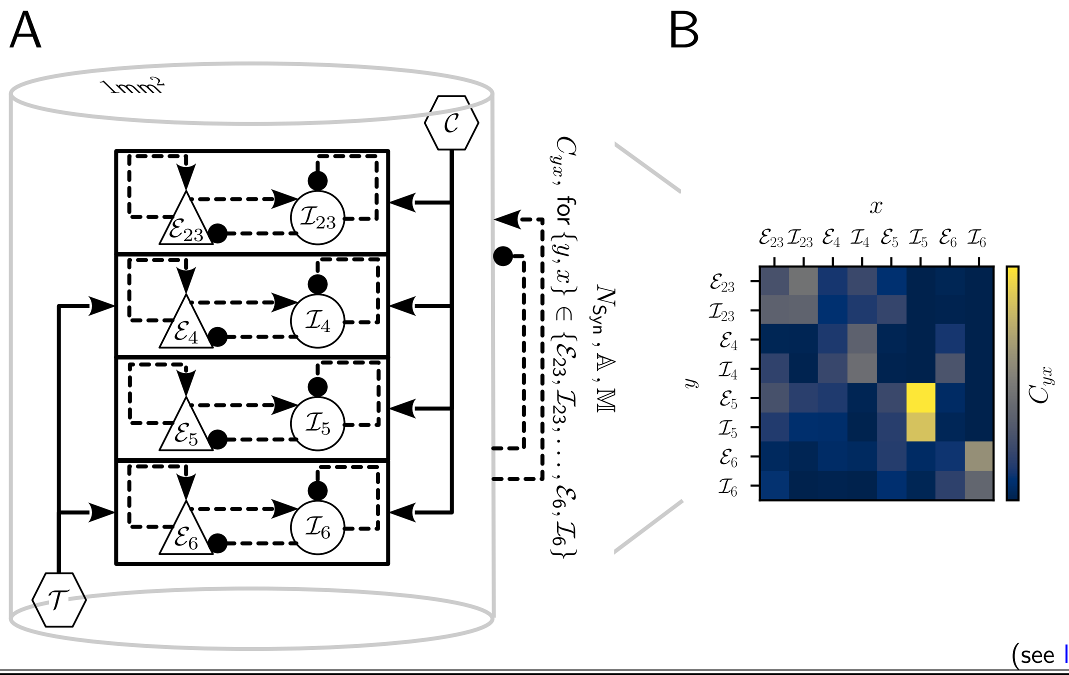
Detailed description of the cortical microcircuit model (Potjans & Diesmann, 2014)

Contents

1 Model description

Summary	
Populations	8 cortical populations in 4 layers (L2/3, L4, L5, L6), driven by a thalamic population ($\mathcal{T}$) and cortico-cortical inputs ($\mathcal{C}$)
Connectivity	random, independent, population-specific
Neuron model	cortex: leaky integrate-and-fire (LIF); thalamus, cortico-cortical inputs: point process
Synapse model	exponential postsynaptic currents with static, normally distributed weights
Predictions	population specific spiking activity



(see legend)

Populations		
Name	**Elements**	**Size**
$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$	LIF	N_x
$\mathcal{P} = \bigcup_{x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}}$	LIF	$N = \sum_x N_x$
$\mathcal{T}$	realizations of Poisson point process	$N_{\mathcal{T}}$
$\mathcal{C} = \bigcup_x \mathcal{C}_x$	realizations of Poisson point process	$N = \sum_x N_x$

Table 1: Description of the network model (continued on next page).

Connectivity		
Source	**Target**	**Pattern**
$x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• random, fixed total number K_{yx} of connections[1] • synaptic weights J_{ij} $(\forall i \in y, j \in x)$ • spike-transmission delays d_{ij} $(\forall i \in y, j \in x)$
$\mathcal{T}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• random, fixed total number $K_{y\mathcal{T}}$ of connections[1] • synaptic weights J_{ij} $(\forall i \in y, j \in \mathcal{T})$ • spike-transmission delays d_{ij} $(\forall i \in y, j \in \mathcal{T})$
$\mathcal{C}_y$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	• one-to-one[2] • synaptic weights J_{ij} $(\forall i \in y, j \in \mathcal{C}_y)$ • spike-transmission delays d_{ij} $(\forall i \in y, j \in \mathcal{C}_y)$

Connectivity patterns:

[1] *random, fixed total number* (N_{Syn}): This rule establishes a total number of

$$K_{yx} = \frac{\ln\left(1 - C_{yx}\right)}{\ln\left(1 - (N_x N_y)^{-1}\right)},$$

connections between a source population x of size N_x and a target population y of size N_y. C_{yx} denotes the connection probability. Sources and targets are randomly and independently drawn from x and y with replacement. Multiple connections between two neurons and self-connections are permitted $(\mathbb{M}, \mathbb{A})$.

