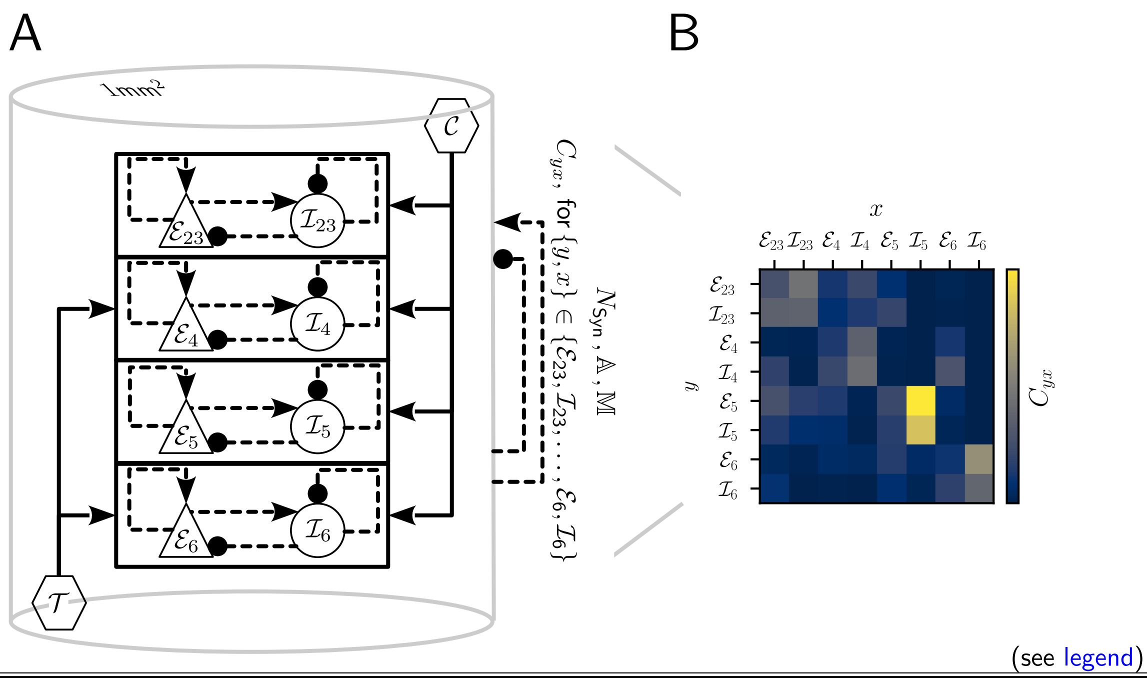
# Detailed description of the cortical microcircuit model (Potjans et al., 2014)

## Contents

# 1 Model description

| Summary | |
|---|---|
| **Populations** | $8$ cortical populations in $4$ layers (L2/3, L4, L5, L6), driven by a thalamic population ($\mathcal{T}$) and cortico-cortical inputs ($\mathcal{C}$) |
| **Connectivity** | random, independent, population specific |
| **Neuron model** | cortex: leaky integrate-and-fire (LIF); thalamus, cortico-cortical inputs: Poisson point process |
| **Synapse model** | exponential postsynaptic currents with static, normally distributed weights and delays |
| **Predictions** | population specific spiking activity |




(see legend)

| Populations | | |
|---|---|---|
| **Name** | **Elements** | **Size** |
| $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$ | LIF | $N_x$ |
| $\mathcal{P} = \bigcup_x x$ | LIF | $N = \sum_x N_x$ (see remark 1) |
| $\mathcal{T}$ | realizations of Poisson point process | $N_{\mathcal{T}}$ |
| $\mathcal{C} = \bigcup_x \mathcal{C}_x$ | population specific currents | $N = \sum_x N_x$ |

Table 1: Description of the network model (continued on next page).

| Connectivity | | |
|---|---|---|
| **Source** | **Target** | **Pattern** |
| $x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ | $y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ | • random, fixed total number $K_{yx}$ of connections[1] (see remark 1)<br>• synaptic weights $J_{ij}$ $(\forall i \in y, j \in x)$<br>• spike-transmission delays $d_{ij}$ $(\forall i \in y, j \in x)$ |
| $\mathcal{T}$ | $y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ | • random, fixed total number $K_{y\mathcal{T}}$ of connections[1]<br>• synaptic weights $J_{ij}$ $(\forall i \in y, j \in \mathcal{T})$<br>• spike-transmission delays $d_{ij}$ $(\forall i \in y, j \in \mathcal{T})$ |
| $\mathcal{C}_y$ | $y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ | • one-to-one[2] |

