Universidade Federal de Santa Maria

SEDES SAPIENTIÆ

1960

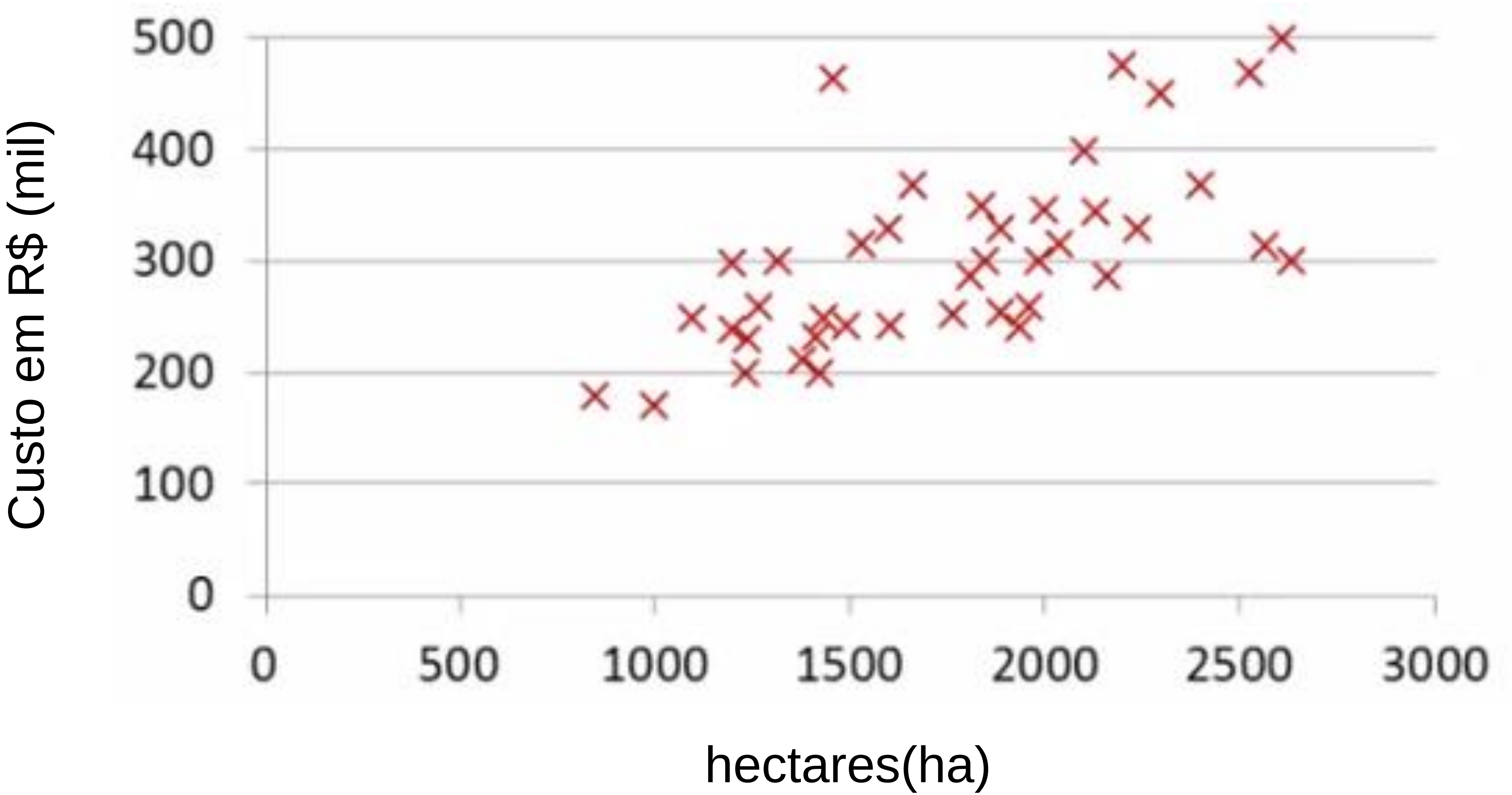
UFVJM

1953

SAPIENTIA ET LIBERTAS

Como repres

Como representar um modelo?



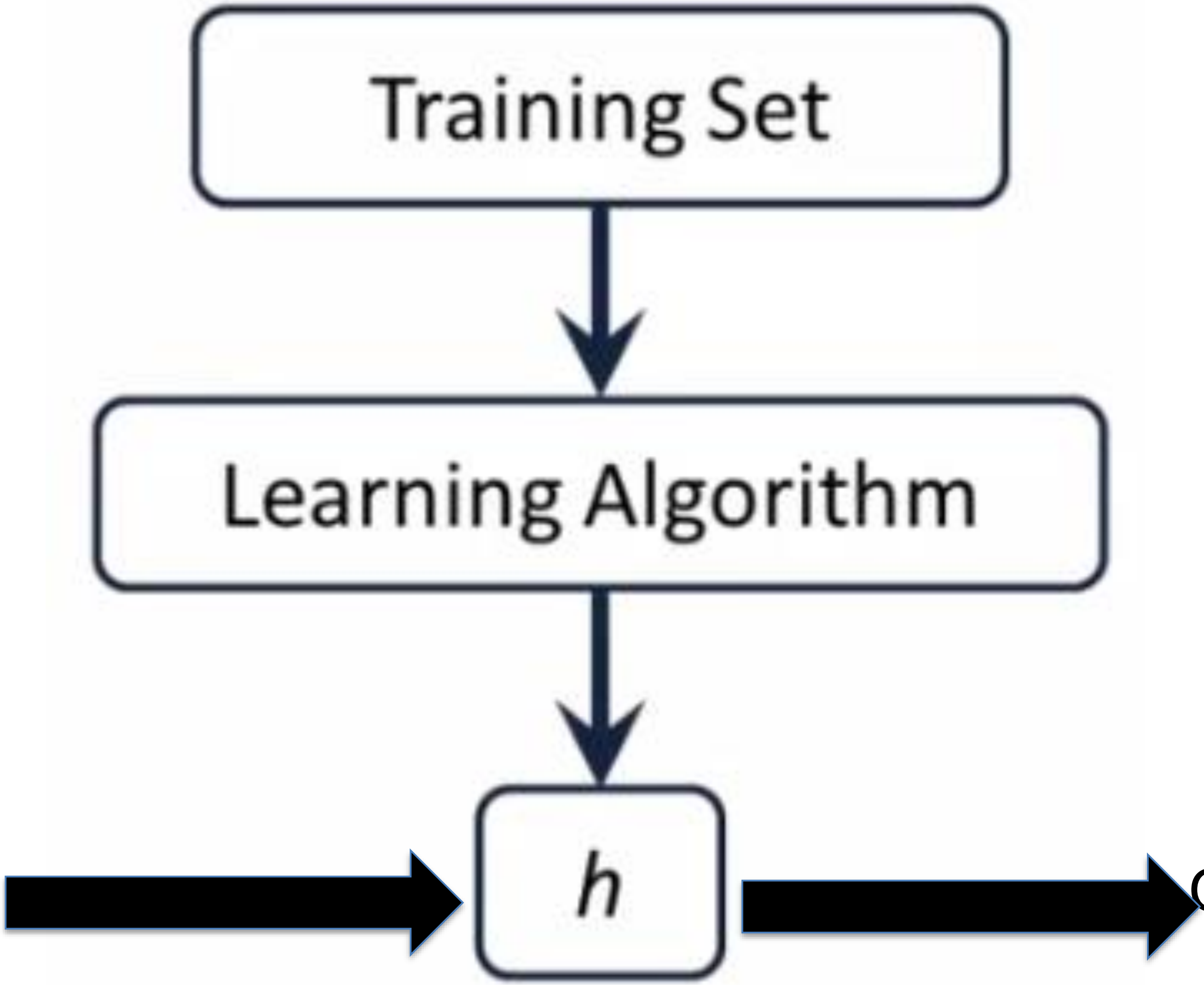
Como representar um modelo?

Tamanho (x)	**Custo R$ (mil)** (y)
2104	460
1416	232
1534	315
852	178
...	...

Notation:

m = Number of training examples
x's = "input" variable / features
y's = "output" variable / "target" variable

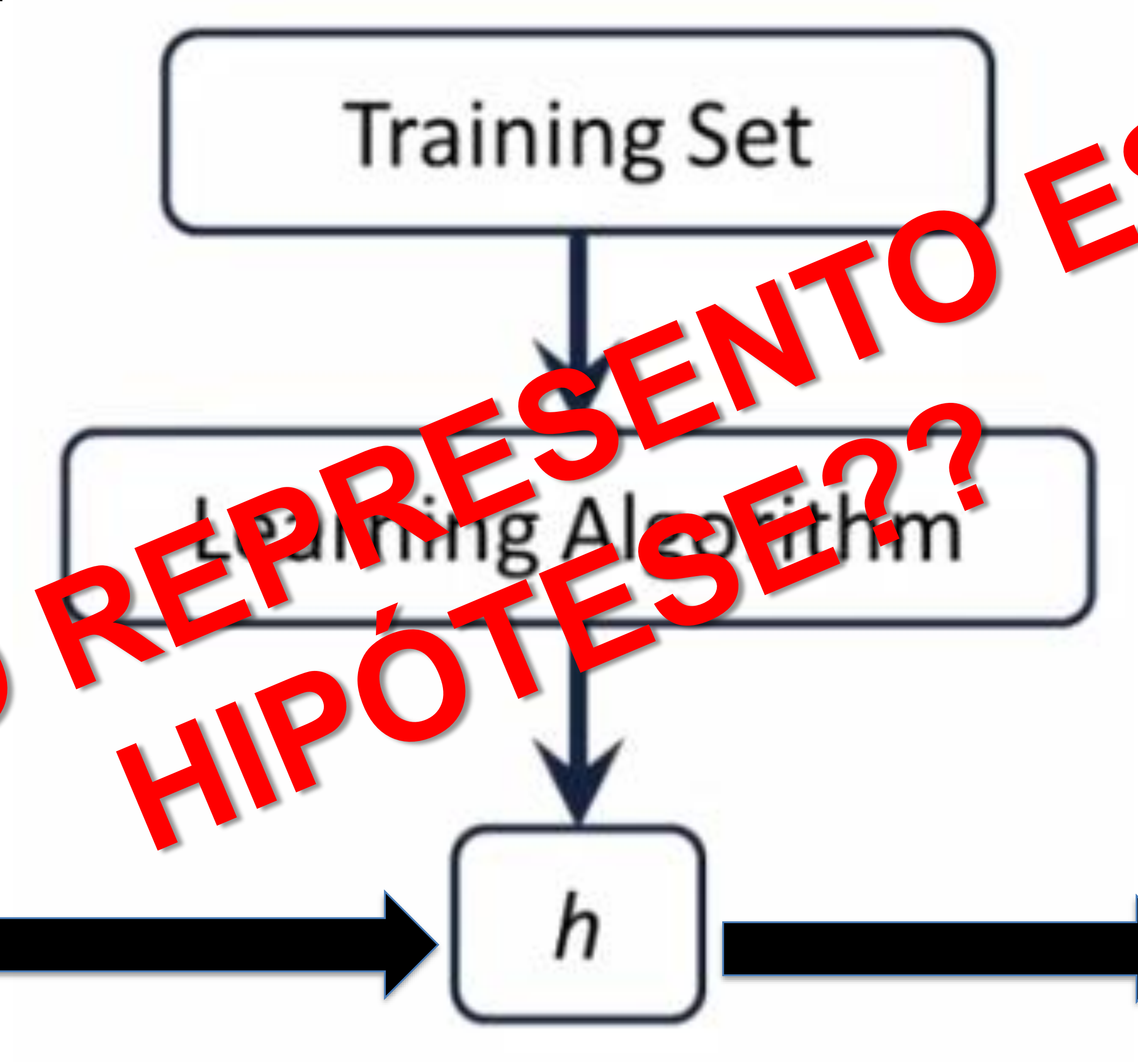
Como representar um modelo?



Tamanho em ha **(x)**

Custo em R$ **(y)**

Como representar um modelo?



COMO REPRESENTO ESSA HIPÓTESE??

Tamanho em ha **(x)** → *h* → Custo em R$ **(y)**

Reconhecimento de padrões e aprendizagem computacional

Regressão Linear

Introdução

Tamanho (x)	Custo R$ (mil) (y)
2104	460
1416	232
1534	315
852	178
...	...

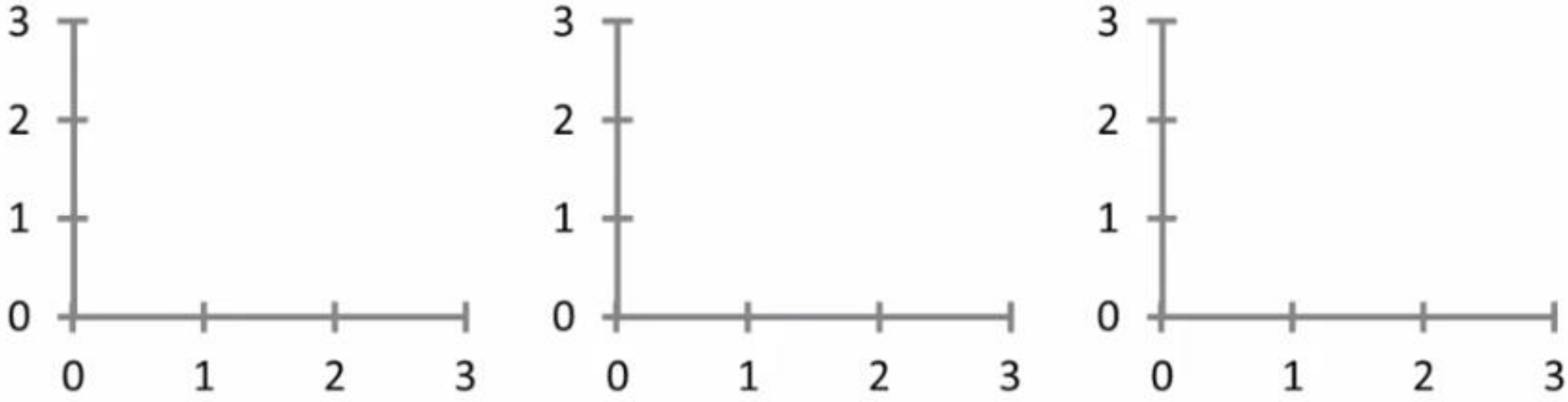
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

Como escolher os parâmetros??

Introdução

$$h_\theta(x) = \theta_0 + \theta_1 x$$



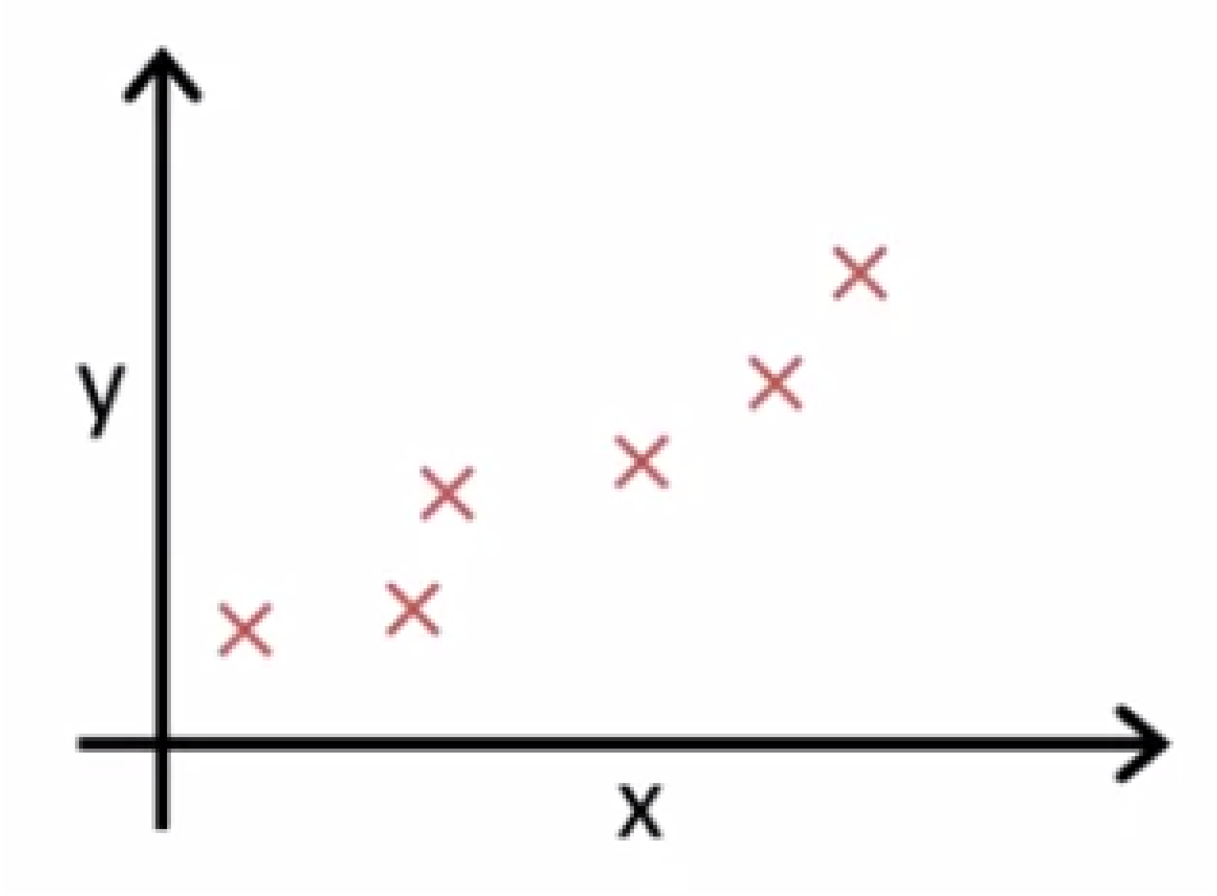
$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

Introdução

Como escolher os parâmetros??