[2] *one-to-one* (δ): Each neuron in the source population is connected to one corresponding neuron in the target population (bijection).

(see "Network sketch" above and Senk et al., 2022)

Table 1: Description of the network model (continued on next page).

<table>
<tr><th colspan="2">Neurons</th></tr>
<tr><th colspan="2">Cortical neurons</th></tr>
<tr><td>Type</td><td>current-based leaky integrate-and-fire with exponential synaptic current</td></tr>
<tr><td>Description</td><td>dynamics of membrane potential $V_i(t)$ and spiking activity $s_i(t)$ of neuron $i \in x$ for $x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$:
• emission of kth ($k = 1, 2, \ldots$) spike of neuron i at time t_i^k if
$$V_i\left(t_i^k\right) \geq \theta$$
with spike threshold θ
• reset and refractoriness:
$$\forall k,\ \forall t \in \left[t_k^i,\, t_k^i + \tau_{\mathsf{ref}}\right]: \quad V_i(t) = V_{\mathsf{reset}}$$
with refractory period τ_{ref} and reset potential V_{reset}
• spike train $s_i(t) = \sum_k \delta(t - t_i^k)$
• subthreshold dynamics of membrane potential $V_i(t)$:
$$\begin{aligned} &\forall k,\ \forall t \notin \left[t_i^k,\, t_i^k + \tau_{\mathsf{ref}}\right): \\ &\tau_{\mathsf{m}} \frac{\mathsf{d}V_i(t)}{\mathsf{d}t} = \left[E_{\mathsf{L}} - V_i(t)\right] + R_{\mathsf{m}} I_i(t) \end{aligned} \tag{1}$$
with membrane time constant τ_{m}, membrane resistance R_{m}, resting potential E_{L}, and total synaptic input current $I_i(t)$</td></tr>
<tr><th colspan="2">Thalamic neurons</th></tr>
<tr><td>Type</td><td>Poisson point process</td></tr>
<tr><td>Description</td><td>spike trains $s_i(t)$ ($i \in \mathcal{T}$) modeled as independent realizations of Poisson point process with piece-wise constant rate
$$\nu_{\mathcal{T}}(t) = \nu_{\mathcal{T}} \cdot (\Theta(t - t_{\mathsf{start}}) - \Theta(t - t_{\mathsf{stop}}))$$</td></tr>
<tr><th colspan="2">Cortico-cortical inputs</th></tr>
<tr><td>Type</td><td>Poisson point process</td></tr>
<tr><td>Description</td><td>independent realizations $s_i(t)$ ($i \in \mathcal{C}_x$ for $x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$) of a Poisson point process with constant rate
$$\nu_{\mathcal{C}_x} = K_{\mathcal{C}_x} \cdot \nu_{\mathcal{C}}\,,$$
where $K_{\mathcal{C}_x}$ is the cortico-cortica in-degree and $\nu_{\mathcal{C}}$ a constant rate</td></tr>
</table>

Table 2: Description of the network model (continued).

<table>
<tr><th colspan="2">Synapses</th></tr>
<tr><td>Type</td><td>exponential synaptic currents with random connectivity</td></tr>
<tr><td>Description</td><td>

- total synaptic input current $I_i(t)$ to neuron i ($\forall i \in y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$) is governed by:
$$\left(\frac{\mathsf{d}}{\mathsf{d}t} + \frac{1}{\tau_\mathsf{s}}\right) I_i(t) = f_i(t) \tag{2}$$
with superposition from all neurons $j \in x$, $\forall x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}$
$$f_i(t) = \sum_x \sum_j f_{ij}(t) = \sum_x \sum_j \hat{I}_{ij} s_j(t - d_{ij})$$
of weighted spike trains with static synaptic weigths $\hat{I}_{ij}$, synaptic time constant τ_s, and spike transmission delays d_{ij}
- solution of (2) for $f_{ij}(t) = \hat{I}_{ij} s_j(t)$ and $I_{ij}(t=0) = 0$:
$$\mathsf{PSC}_{ij}(t) = \hat{I}_{ij} \exp(-t/\tau_\mathsf{s}) \Theta(t)$$
with Heaviside function $\Theta(\cdot)$