Connectivity patterns:

[1] *random, fixed total number* ($N_{\mathsf{Syn}}$): This rule establishes a total number of

$$K_{yx} = \frac{\ln\left(1 - C_{yx}\right)}{\ln\left(1 - (N_x N_y)^{-1}\right)},$$

connections between a source population $x$ of size $N_x$ and a target population $y$ of size $N_y$. $C_{yx}$ denotes the connection probability. Sources and targets are randomly and independently drawn from $x$ and $y$ with replacement. Multiple connections between two neurons and self-connections are permitted ($\mathbb{M}$, $\mathbb{A}$).

[2] *one-to-one* ($\delta$): Each neuron in the source population is connected to one corresponding neuron in the target population (bijection).

(see "Network sketch" above and Senk et al., 2022)

Table 1: Description of the network model (continued).

<table>
<tr><th colspan="2">Neurons</th></tr>
<tr><th colspan="2">Cortical neurons</th></tr>
<tr><td>Type</td><td>leaky integrate-and-fire (LIF)</td></tr>
<tr><td>Description</td><td>dynamics of membrane potential $V_i(t)$ and spiking activity $s_i(t)$ of neuron $i \in x$ for $x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$:<br><br>
• emission of $k$th $(k = 1, 2, \ldots)$ spike of neuron $i$ at time $t_i^k$ if<br>
$$V_i\left(t_i^k\right) \geq \theta$$<br>
with spike threshold $\theta$<br><br>
• reset and refractoriness:<br>
$$\forall k,\ \forall t \in \left[t_k^i,\, t_k^i + \tau_{\text{ref}}\right]: \quad V_i(t) = V_{\text{reset}}$$<br>
with refractory period $\tau_{\text{ref}}$ and reset potential $V_{\text{reset}}$<br><br>
• spike train $s_i(t) = \sum_k \delta(t - t_i^k)$<br><br>
• subthreshold dynamics of membrane potential $V_i(t)$:<br>
$$\begin{aligned} &\forall k,\ \forall t \notin \left[t_i^k,\, t_i^k + \tau_{\text{ref}}\right): \\ &\tau_{\text{m}} \frac{\mathrm{d}V_i(t)}{\mathrm{d}t} = \left[E_{\text{L}} - V_i(t)\right] + R_{\text{m}} I_i(t) \end{aligned} \tag{1}$$<br>
with membrane time constant $\tau_{\text{m}}$, membrane resistance $R_{\text{m}}$, resting potential $E_{\text{L}}$, and total synaptic input current $I_i(t)$</td></tr>
<tr><th colspan="2">Thalamic neurons</th></tr>
<tr><td>Type</td><td>Poisson point process</td></tr>
<tr><td>Description</td><td>spike trains $s_i(t)$ $(i \in \mathcal{T})$ modeled as independent realizations of Poisson point process with piece-wise constant rate<br>
$$\nu_{\mathcal{T}}(t) = \nu_{\mathcal{T}} \cdot \left(\Theta(t - t_{\text{start}}) - \Theta(t - t_{\text{stop}})\right)$$</td></tr>
<tr><th colspan="2">Cortico-cortical inputs</th></tr>
<tr><td>Type</td><td>constant (direct) currents (DC)</td></tr>
<tr><td>Description</td><td>population specific constant input current of magnitude<br>
$$I_{\mathcal{C}_x} = K_{\mathcal{C}_x} \cdot I_{\mathcal{C}} \quad (\forall x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}),$$<br>
with cortico-cortical in-degree $K_{\mathcal{C}_x}$, and mean current<br>
$$I_{\mathcal{C}} = \nu_{\mathcal{C}} \cdot \bar{I}_{y,\mathcal{C}} \cdot \tau_{\text{s}} \quad (\forall y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\})$$<br>
generated by a Poissonian spike train with rate $\nu_{\mathcal{C}}$, convolved with an exponential kernel with amplitude $\bar{I}_{y,\mathcal{C}}$ and time constant $\tau_{\text{s}}$ (see remark 2)</td></tr>
</table>

Table 2: Description of the network model (continued).