Função de Custo – Intuição 1

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Simplified

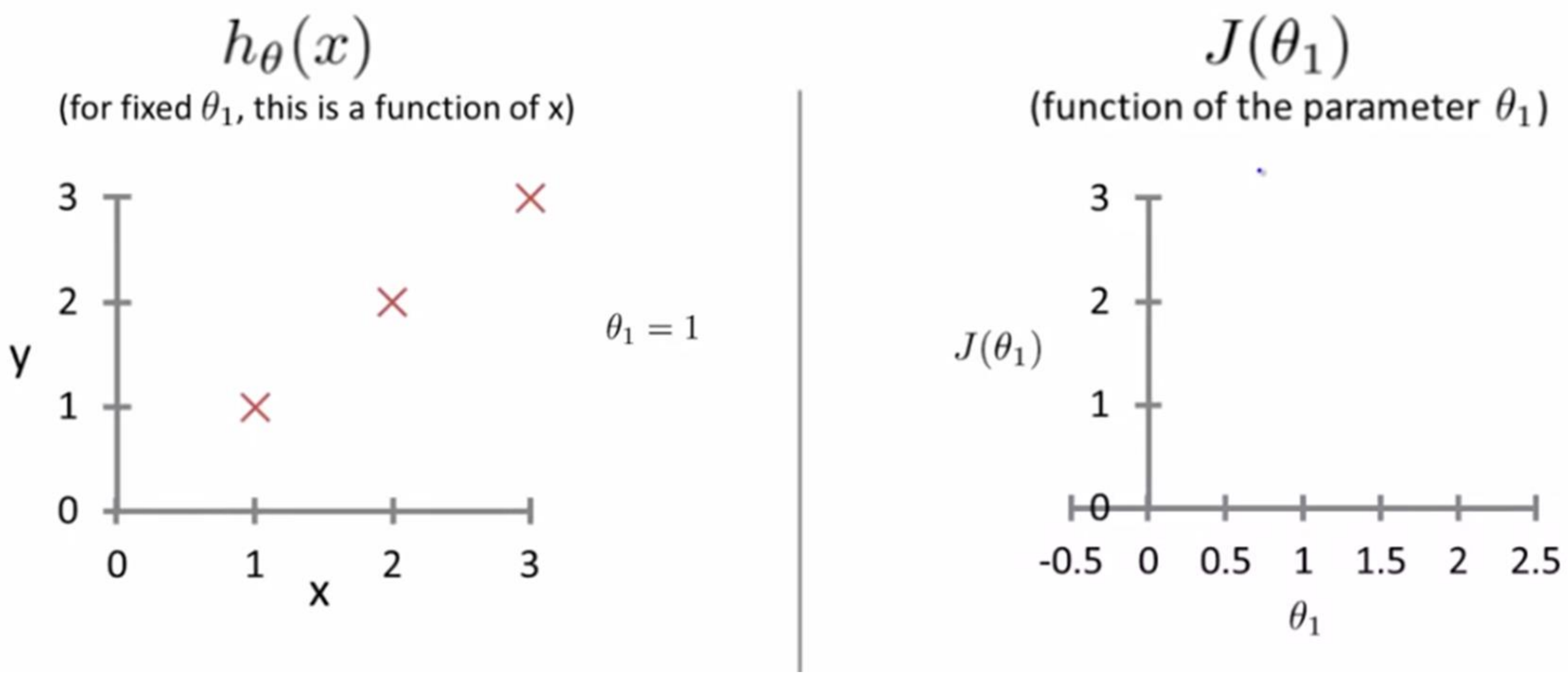
$$h_\theta(x) = \theta_1 x$$

$$\theta_1$$

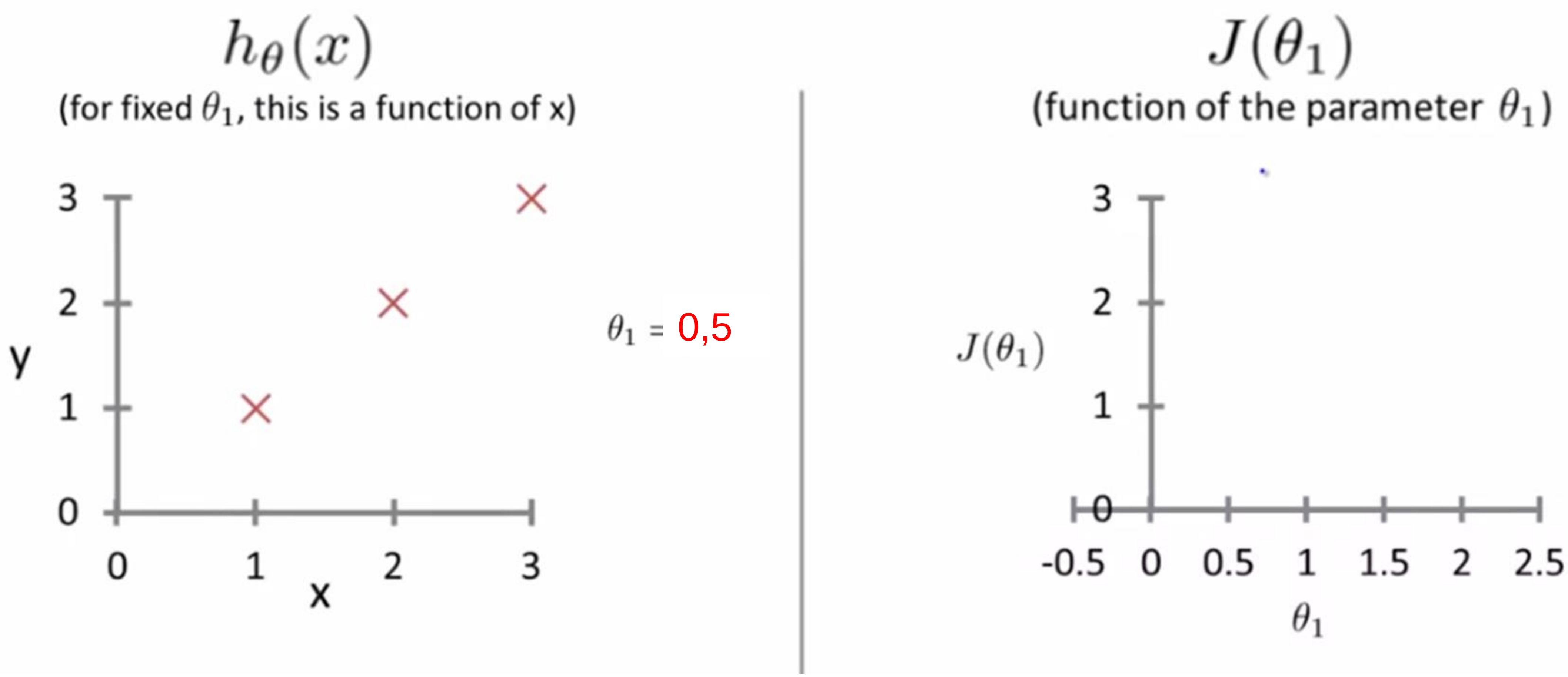
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$

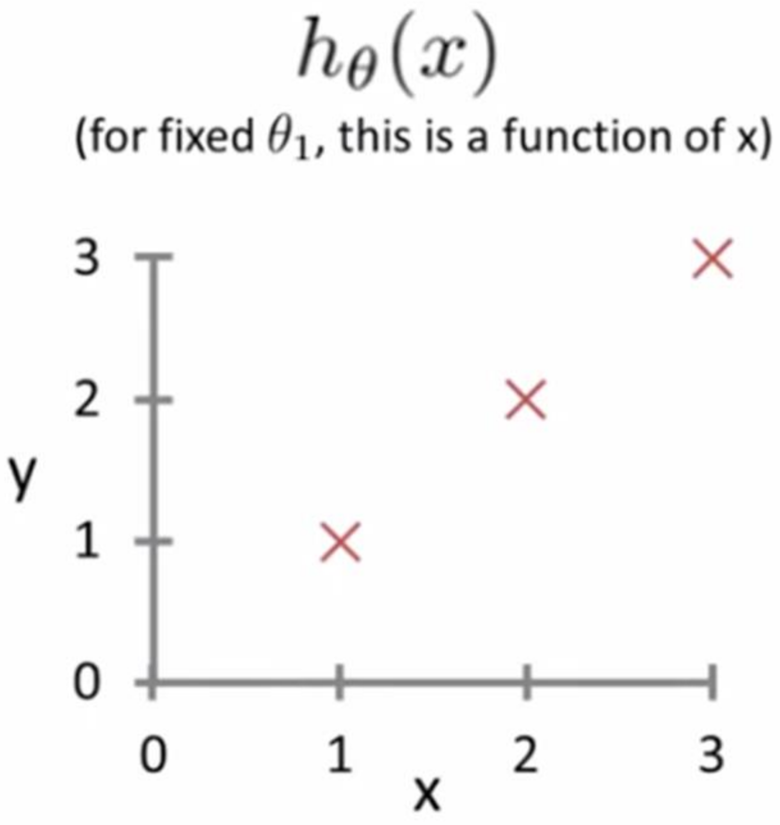
Função de Custo – Intuição 1



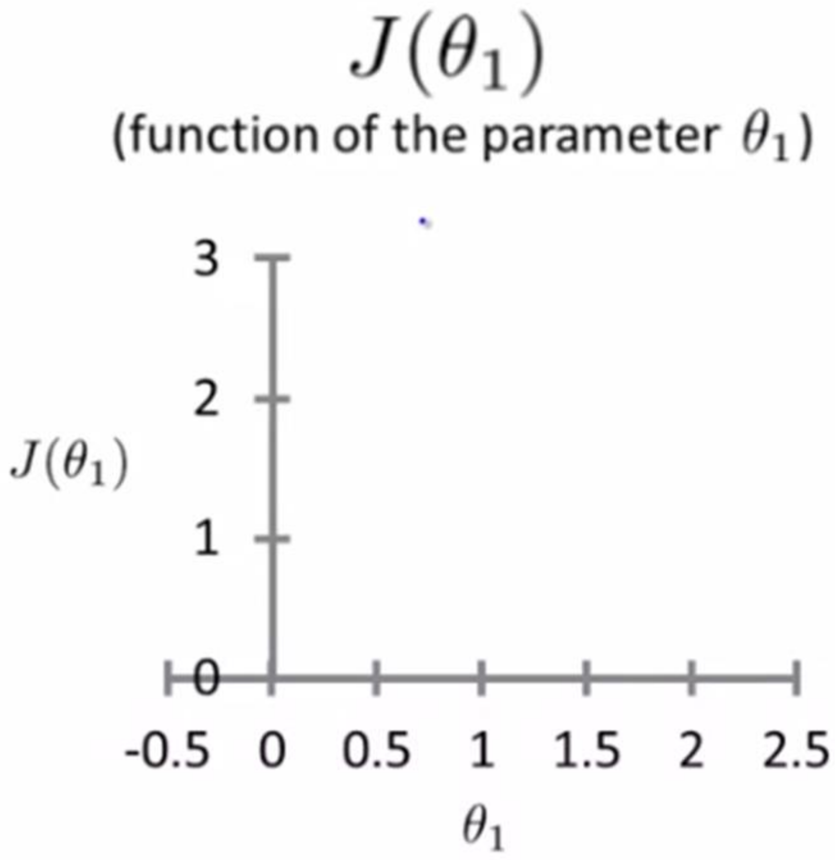
Função de Custo – Intuição 1



Função de Custo – Intuição 1



θ_1 = 0,0



Função de Custo – Intuição 2

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

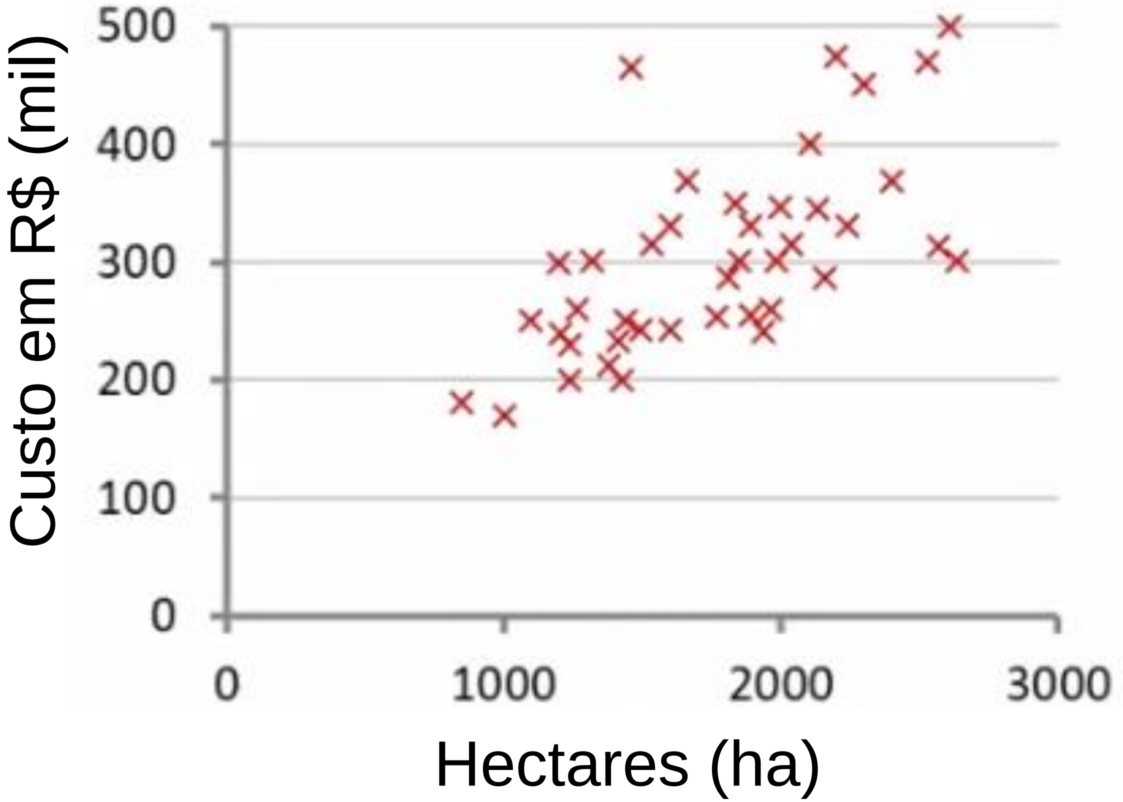
Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\, J(\theta_0, \theta_1)$

Função de Custo – Intuição 2

$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



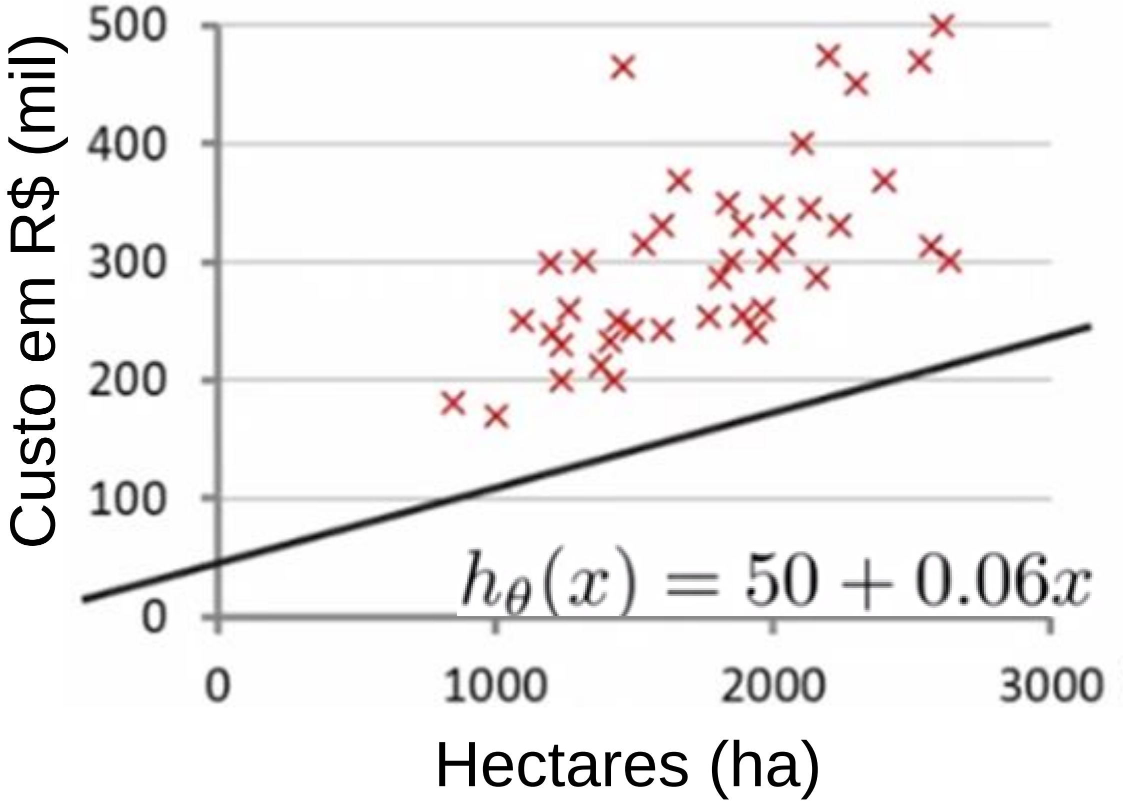
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)

Função de Custo – Intuição 2

$h_\theta(x)$

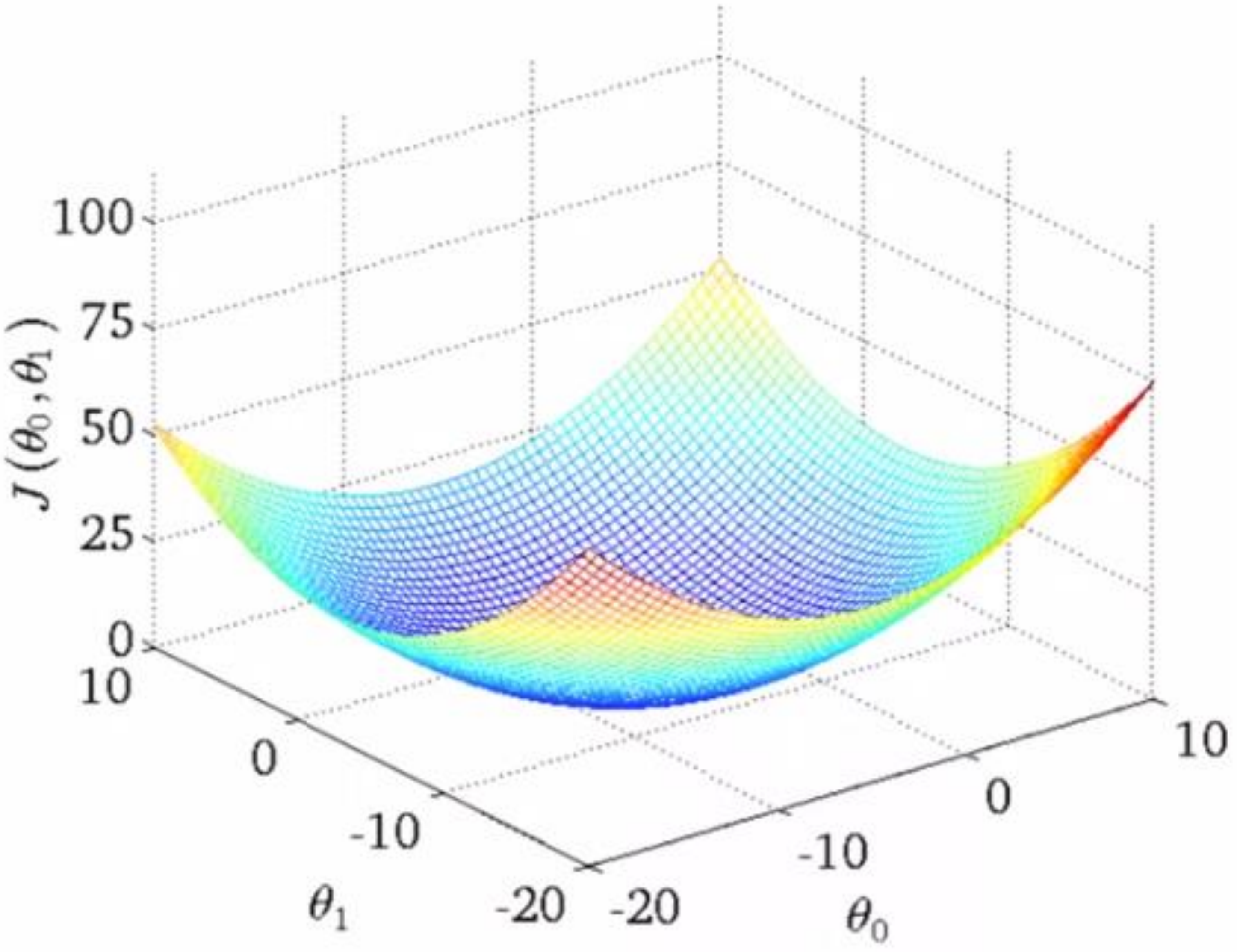
(for fixed θ_0, θ_1, this is a function of x)



