

# Detailed description of the cortical microcircuit model ([Potjans et al., 2014](#))

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# 1 Model description

Summary	
<b>Populations</b>	8 cortical populations in 4 layers (L2/3, L4, L5, L6), driven by a thalamic population ( $\mathcal{T}$ ) and cortico-cortical inputs ( $\mathcal{C}$ )
<b>Connectivity</b>	random, independent, population specific
<b>Neuron model</b>	cortex: leaky integrate-and-fire (LIF); thalamus, cortico-cortical inputs: Poisson point process
<b>Synapse model</b>	exponential postsynaptic currents with static, normally distributed weights and delays
<b>Predictions</b>	population specific spiking activity

**A**

**B**

(see [legend](#))

Populations		
Name	Elements	Size
$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$	LIF	$N_x$
$\mathcal{P} = \bigcup_{x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}}$	LIF	$N = \sum_x N_x$
$\mathcal{T}$	realizations of Poisson point process	$N_{\mathcal{T}}$
$\mathcal{C} = \bigcup_x \mathcal{C}_x$	realizations of Poisson point process	$N = \sum_x N_x$

Table 1: Description of the network model (continued on next page).

Connectivity		
Source	Target	Pattern
$x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> <li>• random, fixed total number <math>K_{yx}</math> of connections<sup>1</sup></li> <li>• synaptic weights <math>J_{ij}</math> (<math>\forall i \in y, j \in x</math>)</li> <li>• spike-transmission delays <math>d_{ij}</math> (<math>\forall i \in y, j \in x</math>)</li> </ul>
$\mathcal{T}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> <li>• random, fixed total number <math>K_{y\mathcal{T}}</math> of connections<sup>1</sup></li> <li>• synaptic weights <math>J_{ij}</math> (<math>\forall i \in y, j \in \mathcal{T}</math>)</li> <li>• spike-transmission delays <math>d_{ij}</math> (<math>\forall i \in y, j \in \mathcal{T}</math>)</li> </ul>
$\mathcal{C}_y$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> <li>• one-to-one<sup>2</sup></li> <li>• synaptic weights <math>J_{ij}</math> (<math>\forall i \in y, j \in \mathcal{C}_y</math>)</li> <li>• spike-transmission delays <math>d_{ij}</math> (<math>\forall i \in y, j \in \mathcal{C}_y</math>)</li> </ul>
<p>Connectivity patterns:</p> <p><sup>1</sup> <i>random, fixed total number</i> (<math>N_{\text{Syn}}</math>): This rule establishes a total number of</p> $K_{yx} = \frac{\ln(1 - C_{yx})}{\ln(1 - (N_x N_y)^{-1})},$ <p>connections between a source population <math>x</math> of size <math>N_x</math> and a target population <math>y</math> of size <math>N_y</math>. <math>C_{yx}</math> denotes the connection probability. Sources and targets are randomly and independently drawn from <math>x</math> and <math>y</math> with replacement. Multiple connections between two neurons and self-connections are permitted (<math>\mathbb{M}, \mathbb{A}</math>).</p> <p><sup>2</sup> <i>one-to-one</i> (<math>\delta</math>): Each neuron in the source population is connected to one corresponding neuron in the target population (bijection).</p> <p>(see “Network sketch” above and <a href="#">Senk et al., 2022</a>)</p>		

Table 1: Description of the network model (continued).