| Synapses | |
|---|---|
| **Type** | exponential postsynaptic currents with static weights and delays |
| **Description** | • total synaptic input current $I_i(t)$ to neuron $i$ ($\forall i \in y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$) is governed by: $$\left(\frac{\mathsf{d}}{\mathsf{d}t} + \frac{1}{\tau_\mathsf{s}}\right) I_i(t) = f_i(t) \qquad (2)$$ with superposition from all neurons $j \in x$, $\forall x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}$ $$f_i(t) = \sum_x \sum_j f_{ij}(t) = \sum_x \sum_j \hat{I}_{ij} s_j(t - d_{ij})$$ of weighted spike trains with static synaptic weigths $\hat{I}_{ij}$, synaptic time constant $\tau_\mathsf{s}$, and spike transmission delays $d_{ij}$<br>• solution of (2) for $f_{ij}(t) = \hat{I}_{ij} s_j(t)$ and $I_{ij}(t = 0) = 0$: $$\mathsf{PSC}_{ij}(t) = \hat{I}_{ij} \exp(-t/\tau_\mathsf{s}) \Theta(t)$$ with Heaviside function $\Theta(\cdot)$<br>$\curvearrowright$ (exponential decaying) posynaptic current triggered by a single presynaptic spike<br>• solution of (1) for $I_i(t) = \mathsf{PSC}_{ij}(t)$, $V_i(t = 0) = 0$, and $E_\mathsf{L} = 0$: $$\mathsf{PSP}_{ij}(t) = \hat{I}_{ij} R_\mathsf{m} \frac{\tau_\mathsf{s}}{\tau_\mathsf{s} - \tau_\mathsf{m}} \left(e^{-t/\tau_\mathsf{s}} - e^{-t/\tau_\mathsf{m}}\right) \Theta(t)$$ • PSC amplitude (synaptic weight): $$\hat{I}_{ij} = \frac{J_{ij}}{J_\mathsf{unit}(\tau_\mathsf{m}, \tau_\mathsf{s}, R_\mathsf{m})}$$ parameterized by PSP amplitude $J_{ij} = \max_t\left(\mathsf{PSP}_{ij}(t)\right)$<br>with unit PSP amplitude (PSP amplitude for $\hat{I}_{ij} = 1$): $$J_\mathsf{unit}(\tau_\mathsf{m}, \tau_\mathsf{s}, R_\mathsf{m}) = R_\mathsf{m} \frac{\tau_\mathsf{s}}{\tau_\mathsf{s} - \tau_\mathsf{m}} \left(\left[\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right]^{-\tau_\mathsf{m}/(\tau_\mathsf{m} - \tau_\mathsf{s})} - \left[\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right]^{-\tau_\mathsf{s}/(\tau_\mathsf{m} - \tau_\mathsf{s})}\right)$$ and time to PSP maximum: $$t_\mathsf{max} = \frac{\tau_\mathsf{s} \tau_\mathsf{m}}{\tau_\mathsf{m} - \tau_\mathsf{s}} \ln\left(\frac{\tau_\mathsf{m}}{\tau_\mathsf{s}}\right)$$ |

Table 3: Description of the network model (continued).

| Synapses (continued) | |
|---|---|
| **Description** | • synaptic weights $$\hat{I}_{ij} = \begin{cases} \mathsf{max}(0, z_{yx}), & j \in x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}\} \\ \mathsf{min}(0, z_{yx}), & j \in x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\} \\ \bar{I}_{yx}, & j \in x = \mathcal{C} \end{cases}$$ with $$z_{yx} \sim \mathcal{N}\left\{\bar{I}_{yx},\, \sigma^2_{\mathsf{s},yx}\right\}$$ drawn from a normal distribution with mean $\bar{I}_{yx}$, variance $\sigma^2_{\mathsf{s},yx}$<br>note: clipping of synaptic weights leads to a deviation of the total number of synapses with non-zero weights from $K_{yx}$ (see "Connectivity")<br>• distributed synaptic delays $$d_{ij} = \begin{cases} \mathsf{max}(d_{\mathsf{min}}, z_x), & j \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\} \\ \bar{d}_x, & j \in x = \mathcal{C} \end{cases}$$ with $$z_x \sim \mathcal{N}\left\{\bar{d}_x,\, \sigma^2_{\mathsf{d},x}\right\}$$ drawn from a normal distribution with mean $\bar{d}_x$, variance $\sigma^2_{\mathsf{d},x}$, and minimal delay $d_{\mathsf{min}} > 0$ |

| Initial conditions | |
|---|---|
| **Type** | random initial membrane potentials and homogeneous initial synaptic currents |
| **Description** | • membrane potentials $$V_i(t=0) \sim \mathcal{N}(\bar{V}_{0,x}, \sigma^2_{\mathsf{v},x})$$ randomly and independently drawn from a normal distribution with mean $\bar{V}_{0,x}$ and variance $\sigma^2_{\mathsf{v},x}$ ($\forall i \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$; see remark 4)<br>• synaptic currents: $I_i(t=0) = 0\,\mathsf{pA}$ ($\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$) |

Table 4: Description of the network model (continued).

