APPENDIX

Derivations for $\left|H(\boldsymbol{x}_{ij,\xi})\right| \leq \tilde{H}(\boldsymbol{x}_{ij}^{s}, \boldsymbol{\delta}_{ij})$

For the brevity of derivation, the AC branch power flow equation reformulated as follows:

$$P(\boldsymbol{x}_{ij}) = g_{ij} v_i^2 - v_i v_j y_{ij} \cos(\theta_{ij} + \varphi_{ij}) \,. \tag{1}$$

Where, y_{ij} is the admittance of branch ij, φ_{ij} is the impedance angle of branch ij, and $g_{ij} = y_{ij} \cos\varphi_{ij}$, $b_{ij} = -y_{ij} \sin\varphi_{ij}$.

Taking the active power flow as an example, the absolute of the Hessian matrix shown as follows:

$$\left|H(\boldsymbol{x}_{ij})\right| = \begin{pmatrix} \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial\theta_{ij}\partial\theta_{ij}}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_i\partial\theta_{ij}}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_j\partial\theta_{ij}}\right| \\ \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial\theta_{ij}\partial v_i}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_i\partial v_i}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_j\partial v_i}\right| \\ \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial\theta_{ij}\partial v_j}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_i\partial v_j}\right| & \left|\frac{\partial^2 P(\boldsymbol{x}_{ij})}{\partial v_j\partial v_j}\right| \end{pmatrix}, \tag{2}$$

$$\left|H(\boldsymbol{x}_{ij})\right| = \begin{pmatrix} \left|v_i v_j y_{ij}\cos(\theta_{ij}+\varphi_{ij})\right| & \left|v_j y_{ij}\sin(\theta_{ij}+\varphi_{ij})\right| & \left|v_i y_{ij}\sin(\theta_{ij}+\varphi_{ij})\right| \\ \left|v_j y_{ij}\sin(\theta_{ij}+\varphi_{ij})\right| & 2\left|g_{ij}\right| & \left|-y_{ij}\cos(\theta_{ij}+\varphi_{ij})\right| \\ \left|v_i y_{ij}\sin(\theta_{ij}+\varphi_{ij})\right| & \left|-y_{ij}\cos(\theta_{ij}+\varphi_{ij})\right| & 0 \end{pmatrix}. \tag{3}$$

Based on the following inequalities in (4)-(5), the corresponding elements in $\left|H(\boldsymbol{x}_{ij,\xi})\right|$ are relaxed.

$$\left|\cos(\theta_{ij,\xi} + \varphi_{ij})\right| \leq 1, \ \left|\sin(\theta_{ij,\xi} + \varphi_{ij})\right| \leq 1 \tag{4}$$

$$\begin{aligned} \left|v_{i,\xi} v_{j,\xi}\right| &= \left|((1-\lambda_{ij}^P)v_i^s + \lambda_{ij}^P v_i)((1-\lambda_{ij}^P)v_j^s + \lambda_{ij}^P v_j)\right| \\ &\leq \left|v_j^s v_i^s\right| + \left|v_i^s v_j\right| + \left|v_j^s v_i\right| + \left|v_i v_j\right| \\ &\Downarrow v_1 \leq \tfrac{1}{2}\delta_{ij,1}, v_j \leq \tfrac{1}{2}\delta_{ij,2} \\ &\leq v_j^s v_i^s + \tfrac{1}{2}(v_i^s \delta_{ij,2} + v_j^s \delta_{ij,1}) + \tfrac{1}{4}\delta_{ij,1}\delta_{ij,2} \end{aligned} \tag{5}$$

Where, $v_{i,\xi}$ and $v_{j,\xi}$ are the elements of $\boldsymbol{x}_{ij,\xi}$; $\delta_{ij,1}$ and $\delta_{ij,2}$ are the elements of $\boldsymbol{\delta}_{ij}$.

Finally, the $\tilde{H}(\boldsymbol{x}_{ij}^{s}, \boldsymbol{\delta}_{ij})$ is obtained as follows:

$$\tilde{H}(\boldsymbol{x}_{ij}^{s}, \boldsymbol{\delta}_{ij}) = \begin{pmatrix} \left|y_{ij}\right| \begin{pmatrix} v_j^s v_i^s + \frac{1}{4}\delta_{ij,1}\delta_{ij,2} + \\ \frac{1}{2}(v_i^s\delta_{ij,2} + v_j^s\delta_{ij,1}) \end{pmatrix} & (v_j^s + \frac{1}{2}\delta_{ij,2})\left|y_{ij}\right| & (v_i^s + \frac{1}{2}\delta_{ij,1})\left|y_{ij}\right| \\ (v_j^s + \frac{1}{2}\delta_{ij,2})\left|y_{ij}\right| & 2\left|g_{ij}\right| & \left|y_{ij}\right| \\ (v_i^s + \frac{1}{2}\delta_{ij,1})\left|y_{ij}\right| & \left|y_{ij}\right| & 0 \end{pmatrix}. \tag{6}$$