$\curvearrowright$ (exponential decaying) posynaptic current triggered by a single presynaptic spike

- solution of (1) for $I_i(t) = \mathsf{PSC}_{ij}(t)$, $V_i(t=0) = 0$, and $E_\mathsf{L} = 0$:
$$\mathsf{PSP}_{ij}(t) = \hat{I}_{ij} R_\mathsf{m} \frac{\tau_\mathsf{s}}{\tau_\mathsf{s} - \tau_\mathsf{m}} \left(e^{-t/\tau_\mathsf{s}} - e^{-t/\tau_\mathsf{m}}\right) \Theta(t)$$
- PSC amplitude (synaptic weight):
$$\hat{I}_{ij} = \frac{J_{ij}}{J_\mathsf{unit}(\tau_\mathsf{m}, \tau_\mathsf{s}, R_\mathsf{m})}$$
parameterized by PSP amplitude $J_{ij} = \max_t\big(\mathsf{PSP}_{ij}(t)\big)$

with unit PSP amplitude (PSP amplitude for $\hat{I}_{ij} = 1$):
$$J_\mathsf{unit}(\tau_\mathsf{m}, \tau_\mathsf{s}, R_\mathsf{m}) = R_\mathsf{m} \frac{\tau_\mathsf{s}}{\tau_\mathsf{s} - \tau_\mathsf{m}} \left(\left[\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right]^{-\tau_\mathsf{m}/(\tau_\mathsf{m} - \tau_\mathsf{s})} - \left[\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right]^{-\tau_\mathsf{s}/(\tau_\mathsf{m} - \tau_\mathsf{s})} \right)$$
and time to PSP maximum:
$$t_\mathsf{max} = \frac{\tau_\mathsf{s}\tau_\mathsf{m}}{\tau_\mathsf{m} - \tau_\mathsf{s}} \ln\left(\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right)$$
</td></tr>
</table>

Table 3: Description of the network model (continued).

Synapses (continued)	
Description	• synaptic weights $$\hat{I}_{ij} = \begin{cases} \mathsf{max}(0, z_{yx}), & j \in x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}\} \\ \mathsf{min}(0, z_{yx}), & j \in x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\} \\ \bar{I}_{yx}, & j \in x = \mathcal{C} \end{cases}$$ with $$z_{yx} \sim \mathcal{N}\left\{\bar{I}_{yx},\, \sigma^2_{\mathsf{s},yx}\right\}$$ drawn from a normal distribution note: clipping of synaptic weights leads to a deviation of the total number of synapses with non-zero weights from K_{yx} (see "Connectivity") • distributed synaptic delays $$d_{ij} = \begin{cases} \mathsf{max}(d_{\mathsf{min}}, z_x), & j \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\} \\ \bar{d}_x, & j \in x = \mathcal{C} \end{cases}$$ with $$z_x \sim \mathcal{N}\left\{\bar{d}_x,\, \sigma^2_{\mathsf{d},x}\right\}$$ drawn from a normal distribution and minimal delay $d_{\mathsf{min}} > 0$

Initial conditions	
Type	random initial membrane potentials and homogeneous initial synaptic currents
Description	• membrane potentials: $V_i(t=0) \sim \mathcal{N}(V^{(y)}_{0,\mathsf{mean}}, V^{(y)}_{0,\mathsf{std}})$ randomly and independently drawn from a normal distribution with population specific mean $V^{(y)}_{0,\mathsf{mean}}$ and population specific standard deviation $V^{(y)}_{0,\mathsf{std}}$ $(\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\})$ • synaptic currents: $I_i(t=0) = 0\,\mathsf{pA}$ $(\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\})$

Table 4: Description of the network model (continued).

2 Model parameters

Network and connectivity

Population sizes

x	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
N_x	$20,683$	$5,834$	$21,915$	$5,479$	$4,850$	$1,065$	$14,395$	$2,948$	902

Connection probabilities C_{yx}

$y \backslash x$	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
$\mathcal{E}_{23}$	0.1009	0.1689	0.0437	0.0818	0.0323	0.0	0.0076	0.0	0.0
$\mathcal{I}_{23}$	0.1346	0.1371	0.0316	0.0515	0.0755	0.0	0.0042	0.0	0.0
$\mathcal{E}_4$	0.0077	0.0059	0.0497	0.1350	0.0067	0.0003	0.0453	0.0	0.0983
$\mathcal{I}_4$	0.0691	0.0029	0.0794	0.1597	0.0033	0.0	0.1057	0.0	0.0619
$\mathcal{E}_5$	0.1004	0.0622	0.0505	0.0057	0.0831	0.3726	0.0204	0.0	0.0
$\mathcal{I}_5$	0.0548	0.0269	0.0257	0.0022	0.0600	0.3158	0.0086	0.0	0.0
$\mathcal{E}_6$	0.0156	0.0066	0.0211	0.0166	0.0572	0.0197	0.0396	0.2252	0.0512
$\mathcal{I}_6$	0.0364	0.0010	0.0034	0.0005	0.0277	0.0080	0.0658	0.1443	0.0196

Neuron

Name	Value	Description
θ	$-50\,\text{mV}$	spike threshold
E_{L}	$-65\,\text{mV}$	resting potential
τ_{m}	$10\,\text{ms}$	membrane time constant
C_{m}	$250\,\text{pF}$	membrane capacitance
R_{m}	$\tau_{\mathsf{m}}/C_{\mathsf{m}} = 40\,\mathrm{M}\Omega$	membrane resistance
V_{reset}	$-65\,\text{mV}$	reset potential
τ_{ref}	$2\,\text{ms}$	absolute refractory period
τ_{s}	$0.5\,\text{ms}$	postsynaptic current time constant
$\nu_{\mathcal{T}}$	$120\,\text{s}^{-1}$	rate of thalamic neurons
t_{start}	$700\,\text{ms}$	start time of thalamic input
$\Delta t_{\mathcal{T}}$	$10\,\text{ms}$	duration of thalamic input
t_{stop}	$t_{\mathsf{start}}+\Delta t_{\mathcal{T}} = 710\,\text{ms}$	stop time of thalamic input
$\nu_{\mathcal{C}}$	$8\,\text{s}^{-1}$	rate of cortico-cortical inputs

Population specific cortico-cortica in-degree $K_{\mathcal{C}_x}$

$\mathcal{C}_x$	$\mathcal{C}_{\mathcal{E}_{23}}$	$\mathcal{C}_{\mathcal{I}_{23}}$	$\mathcal{C}_{\mathcal{E}_4}$	$\mathcal{C}_{\mathcal{I}_4}$	$\mathcal{C}_{\mathcal{E}_5}$	$\mathcal{C}_{\mathcal{I}_5}$	$\mathcal{C}_{\mathcal{E}_6}$	$\mathcal{C}_{\mathcal{I}_6}$	$\mathcal{C}_{\mathcal{T}}$
$K_{\mathcal{C}_x}$	1600	1500	2100	1900	2000	1900	2900	2100	–

Table 5: **Model parameters (continued on next page).**

Synapse		
Name	**Value**	**Description**
J	$0.15\,\text{mV}$	(mean) weight (PSP amplitude) of excitatory synapses
$\bar{I}_{yx}$		synaptic weights:
	$J/J_{\text{unit}} \approx 87.81\,\text{pA}$	$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$
	$-4J/J_{\text{unit}}$	$x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, except for:
	$2J/J_{\text{unit}}$	$(x, y) = (\mathcal{E}_{23}, \mathcal{E}_4)$
$\sigma_{\text{s},yx}$	$0.1 \cdot \bar{I}_{yx}$	standard deviation of weight distribution
$\bar{d}_x$		mean spike transmission delays:
	$1.5\,\text{ms}$	$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$
	$0.75\,\text{ms}$	$x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$
$\sigma_{\text{d},x}$	$0.5 \cdot \bar{d}_x$	standard deviation of spike transmission delays
d_{min}	$0.1\,\text{ms}$	minimal spike transmission delay
Initial conditions		
Original implementation		
Name	**Value**	**Description**
$V_{0,\text{mean}}$	-58.0 mV	homogeneous mean of the distribution of the initial membrane potential ($V_{0,\text{mean}}^{(y)} = V_{0,\text{mean}}\ \forall y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$)
$V_{0,\text{std}}$	10.0 mV	homogeneous standard deviation of the distribution of the initial membrane potential ($V_{0,\text{std}}^{(y)} = V_{0,\text{std}}\ \forall y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$)
Population specific implementation		

y	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$
$V_{0,\text{mean}}^{(y)}$ in mV	-68.28	-63.16	-63.33	-63.45	-63.11	-61.66	-66.72	-61.45
$V_{0,\text{std}}^{(y)}$ in mV	5.36	4.57	4.74	4.94	4.94	4.55	5.46	4.48

Table 6: **Model parameters (continued).**

A Single-neuron dynamics in normal form (subthreshold)

- linear, inhomogeneous dynamics of synaptic input currents and (subthreshold) membrane potential for neuron $i \in y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ (cf. eqs. (1) and (2)):

$$
\begin{aligned}
\dot{I}_i + \frac{1}{\tau_\mathsf{s}} I_i &= f_i(t) \\
\dot{V}_i + \frac{1}{\tau_\mathsf{m}} \Big[V_i - E_\mathsf{L} \Big] - \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i &= 0
\end{aligned} \tag{3}
$$

  with

$$
f_i(t) = \sum_x \sum_{j \in x} \hat{I}_{ij} s_j(t - d_{ij}) \qquad (x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}) \tag{4}
$$

- rescale membrane potential $v_i(t) = V_i(t) - E_\mathsf{L}$ and total current $x_i(t) = \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i(t)$:

$$
\begin{aligned}
\dot{x}_i + \frac{1}{\tau_\mathsf{s}} x_i &= \frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t) \\
\dot{v}_i + \frac{1}{\tau_\mathsf{m}} v_i - x_i &= 0
\end{aligned} \tag{5}
$$

- normal form of neuron-i dynamics (5):

$$
\frac{\mathsf{d}}{\mathsf{d}t} \boldsymbol{y}_i = \boldsymbol{A} \boldsymbol{y}_i + \boldsymbol{f}_i(t) \tag{6}
$$

  with $D = 2$ dimensional state vector

$$
\boldsymbol{y}_i(t) = \Big(x_i(t)\,,\, v_i(t) \Big)^\mathsf{T}, \tag{7}
$$

  with constant $(D \times D)$ matrix

$$
\boldsymbol{A} = \begin{bmatrix} -1/\tau_\mathsf{s} & 0 \\ 1 & -1/\tau_\mathsf{m} \end{bmatrix}, \tag{8}
$$

  and inhomogeneity vector

$$
\boldsymbol{f}_i(t) = \left(\frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t)\,,\, 0 \right)^\mathsf{T} \tag{9}
$$

  (see Sec. 3.2.2 in Rotter & Diesmann, 1999)

- see App. B for an efficient, exact integration scheme of (6)
- back-transform to physical quantities:

$$
\begin{aligned}
V_i(t) &= v_i(t) + E_\mathsf{L} \\
I_i(t) &= \frac{\tau_\mathsf{m}}{R_\mathsf{m}} x_i(t)
\end{aligned} \tag{10}
$$

B Exact integration of single-neuron dynamics (subthreshold)

- exact integration of (6) for spikes arriving at the target neuron i on a time grid $\mathcal{T}_\Delta = \{t_k = k\Delta | k \in \mathbb{N}, \Delta \in \mathbb{R}^+\}$, i.e., for spike trains $s_j(t) = \sum_l \delta(t - t_j^l)$ with $t_j^l \in \mathcal{T}_\Delta$ (Rotter & Diesmann, 1999):

$$\boldsymbol{y}_i(t_{k+1}) = \boldsymbol{P}\boldsymbol{y}_i(t_k) + \boldsymbol{f}_i(t_{k+1}) \tag{11}$$

with $(D \times D)$ propagator matrix (matrix exponential)

$$\boldsymbol{P} = e^{\boldsymbol{A}\Delta} \tag{12}$$

with components

$$\boldsymbol{P} = \begin{bmatrix} e^{-\Delta/\tau_\mathrm{s}} & 0 \\ \frac{e^{-\Delta/\tau_\mathrm{m}} - e^{-\Delta/\tau_\mathrm{s}}}{1/\tau_\mathrm{s} - 1/\tau_\mathrm{m}} & e^{-\Delta/\tau_\mathrm{m}} \end{bmatrix} \tag{13}$$

(see Sec. 3.2.2 in Rotter & Diesmann, 1999)

References

Potjans, T. C., Diesmann, M. (2014). The cell-type specific cortical microcircuit: relating structure and activity in a full-scale spiking network model. Cerebral cortex (New York, N.Y. : 1991), 24(3), 785–806. https://doi.org/10.1093/cercor/bhs358

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