## 2 Model parameters

| Network and connectivity |
|---|
| **Population sizes** |

| $x$ | $\mathcal{E}_{23}$ | $\mathcal{I}_{23}$ | $\mathcal{E}_4$ | $\mathcal{I}_4$ | $\mathcal{E}_5$ | $\mathcal{I}_5$ | $\mathcal{E}_6$ | $\mathcal{I}_6$ | $\mathcal{T}$ |
|---|---|---|---|---|---|---|---|---|---|
| $N_x$ | $20,683$ | $5,834$ | $21,915$ | $5,479$ | $4,850$ | $1,065$ | $14,395$ | $2,948$ | $902$ |

**Connection probabilities $C_{yx}$**

| $y$ \ $x$ | $\mathcal{E}_{23}$ | $\mathcal{I}_{23}$ | $\mathcal{E}_4$ | $\mathcal{I}_4$ | $\mathcal{E}_5$ | $\mathcal{I}_5$ | $\mathcal{E}_6$ | $\mathcal{I}_6$ | $\mathcal{T}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{E}_{23}$ | 0.1009 | 0.1689 | 0.0437 | 0.0818 | 0.0323 | 0.0 | 0.0076 | 0.0 | 0.0 |
| $\mathcal{I}_{23}$ | 0.1346 | 0.1371 | 0.0316 | 0.0515 | 0.0755 | 0.0 | 0.0042 | 0.0 | 0.0 |
| $\mathcal{E}_4$ | 0.0077 | 0.0059 | 0.0497 | 0.1350 | 0.0067 | 0.0003 | 0.0453 | 0.0 | 0.0983 |
| $\mathcal{I}_4$ | 0.0691 | 0.0029 | 0.0794 | 0.1597 | 0.0033 | 0.0 | 0.1057 | 0.0 | 0.0619 |
| $\mathcal{E}_5$ | 0.1004 | 0.0622 | 0.0505 | 0.0057 | 0.0831 | 0.3726 | 0.0204 | 0.0 | 0.0 |
| $\mathcal{I}_5$ | 0.0548 | 0.0269 | 0.0257 | 0.0022 | 0.0600 | 0.3158 | 0.0086 | 0.0 | 0.0 |
| $\mathcal{E}_6$ | 0.0156 | 0.0066 | 0.0211 | 0.0166 | 0.0572 | 0.0197 | 0.0396 | 0.2252 | 0.0512 |
| $\mathcal{I}_6$ | 0.0364 | 0.0010 | 0.0034 | 0.0005 | 0.0277 | 0.0080 | 0.0658 | 0.1443 | 0.0196 |

| Neuron | | |
|---|---|---|
| **Name** | **Value** | **Description** |
| $\theta$ | $-50\,\mathrm{mV}$ | spike threshold |
| $E_\mathsf{L}$ | $-65\,\mathrm{mV}$ | resting potential |
| $\tau_\mathsf{m}$ | $10\,\mathrm{ms}$ | membrane time constant |
| $C_\mathsf{m}$ | $250\,\mathrm{pF}$ | membrane capacitance |
| $R_\mathsf{m}$ | $\tau_\mathsf{m}/C_\mathsf{m} = 40\,\mathrm{M\Omega}$ | membrane resistance |
| $V_\mathsf{reset}$ | $-65\,\mathrm{mV}$ | reset potential |
| $\tau_\mathsf{ref}$ | $2\,\mathrm{ms}$ | absolute refractory period |
| $\tau_\mathsf{s}$ | $0.5\,\mathrm{ms}$ | postsynaptic current time constant |
| $\nu_\mathcal{T}$ | $120\,\mathrm{s}^{-1}$ | rate of thalamic neurons |
| $t_\mathsf{start}$ | $700\,\mathrm{ms}$ | start time of thalamic input |
| $\Delta t_\mathcal{T}$ | $10\,\mathrm{ms}$ | duration of thalamic input |
| $t_\mathsf{stop}$ | $t_\mathsf{start}+\Delta t_\mathcal{T} = 710\,\mathrm{ms}$ | stop time of thalamic input |
| $\nu_\mathcal{C}$ | $8\,\mathrm{s}^{-1}$ | rate of cortico-cortical inputs |
| $I_\mathcal{C}$ | $\nu_\mathcal{C}\bar{I}_{y,\mathcal{C}}\tau_\mathsf{s} = 0.3\,\mathrm{pA}$ | mean amplitude of DC inputs ($\forall y \in \{\mathcal{E}_{23},\ldots,\mathcal{I}_6\}$) |