Neurons	
Cortical neurons	
Type	leaky integrate-and-fire (LIF)
Description	<p>dynamics of membrane potential <math>V_i(t)</math> and spiking activity <math>s_i(t)</math> of neuron <math>i \in x</math> for <math>x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}</math>:</p> <ul style="list-style-type: none"> <li>emission of <math>k</math>th (<math>k = 1, 2, \dots</math>) spike of neuron <math>i</math> at time <math>t_i^k</math> if <math display="block">V_i(t_i^k) \geq \theta</math> <p>with spike threshold <math>\theta</math></p> </li> <li>reset and refractoriness: <math display="block">\forall k, \forall t \in [t_k^i, t_k^i + \tau_{\text{ref}}] : V_i(t) = V_{\text{reset}}</math> <p>with refractory period <math>\tau_{\text{ref}}</math> and reset potential <math>V_{\text{reset}}</math></p> </li> <li>spike train <math>s_i(t) = \sum_k \delta(t - t_i^k)</math></li> <li>subthreshold dynamics of membrane potential <math>V_i(t)</math>: <math display="block">\forall k, \forall t \notin [t_i^k, t_i^k + \tau_{\text{ref}}] : \tau_m \frac{dV_i(t)}{dt} = [E_L - V_i(t)] + R_m I_i(t) \quad (1)</math> <p>with membrane time constant <math>\tau_m</math>, membrane resistance <math>R_m</math>, resting potential <math>E_L</math>, and total synaptic input current <math>I_i(t)</math></p> </li> </ul>
Thalamic neurons	
Type	Poisson point process
Description	<p>spike trains <math>s_i(t)</math> (<math>i \in \mathcal{T}</math>) modeled as independent realizations of Poisson point process with piece-wise constant rate</p> $\nu_{\mathcal{T}}(t) = \nu_{\mathcal{T}} \cdot (\Theta(t - t_{\text{start}}) - \Theta(t - t_{\text{stop}}))$
Cortico-cortical inputs	
Type	Poisson point process
Description	<p>independent realizations <math>s_i(t)</math> (<math>i \in \mathcal{C}_x</math> for <math>x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}</math>) of a Poisson point process with constant rate</p> $\nu_{\mathcal{C}_x} = K_{\mathcal{C}_x} \cdot \nu_{\mathcal{C}},$ <p>where <math>K_{\mathcal{C}_x}</math> is the cortico-cortica in-degree and <math>\nu_{\mathcal{C}}</math> a constant rate</p>

Table 2: Description of the network model (continued).

Synapses	
Type	exponential postsynaptic currents with static weights and delays
Description	<ul style="list-style-type: none"> <li>total synaptic input current <math>I_i(t)</math> to neuron <math>i</math> (<math>\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}</math>) is governed by: <math display="block">\left(\frac{d}{dt} + \frac{1}{\tau_s}\right) I_i(t) = f_i(t) \quad (2)</math> with superposition from all neurons <math>j \in x</math>, <math>\forall x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}</math> <math display="block">f_i(t) = \sum_x \sum_j f_{ij}(t) = \sum_x \sum_j \hat{I}_{ij} s_j(t - d_{ij})</math> of weighted spike trains with static synaptic weights <math>\hat{I}_{ij}</math>, synaptic time constant <math>\tau_s</math>, and spike transmission delays <math>d_{ij}</math> </li> <li>solution of (2) for <math>f_{ij}(t) = \hat{I}_{ij} s_j(t)</math> and <math>I_{ij}(t=0) = 0</math>: <math display="block">\text{PSC}_{ij}(t) = \hat{I}_{ij} \exp(-t/\tau_s) \Theta(t)</math> with Heaviside function <math>\Theta(\cdot)</math>   <math>\curvearrowright</math> (exponential decaying) posynaptic current triggered by a single presynaptic spike </li> <li>solution of (1) for <math>I_i(t) = \text{PSC}_{ij}(t)</math>, <math>V_i(t=0) = 0</math>, and <math>E_L = 0</math>: <math display="block">\text{PSP}_{ij}(t) = \hat{I}_{ij} R_m \frac{\tau_s}{\tau_s - \tau_m} \left( e^{-t/\tau_s} - e^{-t/\tau_m} \right) \Theta(t)</math> </li> <li>PSC amplitude (synaptic weight): <math display="block">\hat{I}_{ij} = \frac{J_{ij}}{J_{\text{unit}}(\tau_m, \tau_s, R_m)}</math> parameterized by PSP amplitude <math>J_{ij} = \max_t (\text{PSP}_{ij}(t))</math> with unit PSP amplitude (PSP amplitude for <math>\hat{I}_{ij} = 1</math>): <math display="block">J_{\text{unit}}(\tau_m, \tau_s, R_m) = R_m \frac{\tau_s}{\tau_s - \tau_m} \left( \left[ \frac{\tau_m}{\tau_s} \right]^{-\tau_m/(\tau_m - \tau_s)} - \left[ \frac{\tau_m}{\tau_s} \right]^{-\tau_s/(\tau_m - \tau_s)} \right)</math> and time to PSP maximum: <math display="block">t_{\max} = \frac{\tau_s \tau_m}{\tau_m - \tau_s} \ln \left( \frac{\tau_m}{\tau_s} \right)</math> </li> </ul>

