Part1: 阶梯型定价

1. 计算分区矩阵 Omega

首先,我们以校园的布局为基础,给出P区、M区、F区和不可停放区域G区,用矩阵Ω表示,元素分别为2、1、0、-1。此外,将P区的长、宽以及中心存储在一个数组里,以便得到车辆的分布状况和产生随机数,从而获得事件矩阵。

【主要思路】

- 1. 首先给定一个矩形区域 Ω (即校园),其长、宽均为整数,分别记为width=74、length=110;
- 2. 再由gly的辛苦劳作得到各个P区、M区、不可停放区域的坐标(左上坐标集合、右下坐标数组,根据这两个数组可以获得P区的中心、长、宽);
- 3. 最后进行图像可视化即可;

```
In [1]: import warnings warnings.filterwarnings('ignore')
import numpy as np import matplotlib.pyplot as plt
from matplotlib.ticker import MultipleLocator, FormatStrFormatter #此模块包含用于配置记号定位和格式的类。提供了通用的记号定位器和格式化程序,以及特定于import pandas as pd import math import numpy.linalg as la
```

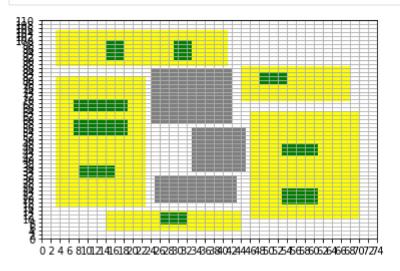
```
In [2]: #首先, 将P、M、G区的左上坐标、右下坐标分别记录在两个数组中,其中numP、numM、numG区分别P_lu = np. array([[14,10],[29,10],[7,40],[7,50],[8,73],[26,97],[48,26],[53,62],[53,85])
P_rd = np. array([[18,20],[33,20],[19,46],[19,58],[16,79],[32,103],[54,32],[61,68],[61,P_lu[:,1] = 110 - P_lu[:,1]
P_rd[:,1] = 110 - P_rd[:,1]
M_lu = np. array([[3,5],[3,28],[14,96],[44,23],[46,46]])
M_rd = np. array([[41,23],[23,94],[44,106],[68,41],[70,100]])
M_lu[:,1] = 110 - M_lu[:,1]
M_rd[:,1] = 110 - M_rd[:,1]
G_lu = np. array([[24,24],[33,54],[25,78]])
G_rd = np. array([[42,52],[45,76],[43,92]])
G_lu[:,1] = 110 - G_lu[:,1]
G_rd[:,1] = 110 - G_rd[:,1]
```

```
#其次,对omega区域进行赋值,如下所示
width = 74
length = 110
numP = 9
numM = 5
numG = 3
Omega = np. zeros([width, length])
for gn in range(numG):
    for gx in range(G_1u[gn, 0], G_rd[gn, 0]):
        for gy in range(G rd[gn, 1], G lu[gn, 1]):
            Omega[gx, gy] = -1;
for mn in range(numM):
    for mx in range(M_lu[mn, 0], M_rd[mn, 0]):
        for my in range (M_rd[mn, 1], M_lu[mn, 1]):
            Omega[mx, my] = 1;
for pn in range(numP):
```

```
for px in range(P_lu[pn, 0], P_rd[pn, 0]):
    for py in range(P_rd[pn, 1], P_lu[pn, 1]):
        Omega[px, py] = 2;
```

```
In [4]:
         #为便于观看,将区域可视化(该步骤目的是为了更直观呈现图像)
         def rectan (width, length, Omega):
             ax=plt. subplot (111)
             #可以使用三个整数,或者三个独立的整数来描述子图的位置信息。如果三个整数是行数、列着
             plt. xlim(0, width) #设定x坐标轴的范围
             plt. ylim(0, length) #设定y坐标轴的范围
             ax. xaxis. set major locator(MultipleLocator(2))#设置x主坐标间隔为 1
             ax. yaxis. set major locator(MultipleLocator(2))#设置y主坐标间隔 1
             ax. xaxis. grid (True, which='major') #major, color='black'
             ax. yaxis. grid (True, which='major') #major, color='black'
             for i in range (width):
                 for j in range(length):
                     if Omega[i, j] == 2:
                         x=np. linspace(i, i+1)
                         y1 = j
                         y2 = j + 1
                         ax. fill_between(x, y1, y2, facecolor='green')
                     elif Omega[i, j] == 1:
                         x=np. linspace(i, i+1)
                         y1=j
                         y2 = j + 1
                         ax. fill_between(x, y1, y2, facecolor='yellow')
                     elif Omega[i, j] == -1:
                         x=np. linspace(i, i+1)
                         y1=j
                         y2 = j + 1
                         ax. fill_between(x, y1, y2, facecolor='gray')
             plt. show()
```

