Please read the following text carefully

An ordinal number (or ordinal in short) is an adjective that describes the numerical position or order of an object with respect to other objects.

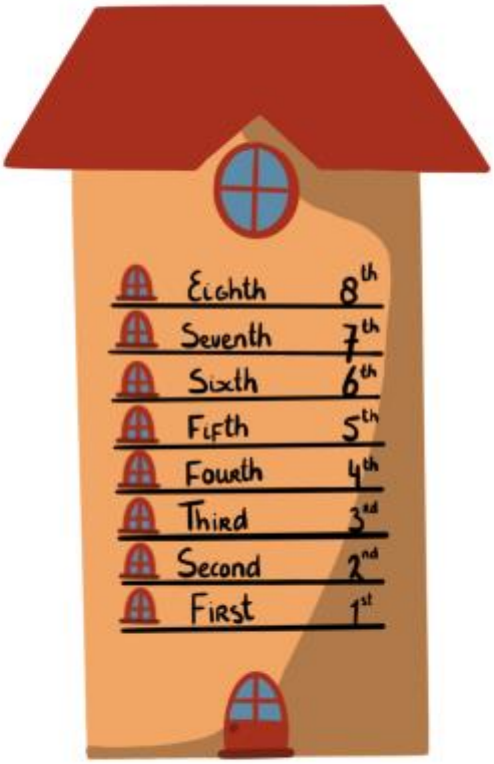


Image 1: The numbers used to describe the different floors in a building are ordinal numbers.

Infinite ordinals

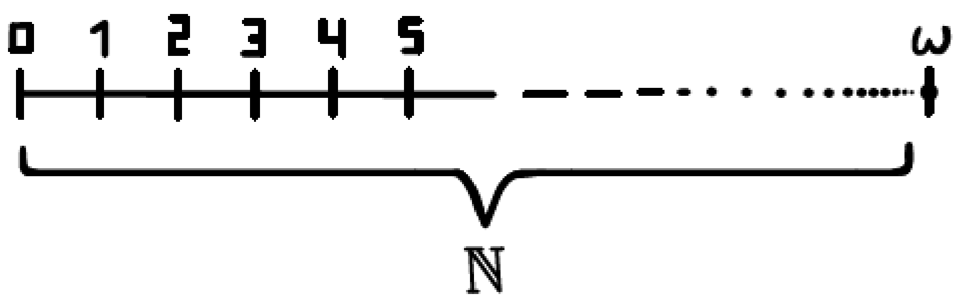


Image 2: *Omega* ω is the first infinite ordinal number that comes **after** all the natural numbers

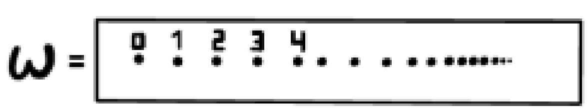


Image 3: ω is the set (i.e., a collection of elements) containing all natural numbers. i.e., ω = {0, 1, 2, 3, ...}

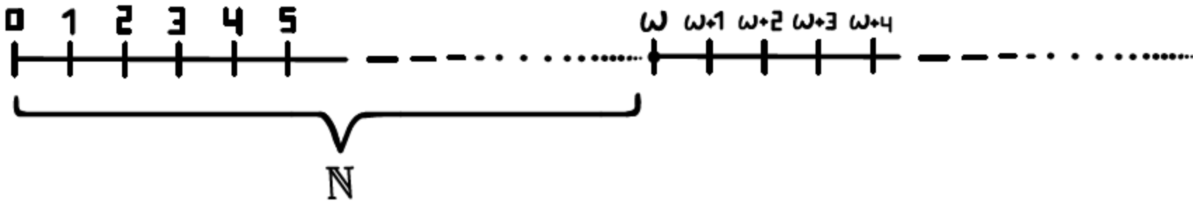


Image 4: Ordinals allow us to continue counting and comparing positions of infinite elements that are beyond the natural numbers