Table 3: Description of the network model (continued).

Synapses (continued)	
<b>Description</b>	<ul style="list-style-type: none"> <li>synaptic weights <math display="block">\hat{I}_{ij} = \begin{cases} \max(0, z_{yx}), &amp; j \in x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}\} \\ \min(0, z_{yx}), &amp; j \in x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\} \\ \bar{I}_{yx}, &amp; j \in x = \mathcal{C} \end{cases}</math> <p>with</p> <math display="block">z_{yx} \sim \mathcal{N}\{\bar{I}_{yx}, \sigma_{s,yx}^2\}</math> <p>drawn from a normal distribution with mean <math>\bar{I}_{yx}</math>, variance <math>\sigma_{s,yx}^2</math></p> <p>note: clipping of synaptic weights leads to a deviation of the total number of synapses with non-zero weights from <math>K_{yx}</math> (see “Connectivity”)</p> </li> <li>distributed synaptic delays <math display="block">d_{ij} = \begin{cases} \max(d_{\min}, z_x), &amp; j \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\} \\ \bar{d}_x, &amp; j \in x = \mathcal{C} \end{cases}</math> <p>with</p> <math display="block">z_x \sim \mathcal{N}\{\bar{d}_x, \sigma_{d,x}^2\}</math> <p>drawn from a normal distribution with mean <math>\bar{d}_x</math>, variance <math>\sigma_{d,x}^2</math>, and minimal delay <math>d_{\min} &gt; 0</math></p> </li> </ul>
Initial conditions	
<b>Type</b>	random initial membrane potentials and homogeneous initial synaptic currents
<b>Description</b>	<ul style="list-style-type: none"> <li>membrane potentials <math display="block">V_i(t=0) \sim \mathcal{N}(\bar{V}_{0,x}, \sigma_{v,x}^2)</math> <p>randomly and independently drawn from a normal distribution with mean <math>\bar{V}_{0,x}</math> and variance <math>\sigma_{v,x}^2</math> (<math>\forall i \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}</math>)</p> </li> <li>synaptic currents: <math>I_i(t=0) = 0</math> pA (<math>\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}</math>)</li> </ul>

Table 4: Description of the network model (continued).