In [5]: rectan(width, length, Omega)



2. 给定最近停车点函数NearestPark

获得分区矩阵后,我们可以直接得到每个点所对应的最近的P区格点,将格点坐标记录在停车点矩阵Park中。或者直接编写一个函数,不需要停车点矩阵,思路如下所示:

【主要思路】

- 如果该格点属于P区域,则输出endPoint;
- 如果该格点属于M区域,则运行NearestPark(2, endPoint),得到最近距离以及P区停车点;
- 如果该格点属于F区域

- 运行NearestPark(2, endPoint),得到最近距离minDist以及P区停车点
- 运行NearestPark(1, endPoint),得到最近距离minDist以及M区停车点
- 1. 输入:目标地区的横坐标x、纵坐标y以及flagvalue (如果flag=1,则代表寻找最近的M区;如果flag=2,则代表寻找最近的P区)
- 2 switch 1:遍历全部格点,计算所有M区到每个格点到(x,y)的距离,输出最小值及其坐标
- 3. switch 2:遍历全部格点,计算所有P区到每个格点到(x,y)的距离,输出最小值及其坐标
- 4. over!

```
In [7]: a1, a2=NearestPark(2, [0, 88]) a2
```

Out[7]: array (E 2 743 dtype=int64)

3. 事件矩阵 Event

理解:事件矩阵的行向量对应某用户在每个时刻所发生的事件,列向量对应某时刻每个用户所发生的事件的起、终点

下面讨论如何随机产生事件矩阵 (二维区域随机投点):

【主要思路】

- 1. 输入事件矩阵的行数m和列数n, 即用户个数m和总时刻数n;
- 2. 再产生2mn个叠加后的正态分布的随机点;
- 3. 将这2mn个随机点赋给事件矩阵。

基本参数假设:

- 2. 在正态分布中, 落在P区范围内服从~原则;

```
In [8]: F = [1/9, 2/9, 3/9, 4/9, 5/9, 6/9, 7/9, 8/9, 9/9]
```

```
In [9]: #首先,给定到达不同p区(共9个)的累积分布向量F、U(0,1)上的随机数eta、起点start、终点er def bisection_search(F, eta, start, end): if (eta <= F[start]):
```

```
return start
n = end - start
if (n <= 0):
    sys. exit()
k = (start + end) // 2
if (eta > F[k]):
    if (eta <= F[k + 1]):
        return k
    else:
        return bisection_search(F, eta, k + 1, end)
else:
    return bisection_search(F, eta, start, k)</pre>
```

```
#其次,尝试生成叠加后的二维正态分布的随机点
#step1: 生成一个服从U(0,1)的随机数,并带入bisection search函数,从而得到随机点所属P区;
#step2: 计算该P区的中心((右下横坐标+左上横坐标)/2,(右下纵坐标+左上纵坐标)/2)、
#step3: 获得随机点
def rand PDF():
   #首先产生服从累积分布F的随机数
   U = np. random. rand()
   X = bisection_search(F, U, 0, numP)
   #再产生对应P区的二维正态分布随机数
   mu_x = (P_rd[X, 0] + P_1u[X, 0])/2
   mu_y = (P_1u[X, 1] + P_rd[X, 1])/2
   sigma_x = (P_rd[X, 0] - P_1u[X, 0])/6
   sigma_y = (P_1u[X, 1] - P_rd[X, 1])/6
   mu = np. array([mu_x, mu_y])
   Sigma = np. array([[sigma_x**2, 0], [0, sigma_y**2]])
   s = np. random. multivariate_normal(mu, Sigma, 1)
   s = s[0]
   if 0 \le s[0] \le width and 0 \le s[1] \le length and 0 \le a[lint(s[0]), int(s[1])]! = -1:
       return s
   else:
       s = rand PDF()
```

```
In [12]: m = 100
 n = 100
```