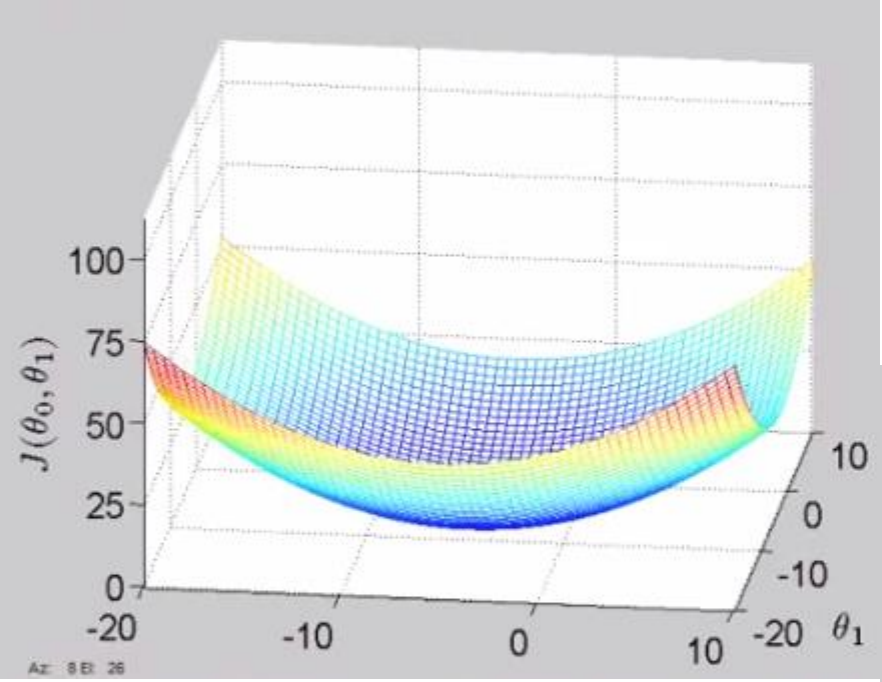
$J(\theta_0, \theta_1)$

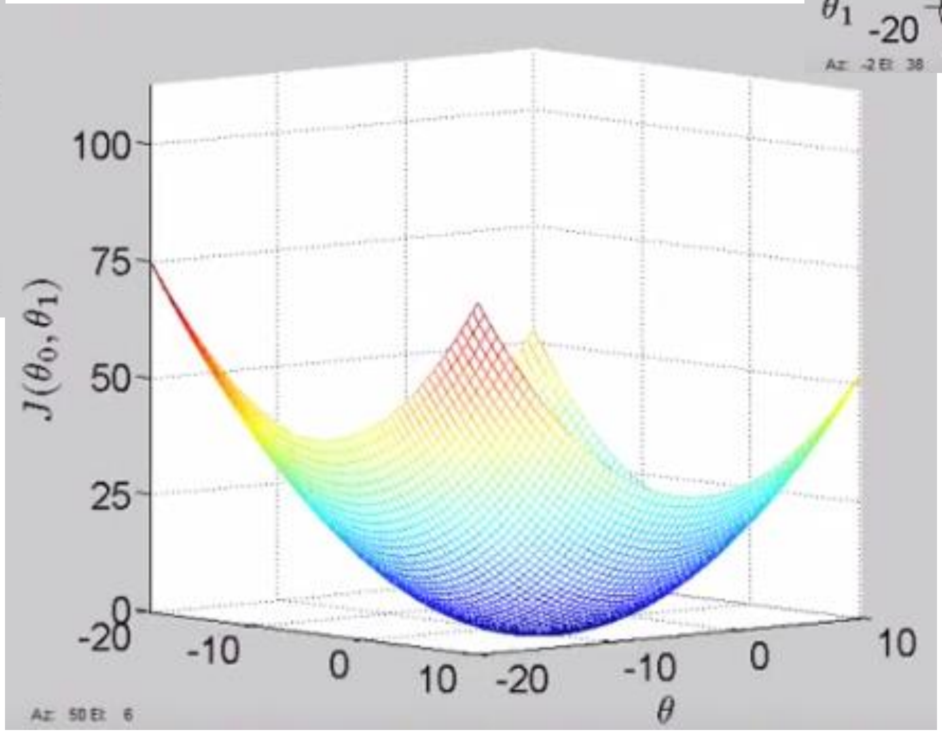
(function of the parameters θ_0, θ_1)

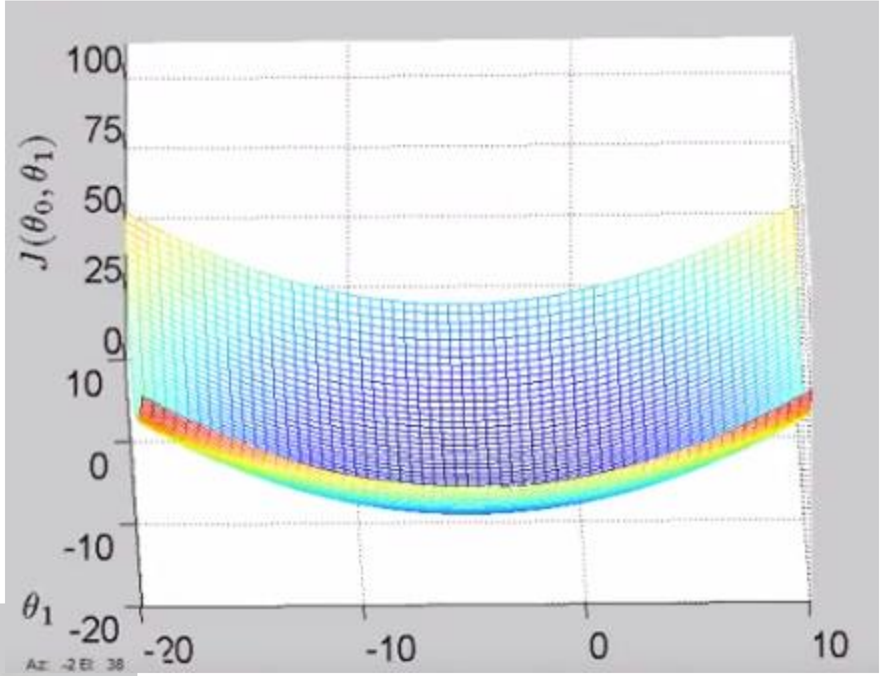
Função de Custo – Intuição 2



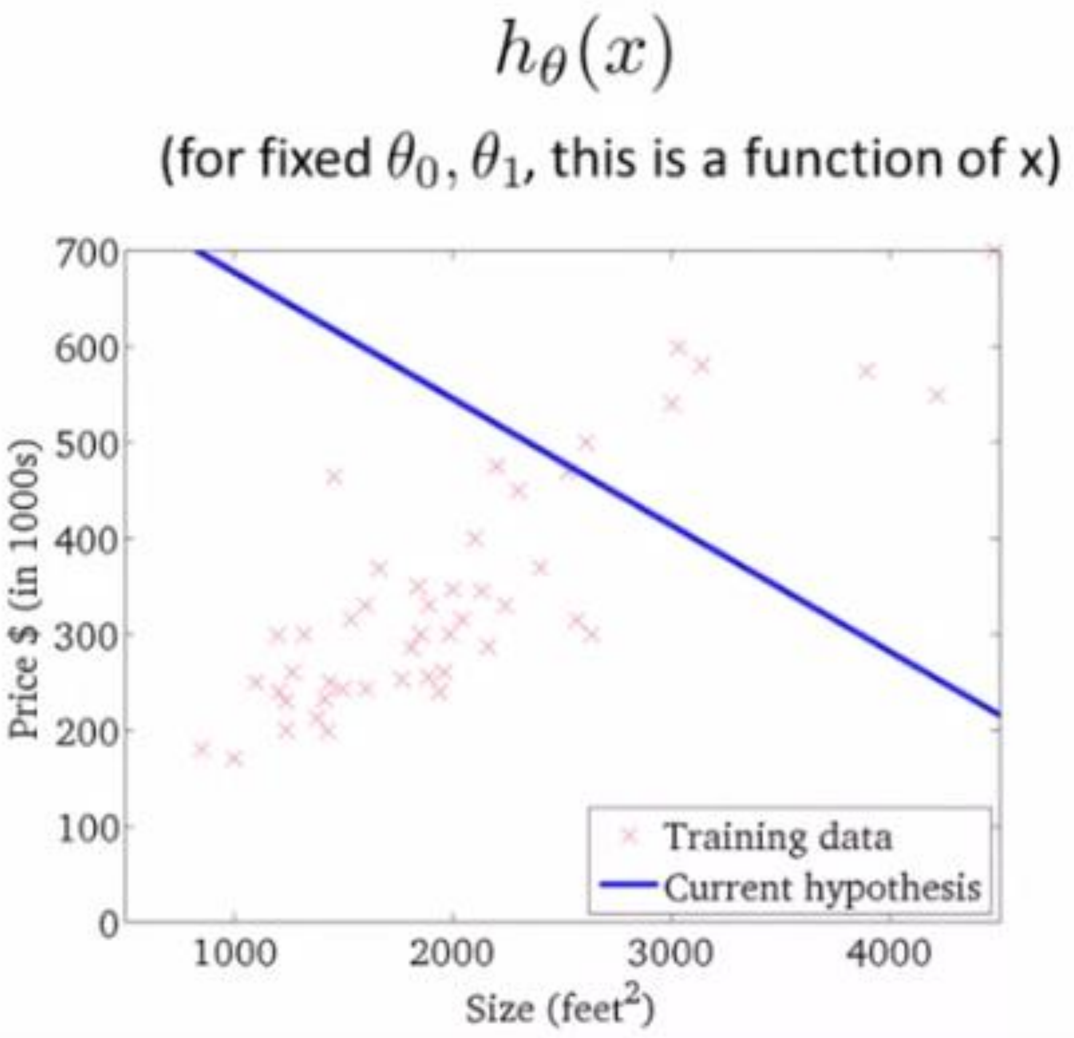
Função de Custo – Intuição 2

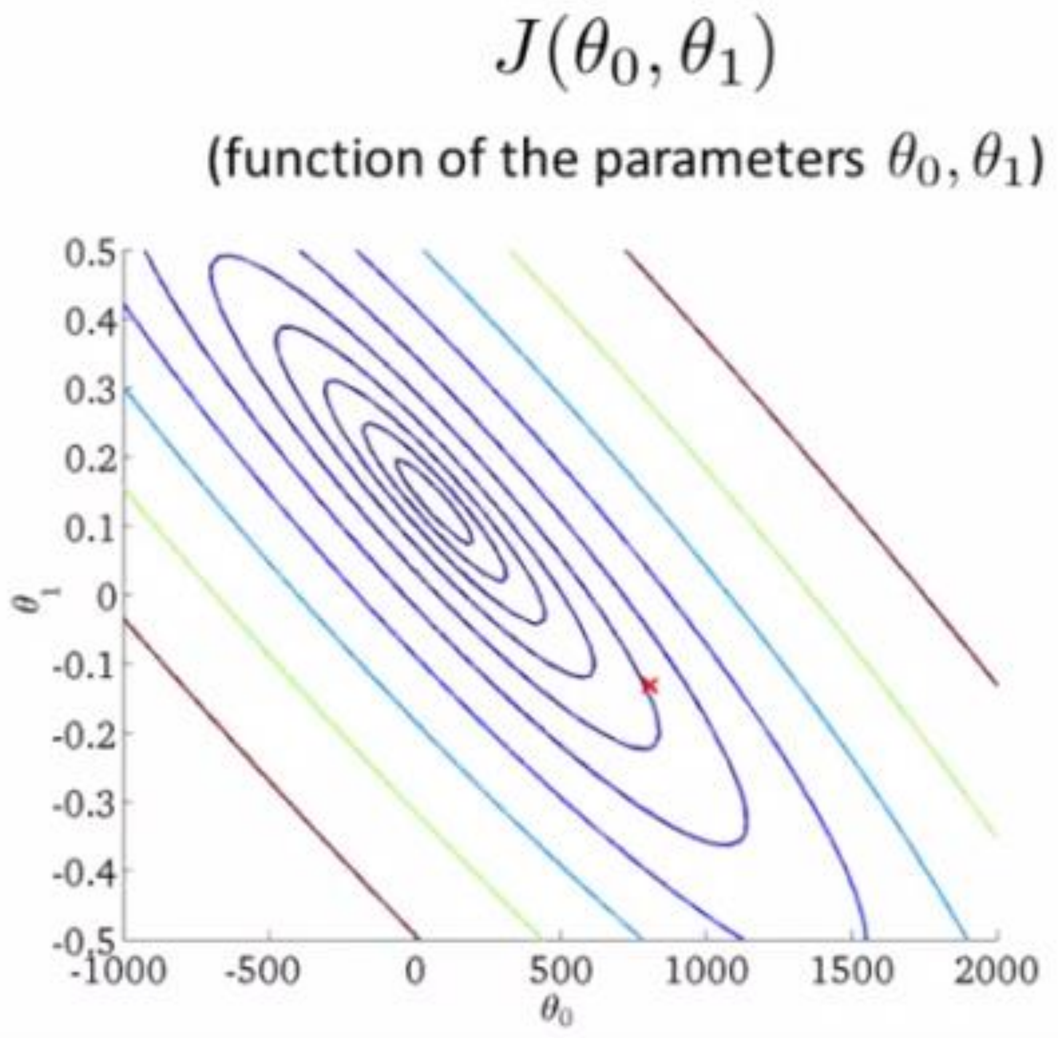






Função de Custo – Intuição 2





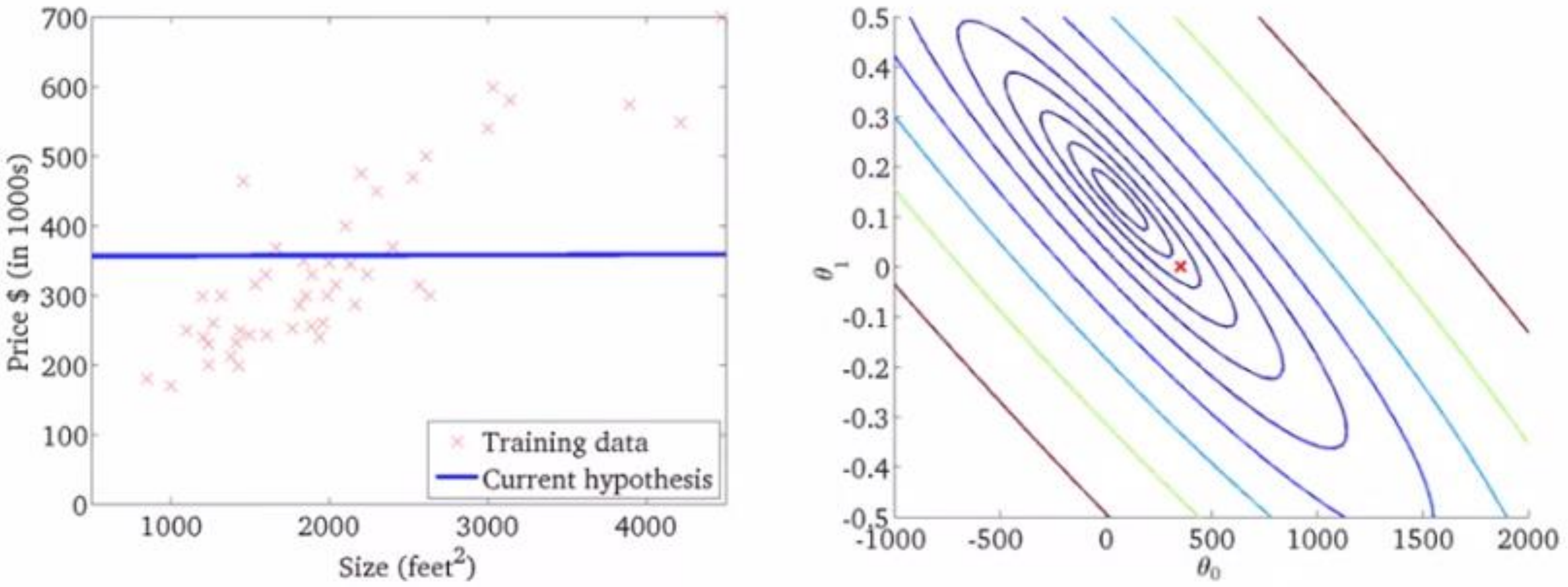
Função de Custo – Intuição 2

$h_\theta(x)$

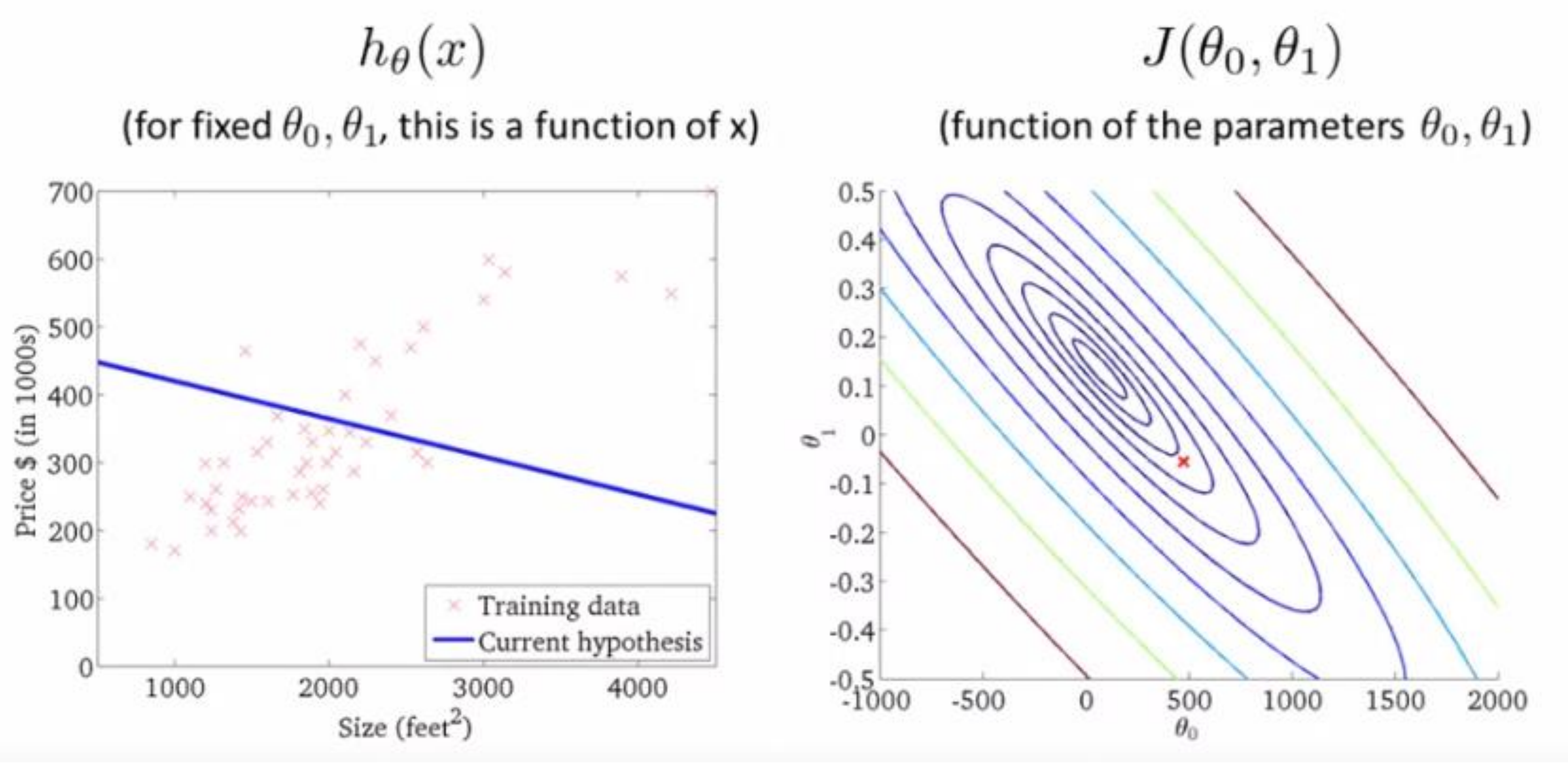
(for fixed θ_0, θ_1, this is a function of x)