## 2 Model parameters

Network and connectivity									
Population sizes									
$x$	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
$N_x$	20,683	5,834	21,915	5,479	4,850	1,065	14,395	2,948	902
Connection probabilities $C_{yx}$									
$\begin{matrix} x \\ y \end{matrix}$	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$	$\mathcal{T}$
$\mathcal{E}_{23}$	0.1009	0.1689	0.0437	0.0818	0.0323	0.0	0.0076	0.0	0.0
$\mathcal{I}_{23}$	0.1346	0.1371	0.0316	0.0515	0.0755	0.0	0.0042	0.0	0.0
$\mathcal{E}_4$	0.0077	0.0059	0.0497	0.1350	0.0067	0.0003	0.0453	0.0	0.0983
$\mathcal{I}_4$	0.0691	0.0029	0.0794	0.1597	0.0033	0.0	0.1057	0.0	0.0619
$\mathcal{E}_5$	0.1004	0.0622	0.0505	0.0057	0.0831	0.3726	0.0204	0.0	0.0
$\mathcal{I}_5$	0.0548	0.0269	0.0257	0.0022	0.0600	0.3158	0.0086	0.0	0.0
$\mathcal{E}_6$	0.0156	0.0066	0.0211	0.0166	0.0572	0.0197	0.0396	0.2252	0.0512
$\mathcal{I}_6$	0.0364	0.0010	0.0034	0.0005	0.0277	0.0080	0.0658	0.1443	0.0196
Neuron									
Name	Value	Description							
$\theta$	$-50\text{ mV}$	spike threshold							
$E_{\text{L}}$	$-65\text{ mV}$	resting potential							
$\tau_{\text{m}}$	$10\text{ ms}$	membrane time constant							
$C_{\text{m}}$	$250\text{ pF}$	membrane capacitance							
$R_{\text{m}}$	$\tau_{\text{m}}/C_{\text{m}} = 40\text{ M}\Omega$	membrane resistance							
$V_{\text{reset}}$	$-65\text{ mV}$	reset potential							
$\tau_{\text{ref}}$	$2\text{ ms}$	absolute refractory period							
$\tau_{\text{s}}$	$0.5\text{ ms}$	postsynaptic current time constant							
$\nu_{\mathcal{T}}$	$120\text{ s}^{-1}$	rate of thalamic neurons							
$t_{\text{start}}$	$700\text{ ms}$	start time of thalamic input							
$\Delta t_{\mathcal{T}}$	$10\text{ ms}$	duration of thalamic input							
$t_{\text{stop}}$	$t_{\text{start}} + \Delta t_{\mathcal{T}} = 710\text{ ms}$	stop time of thalamic input							
$\nu_{\mathcal{C}}$	$8\text{ s}^{-1}$	rate of cortico-cortical inputs							
Population specific cortico-cortica in-degree $K_{C_x}$									
$C_x$	$C_{\mathcal{E}_{23}}$	$C_{\mathcal{I}_{23}}$	$C_{\mathcal{E}_4}$	$C_{\mathcal{I}_4}$	$C_{\mathcal{E}_5}$	$C_{\mathcal{I}_5}$	$C_{\mathcal{E}_6}$	$C_{\mathcal{I}_6}$	$C_{\mathcal{T}}$
$K_{C_x}$	1600	1500	2100	1900	2000	1900	2900	2100	—

Table 5: Model parameters (continued on next page).

Synapse								
Name	Value	Description						
$J$	0.15 mV	(mean) weight (PSP amplitude) of excitatory synapses						
$\bar{I}_{yx}$	$J/J_{\text{unit}} \approx 87.81 \text{ pA}$ $-4J/J_{\text{unit}}$ $2J/J_{\text{unit}}$	synaptic weights: $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$ , except for: $(x, y) = (\mathcal{E}_{23}, \mathcal{E}_4)$						
$\sigma_{s, yx}$	$0.1 \cdot \bar{I}_{yx}$	standard deviation of weight distribution						
$\bar{d}_x$	1.5 ms 0.75 ms	mean spike transmission delays: $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$						
$\sigma_{d, x}$	$0.5 \cdot \bar{d}_x$	standard deviation of spike transmission delays						
$d_{\text{min}}$	0.1 ms	minimal spike transmission delay						
Initial conditions								
Population specific mean and standard deviation of initial membrane-potential distributions								
population $x$	$\mathcal{E}_{23}$	$\mathcal{I}_{23}$	$\mathcal{E}_4$	$\mathcal{I}_4$	$\mathcal{E}_5$	$\mathcal{I}_5$	$\mathcal{E}_6$	$\mathcal{I}_6$
$\bar{V}_{0, x}$ (mV)	-68.28	-63.16	-63.33	-63.45	-63.11	-61.66	-66.72	-61.45
$\sigma_{v, x}$ (mV)	5.36	4.57	4.74	4.94	4.94	4.55	5.46	4.48

Table 6: Model parameters (continued).