Successor and predecessor of ordinals

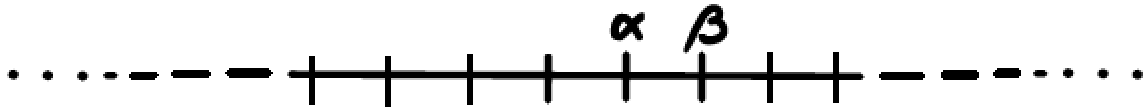
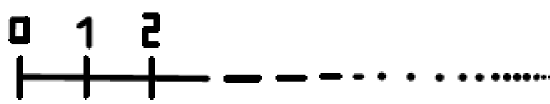


Image 5: we say β is the *successor* of α (S(α) = β) when α < β and there is no other number in between them (i.e., β = α + 1).

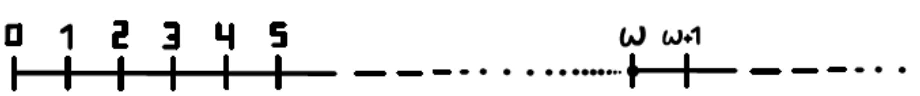
For example:



- Image 6: S(1) = 2
- Image 7: S(973) =

974

- Image 8: S(ω) = ω+1



For an ordinal number α, a *predecessor* of α is some ordinal γ, such that γ < α and there is no other ordinal number between them (i.e., α = γ +1).

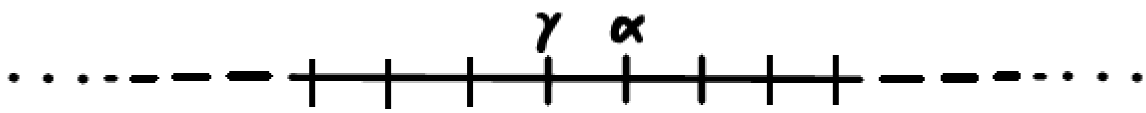


Image 9: we say γ is the *predecessor* of α (S(γ) = α) when γ < α and there is no other number in between them

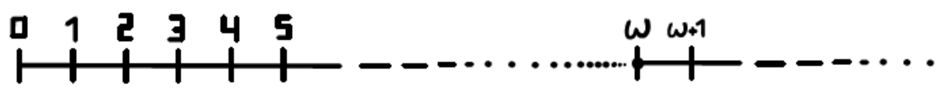
For example:

- Image 10: S(5) =

  6
- Image 11: S(2965) =

  2966

- Image 12: S(ω) =
  ω+1



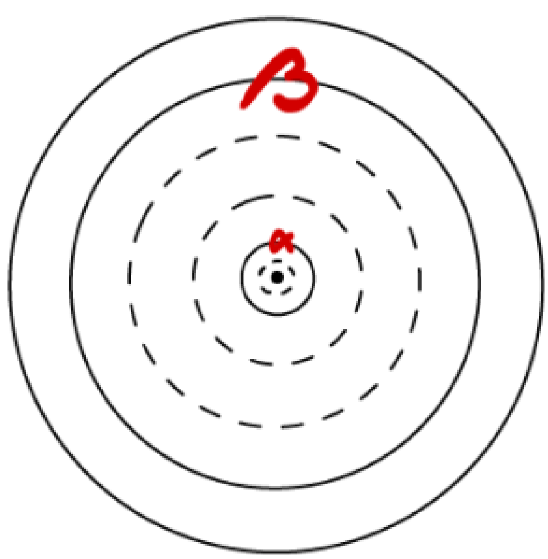
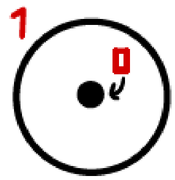
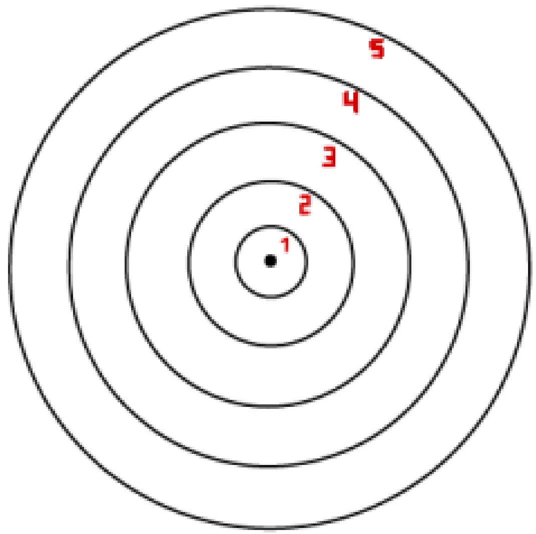


Image 13: Every ordinal is a set that contains all of its predecessors; for any pair α and β of ordinal numbers, if α < β, then α is 'contained' within β (α ∈ β)

For example:



- Image 14: 1 = {0}



- Image 15: 5 = {0, 1, 2, 3, 4}

Addition of ordinal numbers

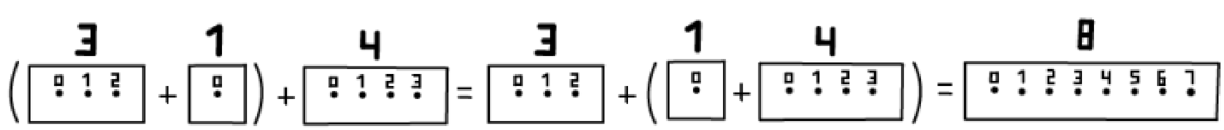


Image 16: Addition of ordinals is associative; e.g., (3 + 1) + 4 = 3 + (1 + 4) = 8

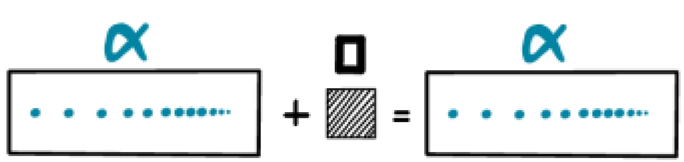
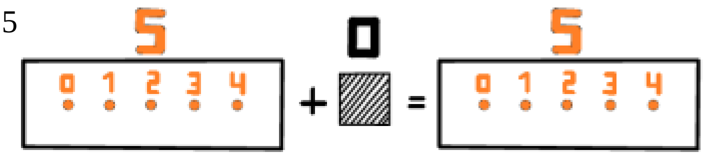


Image 17: For any ordinal α, α + 0 = α

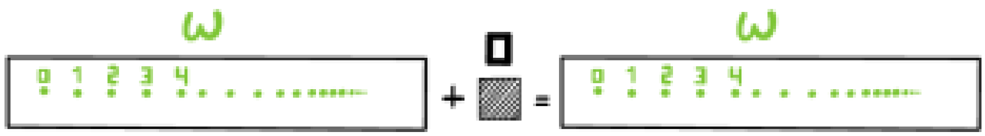
For example:

- Image 18: 5



5

- Image 19: ω + 0
= ω



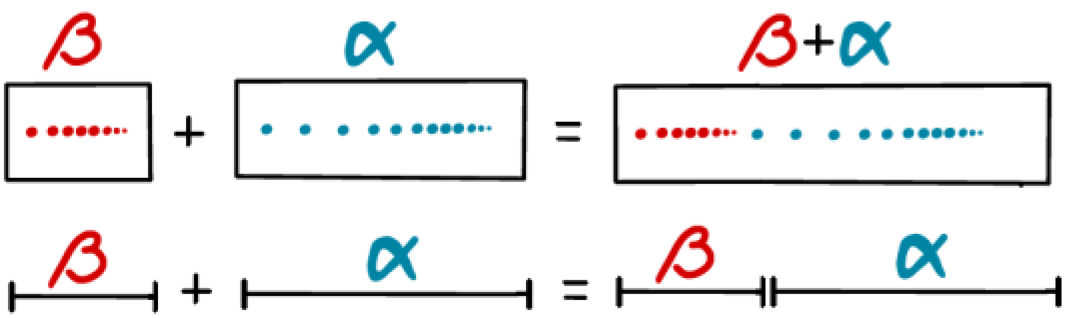
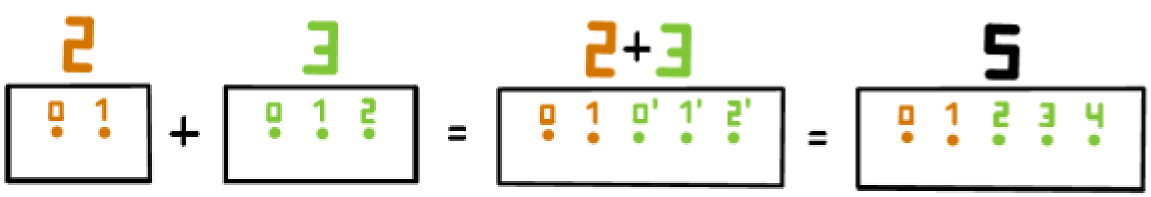
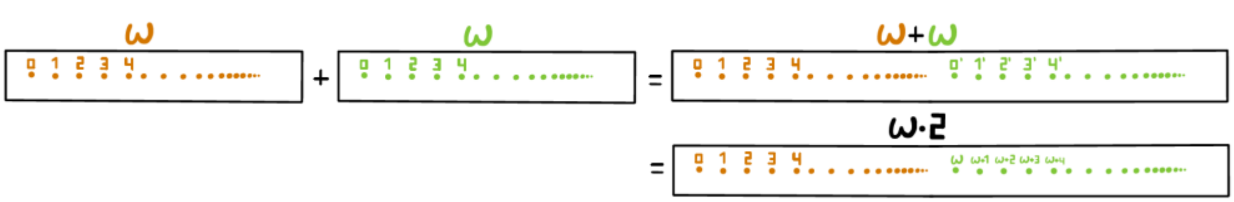


Image 20: Addition between two ordinals β, α (β + α)

For example:

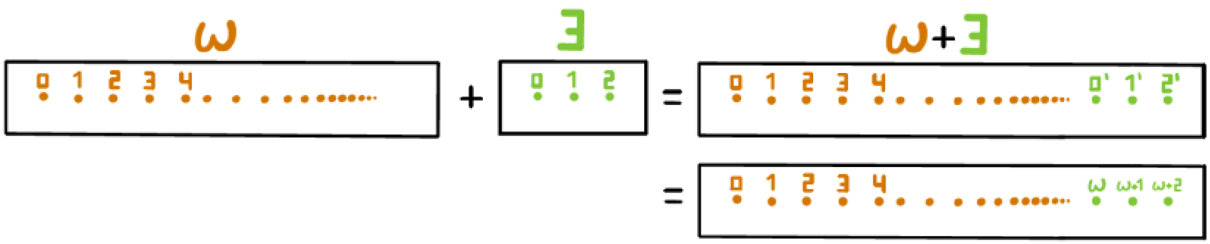


- Image 21: 2 + 3 = {0, 1} + {0, 1, 2} = {0, 1, 0', 1', 2'} = {0, 1, 2, 3, 4} = 5

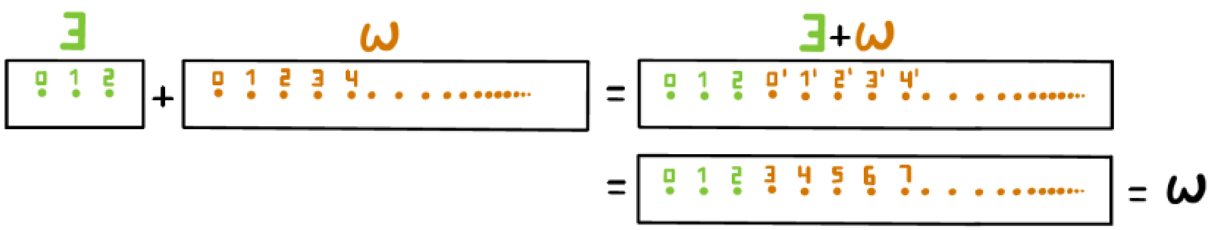


- Image 22: $\omega + \omega = \{0, 1, 2, 3, ...\} + \{0, 1, 2, 3, ...\} = \{0, 1, 2, 3, ..., 0', 1', 2', 3', ...\} = \{0, 1, 2, 3, ..., \omega, \omega+1, \omega+2, \omega+3, ...\} = \omega * 2$.

However, let's consider the ordinal numbers $\omega+3$ and $3+\omega$. Following the same idea, we can order these two ordinals as follows:



- Image 23: $\omega + 3 := \{0, 1, 2, 3, ...\} + \{0, 1, 2\} = \{0, 1, 2, 3, ..., 0', 1', 2'\} = \{0, 1, 2, 3, ..., \omega, \omega+1, \omega+2\} = \omega+3$



- Image 24: $3 + \omega := \{0, 1, 2\} + \{0, 1, 2, 3, ...\} = \{0, 1, 2, 0', 1', 2', 3', ...\} = \{0, 1, 2, 3, 4, ...\} = \omega$

Therefore, we can conclude

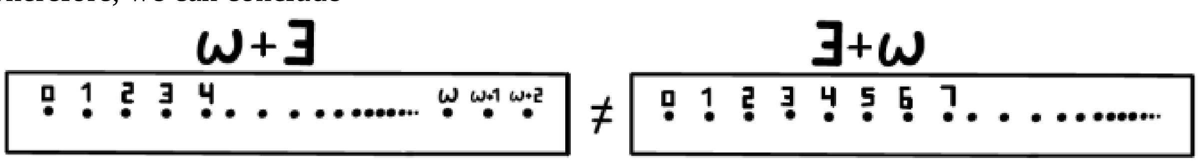


Image 25: $\omega+3 \neq 3+\omega$

Multiplication of ordinal numbers

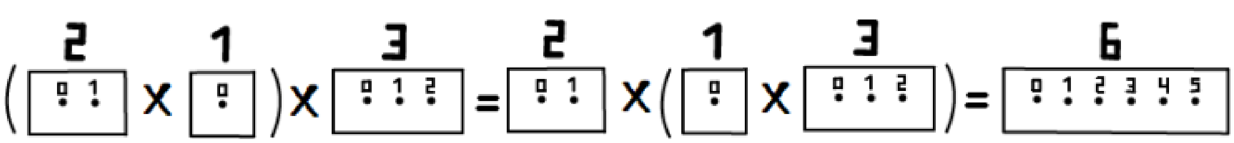


Image 26: Addition of ordinals is associative; e.g., $(2 * 1) * 3 = 2 * (1 * 3) = 6$

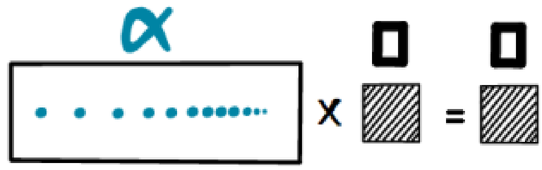
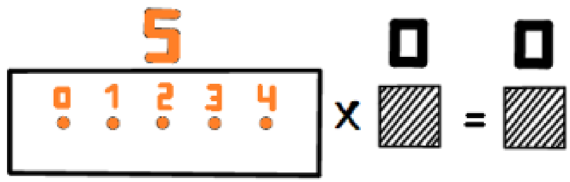


Image 27: For any ordinal α, α * 0 = 0

For example:



- Image 28: 5 * 0 = 0
- Image 29: ω * 0 =

0

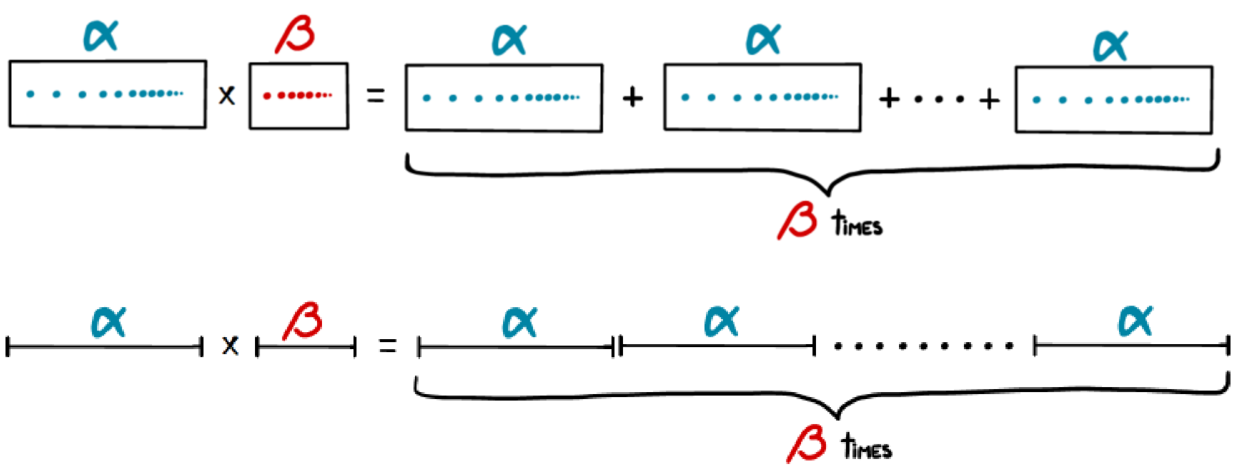
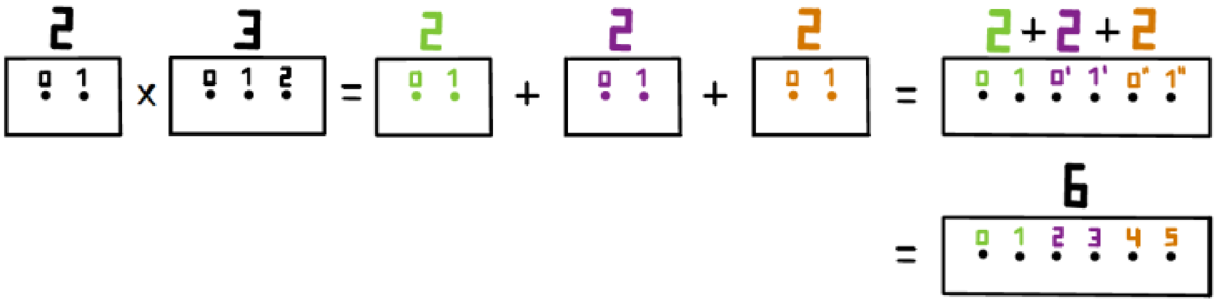
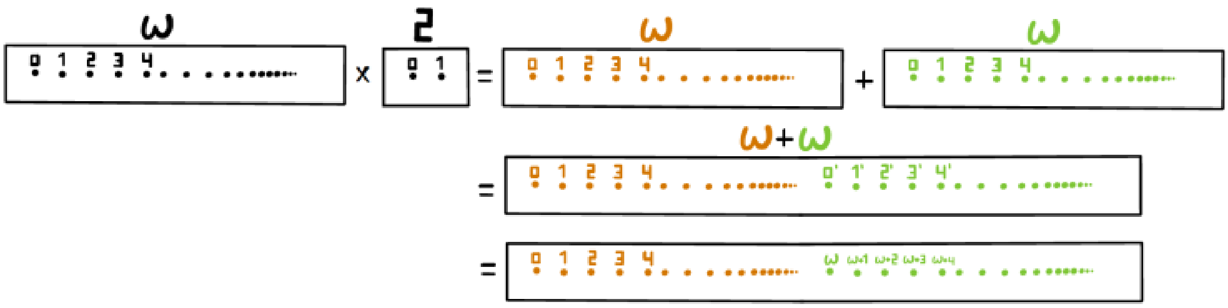


Image 30: Multiplication between two ordinals α, β (α * β)

For example:

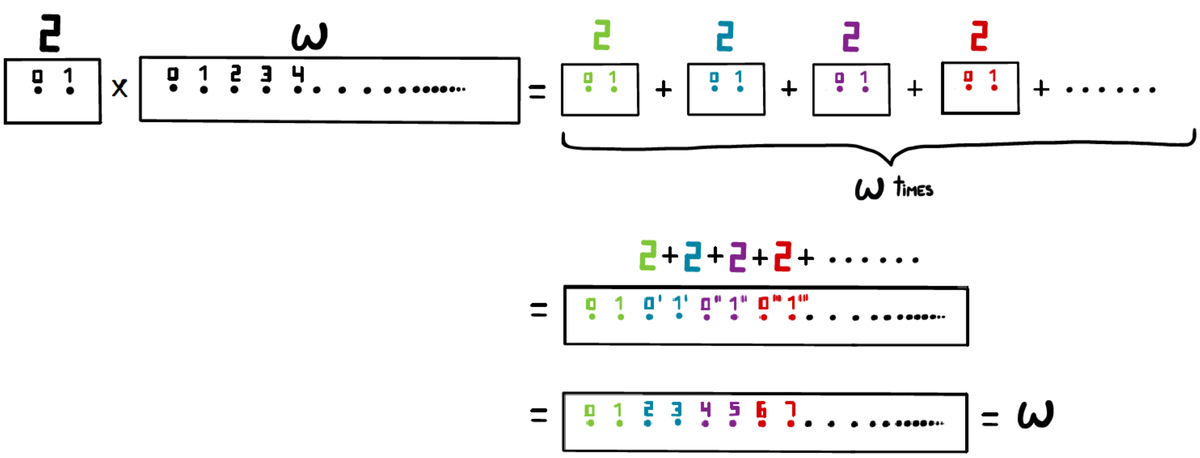


- Image 31: 2 * 3 = 2 + 2 + 2 = {0, 1} + {0, 1} + {0, 1} = {0, 1, 0', 1', 0", 1"} = 6



- Image 32: ω * 2 = {0, 1, 2, 3, ...} * 2 = {0, 1, 2, 3, ...} + {0, 1, 2, 3, ...} = {0, 1, 2, 3, ..., 0', 1', 2', 3', ...} = {0, 1, 2, 3, ..., ω, ω+1, ω+2, ω+3, ...} = ω + ω

However, let's consider the ordinal numbers 2*ω. Following the same idea, we can order this ordinal as follows:



- Image 33: 2 * ω := {0, 1} + {0, 1} + {0, 1} + ... ω many times = {0, 1, 0', 1', 0", 1", 0"', 1"', 0"", 1"", ...} = {0, 1, 2, 3, 4, 5, 6, ...} = ω

Therefore, we can conclude

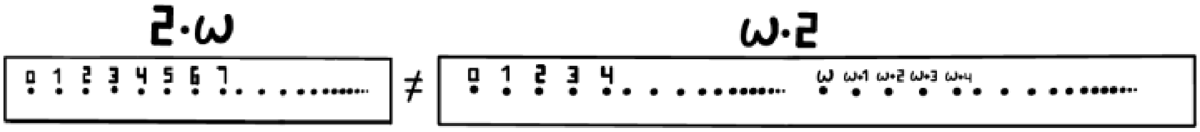


Image 34: 2*ω ≠ ω*2