In [13]: Event = geEvent(m, n)

4. 给定方格矩阵 Cell

方格矩阵代表了各个小方格上的自行车数量。

- 方格矩阵初始化;
- 方格矩阵的更新: 在每个时刻事件发生过程中都要更新一次;

【主要思路】

- 1. 方格矩阵的初始化: 所有P区的方格赋值为2(比如说), 其他区域全部为0。
- 2. 方格矩阵的变化: (自上至下,自左至右)每个时刻 t_i 下,每个人从起点前往终点,只有在确定该人是否骑车以及停车位置后才能改变方格矩阵。

- 利用方格矩阵,首先判断是否该地区是否有单车;
- 再考虑距离因素,过大或者过小都不适合骑车;
- 其次,根据停车点矩阵和内部判定条件确定是否更改终点;
- 若此人骑车,则改变方格矩阵;否则方格矩阵不变。

```
# 判定是否有车
## 作用: 判定方格矩阵中某方格周围共9个方格内是否有车,没车就直接判定为不骑车,有车再骑
## 输入: 某个点
## 输出:周围方格是否有车以及新的起点
## 查阅可得,紫金港占地面积5740000平方米,我们的矩阵是74*110,相当于缩小了700倍,依据过
def judgeCell(point):
   px = int(point[0])
   py = int(point[1])
   if Cell[px, py]!=0:
      return [px, py], 1
                      #起点不变,且该方格内有车
   else:
      for i in range (px-1, px+2):
         for j in range (py-1, py+2):
             if Cell[i, j]!=0:
                return [i, j],1 #起点改变,周围方格内有车
             else:
                continue
      return [px,py],0 #起点不变,该方格和周围方格都无车。判定为不骑车!
# 初始化方格矩阵
## 作用: 每次搬车后, 车子回到初始状态——P区(2)1辆车, M区(1)和F区(0)都是0辆车
def initialCell():
   Cell = np. zeros([74, 110])
   Cell[Omega==2] = 1
   Ce11[Omega==1] = 0
   Cell[Omega==0] = 0
   return Cell
#更新该方格的车
#作用:每一件事件发生后都需要重新更新方格矩阵
#输入:起点和终点
# 无输出,仅对全局变量Cell做更改
def updateCell(startpoint, endpoint):
   sx = int(startpoint[0])
   sy = int(startpoint[1])
   Cell[sx, sy] = Cell[sx, sy] + 1
   ex = int(endpoint[0])
   ey = int(endpoint[1])
   Cell[ex, ey] = Cell[ex, ey] + 1
```

```
#给定各个参数
park=np. zeros([m, n, 2])
distance=np. zeros([m, n])
#定义a<Cp<C<Cm<Cf<Df<Dm<D<Dp<b
#定义p和m
C = 20
C_{D} = 12
Cm=40
Cf = 60
Df=800
Dm = 800
Dp = 800
D = 800
b = 800
1 imitp=12
1 imitm=4
```

```
In [16]: | ##计算距离
          Cell=initialCell()
          for i in range(m):
              for j in range(n):
                  distance[i, j] = np. linalg. norm(Event[i, j, 1]-Event[i, j, 0])#事件矩阵起点和终点当
          ##判定是否骑车及其起点和终点
          for i in range(m):
              for j in range(n):
                  if a distance[i, j] and distance[i, j] b:#(排除极端情况)#if 这里有车才能进行:
                      vector0 = Event[i, j, 0]
                      vector1, flag = judgeCell(vector0)
                      vector2 = Event[i, j, 1]
                      if flag == 1:
                          if Omega[int(vector2[0]), int(vector2[1])] == 2: #(终点在p区内)
                              distance[i, j] = np. linalg. norm(vector2 - vector1)
                              park[i,j] = vector2 # (停车点距离相应的就等于终点的位置,录入到矩
                              if Cp \ge distance[i, j] or distance[i, j] \ge Dp:
                                  distance[i, j] = 0
                              else:
                                  updateCell(vector1, vector2)
                          elif Omega[int(vector2[0]), int(vector2[1])] == 1:#(终点在m区)