## A Single-neuron dynamics in normal form (subthreshold)

- linear, inhomogeneous dynamics of synaptic input currents and (subthreshold) membrane potential for neuron  $i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$  (cf. eqs. (1) and (2)):

$$\begin{aligned} \dot{I}_i + \frac{1}{\tau_s} I_i &= f_i(t) \\ \dot{V}_i + \frac{1}{\tau_m} [V_i - E_L] - \frac{R_m}{\tau_m} I_i &= 0 \end{aligned} \quad (3)$$

with

$$f_i(t) = \sum_x \sum_{j \in x} \hat{I}_{ij} s_j(t - d_{ij}) \quad (x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}) \quad (4)$$

- rescale membrane potential  $v_i(t) = V_i(t) - E_L$  and total current  $x_i(t) = \frac{R_m}{\tau_m} I_i(t)$ :

$$\begin{aligned} \dot{x}_i + \frac{1}{\tau_s} x_i &= \frac{R_m}{\tau_m} f_i(t) \\ \dot{v}_i + \frac{1}{\tau_m} v_i - x_i &= 0 \end{aligned} \quad (5)$$

- normal form of neuron- $i$  dynamics (5):

$$\frac{d}{dt} \mathbf{y}_i = \mathbf{A} \mathbf{y}_i + \mathbf{f}_i(t) \quad (6)$$

with  $D = 2$  dimensional state vector

$$\mathbf{y}_i(t) = \begin{pmatrix} x_i(t), v_i(t) \end{pmatrix}^\top, \quad (7)$$

with constant  $(D \times D)$  matrix

$$\mathbf{A} = \begin{bmatrix} -1/\tau_s & 0 \\ 1 & -1/\tau_m \end{bmatrix}, \quad (8)$$

and inhomogeneity vector

$$\mathbf{f}_i(t) = \begin{pmatrix} \frac{R_m}{\tau_m} f_i(t), 0 \end{pmatrix}^\top \quad (9)$$

(see Sec. 3.2.2 in [Rotter & Diesmann, 1999](#))

- see App. B for an efficient, exact integration scheme of (6)
- back-transform to physical quantities:

$$\begin{aligned} V_i(t) &= v_i(t) + E_L \\ I_i(t) &= \frac{\tau_m}{R_m} x_i(t) \end{aligned} \quad (10)$$

## B Exact integration of single-neuron dynamics (subthreshold)

- exact integration of (6) for spikes arriving at the target neuron  $i$  on a time grid  $\mathcal{T}_\Delta = \{t_k = k\Delta | k \in \mathbb{N}, \Delta \in \mathbb{R}^+\}$ , i.e., for spike trains  $s_j(t) = \sum_l \delta(t - t_j^l)$  with  $t_j^l \in \mathcal{T}_\Delta$  (Rotter & Diesmann, 1999):

$$\mathbf{y}_i(t_{k+1}) = \mathbf{P}\mathbf{y}_i(t_k) + \mathbf{f}_i(t_{k+1}) \quad (11)$$

with  $(D \times D)$  propagator matrix (matrix exponential)

$$\mathbf{P} = e^{\mathbf{A}\Delta} \quad (12)$$

with components

$$\mathbf{P} = \begin{bmatrix} e^{-\Delta/\tau_s} & 0 \\ \frac{e^{-\Delta/\tau_m} - e^{-\Delta/\tau_s}}{1/\tau_s - 1/\tau_m} & e^{-\Delta/\tau_m} \end{bmatrix} \quad (13)$$

(see Sec. 3.2.2 in Rotter & Diesmann, 1999)

## **C   Remarks**

1. The initial voltages...
2. DC stimulation...
3. The initial conditions...

## References

- Potjans, T. C., Diesmann, M. (2014). The cell-type specific cortical microcircuit: relating structure and activity in a full-scale spiking network model. *Cerebral cortex* (New York, N.Y. : 1991), 24(3), 785–806. <https://doi.org/10.1093/cercor/bhs358>
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