Orbital Project

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1 Notes

The mission statement is to define an initial radius and initial velocity for the moon, where the earth is fixed at the origin of our 2-D plane. This will allow for the implementation of the Verlet Velocity integration method. From wikipedia, "Verlet integration is a numerical method used to integrate Newton's equations of motion."

Will simulate a static earth where moon orbits elliptically, I will have some equation relating change in radius over change in time. We are given that Kepler's equation relates the mean anomaly $M(t)$, which is the fraction of the orbit's period that has elapsed, to the eccentric anomaly $E(t)$, which in turn can be used to find the true anomaly $\theta(t)$ which defines the moon's position. Eccentricity is a measure of degree in which the orbit differs from a perfect circular orbit. The most important equation so far is the Vis-Viva equation:

$$v(t) = \sqrt{\mu\left(\frac{2}{r(t)} - \frac{1}{a}\right)}$$

I first give an inital radius and velocity of the moon as

$$V_i = 1023.056m/s$$

$$r_i = 3844000m$$

then using vis-viva equation we re-write to solve for semi-major axis, which is half the length of the longest diameter of the ellipse, denoted a

$$a = \left(-\frac{v_i^2}{GM_E} + \frac{2}{r_i}\right)^{-1}$$

$$a = 388059385.1m$$

We now need to solve for eccentricity. Orbital energy is now considered as

$$\epsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$