$J(\theta_0, \theta_1)$

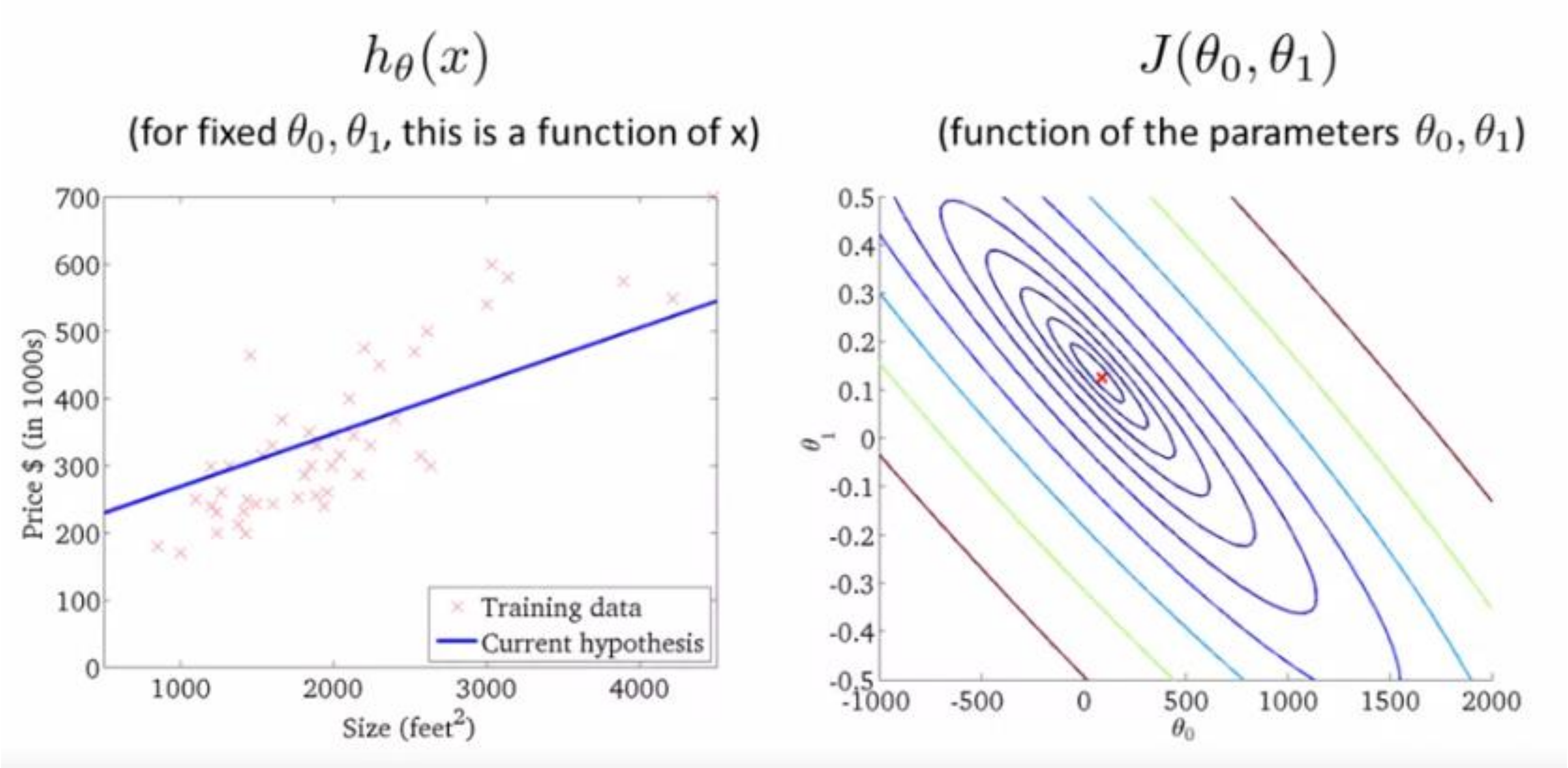
(function of the parameters θ_0, θ_1)



Função de Custo – Intuição 2



Função de Custo – Intuição 2



Gradiente descendente

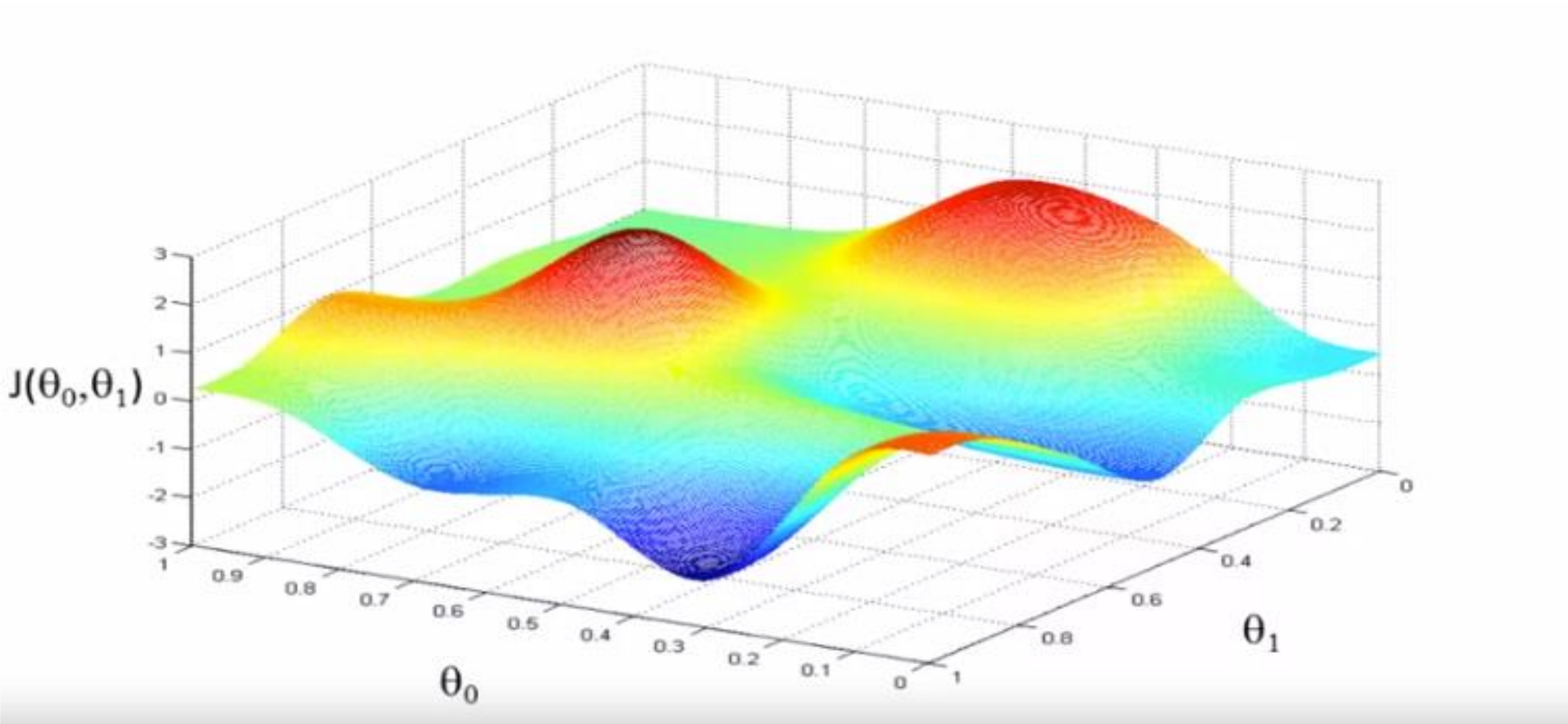
Have some function $J(\theta_0, \theta_1)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

Outline:

- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum

Gradiente descendente



Gradiente descendente

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$

}

Correct: Simultaneous update

$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

$\theta_0 := \text{temp0}$

$\theta_1 := \text{temp1}$

Incorrect:

→ $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

→ $\theta_0 := \text{temp0}$

→ $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

→ $\theta_1 := \text{temp1}$

Gradiente descendente - Intuição

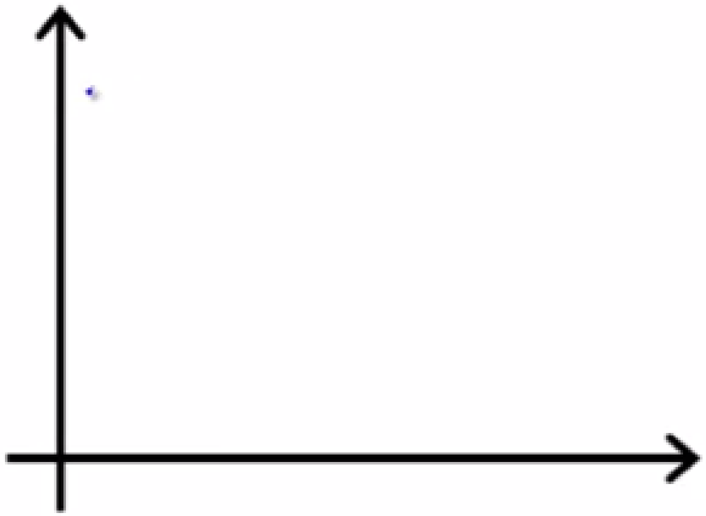
Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad \text{(simultaneously update } j = 0 \text{ and } j = 1\text{)}$$

}

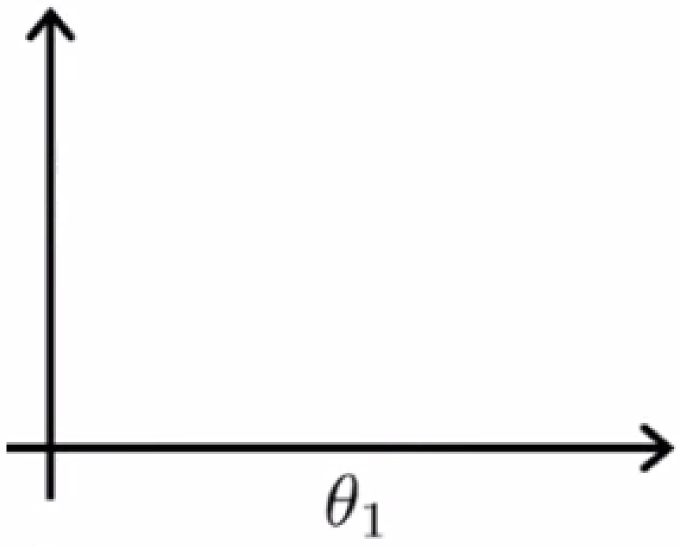
Gradiente descendente - Intuição



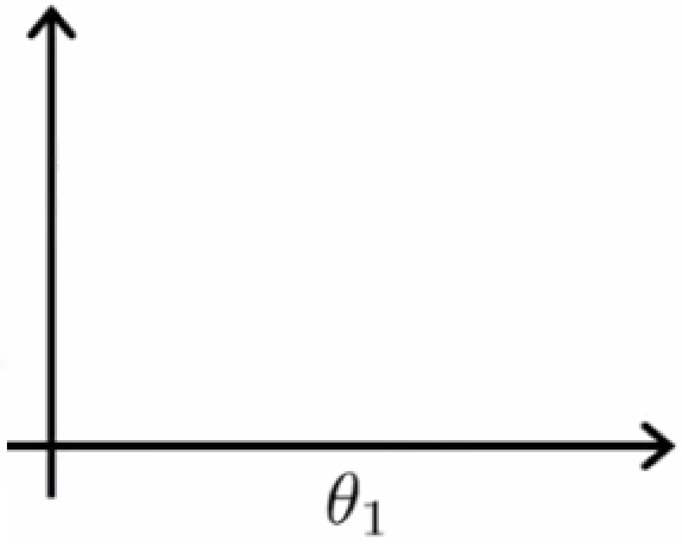
Gradiente descendente - Intuição

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

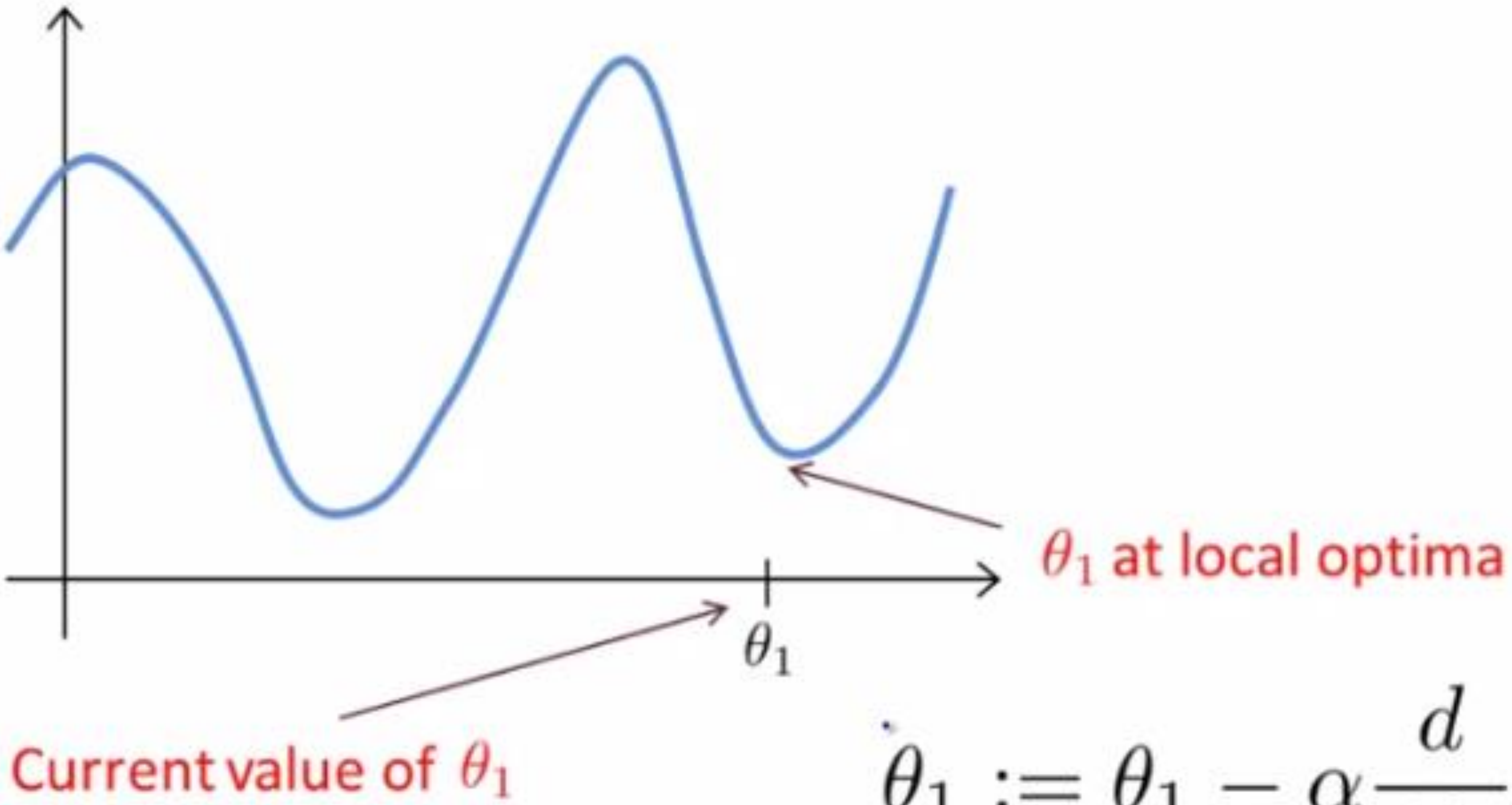
If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



Gradiente descendente - Intuição



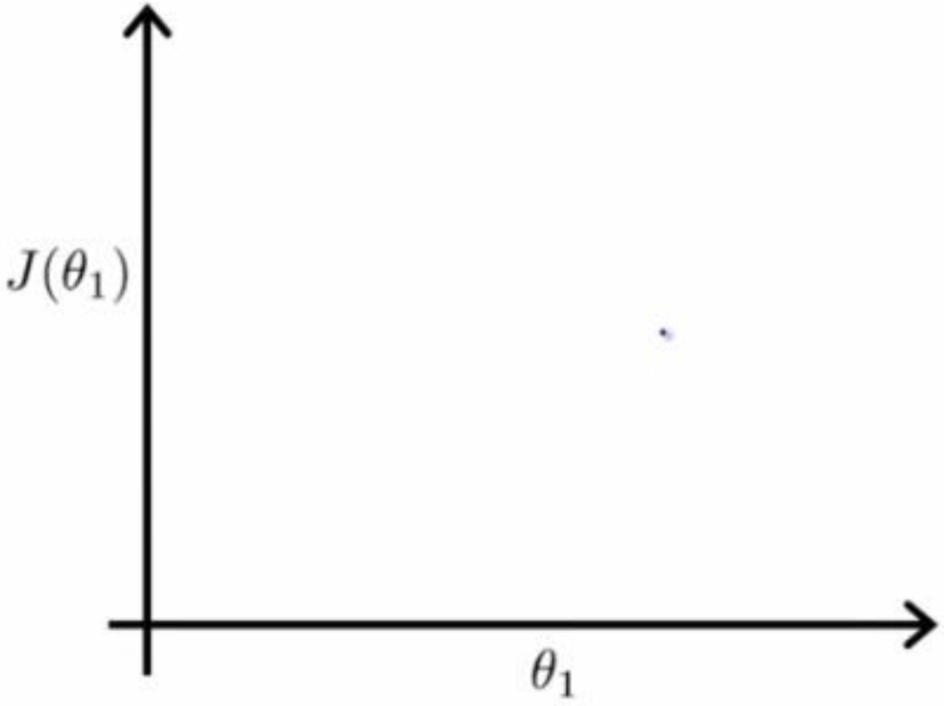
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradiente descendente - Intuição

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.