**Population specific cortico-cortical in-degree $K_{\mathcal{C}_x}$**

| $\mathcal{C}_x$ | $\mathcal{C}_{\mathcal{E}_{23}}$ | $\mathcal{C}_{\mathcal{I}_{23}}$ | $\mathcal{C}_{\mathcal{E}_4}$ | $\mathcal{C}_{\mathcal{I}_4}$ | $\mathcal{C}_{\mathcal{E}_5}$ | $\mathcal{C}_{\mathcal{I}_5}$ | $\mathcal{C}_{\mathcal{E}_6}$ | $\mathcal{C}_{\mathcal{I}_6}$ | $\mathcal{C}_\mathcal{T}$ |
|---|---|---|---|---|---|---|---|---|---|
| $K_{\mathcal{C}_x}$ | 1600 | 1500 | 2100 | 1900 | 2000 | 1900 | 2900 | 2100 | – |

Table 5: Model parameters (continued on next page).

| Synapse | | |
|---|---|---|
| **Name** | **Value** | **Description** |
| $J$ | $0.15\,\mathrm{mV}$ | (mean) weight (PSP amplitude) of excitatory synapses |
| $\bar{I}_{yx}$ | | synaptic weights: |
| | $J/J_{\mathsf{unit}} \approx 87.81\,\mathrm{pA}$ | $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ |
| | $-4J/J_{\mathsf{unit}}$ | $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, except for: |
| | $2J/J_{\mathsf{unit}}$ | $(x, y) = (\mathcal{E}_{23}, \mathcal{E}_4)$ |
| $\sigma_{\mathsf{s},yx}$ | $0.1 \cdot \bar{I}_{yx}$ | standard deviation of weight distribution |
| $\bar{d}_x$ | | mean spike transmission delays: |
| | $1.5\,\mathrm{ms}$ | $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ |
| | $0.75\,\mathrm{ms}$ | $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$ |
| $\sigma_{\mathsf{d},x}$ | $0.5 \cdot \bar{d}_x$ | standard deviation of spike transmission delays |
| $d_{\mathsf{min}}$ | $0.1\,\mathrm{ms}$ | minimal spike transmission delay |

**Initial conditions**

**Population specific mean and standard deviation of initial membrane-potential distributions**

| population $x$ | $\mathcal{E}_{23}$ | $\mathcal{I}_{23}$ | $\mathcal{E}_4$ | $\mathcal{I}_4$ | $\mathcal{E}_5$ | $\mathcal{I}_5$ | $\mathcal{E}_6$ | $\mathcal{I}_6$ |
|---|---|---|---|---|---|---|---|---|
| $\bar{V}_{0,x}$ ( mV) | $-68.28$ | $-63.16$ | $-63.33$ | $-63.45$ | $-63.11$ | $-61.66$ | $-66.72$ | $-61.45$ |
| $\sigma_{\mathsf{v},x}$ ( mV) | $5.36$ | $4.57$ | $4.74$ | $4.94$ | $4.94$ | $4.55$ | $5.46$ | $4.48$ |

Table 6: Model parameters (continued).