                              Kp, vector3 = NearestPark(2, vector2)#此处用x11的方法找到最近的p区[
                              if Kp >= 1imitp:
                                  distance[i, j] = np. linalg. norm(vector2 - vector1)
                                  park[i,j] = vector2 # (停车点距离相应的就等于终点的位置,录入)
                                  if Cm>=distance[i, j] or distance[i, j]>=Dm:
                                     distance[i, j] = 0
                                  else:
                                     updateCell(vector1, vector2)
                              elif Kp < limitp:
                                  distance[i, j] = np. linalg. norm(vector3-vector1)# 就是那个p点
                                  park[i, j] = vector3
                                  if Cp>=distance[i, j] or distance[i, j]>=Dp:
                                     distance[i, j]=0
                                  else:
                                     updateCell(vector1, vector3)
                          elif Omega[int(vector2[0]), int(vector2[1])] == 0: # (终点在f区)
                              Kp, vector3 = NearestPark(2, vector2)#此处用x11的方法找到最近的p区|
                              Km, vector4 = NearestPark(1, vector2)#此处用x11的方法找到最近的m区
                              if Kp<limitp:
                                 distance[i, j] = np. linalg. norm(vector3-vector1) # 就np. linal
                                  park[i, j] = vector3
                                  if Cp>=distance[i, j] or distance[i, j]>=Dp:
                                     distance[i, j] = 0
                                  else:
                                     updateCell(vector1, vector3)
                              elif Kp \ge = limitp and Km \le limitm:
                                  distance[i, j] = np. linalg. norm(vector4-vector1) # 就是那个m点
                                  park[i, j] = vector4
                                  if Cm>=distance[i, j] or distance[i, j]>=Dm:
                                     distance[i, j] = 0
                                 else:
                                     updateCell(vector1, vector4)
                              else:#就是此时Kp比p大,而且Km比m大
                                  distance[i, j] = np. linalg. norm(vector2-vector1)# 就是终点减去
                                  park[i, j] = vector2
                                  if Cf \ge distance[i, j] or distance[i, j] \ge Df:
                                     distance[i, j] = 0
                                  else:
                                     updateCell(vector1, vector2)
                      else:
                          distance[i, j] = 0
                  else:
```

```
distance[i,j] = 0
#这一部分实际上就是为了计算相应的费用,需要结合5,如下所示:
```