Gradiente descendente → Regressão Linear

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Gradiente descendente → Regressão Linear

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) =$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) =$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) =$$

Gradiente descendente → Regressão Linear

Gradient descent algorithm

repeat until convergence {

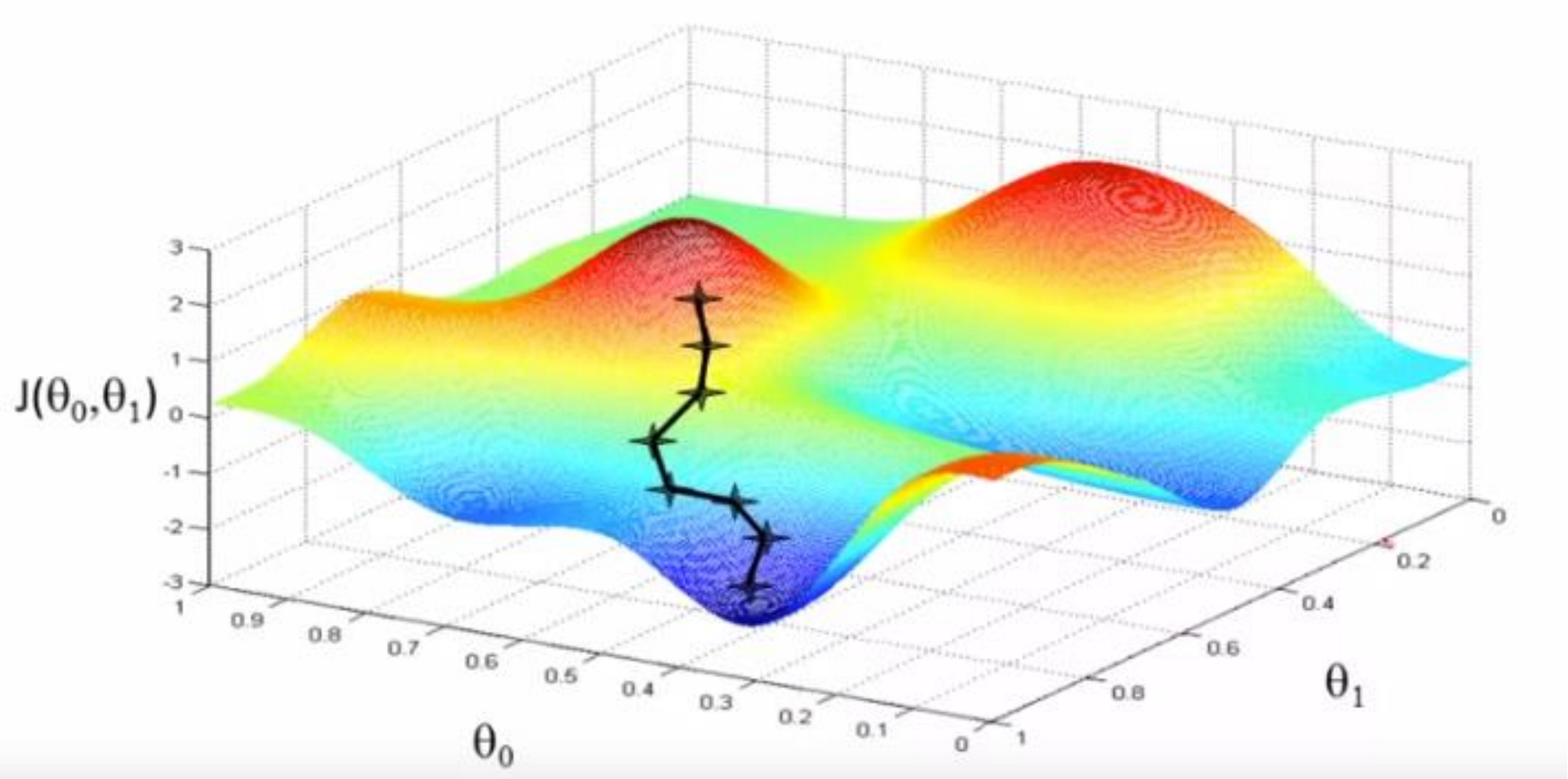
$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

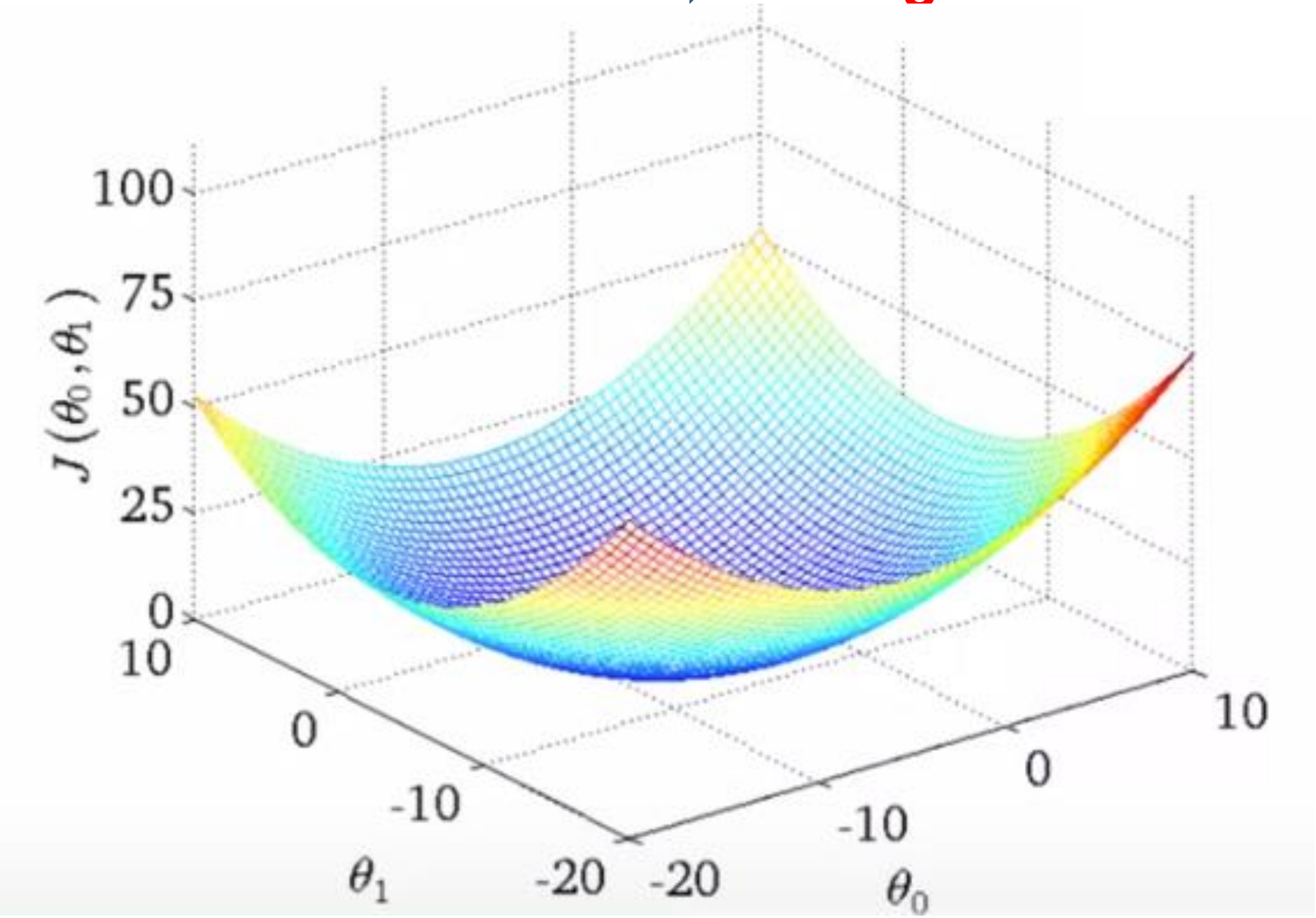
}

update θ_0 and θ_1 simultaneously

Gradiente descendente → Regressão Linear



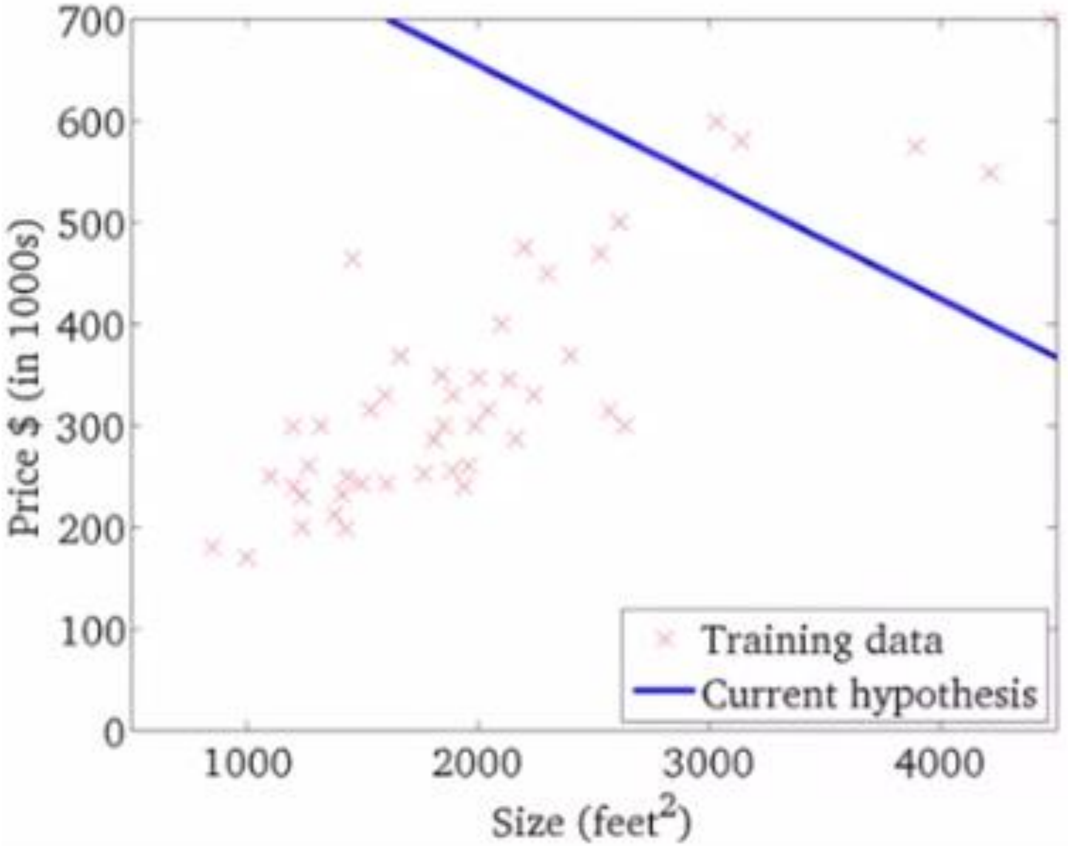
Gradiente descendente → Regressão Linear



Gradiente descendente → Regressão Linear

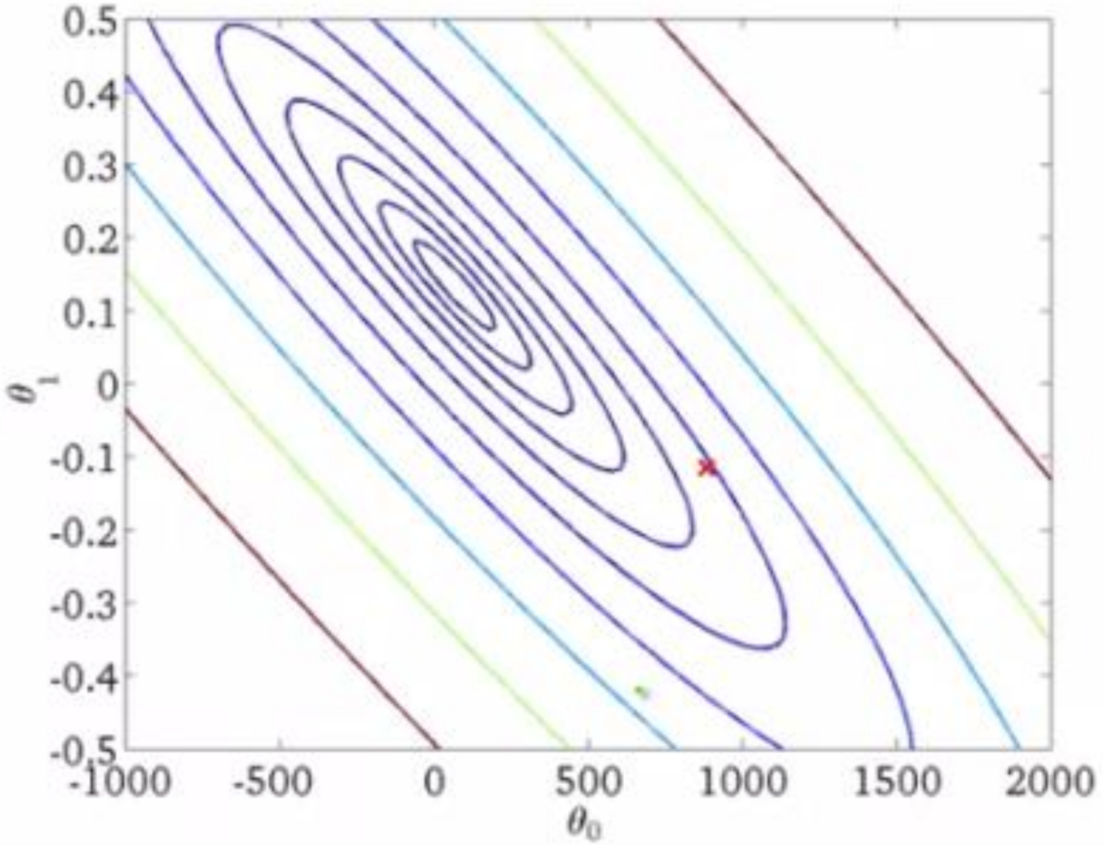
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



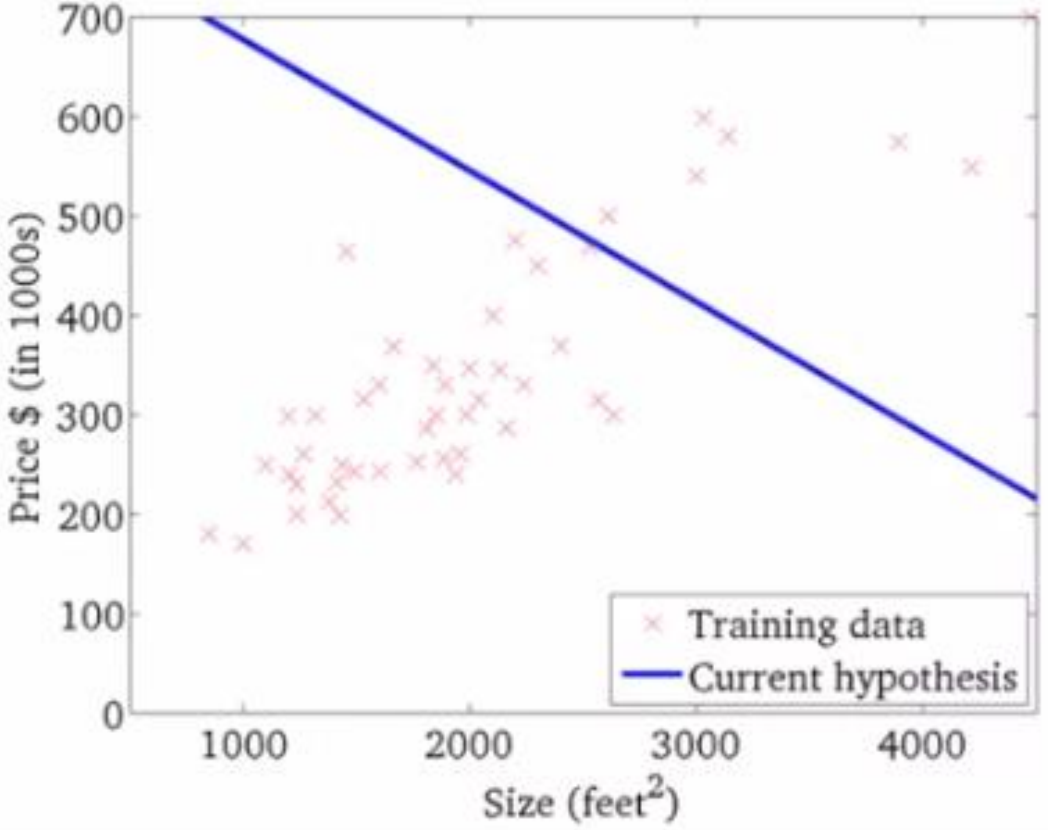
Gradiente descendente → Regressão Linear

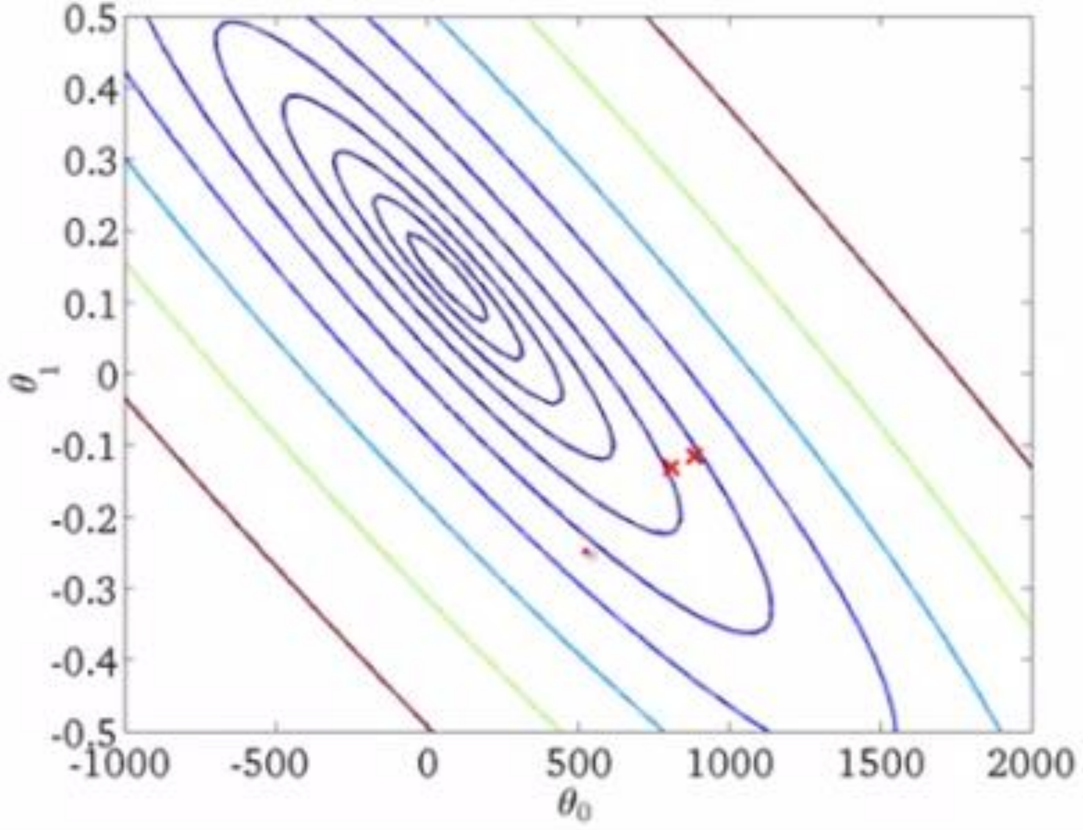
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)