# A Single-neuron dynamics in normal form (subthreshold)

- linear, inhomogeneous dynamics of synaptic input currents and (subthreshold) membrane potential for neuron $i \in y \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6\}$ (cf. eqs. (1) and (2)):

$$\begin{aligned} \dot{I}_i + \frac{1}{\tau_\mathsf{s}} I_i &= f_i(t) \\ \dot{V}_i + \frac{1}{\tau_\mathsf{m}} \Big[ V_i - E_\mathsf{L} \Big] - \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i &= 0 \end{aligned} \tag{3}$$

  with

$$f_i(t) = \sum_x \sum_{j \in x} \hat{I}_{ij} s_j(t - d_{ij}) \qquad (x \in \{\mathcal{E}_{23}, \ldots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}) \tag{4}$$

- rescale membrane potential $v_i(t) = V_i(t) - E_\mathsf{L}$ and total current $x_i(t) = \frac{R_\mathsf{m}}{\tau_\mathsf{m}} I_i(t)$:

$$\begin{aligned} \dot{x}_i + \frac{1}{\tau_\mathsf{s}} x_i &= \frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t) \\ \dot{v}_i + \frac{1}{\tau_\mathsf{m}} v_i - x_i &= 0 \end{aligned} \tag{5}$$

- normal form of neuron-$i$ dynamics (5):

$$\frac{\mathsf{d}}{\mathsf{d}t} \boldsymbol{y}_i = \boldsymbol{A} \boldsymbol{y}_i + \boldsymbol{f}_i(t) \tag{6}$$

  with $D = 2$ dimensional state vector

$$\boldsymbol{y}_i(t) = \Big( x_i(t)\,,\, v_i(t) \Big)^\mathsf{T}, \tag{7}$$

  with constant $(D \times D)$ matrix

$$\boldsymbol{A} = \begin{bmatrix} -1/\tau_\mathsf{s} & 0 \\ 1 & -1/\tau_\mathsf{m} \end{bmatrix}, \tag{8}$$

  and inhomogeneity vector

$$\boldsymbol{f}_i(t) = \left( \frac{R_\mathsf{m}}{\tau_\mathsf{m}} f_i(t)\,,\, 0 \right)^\mathsf{T} \tag{9}$$

  (see Sec. 3.2.2 in Rotter & Diesmann, 1999)

- see App. B for an efficient, exact integration scheme of (6)
- back-transform to physical quantities:

$$\begin{aligned} V_i(t) &= v_i(t) + E_\mathsf{L} \\ I_i(t) &= \frac{\tau_\mathsf{m}}{R_\mathsf{m}} x_i(t) \end{aligned} \tag{10}$$

# B Exact integration of single-neuron dynamics (subthreshold)

- exact integration of (6) for spikes arriving at the target neuron $i$ on a time grid $\mathcal{T}_\Delta = \{t_k = k\Delta | k \in \mathbb{N}, \Delta \in \mathbb{R}^+\}$, i.e., for spike trains $s_j(t) = \sum_l \delta(t - t_j^l)$ with $t_j^l \in \mathcal{T}_\Delta$ (Rotter & Diesmann, 1999):

$$\boldsymbol{y}_i(t_{k+1}) = \boldsymbol{P}\boldsymbol{y}_i(t_k) + \boldsymbol{f}_i(t_{k+1}) \tag{11}$$

with $(D \times D)$ propagator matrix (matrix exponential)

$$\boldsymbol{P} = e^{\boldsymbol{A}\Delta} \tag{12}$$

with components

$$\boldsymbol{P} = \begin{bmatrix} e^{-\Delta/\tau_\mathsf{s}} & 0 \\ \frac{e^{-\Delta/\tau_\mathsf{m}} - e^{-\Delta/\tau_\mathsf{s}}}{1/\tau_\mathsf{s} - 1/\tau_\mathsf{m}} & e^{-\Delta/\tau_\mathsf{m}} \end{bmatrix} \tag{13}$$

(see Sec. 3.2.2 in Rotter & Diesmann, 1999)

## C Remarks

1. The implementation contains, besides the original full-scale model of Potjans et al. (2014), a downscaled version, which can run on a desktop computer. This is controlled by the parameters `N_scaling` and `K_scaling` in the network parameters file. The first parameter scales the number of neurons, whereas the second parameter scales the number of synapses per neuron or in-degree. The scaling happens on both, inter-population and external connectivity.

   Downscaling the in-degree $K$ changes the mean and variance of the synaptic input currents. In order to avoid this, one can choose the new synaptic weights $\bar{I}^*_{i,x}$ $(\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}, x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{C}_y\})$ in such a way that together with a compensation current these effects are compensated. For the full-scale model we have

$$\mu_i = I_{\text{rec},i} + I_{\text{ext},i} = \tau_{\text{s}} \sum_x K_{i,x} \bar{I}_{i,x} \nu_x \,,$$
$$\sigma_i^2 = \sigma^2_{\text{rec},i} + \sigma^2_{\text{ext},i} = \tau_{\text{s}} \sum_x K_{i,x} \bar{I}^2_{i,x} \nu_x \,,$$

   where $K_{i,x}$ is the in-degree, $\bar{I}_{i,x}$ the synaptic weight and $\nu_x$ the mean firing rate. For the downscaled model we have

$$\mu_i^* = \tau_{\text{s}} \sum_x K^*_{i,x} \bar{I}^*_{i,x} \nu_x + \mu_{i,0} \,,$$
$$(\sigma^*)^2 = \tau_{\text{s}} \sum_x K^*_{i,x} (\bar{I}^*_{i,x})^2 \nu_x \,.$$

   Here, $K^*_{i,x} = f K_{i,x}$ with some factor $0 < f < 1$. Comparing the equations of the variance we find $\bar{I}^*_{i,x} = \bar{I}_{i,x}/\sqrt{f}$, if we want to leave fluctuations invariant. We have also a constant compensation current $\mu_{i,0}$ to leave also the mean input current invariant. Inserting this in the equation of the mean and solving for the compensation current $\mu_{i,0}$ we find

$$\mu_{i,0} = \tau_{\text{s}} \left(1 - \sqrt{f}\right) \sum_x K_{i,x} \bar{I}_{i,x} \nu_x \,.$$

   There are other ways to downscale the model, for more details see (van Albada et al., 2015). Note that this derivation assumes that all spike-trains are stationary Poissonian inputs (for downscaling with DC inputs see remark 3).
   The scaling and compensation current are implemented in the PyNEST implementation provided here.

2. In the original model of Potjans et al. (2014), the cortico-cortical inputs are modeled as independent realizations $s_i(t)$ $(i \in \mathcal{C}_x$ for $x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\})$ of a Poisson point process with constant rate $\nu_{\mathcal{C}_x} = K_{\mathcal{C}_x} \nu_{\mathcal{C}}$, filtered by an exponential kernel PSC$(t)$ with time constant $\tau_{\text{s}}$ and amplitude $\bar{I}_{y,\mathcal{C}}$ $(\forall y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\})$. Here, $K_{\mathcal{C}_x}$ denotes the cortico-cortical in-degree and $\nu_{\mathcal{C}}$ a constant rate. DC inputs are computationally less expensive, exactly reproducible, and lead to similar network activity statistics. When replacing cortico-cortical input spikes $s(t)$ by DC inputs, the current implementation preserves the mean input current

$$I_{\mathcal{C}} = (\langle s \rangle * \text{PSC})(t) = \bar{I}_{y,\mathcal{C}} \nu_{\mathcal{C}} \tau_{\text{s}} \,.$$

3. In remark 2 one can find the expression for the DC amplitude to keep the same mean input current as in the Poissonian case. With cortico-cortical inputs modeled as DC currents, the current amplitude of the downscaled model is given by

$$I^*_{\text{DC},i} = I_{\text{DC},i} + (1 - \sqrt{f}) I_{\text{rec},i} \,,$$

   (see remark 1). To ensure the downscaled network is activated by the cortico-cortical input, the current amplitude $I^*_{\text{DC},i}$ of the downscaled model needs to exceed the rheobase current, i.e.

$$I_{\text{rh},i} = \frac{\Theta - E_{\text{L}}}{R_{\text{m}}} \leq I^*_{\text{DC},i} \,.$$

   which allows us to compute the minimal downscaling factor $f_{\text{min}}$ as

$$f_{\text{min}} = \left(1 - \frac{I_{\text{rh},i} - I_{\text{ext},i}}{-|I_{\text{rec},i}|}\right)^2 \,.$$

4. The original model of Potjans et al. (2014) uses population *unspecific* normal distributions of initial membrane potentials. By default, the current implementation uses population *specific* initial membrane potential distributions instead to speed up convergence to the stationary state. In the reference implementation, the type of initial conditions can be set by the parameter `V0_type` (`"optimized"` [default] or `"original"`). In (Senk et al., 2025), the population specific initial conditions are refered to as *amended* initial conditions.

## References

Potjans, T. C., Diesmann, M. (2014). The cell-type specific cortical microcircuit: relating structure and activity in a full-scale spiking network model. Cerebral cortex (New York, N.Y. : 1991), 24(3), 785–806. https://doi.org/10.1093/cercor/bhs358

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