5. 计算盈利情况

• 成本:搬运费

• 收入: 单车使用费、广告费

```
In [18]:
          ##计算距离
          Cell=initialCell()
          for i in range(m):
              for j in range(n):
                  distance[i, j] = np. linalg. norm(Event[i, j, 1]-Event[i, j, 0])#事件矩阵起点和终点当
          ##判定是否骑车及其起点和终点
          for i in range(m):
              for j in range(n):
                  if a distance[i, j] and distance[i, j] b:#(排除极端情况)#if 这里有车才能进行:
                     vector0 = Event[i, j, 0]
                     vector1, flag = judgeCell(vector0)
                     vector2 = Event[i, j, 1]
                      if flag == 1:
                         if Omega[int(vector2[0]), int(vector2[1])] == 2: #(终点在p区内)
                             distance[i, j] = np. linalg. norm(vector2 - vector1)
                             park[i,j] = vector2 #(停车点距离相应的就等于终点的位置,录入到矩
                             if Cp \ge distance[i, j] or distance[i, j] \ge Dp:
                                 distance[i, j] = 0
                             else:
                                 updateCell(vector1, vector2)
                         elif Omega[int(vector2[0]), int(vector2[1])] == 1:#(终点在m区)
                             Kp, vector3 = NearestPark(2, vector2)#此处用x11的方法找到最近的p区|
                             if Kp >= 1imitp:
                                 distance[i, j] = np. linalg. norm(vector2 - vector1)
                                 park[i, j] = vector2 # (停车点距离相应的就等于终点的位置, 录入)
                                 if Cm>=distance[i, j] or distance[i, j]>=Dm:
                                    distance[i, j] = 0
                                 else:
                                    updateCell(vector1, vector2)
                             elif Kp < limitp:
                                 distance[i, j] = np. linalg. norm(vector3-vector1)# 就是那个p点
                                 park[i, j] = vector3
                                 if Cp>=distance[i, j] or distance[i, j]>=Dp:
                                    distance[i, j]=0
                                 else:
                                     updateCell(vector1, vector3)
                         elif Omega[int(vector2[0]), int(vector2[1])] == 0: # (终点在f区)
                             Kp, vector3 = NearestPark(2, vector2)#此处用xll的方法找到最近的p区
```

if Kp<limitp:

```
park[i, j] = vector3
                                 if Cp>=distance[i, j] or distance[i, j]>=Dp:
                                    distance[i, j] = 0
                                 else:
                                    undateCell(vector1, vector3)
                             elif Kp>=limitp and Km<limitm:</pre>
                                 distance[i, j] = np. linalg. norm(vector4-vector1) # 就是那个m点
                                 park[i, j] = vector4
                                 if Cm>=distance[i, j] or distance[i, j]>=Dm:
                                    distance[i, j] = 0
                                 else:
                                    updateCell(vector1, vector4)
                             else:#就是此时Kp比p大,而且Km比m大
                                 distance[i, j] = np. linalg. norm(vector2-vector1)# 就是终点减去
                                 park[i, j] = vector2
                                 if Cf>=distance[i, j] or distance[i, j]>=Df:
                                    distance[i, j] = 0
                                 else:
                                    updateCell(vector1, vector2)
                     else:
                         distance[i, j] = 0
                  else:
                     distance[i, j] = 0
                  countcost (Cell, i+1) #计算是否需要重新搬运,对cell矩阵进行初始化
In [19]:
         sumcost = sum(cost)
          # 单车骑行收费和广告费用
          k1=0.5
                  #一个参数p区收费和距离的参数
          k2=1.5 #一个参数m区收费和距离的参数
          k3=2.5 #一个参数f区收费和距离的参数
          g1 = 50
                  #一个参数广告和人数的参数
          def countincome():
              income1 = 0
              income2 = 0
              for i in range(m):
                  for j in range(n):
                     feetype = Omega[int(park[i, j, 0]), int(park[i, j, 1])]
                      if feetype == 2:
                         income1 = income1 + (int(distance[i, j]/120)+1)*k1
                     elif feetype == 1:
                         income1 = income1 + (int(distance[i, j]/120)+1)*k2
                     elif feetype == 0:
                         income1 = income1 + (int(distance[i, j]/120)+1)*k3
                  income2 = np. sum(distance!=0)*g1 #广告费
              return income1, income2
          inc1, inc2 = countincome()
          sumincome = inc1 + inc2
          # 综上所述, 共享单车公司的总利润为: 收入 - 成本
          sumbenifit = sumincome - sumcost
          sumbenifit1=sumbenifit
          sumbenifit1
Out[24]: 377838.0
```