$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

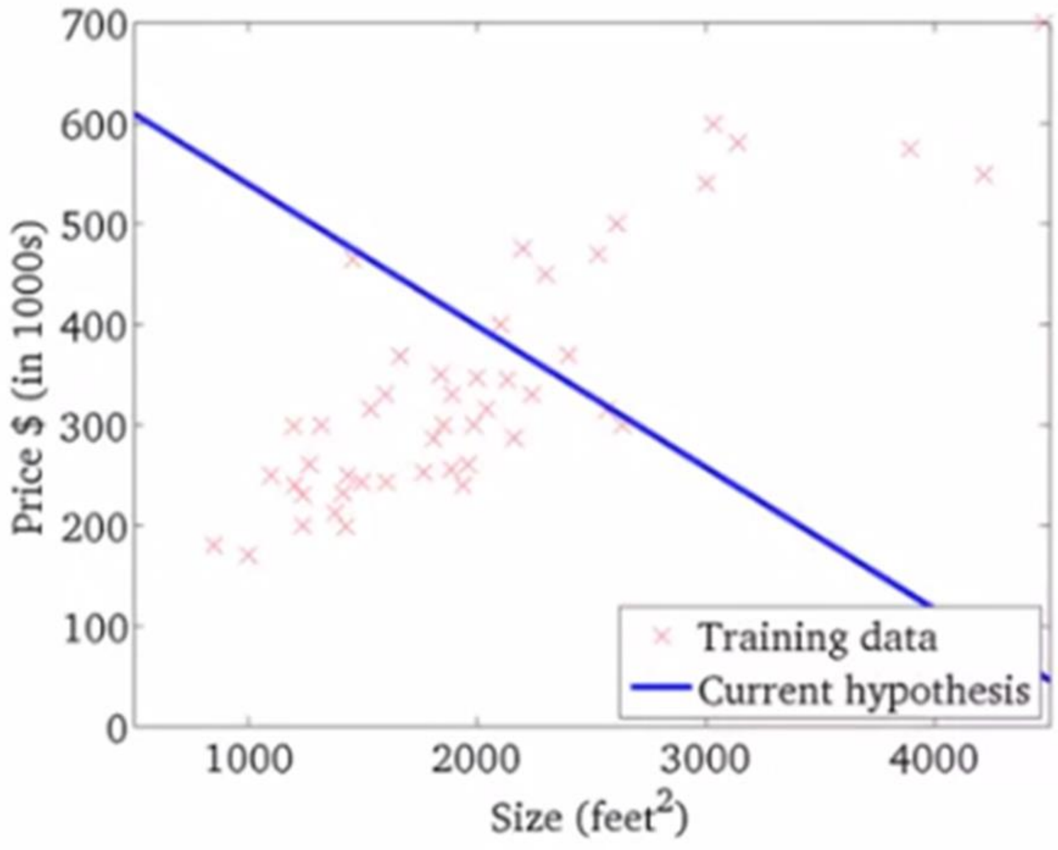




Gradiente descendente → Regressão Linear

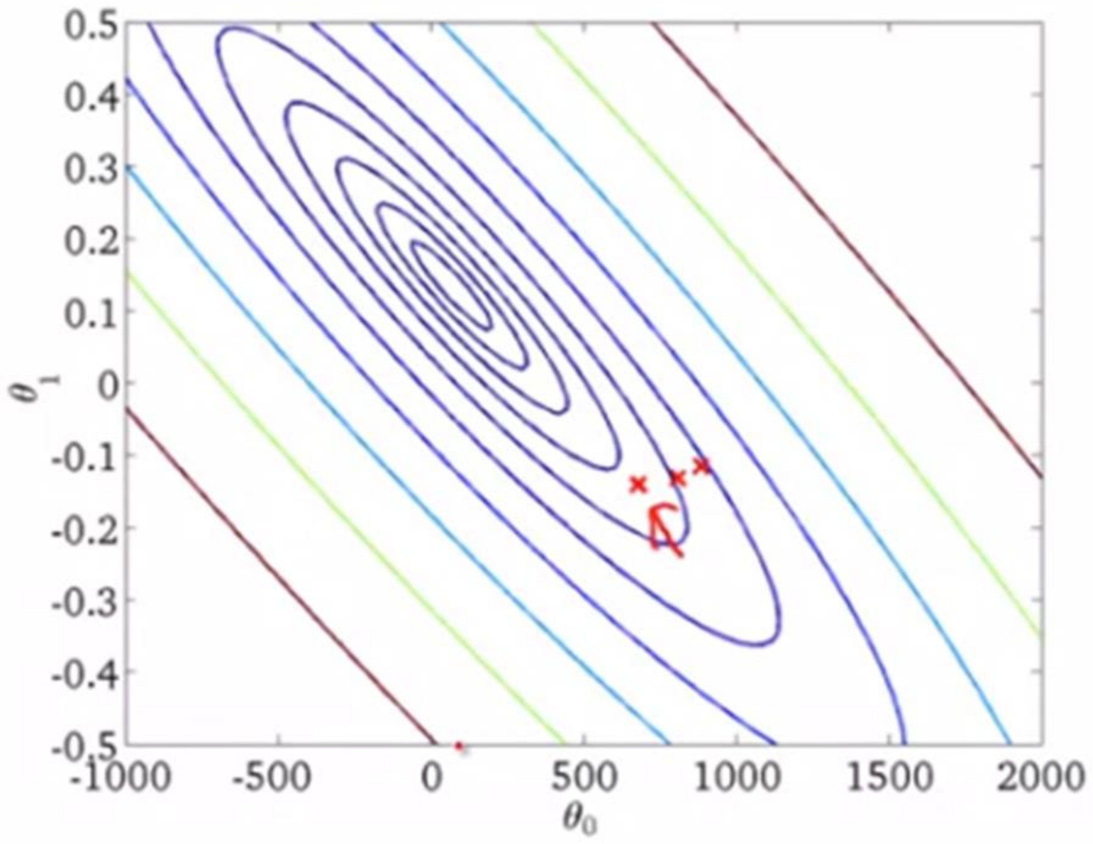
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

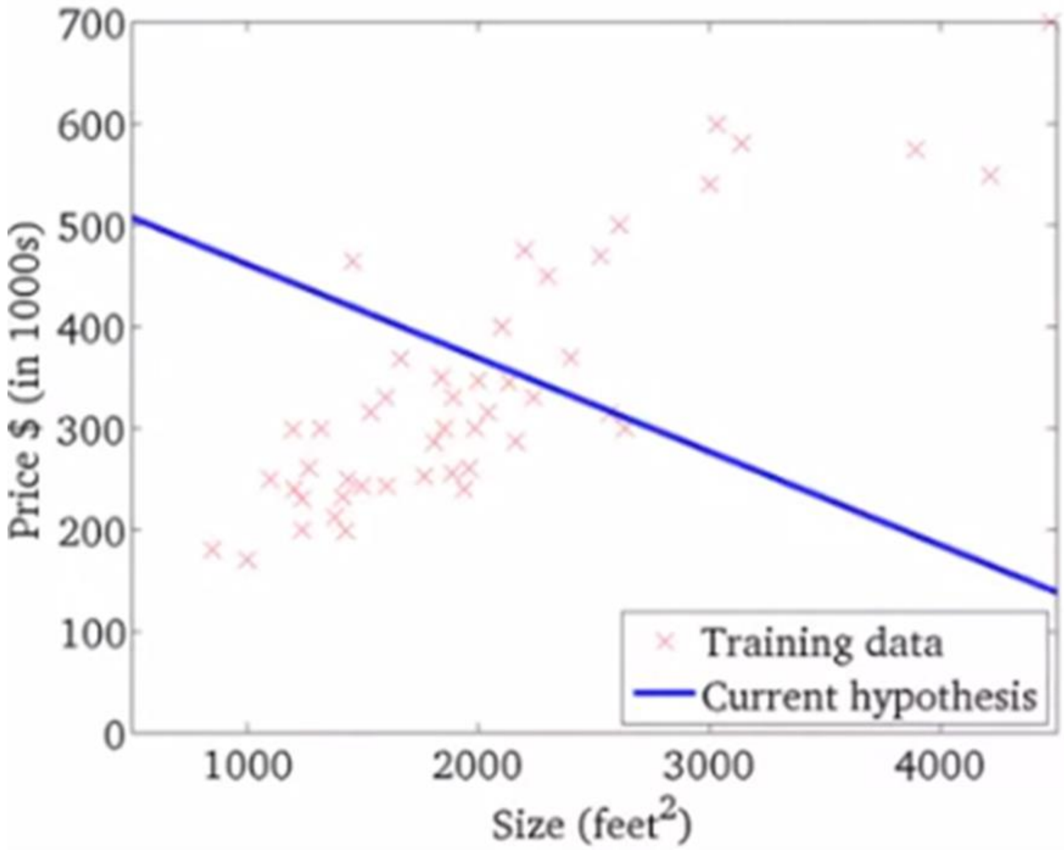
(function of the parameters θ_0, θ_1)



Gradiente descendente → Regressão Linear

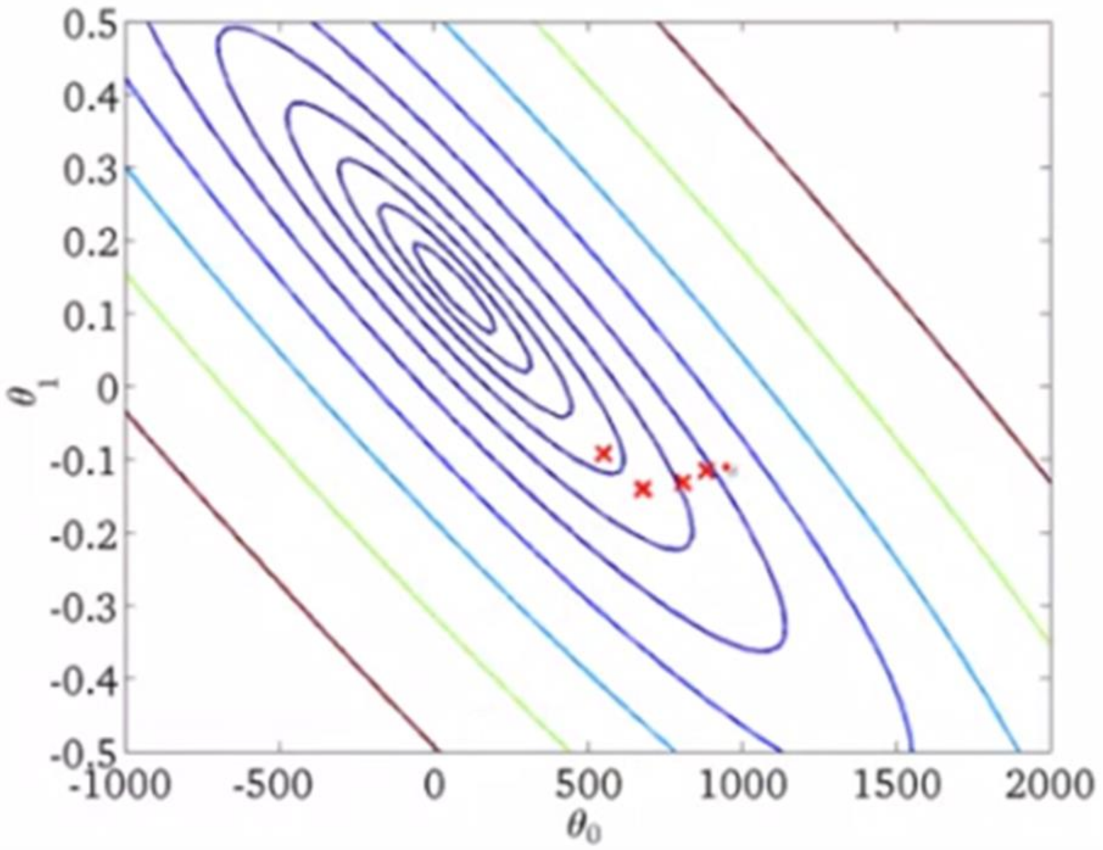
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)

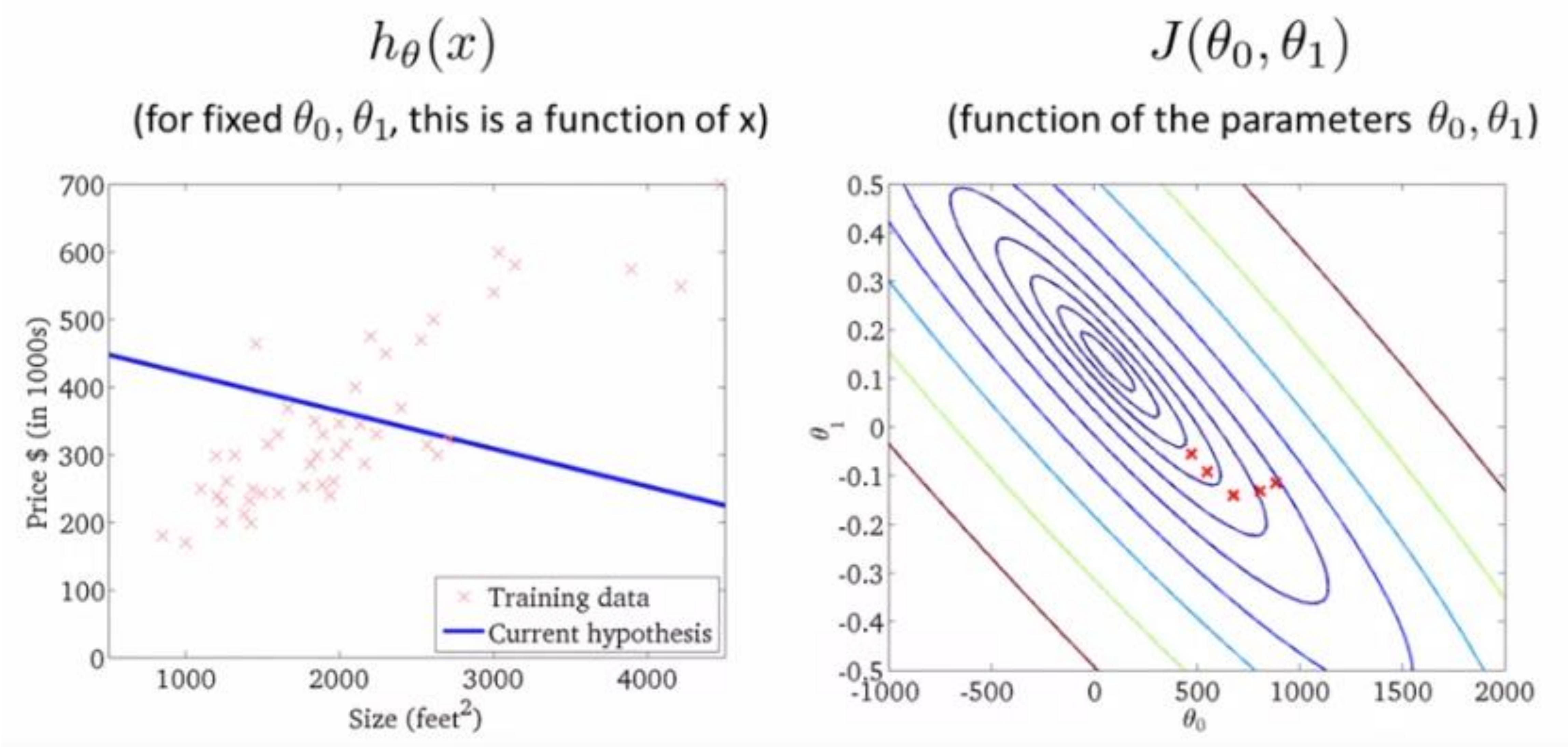


$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



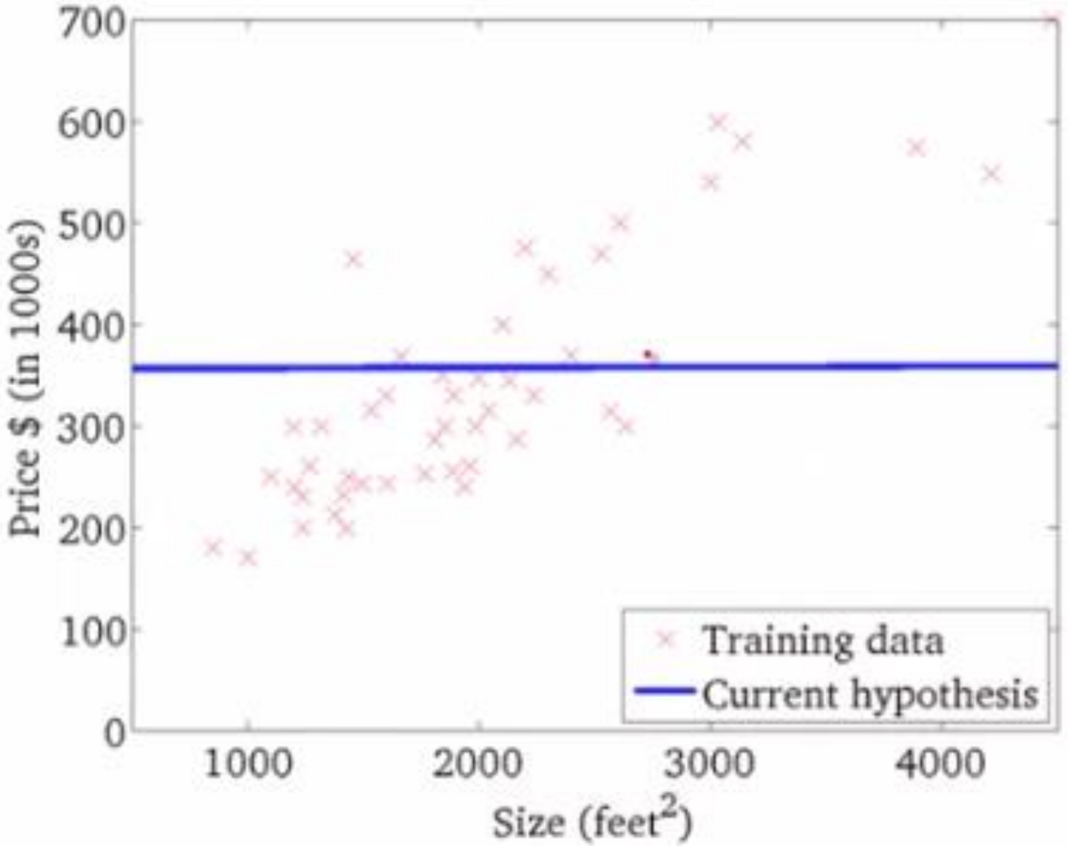
Gradiente descendente → Regressão Linear



Gradiente descendente → Regressão Linear

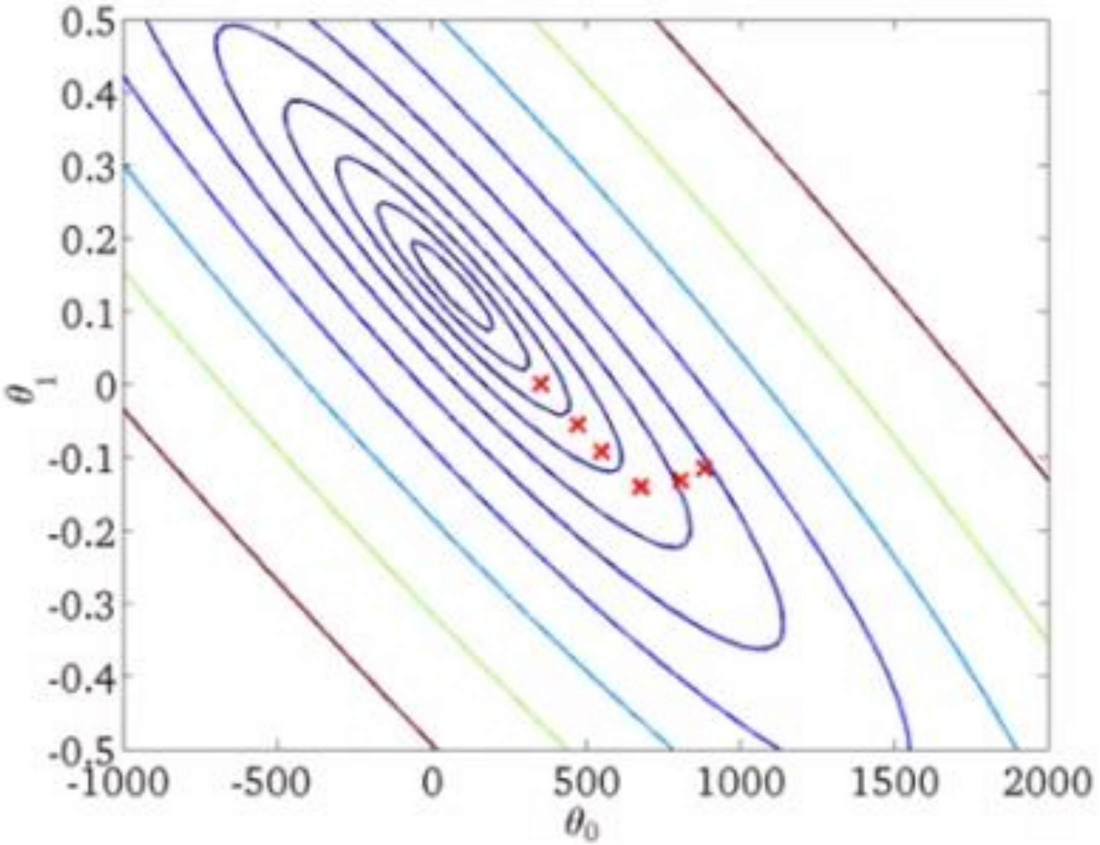
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

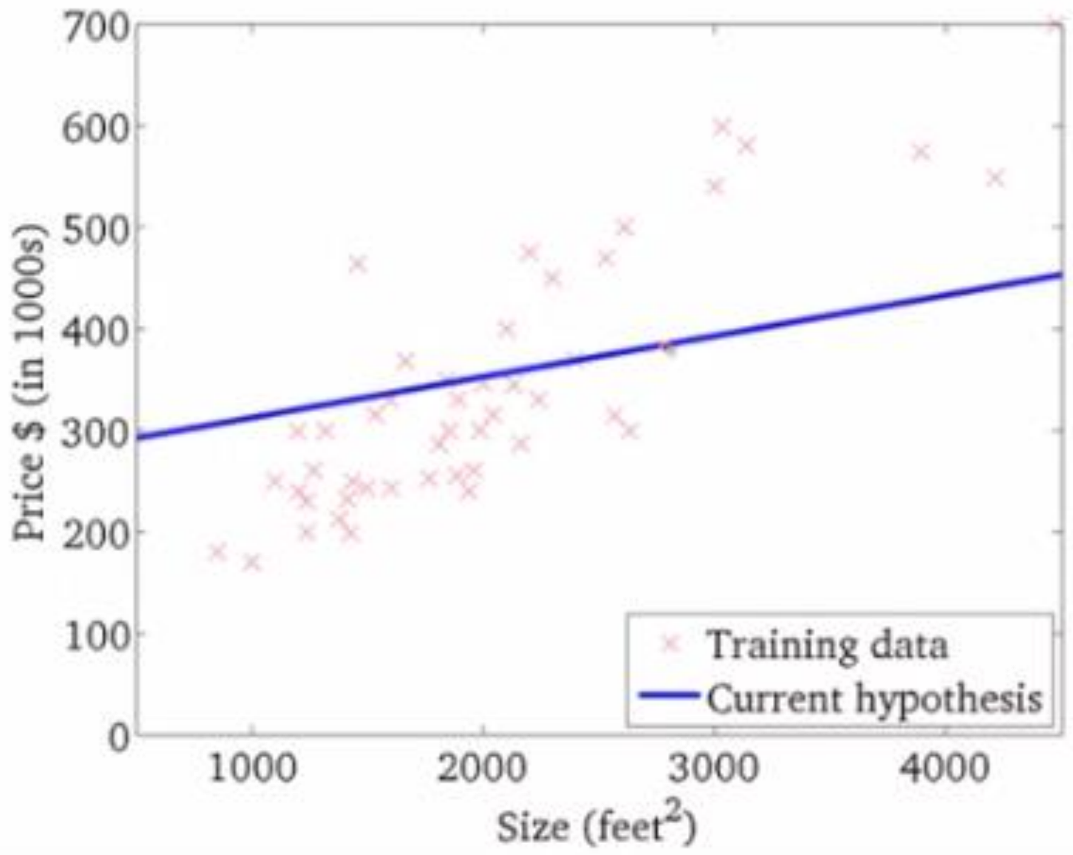
(function of the parameters θ_0, θ_1)



Gradiente descendente → Regressão Linear

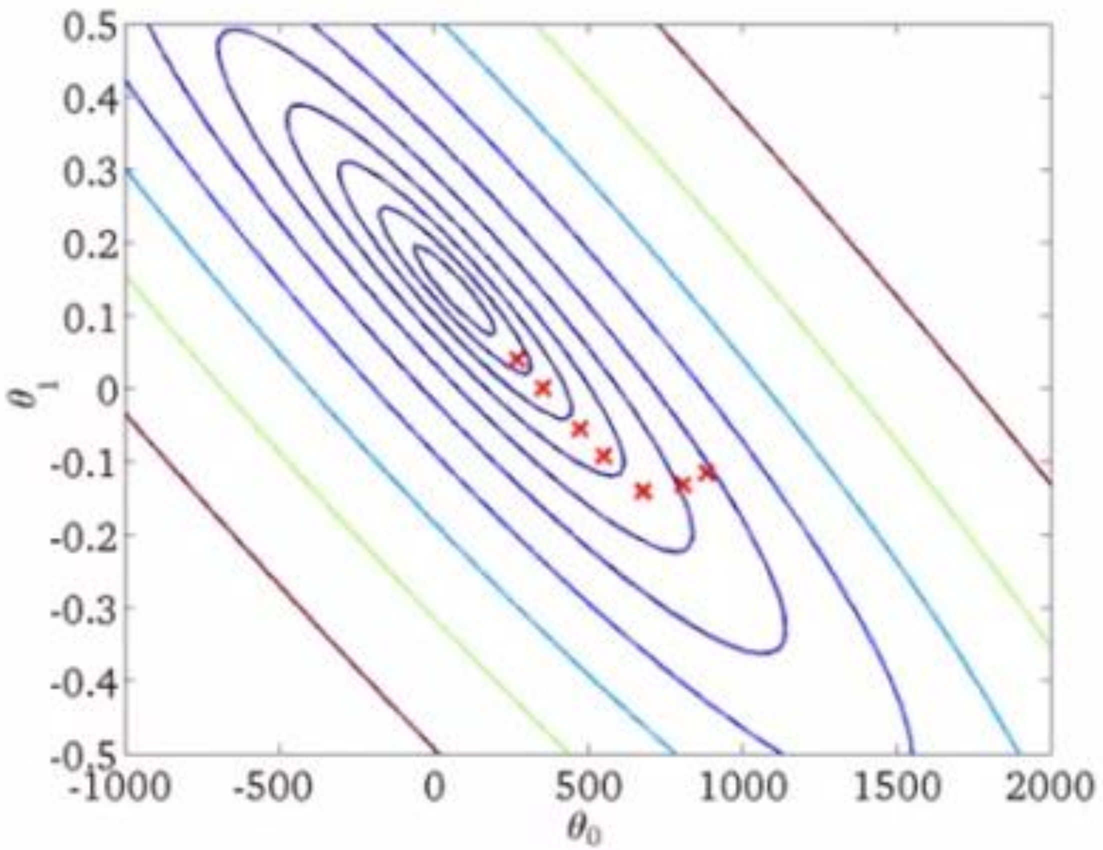
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

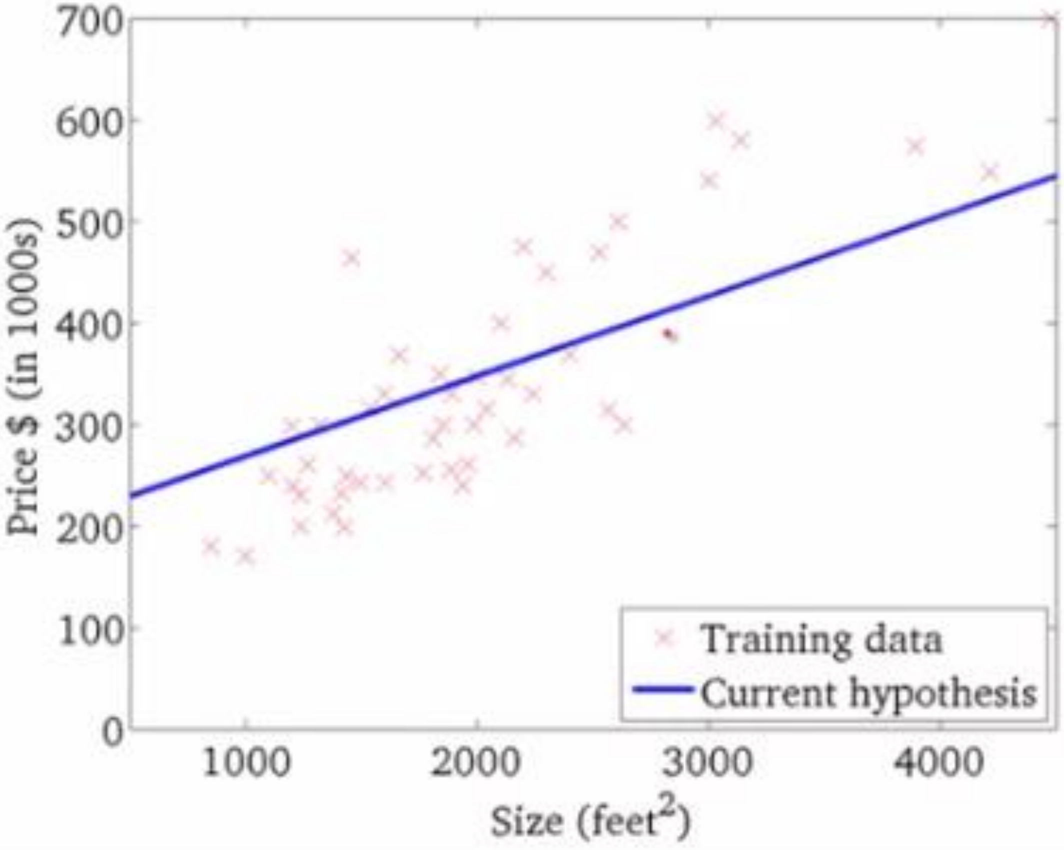
(function of the parameters θ_0, θ_1)



Gradiente descendente → Regressão Linear

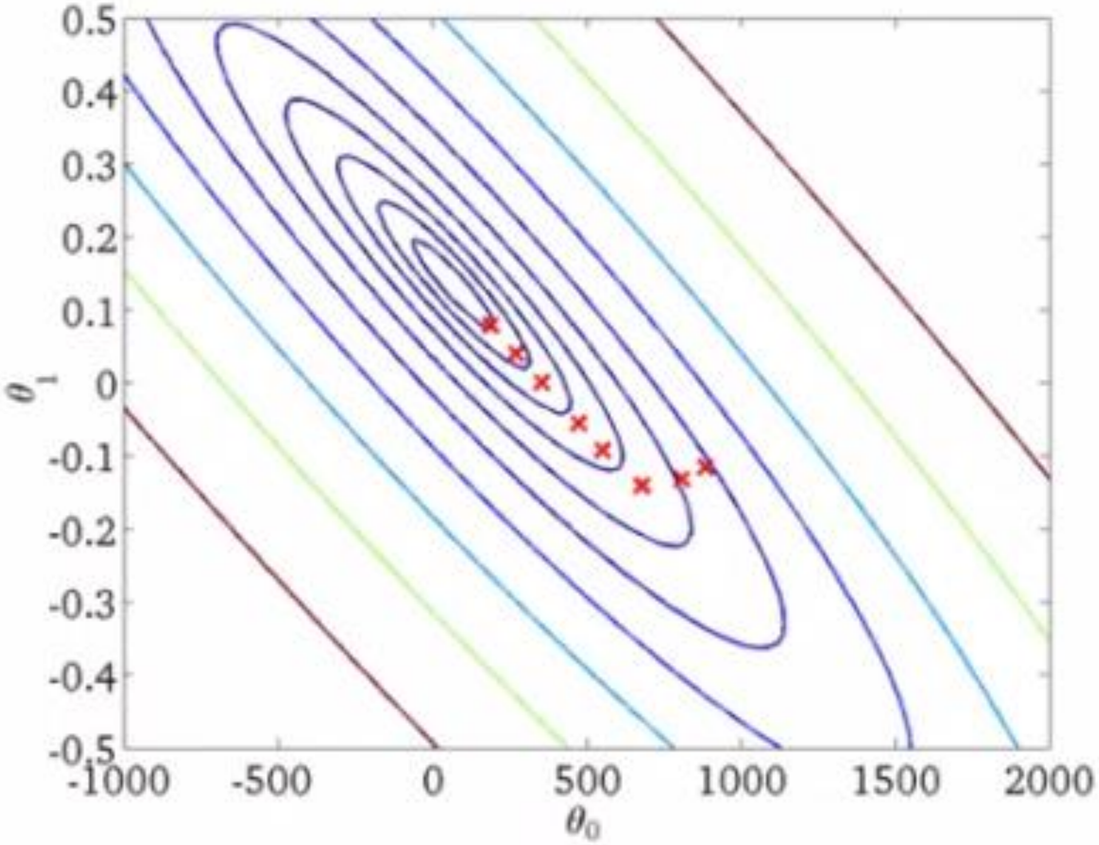
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

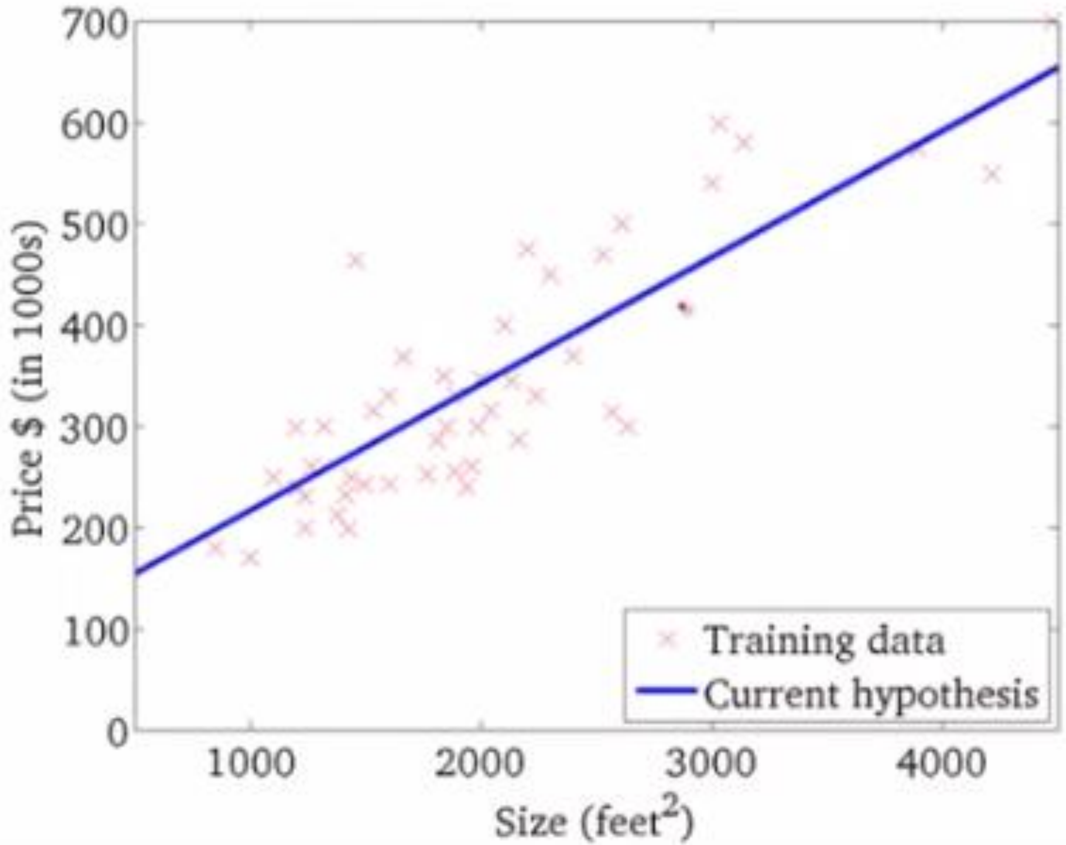
(function of the parameters θ_0, θ_1)



Gradiente descendente → Regressão Linear

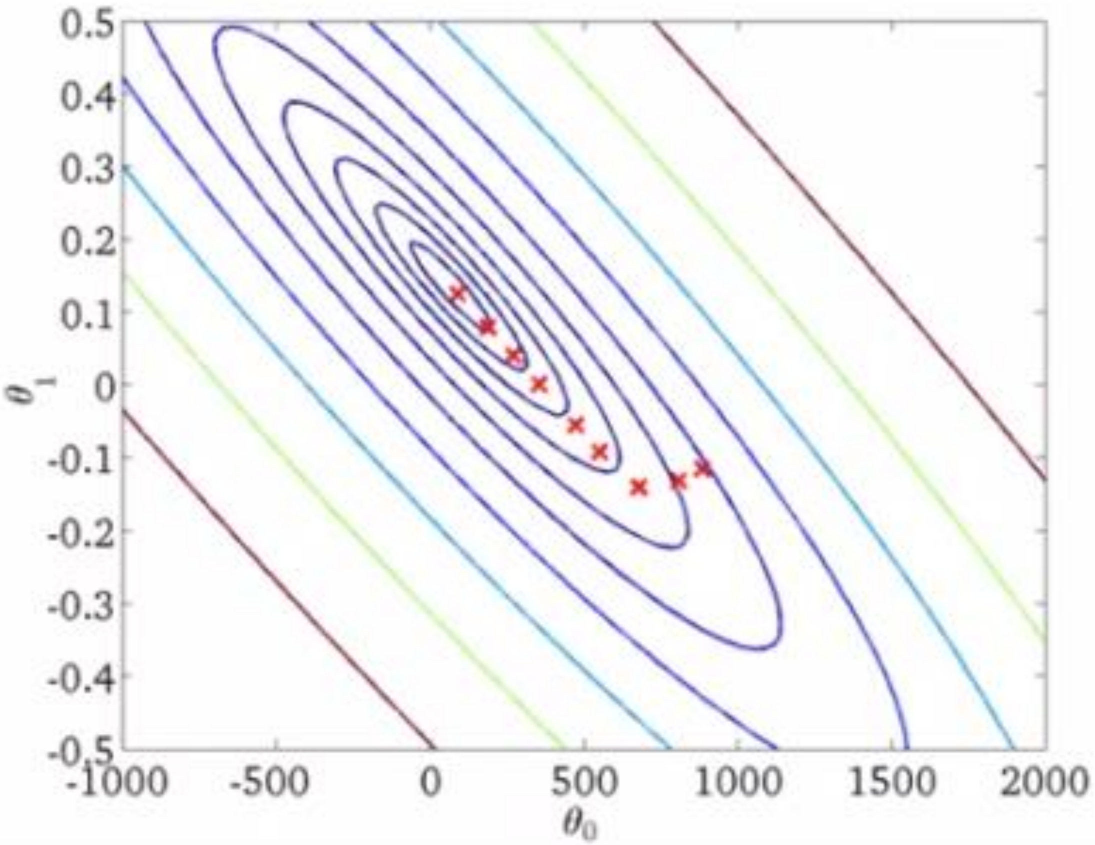
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



Reconhecimento de padrões e aprendizagem computacional

Regressão Linear Múltipla

Múltiplas variáveis

Tamanho (x)	Custo R$ (mil) (y)
2104	460
1416	232
1534	315
852	178
...	...

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Múltiplas variáveis – Exemplo do preço de uma casa

Size (feet²)	Number of bedrooms	Number of floors	Age of home (years)	Price ($1000)
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
...	...	...	...	...

Notation:

n = number of features

$x^{(i)}$ = input (features) of i^{th} training example.

$x_j^{(i)}$ = value of feature j in i^{th} training example.

Múltiplas variáveis

Hypothesis:

Previously: $\cancel{h_\theta(x) = \theta_0 + \theta_1 x}$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

Para simplificar e facilitar o uso da equação: $x_0 = 1.$

$$h_\theta(x) = \begin{bmatrix} \theta_0 & \theta_1 & \ldots & \theta_n \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \theta^T x$$

Múltiplas variáveis – Gradiente descendente

Hypothesis: $h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$

Parameters: $\theta_0, \theta_1, \ldots, \theta_n$

Cost function:

$$J(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Gradient descent:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \ldots, \theta_n)$$

$\}$ (simultaneously update for every $j = 0, \ldots, n$)

Múltiplas variáveis – Gradiente descendente

Gradient Descent

Previously (n=1):

Repeat {

$$\theta_0 := \theta_0 - \alpha \underbrace{\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})}_{\frac{\partial}{\partial \theta_0} J(\theta)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New algorithm $(n \geq 1)$:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \ldots, n$)

}

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$$\ldots$$

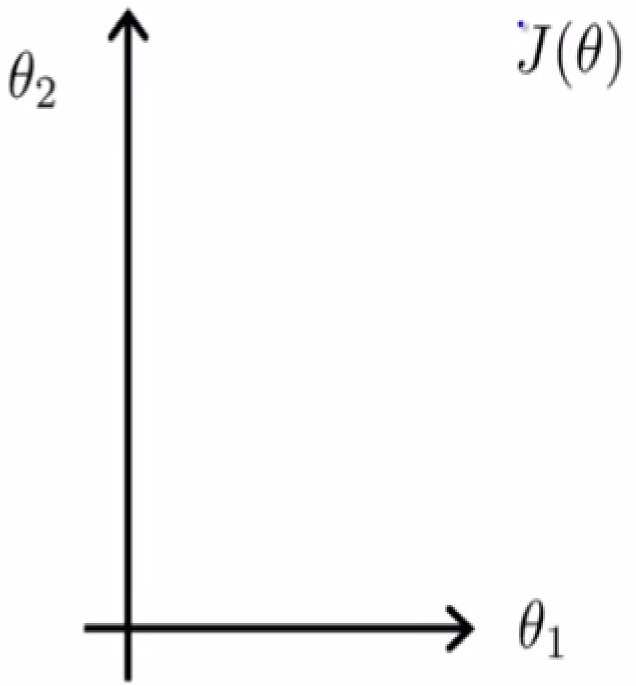
Gradiente descendente – “Truques para melhorá-lo”

Feature Scaling

Idea: Make sure features are on a similar scale.

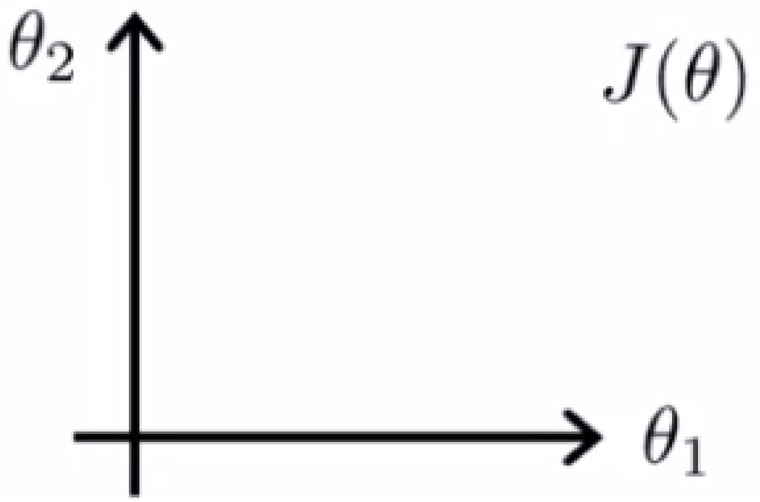
E.g. x_1 = size (0-2000 feet^2)

x_2 = number of bedrooms (1-5)



$$x_1 = \frac{\text{size (feet}^2)}{2000}$$

$$x_2 = \frac{\text{number of bedrooms}}{5}$$



Gradiente descendente – “Truques para melhorá-lo”

Mean normalization

Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean (Do not apply to $x_0 = 1$).

E.g. $x_1 = \dfrac{size - 1000}{2000}$

$x_2 = \dfrac{\#bedrooms - 2}{5}$

$$-0.5 \le x_1 \le 0.5, -0.5 \le x_2 \le 0.5$$

Para ficar mais próximo a |0,5| pode-se utilizar o “*range*” no quociente da normalização.

$$\boxed{x_i := \frac{x_i - \mu_i}{s_i}}$$

Gradiente descendente – “Truques para melhorá-lo”

Gradient descent

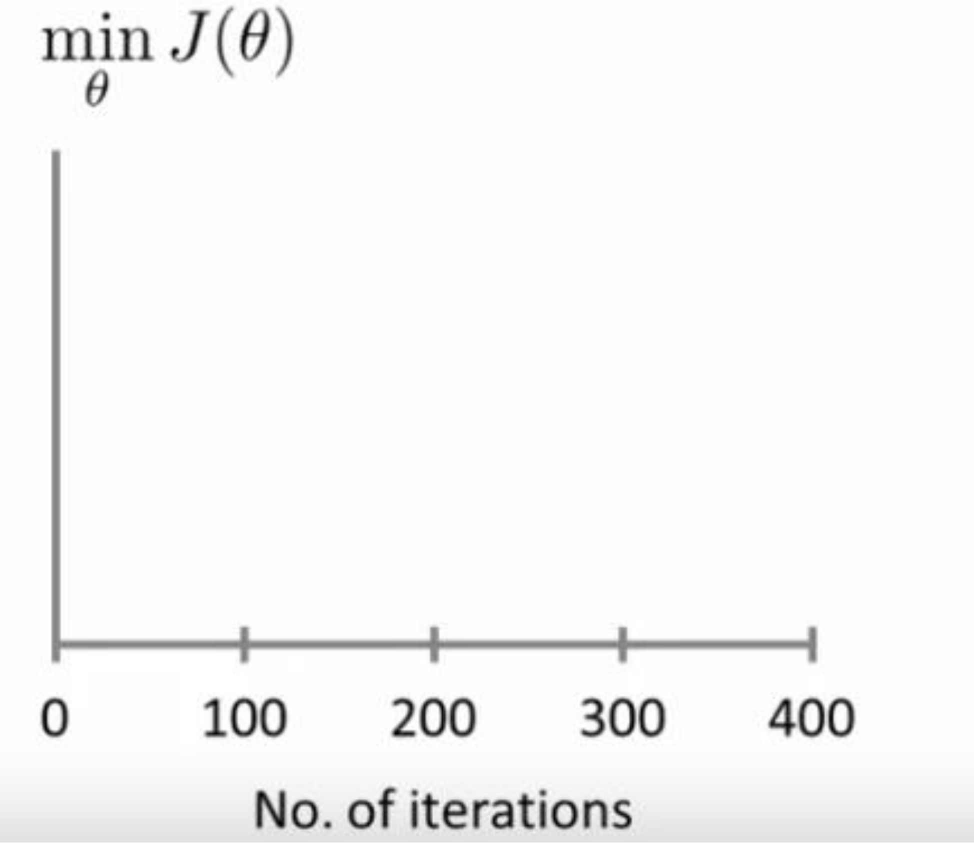
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

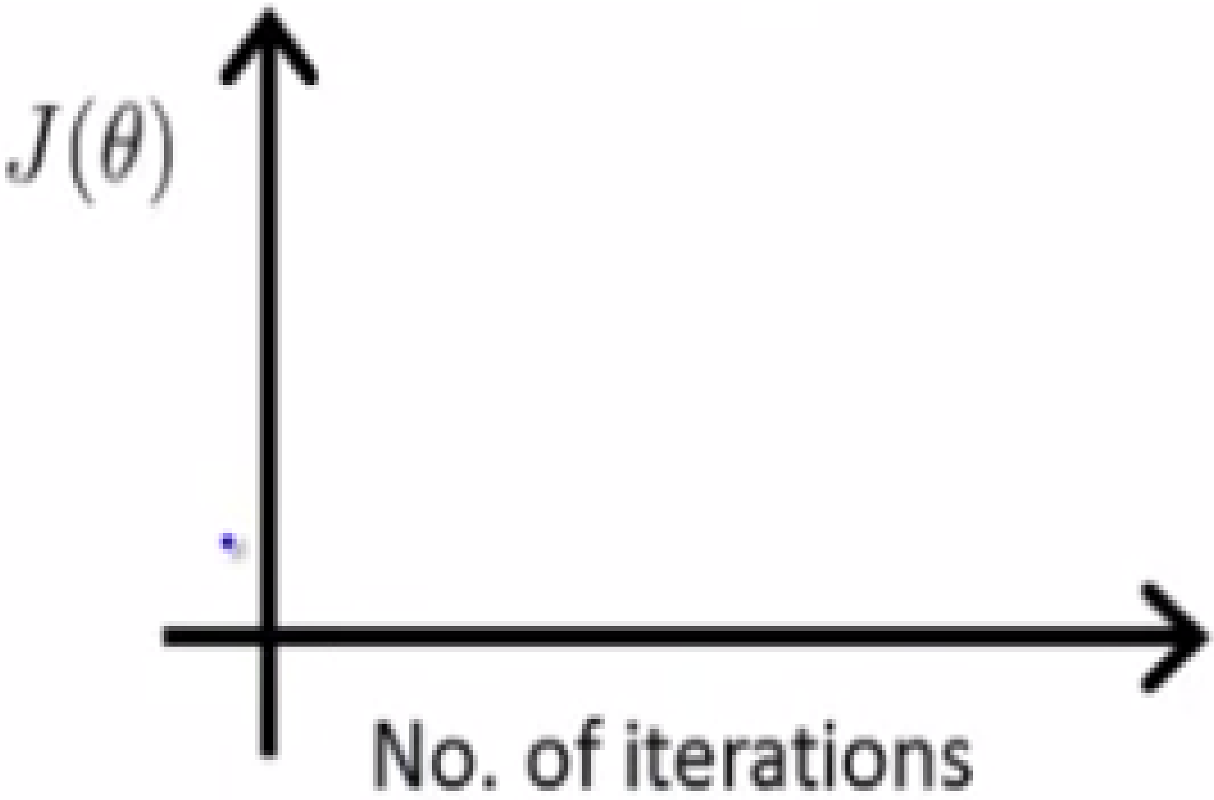
“Debugging”: como garantir que a descida do gradiente esteja funcionando corretamente?

Como escolher a taxa de aprendizado “α”?

Gradiente descendente – “Truques para melhorá-lo”

Garantir que a descida do gradiente esteja funcionando corretamente.





Gradiente descendente – “Truques para melhorá-lo”

Para resumir:

Se α é muito pequeno: convergência lenta.

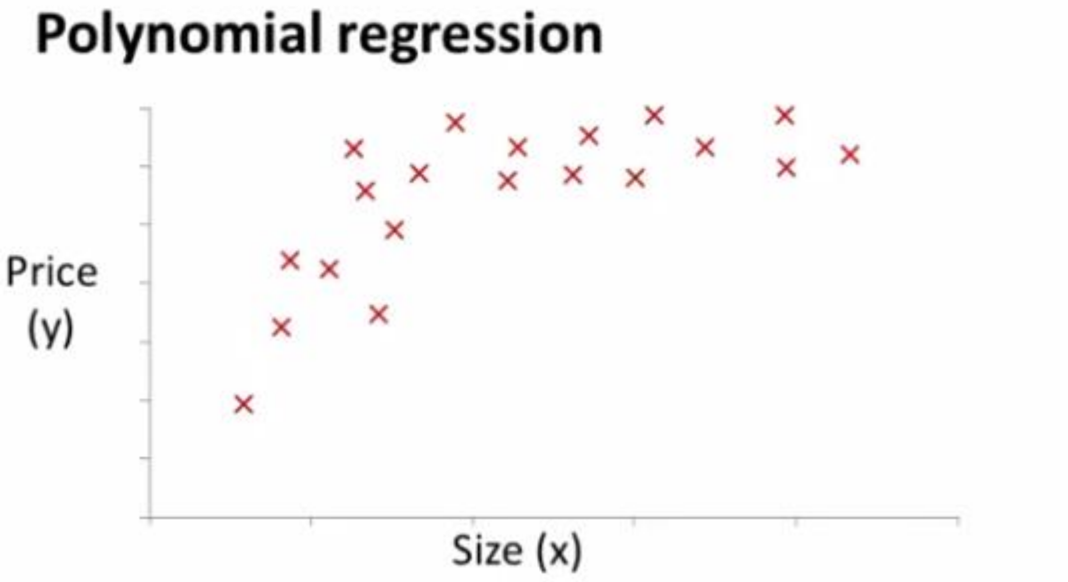
Se α é muito grande: pode não diminuir a cada iteração e, portanto, pode não convergir.

Múltiplas variáveis – Regressão Polinomial

Housing prices prediction

$$h_\theta(x) = \theta_0 + \theta_1 \times frontage + \theta_2 \times depth$$

Múltiplas variáveis – Regressão Polinomial



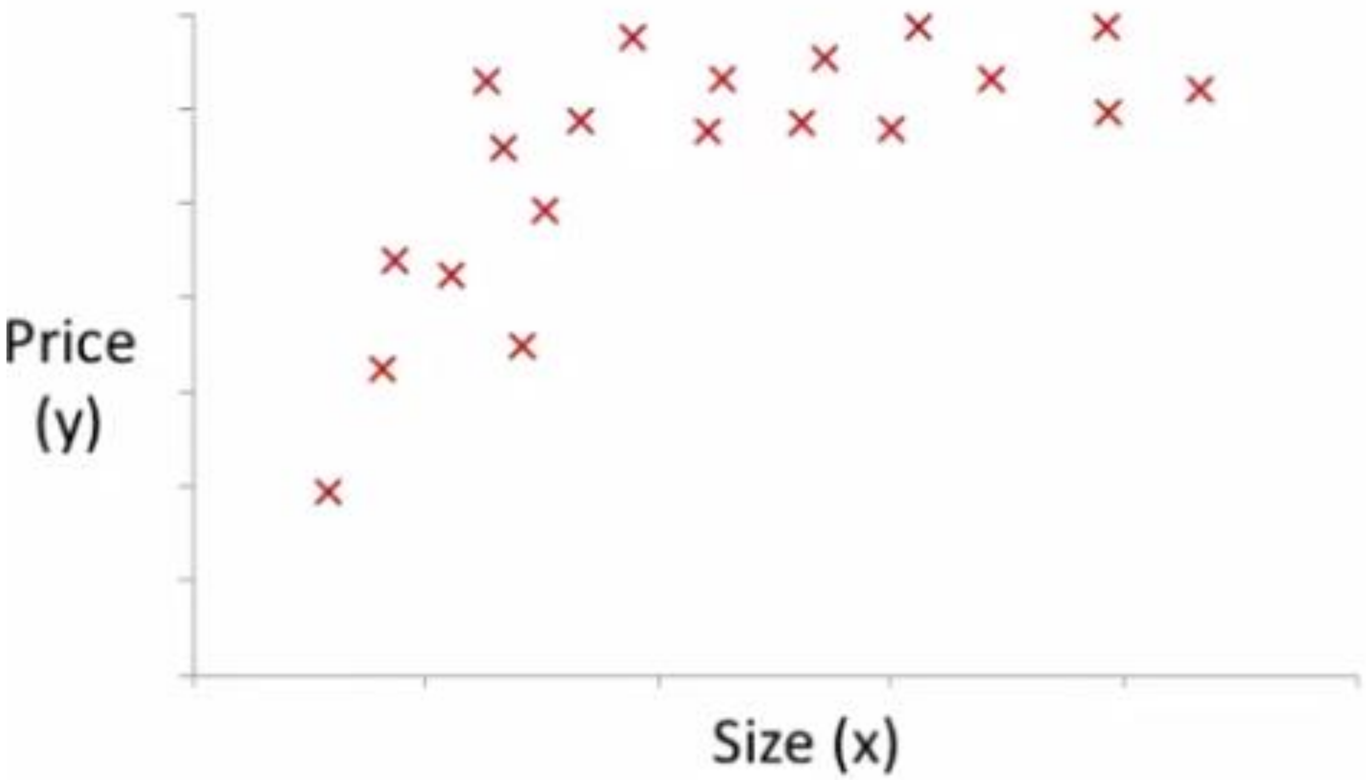
$$\theta_0 + \theta_1 x + \theta_2 x^2$$

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$$

$$\begin{aligned} h_\theta(x) &= \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 \\ &= \theta_0 + \theta_1(size) + \theta_2(size)^2 + \theta_3(size)^3 \end{aligned}$$

$$x_1 = (size)$$
$$x_2 = (size)^2$$
$$x_3 = (size)^3$$

Múltiplas variáveis – Regressão Polinomial



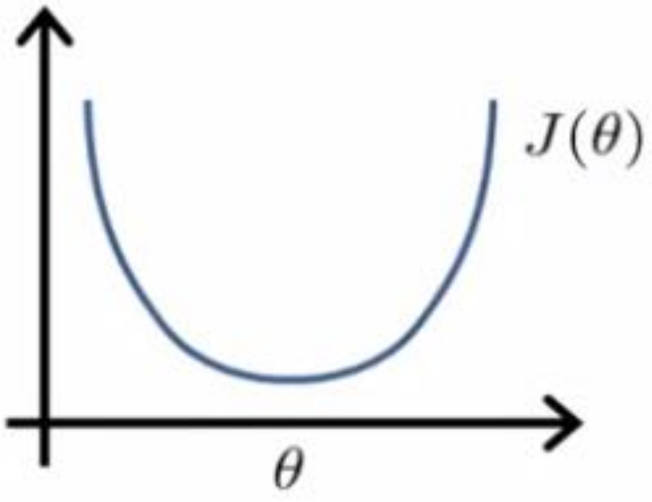
$$h_\theta(x) = \theta_0 + \theta_1(size) + \theta_2(size)^2$$

$$h_\theta(x) = \theta_0 + \theta_1(size) + \theta_2\sqrt{(size)}$$

Múltiplas variáveis – Regressão Polinomial

Intuition: If 1D $(\theta \in \mathbb{R})$

$$J(\theta) = a\theta^2 + b\theta + c$$



$\theta \in \mathbb{R}^{n+1}$ $$J(\theta_0, \theta_1, \ldots, \theta_m) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta) = \cdots = 0$$ (for every j)

Solve for $\theta_0, \theta_1, \ldots, \theta_n$

Múltiplas variáveis – Regressão Polinomial

$m = 4.$

Size (feet²) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix} \qquad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$$\theta = (X^T X)^{-1} X^T y$$

Múltiplas variáveis – Regressão Polinomial

RESUMINDO:

Gradiente descendente	Equação normal
Precisa escolher alfa	Não há necessidade de escolher alfa
Precisa de muitas iterações	Não há necessidade de iterar
O ($k\ n^2$)	O (n^3), precisa calcular inverso de $X^T\ X$
Funciona bem quando n é grande	Lento se n for muito grande

Com a equação normal, o cálculo da inversão é complexo...

Portanto, se tivermos um número muito grande de recursos, a equação normal será lenta.

Na prática, quando n excede 10.000, pode ser um bom momento para passar de uma solução normal para um processo iterativo.

Tarefa COLAB

1. Próxima aula