depends on demand, due to the uneven distribution of demand, fleet size will need to be larger than the demand. Without a careful analysis of fleet size and system operation, the cost of operating such a system will be higher and the demand may not be met, resulting in customer complaints and decreased viability of the system.

This paper attempts to demonstrate that mathematical programming can be applied to effectively plan for bike distribution for a bike-sharing system for circulation inside small communities, or to extend public transit service areas to serve access or egress trips. The goal of this paper is to show that the design of a bike-sharing system can be optimized by determining the minimum required bike fleet size that minimizes simultaneously unmet demand, unutilized bikes, and the need to transport empty bikes between rental stations to meet demand.

This paper is organized as follows. Section 2 provides a brief literature review summarizing the current state of the art and highlighting deficiencies to be addressed by the proposed work. The formulation of a mathematical model for the problems is detailed in Section 3. This includes a thorough discussion of the design variables, objective functions, and constraints. Section 4 demonstrates the proposed mathematical model through an example problem. Finally, Section 5 summarizes the main contributions of this study, discusses the broader implications of the work, and identifies potential areas for future work.

## 2. Literature review

Fleet planning models have been applied in a number of areas in transportation systems and industries. It is related to overall service design [8], trucking (e.g., [9–11]), railway industry service [12–15], and airline express package service [16]. Fleet sizing is also important in material handling systems used for manufacturing operations (e.g., [17–19]).

An extensive and rapidly growing literature suggests the need to facilitate bicycling through appropriate infrastructure (such as bike paths and bike parking), traffic calming, training and education programs, and other supportive measures. Countries and cities with high levels of bicycling and good safety records tend to have extensive infrastructure, as well as proactive bicycle policies and programs, whereas those with low bicycle use rates and poor safety records generally have done much less [20–23].

The papers by a number of authors have investigated preferences of cyclists and the bicycling environment as well as the relationship between the supply and use of facilities. Availability of cycling facilities and the type and quality of a cycling facility are important determinants of how well they are used. Studies by Dill and Carr [24] and Nelson and Allen [25] have shown that there is a positive correlation between the number of facilities that are provided and the percentage of people that use bicycling for commuting purposes. Bovy and Bradley [26] work found that travel time was the most important factor in route choice followed by surface type. Another study by Hopkinson and Wardman [27] investigated the demand for cycling facilities using stated preference in a route choice context.

Currently, there are no optimization models that combine the following capabilities: (1) incorporation of multi-periodic optimization formulation for planning and analysis of bike-sharing systems, (2) consideration of both bike planning and operation, and (3) meet all demands at the end of planning time period.

## 3. The mathematical model

This section details a multi-periodic optimization formulation for bike planning problems. In this system, bike rental facilities are distributed throughout a region in multiple locations within communities. Customers arrive at rental stations, utilize the bicycle for some amount of time, and then return the bike to the same or a different bike station. The system charges a fee for the use of the bicycles based on the duration of time of usage. The system monitors the bicycle rental operation by tracking the number of bicycles available at each facility and the number of bicycles rented out. Due to uneven demand distribution, there may be bicycles at certain locations that are not in use, where there is a shortage at other locations. To minimize the fleet size and maximize the use, the proposed model will determine how to distribute bicycles by transporting them between different facilities over a specified period of time.

### 3.1. Design variables

The design variables for this problem are tied directly to bikes and to bike stations among which the bikes must be distributed. The set of bike stations is denoted by $N$ and is divided into two subsets $N_1$ and $N_2$. $N_1$ represents the number of origin points, and $N_2$ represents the number of destination points. Because a station can serve as both origin and destination, typically there is $N_1 = N_2$, but the model can propose a plan for different $N_1$ and $N_2$. The planning horizon is divided into discrete time steps, and $t$ denotes any given time period. Thus, $t = 1, 2, \ldots, T$, where $T$ is the total number of periods. The demand information obtained based on actual usage and from an Internet-based reservation system. These demands are represented as $db_{ij}(t)$. The demands induce bike movements, which are represented by the first set of design variables, $LB_{ij}(t)$. $LB_{ij}(t)$ is the number of bikes that travel from origin $i$ to destination $j$ during period $t$. Because the demand for bikes at any point $i$ may exceed the bikes available, movement of empty bikes from locations that have a surplus supply to those where there is a shortage of bicycles will be necessary to meet all of the demand. The movement of empty bikes represents additional design variables that are denoted by $EB_{ij}(t)$. $EB_{ij}(t)$ is the number of empty or unutilized bikes that are dispatched from destination $j$ to origin $i$ during period $t$.

The design variables are all integers and are summarized as follows:

$LB_{ij}(t)$ number of rented bikes dispatched from $i \in N_1$ to $j \in N_2$ during period $t \in T$.
$EB_{ij}(t)$: number of unutilized bikes dispatched from $i \in N_1$ to $j \in N_2$ during period $t \in T$.
$OB_{ij}(t)$: number of bikes present at origin $i \in N_1$ at the beginning of period $t \in T$.
$DB_{ij}(t)$: number of bikes present at destination $j \in N_2$ at the beginning of period $t \in T$.
$UB_{ij}(t)$: unmet demand (in terms of number of bikes) from $i \in N_1$ to $j \in N_2$ at the end of period $t \in T$.

Fig. 1 illustrates the concepts of origins and destinations and the variables $LB_{ij}(t)$ and $EB_{ij}(t)$ for a two station system. Note that because a station can serve both as origin and destination, $EB_{ii}(t)$ occurs within the same station and is not associated with any transport cost.

Given these design variables, the total number of bikes or fleet size, $BS$, can be defined as the sum of $OB_i(1)$ for all origins in the first time period and $DB_i(1)$ for all destinations in the end of time period. Therefore, $BS$ is defined as follows:

$$BS = \sum_i OB_i(1). \tag{1}$$

Because transporting empty bikes can be costly and wasteful, it is helpful to quantify how many empty bikes may need to be transported throughout the entire planning period. The total number of empty bikes moved during all time periods is calculated as follows:

$$EB = \sum_i \sum_j \sum_t EB_{ij}(t). \tag{2}$$

### 3.2. Input parameters

$db_{ij}(t)$: demand for transportation service between $i \in N_1$ and $j \in N_2$ in period $t \in T$
$r_{ij}$: revenue per utilized bike sent from $i \in N_1$ to $j \in N_2$.
$\tau_{ij}$: rental operating cost per utilized bikes $i \in N_1$ to $j \in N_2$. This is associated with processing the rental request and is on a per request basis.
$\pi_{ji}$: cost of transporting a empty bike from $j \in N_2$ to $i \in N_1 (\forall i \neq j)$.
$p$: the purchasing cost per bike per day.
$h_i$: cost of holding a bike for one period at station $i \in N_1$.
$\theta_{ij}$: penalty cost per period for one unit of unmet demand (one bike trip) from $i \in N_1$ to $j \in N_2$.
$\lambda_{ij}(\eta, t)$: proportion of all utilized bikes that are dispatched in period $\eta \in T$ from $i \in N_1$ and arrive in period $t \in T$ $(\eta \leqslant t)$ at destination $j \in N_2$. There is

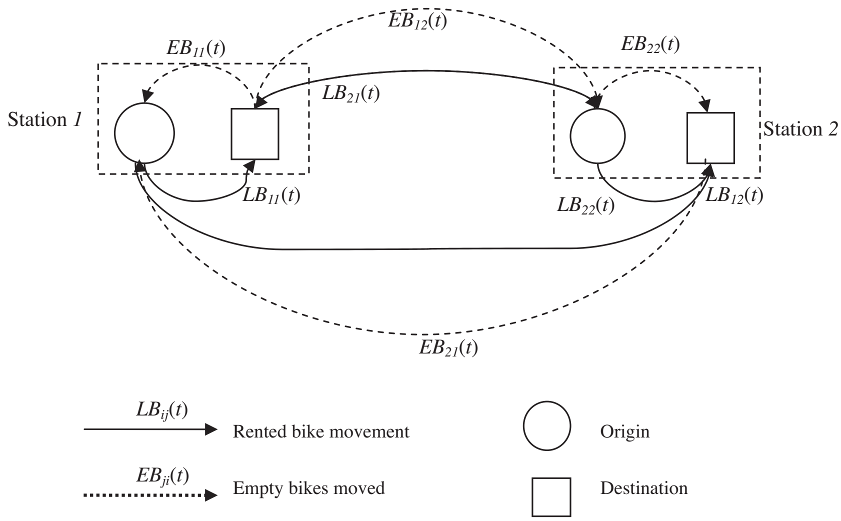


**Fig. 1.** Illustration of a bike-sharing system and movements of rented and empty bikes.

$$\sum_{\eta<t} \lambda_{ij}(\eta, t) = 1 \quad \forall\ i, j, t. \tag{3}$$

$\mu_{ji}(\eta, t)$: proportion of empty bikes that are transported from $i \in N_1$ to $j \in N_2$ in period $\eta \in T$, which arrive in period $t \in T$ There is:

$$\sum_{\eta<t} \mu_{ji}(\eta, t) = 1 \quad \forall\ i, j, t. \tag{4}$$

Please note that travel time is $t - \eta$.

### 3.3. Optimization formulation

The mathematical model, expressed in Eq. (5), maximizes the total benefit to company, $\varphi$, which consists of six terms: (a) the revenue from rented bikes traveling between origin $i$ to a destination $j$, (b) the cost of moving empty bikes from destination $j$ to origin $i$, (c) the operating cost (cost of processing and maintenance) of renting bikes, (d) the cost of holding a bike at station $i$, (e) the capital cost per period of a bike, and (f) the penalty cost of unmet demands:

$$\begin{aligned} Max\varphi = {} & \sum_i \sum_j \sum_t r_{ij} LB_{ij}(t) - \sum_i \sum_{j \neq i} \sum_t \pi_{ji} EB_{ji}(t) - \sum_i \sum_j \sum_t \tau_{ij} LB_{ij}(t) - \sum_i \sum_t hOB_i(t) - \sum_i pOB_i(1) \\ & - \sum_i \sum_j \sum_t \theta_{ij} UB_{ij}(t), \end{aligned} \tag{5}$$

$$UB_{ij}(t) = UB_{ij}(t-1) + db_{ij}(t) - LB_{ij}(t) \quad \forall\ i, j, t, \tag{6}$$

$$OB_i(t) = OB_i(t-1) + \sum_j \sum_{\eta<t} \mu_{ji}(\eta, t) EB_{ji}(\eta) - \sum_j LB_{ij}(t-1) \quad \forall\ i, t, \tag{7}$$

$$DB_j(t) = DB_j(t-1) + \sum_i \sum_{\eta<t} \lambda_{ij}(\eta, t) LB_{ij}(\eta) - \sum_i EB_{ji}(t-1) \quad \forall\ j, t, \tag{8}$$

$$LB_{ij}(t) \leqslant db_{ij}(t) \quad \forall\ i, j, t, \tag{9}$$

$$\sum_j LB_{ij}(t) \leqslant OB_i(t) \quad \forall\ i, t, \tag{10}$$

$$\sum_i EB_{ji}(t) \leqslant DB_j(t) \quad \forall\ j, t, \tag{11}$$

$$LB_{ij}(t) \geqslant 0, \quad EB_{ji}(t) \geqslant 0, \quad UB_{ij}(t) \geqslant 0, \quad OB_i(t) \geqslant 0, \quad DB_j(t) \geqslant 0 \quad \forall\ i, j, t. \tag{12}$$

Constraints (6) are the unmet demand in period $t$, and it is equal to the unmet demand from the previous period plus the new demand minus utilized movements. Constraints (7) and (8) represent the conservation of flow for bikes at each location in each time period, and include the effects of travel times for bike movements through the $\alpha$ and $\beta$ terms, representing the arrival times of bike at their destinations. Constraints (9) represent the balance between demands and utilize bikes. The constraints in (10) and (11) are balancing constraints for bikes at each location, at the beginning of each period. The limits in (12) ensure that the design variables are nonnegative. The complete feasible space defined by (6)–(12) is denoted as **X**. The total number of limits is equal to the number of design variables: $T(3N_1N_2 + N_1 + N_2)$. The total number of constraints is equal to $2T(N_1 + N_2)$. Note that the input parameters, $N_1$, $N_2$, and $T$ govern the size of the problem with respect to both the design variables and the constraints.

## 4. Numerical example

### 4.1. Problem description

The proposed model was designed and run according to the data of Tehran transportation system. In Tehran city, there are bike houses in most of the main centers. The demand information obtained based on actual usage and from an Internet-based reservation system. For testing the model two main squares in the center of Tehran city were chosen. This example includes two origins and two destinations ($N_1 = 2$ and $N_2 = 2$).The time horizon for this example is 6 h. The revenue for one unit of bike is \$1.5 per time period. The penalty for one unit of unmet demand is \$0.01 per time period (see Table 2). Table 1 shows the demands for the transportation service between origins and destinations as well as the proportions of bikes arriving at various destinations. The bike holding costs $h_i$ and $s_j$ (at all origins and all destinations) are \$0.01 per time

**Table 1**
Demand scenarios for the example.

| Origin | Destination | Periods | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 175 | 176 | 161 | 155 | 170 | 0 |
| 1 | 2 | 167 | 180 | 160 | 158 | 165 | 0 |
| 2 | 1 | 153 | 169 | 177 | 170 | 172 | 0 |
| 2 | 2 | 195 | 151 | 161 | 150 | 163 | 0 |

**Table 2**
Penalty costs per planning period for unmet demand for the example.

| Origin | Destination | $\theta_{ij}$ ($) |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 2 | 0.01 |
| 2 | 1 | 0.01 |
| 2 | 2 | 0 |

period. The cost $p$ to lease a bike is $0.05 per time period. This problem is solved using LINDO 8, which utilizes the branch and bound approach to integer problems.

## 4.2. Results

### 4.2.1. Experimentation with the solution procedure

The model gives the movements bikes in the network, customers demand satisfied, and the system costs, including bike holding costs and transportation cost. Moreover, the values of the model provide network information such as the number of bikes present at origins and destinations at the end of each period, unmet demands, and number of utilized movements and empty bikes at any given time and location. The number of bikes at origin $i(i = 1,2)$ at the initial period $(OB_1(1) = 724, OB_2(1) = 648)$ is determined, therefore, a fleet consisting of 1372(724 + 648) bikes is required for the proper functioning of the described system. The system configuration and the movement of rented and empty bikes by time periods are shown in Fig. 2, where the solid lines represent the bike travel between two stations during one planning time period, and dashed lines represent the movement of empty bikes between stations to meet the demand that exceed the number of bikes available at a station. Note that Fig. 2 reflects the states of the design variables during different planning period, and empty bike movement between destination $i$ to origin $i$ $(i = 1,2)$ does not represent actual movement and there is no cost associated with the movement.

The results from the model are given in Table 3. In the table, the first column identifies each planning period. For each planning period, the second column gives the number of trips take place between two stations, the third column empty bikes

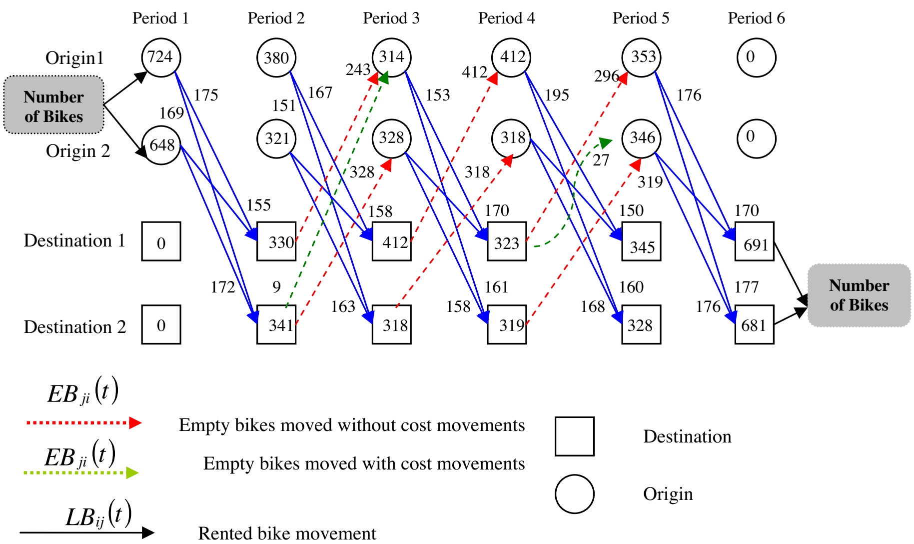


**Fig. 2.** Control actions.

**Table 3**
Optimal values for the design variables in the illustrative example.

| Periods | LB($i$,$j$) | EB($j$,$i$) | UB($i$,$j$) | OB($i$) | DB($j$) |
|---|---|---|---|---|---|
| 1 | LB(1,1) = 175 | EB(1,1) = 0 | UB(1,1) = 0 | OB(1) = 724 | DB(1) = 0 |
| | LB(1,2) = 169 | EB(1,2) = 0 | UB(1,2) = 0 | | |
| | LB(2,1) = 155 | EB(2,1) = 0 | UB(2,1) = 0 | OB(2) = 648 | DB(2) = 0 |
| | LB(2,2) = 172 | EB(2,2) = 0 | UB(2,2) = 0 | | |
| Total | 671 | 0 | 0 | 1372 | 0 |
| 2 | LB(1,1) = 167 | EB(1,1) = 243 | UB(1,1) = 0 | OB(1) = 380 | DB (1) = 330 |
| | LB(1,2) = 151 | EB(1,2) = 0 | UB(1,2) = 0 | | |
| | LB(2,1) = 158 | EB(2,1) = 9 | UB(2,1) = 0 | OB(2) = 321 | DB(2) = 341 |
| | LB(2,2) = 163 | EB(2,2) = 328 | UB(2,2) = 0 | | |
| Total | 639 | 580 | 0 | 701 | 671 |
| 3 | LB(1,1) = 153 | EB(1,1) = 412 | UB(1,1) = 0 | OB(1) = 314 | DB(1) = 412 |
| | LB(1,2) = 161 | EB(1,2) = 0 | UB(1,2) = 0 | | |
| | LB(2,1) = 170 | EB(2,1) = 0 | UB(2,1) = 0 | OB(2) = 328 | DB(2) = 318 |
| | LB(2,2) = 158 | EB(2,2) = 318 | UB(2,2) = 0 | | |
| Total | 642 | 730 | 0 | 642 | 730 |
| 4 | LB(1,1) = 195 | EB(1,1) = 296 | UB(1,1) = 0 | OB(1) = 412 | DB(1) = 323 |
| | LB(1,2) = 160 | EB(1,2) = 27 | UB(1,2) = 0 | | |
| | LB(2,1) = 150 | EB(2,1) = 0 | UB(2,1) = 0 | OB(2) = 318 | DB(2) = 319 |
| | LB(2,2) = 168 | EB(2,2) = 319 | UB(2,2) = 0 | | |
| Total | 673 | 642 | 0 | 730 | 642 |
| 5 | LB(1,1) = 176 | EB(1,1) = 0 | UB(1,1) = 0 | OB(1) = 353 | DB(1) = 345 |
| | LB(1,2) = 177 | EB(1,2) = 0 | UB(1,2) = 0 | | |
| | LB(2,1) = 175 | EB(2,1) = 0 | UB(2,1) = 0 | OB(2) = 346 | DB(2) = 328 |
| | LB(2,2) = 176 | EB(2,2) = 0 | UB(2,2) = 0 | | |
| Total | 704 | 0 | 0 | 699 | 673 |
| 6 | LB(1,1) = 0 | EB(1,1) = 0 | UB(1,1) = 0 | OB(1) = 0 | DB(1) = 691 |
| | LB(1,2) = 0 | EB(1,2) = 0 | UB(1,2) = 0 | | |
| | LB(2,1) = 0 | EB(2,1) = 0 | UB(2,1) = 0 | OB(2) = 0 | DB(2) = 681 |
| | LB(2,2) = 0 | EB(2,2) = 0 | UB(2,2) = 0 | | |
| Total | 0 | 0 | 0 | 0 | 1372 |

**Table 4**
Example problems – larger scale problems.

| Problem | $N_1$ | $N_2$ | $T$ | $BS$ | $db$ | $BS/db$ | $\varphi$ |
|---|---|---|---|---|---|---|---|
| 1 | 6 | 6 | 15 | 1637 | 7995 | 0.20 | 11,860 |
| 2 | 8 | 8 | 25 | 4563 | 22,146 | 0.20 | 16,532 |
| 3 | 4 | 4 | 30 | 1598 | 6985 | 0.22 | 20,123 |
| 4 | 5 | 5 | 50 | 1480 | 16,843 | 0.08 | 23,135 |
| 5 | 30 | 30 | 15 | 106,425 | 195,142 | 0.54 | 30,565 |
| 6 | 35 | 35 | 7 | 77,899 | 120,259 | 0.64 | 38,868 |
| 7 | 40 | 40 | 15 | 198,358 | 340,105 | 0.58 | 41,854 |
| 8 | 50 | 50 | 7 | 173,986 | 258,193 | 0.67 | 46,965 |

moved from one station to another, the fourth column the unmet demand, and the last two columns the number of bikes present at the beginning of the planning period. The unmet demands are zero at the end of the planning time period, but the system cannot response to all demands during each time period. For example, at the end of the first planning period, there is a shortage of 324 bikes at stations 1 and 2 (see Table 3, column 4). This demand is met in the next planning period, and there is no more unmet demand in the remaining planning periods. Only nine empty bikes have to be transported. The model is able to minimize both unmet demand and the need of moving empty bikes between stations.

Table 4 shows the results with $N_1$, $N_2$, and $T$ increased substantially. $BS$ is the total number of bikes calculated in (1). $db$ represents the total demand over all segments and all time periods. Although demand is a fixed parameter for the optimization problem, changing $N_1$, $N_2$, and/or $T$ alters the problem and thus the average demand, which depends on the number of time steps, as well as the number of origins and destinations. This study demonstrates the applicability of the proposed approach to problems of varying sizes and time discretization schemes. Whereas the initial example problem discussed in this paper involves 231 design variables, problem 7 in Table 4 involves 73,200 design variables.

**Table 5**
Example problems – effects of changes in *T*.

| Problem | $N_1$ | $N_2$ | $T$ | $BS$ | $db$ | $BS/db$ | $EBS/db$ |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 6 | 4352 | 6400 | 0.68 | 0.32 |
| 2 | 3 | 3 | 8 | 4454 | 7680 | 0.57 | 0.43 |
| 3 | 3 | 3 | 10 | 4700 | 9216 | 0.51 | 0.49 |
| 4 | 3 | 3 | 12 | 5308 | 11,060 | 0.47 | 0.52 |
| 5 | 3 | 3 | 14 | 6100 | 12,451 | 0.48 | 0.51 |
| 6 | 3 | 3 | 16 | 7412 | 14,825 | 0.50 | 0.50 |
| 7 | 3 | 3 | 18 | 7831 | 15,356 | 0.51 | 0.49 |
| 8 | 3 | 3 | 20 | 9245 | 18,128 | 0.51 | 0.49 |

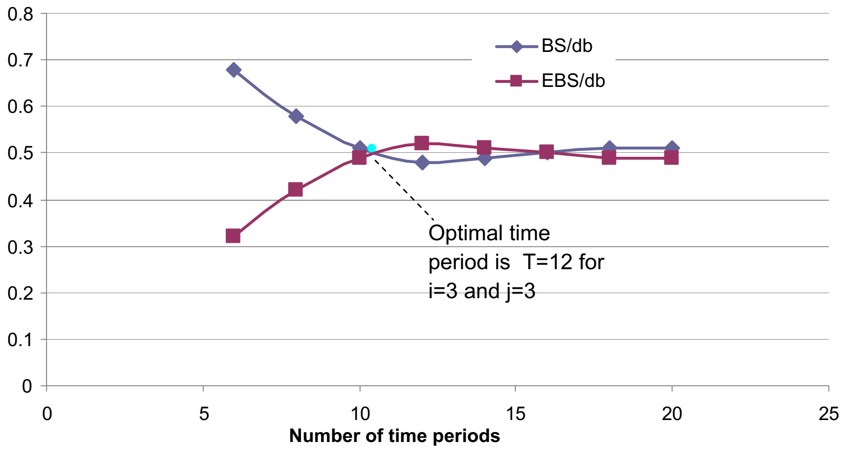


**Fig. 3.** Effect of changing *T*.

Table 5 shows the effects of changing the number of time steps and number of stations, a series of additional problems is solved for a system with three stations. The value of *T* is varied between 6 and 20, while the values of other parameters are generated randomly remain unchanged during the optimization. Fig. 3 shows that optimal time periods can be found for any *i* and *j* by sensitivity analysis. In the figure, one curve, labeled as *BS/db*, represents the number of bikes per demand trip needed for different number of planning periods. The other curve, labeled *EBS/db*, depicts the change in the number of empty bikes that need to be moved. The best system should minimize both *BS/db* and *EBS/db*. In this example, the optimal number of planning time periods is found to be 12. This solution is able to respond to all demands with the smallest bike fleet size and minimum movements of empty bikes to meet the unmet demand.

## 5. Conclusions

This paper proposed an optimization formulation to design a bike-sharing system for short-distance travel within small communities or as an extension of a public transit system to help transit users to reach their final destination in a shorter time. This new approach provides an optimization formulation for bike-sharing system planning that utilizes vehicle network information such as bike station capacity, unmet demand, and the number of utilized bikes at any given time and location. The model is able to minimize unmet demand and the cost of operating such a system by minimizing the required fleet size and the cost for transporting empty bikes between stations to respond to demand. Consequently, the model provides a tool for helping managers in the planning of a bike-sharing system. In this model, it is assumed that demand by planning periods is known. In practice, the demand information may be obtained based on actual usage and from an Internet-based reservation system. The demand information will change as the system matures, and there will be some fluctuations in demand, which will require the fleet size to be larger than the minimum estimated by the model. The proposed approach is applied to an example problem and is shown to be successful, ultimately providing a simulation tool by system dynamic for planning and analyzing bicycle use effectively. Future research will involve analyzing the sensitivity of the model to other input parameters, such as rental costs and penalties, creating a multi-objective optimization model that minimizes costs and maximizes service quality for bike planning. The model can be extended to consider a combination of transportation modes.

## References


[1] K. Martens, The bicycle as a feedering mode: experiences from three European countries, Transport. Res. D. Trans. Environ. 9 (4) (2004) 281–294.
[2] Department of Environment, Transport and the Regions, 1999. Bikerail: combined journeys by cycle and rail, DETR.∗∗∗
[3] Bundesministerium fuer Verkehr, Bau-und Wohnungswesen, 2002. Fahrrad! Nationaler Radverkehrsplan 2002–2012: Massnahmen zur Foerderung des Radverkehrs in Deutschland. Bericht der Bundesregierung, Berlin.
[4] J.T. Doolittle, E.K. Porter, Integration of Bicycles and Transit, National Academy Press, Washington, DC, 1994.
[5] C.A. Hagelin, A Return on Investment Analysis of Bikes-on-Bus Programs, State of Florida Department of Transportation/National Center for Transit Research, Tallahassee, FL/Tampa, FL, 2005.
[6] M.J.N. Keijer, P. Rietveld, How do people get to the railway station? The Dutch experience, Transport. Plan. Technol. 23 (2000) 215–235.
[7] Velib. <http://www.velib.paris.fr>.
[8] T.G. Crainic, Service network design in freight transportation, Eur. J. Oper. Res. 122 (2) (2000) 272–288.
[9] R.W. Hall, M. Racer, Transportation with common carrier and private fleets: system assignment and shipment frequency optimization, IIE Trans. 27 (1995) 217–225.
[10] Y. Du, R. Hall, Fleet sizing and empty equipment redistribution for center-terminal transportation networks, Manag. Sci. 43 (2) (1997) 145–157.
[11] L. Ozdamar, T. Yazgac, Hierarchical planning approach for a production–distribution system, Int. J. Prod. Res. 37 (16) (1999) 3759–3772.
[12] H.D. Sherali, C.H. Tuncbilek, Static and dynamic time–space strategic models and algorithms for multilevel rail–car fleet management, Manage. Sci. 43 (2) (1997) 235–250.
[13] H.R. Sayarshad, R. Tavakoli-Moghadam, Solving a multi periodic stochastic model of the rail–car fleet sizing by two-stage optimization formulation, Appl. Math. Model. 34 (2) (2010) 1164–1174.
[14] H.R. Sayarshad, N. Javadian, R. Tavakoli-Moghadam, N. Forghani, Solving multi-objective optimization formulation for fleet planning in a railway industry, Ann. Oper. Res. (2010), doi:10.1007/s10479-010-0714-1.
[15] H.R. Sayarshad, K. Ghoseiri, A simulated annealing approach for the multi-periodic rail–car fleet sizing problem, Comput. Oper. Res. 36 (2009) (2010) 1789–1799.
[16] C. Barnhart, R.R. Schneur, Air network design for express shipment service, Oper. Res. 44 (6) (1996) 852–863.
[17] B.M. Beamon, A.N. Deshpande, Mathematical programming approach to simultaneous unit-load and fleet-size optimization in material handling systems design, Int. J. Adv. Manuf. Technol. 14 (11) (1998) 858–863.
[18] B.M. Beamon, V.C.P. Chen, Performability-based fleet sizing in a material handling system, Int. J. Adv. Manuf. Technol. 14 (6) (1998) 441–449.
[19] H.R. Sayarshad, Using bees algorithm for material handling equipment planning in manufacturing systems, Int. J. Adv. Manuf. Technol. (2010), doi:10.1007/s00170-009-2363-6.
[20] J. Pucher, L. Dijkstra, Promoting safe walking and cycling to improve public health: lessons from the Netherlands and Germany, Am. J. Public Health 93 (9) (2003) 1509–1516.
[21] Fietsberaad, Continuous and integral: the cycling policies of Groningen and other European cycling cities. Fietsberaad, Amsterdam. Accessible at: <http://www.fietsberaad.nl/library/>, 2006
[22] J. Pucher, R. Buehler, Making cycling irresistible: lessons from the Netherlands, Denmark, and Germany, Transport Rev. 28 (4) (2008) 495–528.
[23] J. Pucher, R. Buehler, Integration of bicycling with public transport, J. Public Transport 12 (3) (2009) 79–104.
[24] J. Dill, T. Carr, Bicycle commuting and facilities in major US cities: if you build them, commuters will use them, Transportation Research Board, National Research Council, Washington, DC, 2003.
[25] A.C. Nelson, D. Allen, If you build them, commuters will use them. Transportation Research Record: Journal of the Transportation Research Board, No. 1578, TRB, National Research Council, Washington, DC, 1997, pp. 79–83.
[26] P. Bovy, P. Bradley, Route choice analyzed with stated preference approaches. Transportation Research Record 1037, TRB, National Research Council, Washington, DC, 1985.
[27] P. Hopkinson, M. Wardman, Evaluating the demand for new cycle facilities, Transport Policy 2 (4) (1996) 241–249.