Km, vector4 = NearestPark(1, vector2)#此处用xll的方法找到最近的m区。

distance[i, j] = np. linalg. norm(vector3-vector1) # 就np. linal

Part2:统一型定价

```
park=np. zeros([m, n, 2])
distance=np. zeros([m, n])
a=4
#定义a<Cp<C<Cm<Cf<Df<Dm<D<Dp<b
#定义p和m
C=20
Cp=12
Cm=40
Cf = 60
Df=800
Dm = 800
D_{D} = 800
D = 800
b = 800
1 imitp=12
1 \text{imitm} = 4
##计算距离
for i in range(m):
    for j in range(n):
        distance[i, j] = np. linalg. norm(Event[i, j, 1]-Event[i, j, 0])#事件矩阵起点和终点当
##判定是否骑车及其起点和终点
for i in range(m):
    for j in range(n):
        if a distance[i, j] and distance[i, j] b:#(排除极端情况)#if 这里有车才能进行:
            vector0 = Event[i, j, 0]
            vector1, flag = judgeCell(vector0)
            vector2 = Event[i, j, 1]
            if flag == 1:
                if Omega[int(vector2[0]), int(vector2[1])] <= 2 and Omega[int(vector2[
                    distance[i, j] = np. linalg. norm(vector2 - vector1)
                    park[i,j] = vector2 #(停车点距离相应的就等于终点的位置,录入到矩
                    if C \ge distance[i, j] or distance[i, j] \ge D:
                        distance[i, j] = 0
                    else:
                        updateCell(vector1, vector2)
            else:
                distance[i, j] = 0
        else:
            distance[i, j] = 0
```

```
In [27]: ##计算距离
Cell=initialCell()
for i in range(m):
    for j in range(n):
        distance[i, j] = np. linalg. norm(Event[i, j, 1]-Event[i, j, 0])#事件矩阵起点和终点
```

##判定是否骑车及其起点和终点

for j in range(n):

vector0 = Event[i, j, 0]

for i in range(m):

```
vector1, flag = judgeCell(vector0)
                     vector2 = Event[i, j, 1]
                     if flag == 1:
                         if Omega[int(vector2[0]), int(vector2[1])] <= 2 and Omega[int(vector2[
                            distance[i, j] = np. linalg. norm(vector2 - vector1)
                            park[i,j] = vector2 #(停车点距离相应的就等于终点的位置,录入到矩
                            if C>=distance[i, j] or distance[i, j]>=D:
                                distance[i, j] = 0
                            else:
                                updateCell(vector1, vector2)
                     else:
                         distance[i, j] = 0
                 else:
                     distance[i, j] = 0
                 countcost (Cell, i+1) #计算是否需要重新搬运,对cell矩阵进行初始化
In [28]:
         sumcost = sum(cost)
In [29]:
          # 单车骑行收费和广告费用
          k1=0.5 #一个参数p区收费和距离的参数
          k2=1.5 #一个参数m区收费和距离的参数
          k3=2.5 #一个参数f区收费和距离的参数
                  #一个参数广告和人数的参数
          g1 = 50
          def countincome():
              income1 = 0
              income2 = 0
              for i in range(m):
                 for j in range(n):
                     feetype = Omega[int(park[i, j, 0]), int(park[i, j, 1])]
                     if feetype == 2:
                         income1 = income1 + (int(distance[i, j]/120)+1)*k1
                     elif feetype == 1:
                         income1 = income1 + (int(distance[i, j]/120)+1)*k2
                     elif feetype == 0:
                         incomel = incomel + (int(distance[i, j]/120)+1)*k3
                 income2 = np. sum(distance!=0)*g1 #广告费
             return income1, income2
          inc1, inc2 = countincome()
          sumincome = inc1 + inc2
          # 综上所述, 共享单车公司的总利润为: 收入 - 成本
          sumbenifit = sumincome - sumcost
          sumbenifit2=sumbenifit
          sumbenifit2
Out[33]: 351600.5
```

if a distance[i, j] and distance[i, j] b:#(排除极端情况)#if 这里有车才能进行:

Part3: 二者比较

在m=100, n=100下, 阶梯型和统一型定价结果如下所示:

benifit1: 377838.0benifit2: 351600.5

在人工参数下,阶梯型比统一型更佳。

In []: