Analysis, Modeling and Control of a 2-DOF Helicopter

Project Report

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Contents

1 Introduction

This report presents a comprehensive summary of the work conducted during the Non-Linear Control Systems course at the Center of Scientific Innovation and Education. The project focuses on the analysis, modeling, and control of a 2-DOF (Degrees of Freedom) helicopter system, as depicted in Figure 1.

The primary objectives of this project are:

- To examine the physical principles and develop implementations of the mathematical models that describe the dynamics of the 2-DOF helicopter.
- To design and implement a control system for the model.
- To evaluate the controller's performance under various operating conditions, identifying operating ranges, robustness and areas for improvment.

The insights gained from the initial modeling and control design will serve as a foundation for further exploration of advanced control strategies. In particular, the project aims to investigate the implementation and effectiveness of Model Reference Adaptive Control (MRAC) to overcome the limitations observed with the currently used Linear Quadratic Regulator (LQR) controller, thereby enhancing the system's robustness and adaptability.

The Simulink code and further resources can be found in [2].

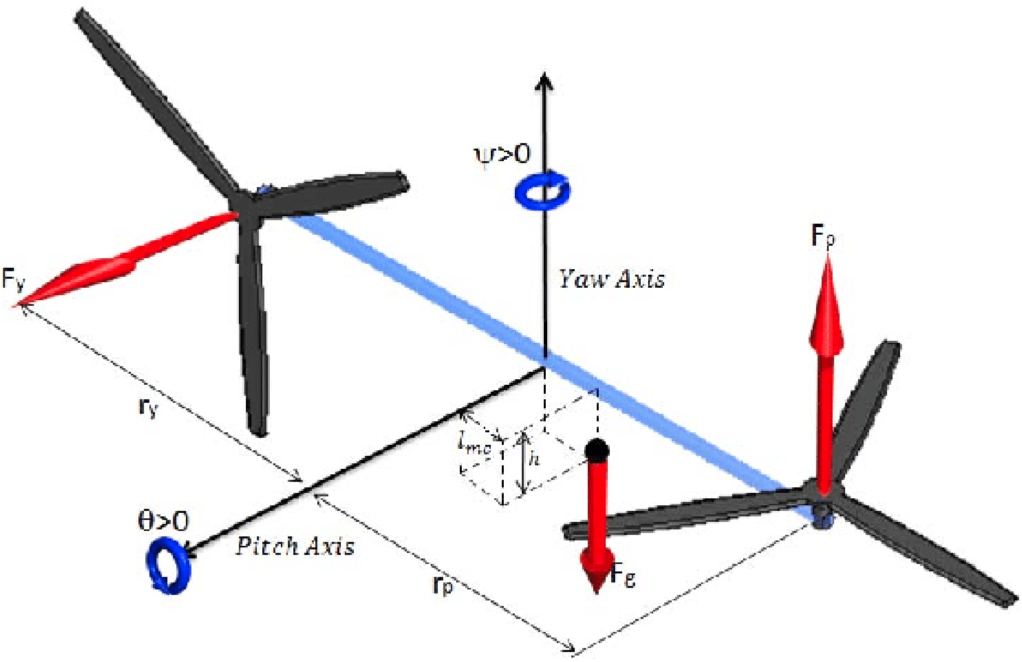

Figure 1: 2-DOF Helicopter Model

2 Modeling Dynamics

The 2-DOF helicopter system serves as a simplified example of a highly non-linear Multiple Input Multiple Output (MIMO) system. The model used in this study is based on the Quanser helicopter, which operates with two degrees of freedom: the pitch angle (θ) and the yaw angle (ψ). These angles are controlled by two propellers, each driven by a DC motor. [5]

The control inputs for the system are the voltages applied to the pitch and yaw motors, ranging from $\pm 24V$ for the pitch motor and $\pm 15V$ for the yaw motor. A positive pitch angle ($\theta > 0$) occurs when the nose of the helicopter moves upwards, while a positive yaw angle ($\psi > 0$) is observed when the helicopter rotates clockwise.

The primary forces acting on the system are the thrust forces F_p and F_y, generated by the pitch and yaw propellers, and the gravitational force F_g, which acts downward on the helicopter's nose. The center of mass is located at a distance l from the pitch axis along the length of the helicopter body.

Key parameters of the system are summarized in Table 1 [5].

Table 1: 2 DOF Helicopter System Specifications

Symbol	Description	Value	Unit
K_{pp}	Thrust force constant of yaw motor/propeller	0.204	$\mathrm{N \cdot m/V}$
K_{yy}	Thrust torque constant of yaw axis from yaw motor/propeller	0.072	$\mathrm{N \cdot m/V}$
K_{py}	Thrust torque constant acting on pitch axis from yaw motor/propeller	0.0068	$\mathrm{N.m/V}$
K_{yp}	Thrust torque constant acting on yaw axis from pitch motor/propeller	0.0219	$\mathrm{N.m/V}$
B_p	Equivalent viscous damping about pitch axis	0.800	$\mathrm{N/N}$
B_y	Equivalent viscous damping about yaw axis	0.318	$\mathrm{N/N}$
m	Total moving mass of the helicopter (body, two propeller assemblies, etc.)	1.3872	kg
l	Center of mass length along helicopter body from pitch axis	0.186	m
$J_{eq,p}$	Total moment of inertia about pitch axis	0.0384	$\mathrm{kg \cdot m^2}$
$J_{eq,y}$	Total moment of inertia about yaw axis.	0.0432	$\mathrm{kg \cdot m^2}$

The dynamical equations describing the 2-DOF helicopter system are derived using the Euler-Lagrange equations, following the detailed methodology presented in [1]. The two second-order differential equations characterize the rotational dynamics about the pitch (θ) and yaw (ψ) axes:

$$\ddot{\theta} = \frac{1}{J_{\text{eq},p} + ml^2} \left[(K_{pp}V_{mp} + K_{py}V_{my}) - \left(B_p\dot{\theta} + mgl\cos\theta + ml^2 \sin\theta\cos\theta\dot{\psi}^2 \right) \right] \tag{1}$$

$$\ddot{\psi} = \frac{1}{J_{\text{eq},y} + ml^2\cos^2\theta} \left[(K_{yp}V_{mp} + K_{yy}V_{my}) - B_y\dot{\psi} + 2ml^2 \sin\theta\cos\theta\dot{\psi}\dot{\theta} \right] \tag{2}$$

These equations can be rewritten in state-space form, where the state of the system is represented by vector:

$$\mathbf{x} = \begin{bmatrix} \theta & \psi & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \tag{3}$$

By defining each state component, we can derive the state-space model of the system as follows:

$$
\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{1}{J_{\mathsf{eq},p} + ml^2} \left[(K_{pp}u_1 + K_{py}u_2) - \left(B_p x_3 + ml^2 x_4^2 \sin x_1 \cos x_1 + mgl \cos x_1 \right) \right] \\
\dot{x}_4 &= \frac{(K_{yp}u_1 + K_{yy}u_2) - B_y x_4 + 2ml^2 x_3 x_4 \sin x_1 \cos x_1}{J_{\mathsf{eq},y} + ml^2 \cos^2 x_1}
\end{aligned}
$$

where $u_1 = V_{mp}$ and $u_2 = V_{my}$ are the control voltages.

The Simulink implementation of the model can be found in [2].

3 Control System

The objective of the control system was to bring and maintain the helicopter at a zero pitch and zero yaw angle. The initial conditions for evaluating the control system were defined as follows:

- **Initial Pitch Angle (x_1)**: Between $-10°$ and $10°$.
- **Initial Yaw Angle (x_2)**: Between $-10°$ and $10°$.
- **Initial Pitch Rate (x_3)**: Between $-15\,\text{deg/s}$ and $15\,\text{deg/s}$.
- **Initial Yaw Rate (x_4)**: Between $-15\,\text{rad/s}$ and $15\,\text{rad/s}$.

For evaluating the performance of the controller, the following time-domain specifications were devised:

1. **Settling Time (T_s)**: The system should settle to within $\pm 0.2°$ of the 0 pitch and 0 yaw within 4 seconds.
2. **Fall Time (T_r)**: The fall time, defined as the time to go from 90% to 10% of the initial value, should be less than 2 seconds.
3. **Overshoot/undershoot (M_p)**: The maximum overshoot/undershoot should be less than $\pm 1°$ for both pitch and yaw angles.
4. **Steady-State Error**: The steady-state error for both pitch and yaw should be zero or within $\pm 0.1°$.
5. **Control Effort**: Control inputs should be minimized to reduce wear on actuators and maintain efficiency.
6. **Disturbance Rejection**: The system should return to within the 1% tolerance band of zero pitch and yaw within 2 seconds after encountering a disturbance.

Given this requirements and the complex MIMO dynamics of the system, the **Linear Quadratic Regulator (LQR)** was selected as a starting control strategy. [6]

LQR was chosen because it provides a systematic way to manage the interconnections between multiple inputs and multiple outputs and it gives an optimal solution to a cost function that incorporates both the error factor and the control effort factor.

However, it is important to note that LQR requires the linearization of the system, which means that certain intrinsic nonlinear characteristics of the helicopter will be approximated or neglected. This linearization reduces the robustness of the control system and limits its applicability to a range of values within the neighborhood of the linearization point. This range of applicability should be further experimentally estimated and compared with the ranges that we have defined for the initial values.

As the desired state for the model is $\mathbf{x} = [0, 0, 0, 0]^T$, we must linearize and design the control system around this point. However, the zero state vector does not naturally serve as an equilibrium point for the system. Therefore, we need to adjust the control inputs u_1 and u_2 to shift the equilibrium to this zero point.

By setting the right-hand sides of the system's differential equations to zero, we can determine the necessary values for u_{10} and u_{20} as follows:

$$u_{10} = -\frac{K_{yy}}{K_{yp}} u_{20} \tag{4}$$

$$u_{20} = \frac{mgl}{K_{py} - \frac{K_{pp}K_{yy}}{K_{yp}}} \tag{5}$$

Fixing the operating point and linerizing we get the following linear model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{6}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-B_p}{J_{\text{eq},p}+ml^2} & 0 \\ 0 & 0 & 0 & \frac{-B_y}{J_{\text{eq},y}+ml^2} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{\text{eq},p}+ml^2} & \frac{K_{py}}{J_{\text{eq},p}+ml^2} \\ \frac{K_{yp}}{J_{\text{eq},y}+ml^2} & \frac{K_{yy}}{J_{\text{eq},y}+ml^2} \end{bmatrix}$$

Observing the controllability matrix, obtained using the MATLAB `ctrb()` command, shows that it is full-rank which means that the liniearized system is controllable.

Later, appropriate Q and R matrices were defined for the LQR design as described in the next section. The LQR gains were calculated based on these matrices, and the controller was then implemented on the nonlinear system.

4 Simulation

The simulation and performance evaluation were conducted in multiple stages to assess the controller's effectiveness under various conditions:

Stage 1: Baseline Testing Without Disturbances This stage aimed to evaluate the controller's performance under ideal conditions, including scenarios with initial velocities. Various combinations of initial yaw and pitch angles within the specified ranges were used, with the initial velocities for yaw and pitch set to either zero or varied within their respective ranges.

The initial iterations started with $\mathbf{Q}$ and $\mathbf{R}$ matrices set as identity matrices. Adjustments were then made to these matrices through subsequent iterations to tune the controller's response and ensure the model met the desired performance parameters. Table 2 presents the results of the simulations for adjusted $\mathbf{Q}$ and $\mathbf{R}$ matrices with initial velocities set to 0. Steady-state errors and overshoot/undershoots were not included in the table, as they consistently remained close to zero.

$$Q = \begin{bmatrix} 35 & 0 & 0 & 0 \\ 0 & 35 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Table 2: Performance results for experiments without disturbances

Ex	**Initial**(deg)		**Settling Time**(sec)		**Falling Time**(sec)	
	Pitch	**Yaw**	**Pitch**	**Yaw**	**Pitch**	**Yaw**
1	3	-3	2.47	2.75	1.8	1.98
2	6	-6	3.15	3.34	1.93	2.03
3	10	-10	3.65	3.8	1.98	2.07

The experiments with non zero initial velocities were also passed successfully and the controller was able to deal with different values of velocities within the predefined range, even when the direction of the initial velocity was in the opposite direction.

The experiments have shown that increasing the diagonal elements of the $\mathbf{R}$ matrix, i.e., trying to limit the control effort spent, significantly affected the performance specifications of the system. Therefore, these elements were kept at one.

Stage 2: Testing with Disturbances and Parameter Variations In the second stage, disturbances were introduced to the system, including white Gaussian noise with a mean of 0 and a variance of 0.001 added to the control inputs, as well as simulated effects of sudden wind gusts. Additionally, experiments were conducted to assess the impact of mass variations, simulating the effects of fuel consumption or possible payloads. In this stage, the $\mathbf{Q}$ and $\mathbf{R}$ matrices determined in the first stage were kept fixed.

The experiments demonstrated that with the same Q and R matrices, the system could tolerate short, abrupt disturbances with amplitudes up to 1.7 in yaw angular velocity and 0.5 in pitch velocity.

Figures 1 and 2 present the system's performance before and after adding these disturbances.

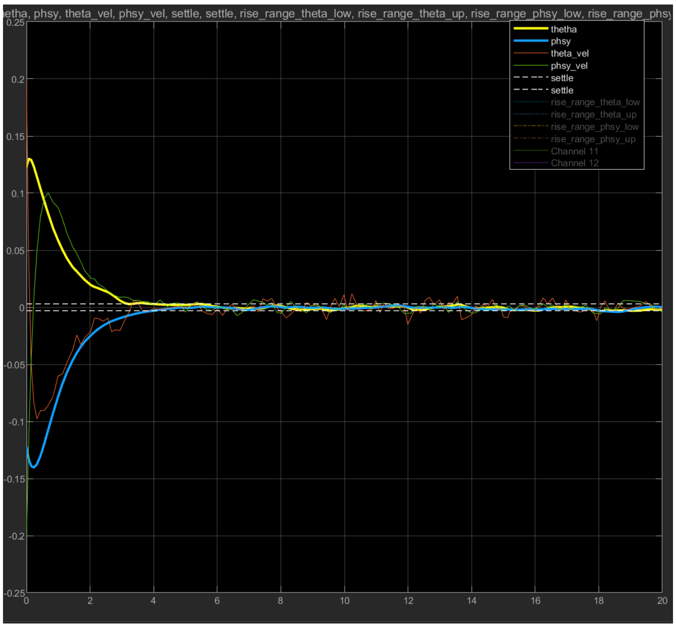

Figure 2: Effect of White noise on the control inputs.

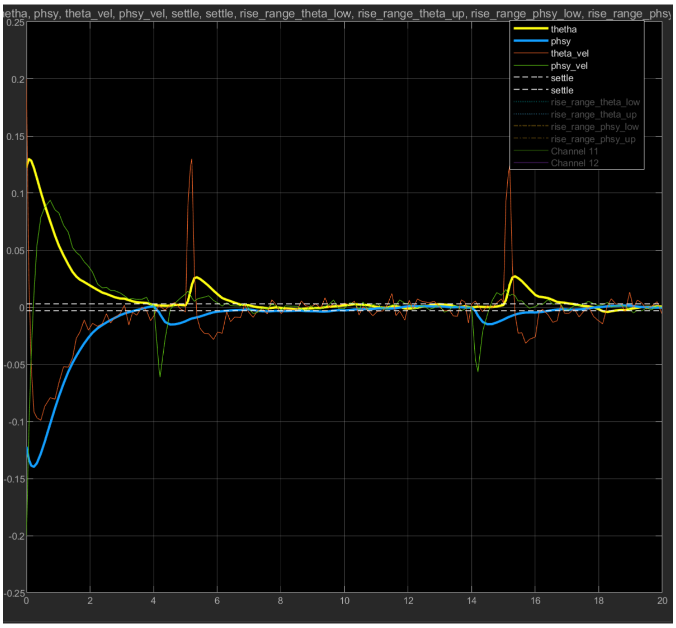

Figure 3: Effects of the short disturbances at the yaw and pitch angles.

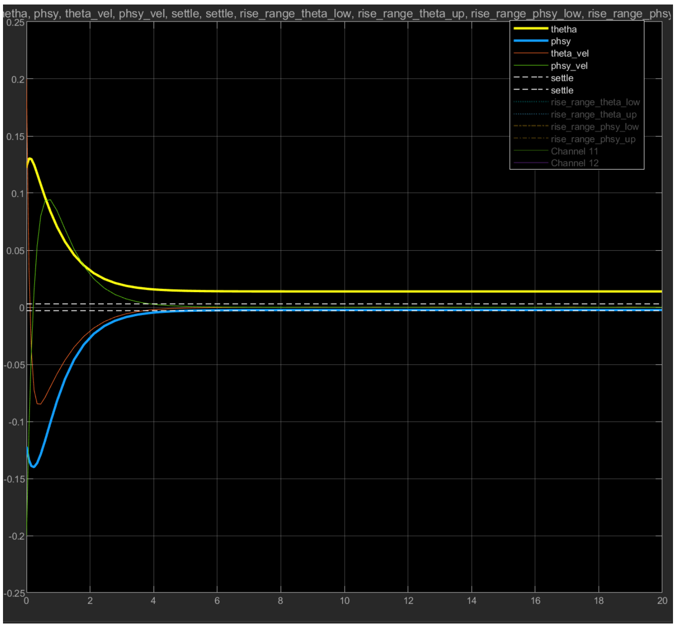

Figure 4: System behavior after 1% drop in the mass.

The next experiment evaluated the performance of the helicopter under various weight changes. However, the controller failed dramatically under these conditions, producing large steady-state errors even with mass changes of just 1% of the total mass [Figure 4]. Note that helicopters in general may encounter more than 20% mass changes relative to their takeoff mass across different operational conditions.

5 Conclusion

The experiments and simulations have demonstrated that the Linear Quadratic Regulator (LQR) can effectively manage the control of a 2-DOF helicopter within a specified operational range under ideal conditions, handling both noise and disturbances while meeting performance specifications. The controller was capable of tolerating moderate wind forces and noise with low variance. For that, it required high penalties for errors in state variables and did not focus on minimizing control effort effectively.

However, the controller struggled with even small parameter changes, such as variations in mass, which are inevitable in real-life scenarios. To address a broader range of operational conditions—including stronger disturbances and higher noise - more advanced control techniques may be necessary.

6 Further Work

To further study the model and enhance the control system, several key areas require more investigation:

- **Development of Metrics:** It is essential to develop and utilize metrics for measuring the control effort expended. This will provide a clearer understanding of the trade-offs between control precision and energy consumption.
- **Updating System Specifications:** The system specifications should be revised to reflect more realistic operating scenarios. This includes gaining a comprehensive understanding of the effects of wind and other disturbances that the helicopter may encounter in real-world environments.
- **Advanced Control Techniques:** The implementation of Model Reference Adaptive Control (MRAC) should be considered, as it offers the potential to dynamically adjust the control strategy in response to changing conditions.

7 Bibliography

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