

Detailed description of the cortical microcircuit model ([Potjans et al., 2014](#))

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1 Model description

Summary	
Populations	8 cortical populations in 4 layers (L2/3, L4, L5, L6), driven by a thalamic population (\mathcal{T}) and cortico-cortical inputs (\mathcal{C})
Connectivity	random, independent, population specific
Neuron model	cortex: leaky integrate-and-fire (LIF); thalamus, cortico-cortical inputs: Poisson point process
Synapse model	exponential postsynaptic currents with static, normally distributed weights and delays
Predictions	population specific spiking activity

A

B

(see [legend](#))

Populations		
Name	Elements	Size
$x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$	LIF	N_x
$\mathcal{P} = \bigcup_{x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}}$	LIF	$N = \sum_x N_x$
\mathcal{T}	realizations of Poisson point process	$N_{\mathcal{T}}$
$\mathcal{C} = \bigcup_x \mathcal{C}_x$	realizations of Poisson point process	$N = \sum_x N_x$

Table 1: Description of the network model (continued on next page).

Connectivity		
Source	Target	Pattern
$x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> • random, fixed total number K_{yx} of connections¹ • synaptic weights J_{ij} ($\forall i \in y, j \in x$) • spike-transmission delays d_{ij} ($\forall i \in y, j \in x$)
\mathcal{T}	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> • random, fixed total number $K_{y\mathcal{T}}$ of connections¹ • synaptic weights J_{ij} ($\forall i \in y, j \in \mathcal{T}$) • spike-transmission delays d_{ij} ($\forall i \in y, j \in \mathcal{T}$)
\mathcal{C}_y	$y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$	<ul style="list-style-type: none"> • one-to-one² • synaptic weights J_{ij} ($\forall i \in y, j \in \mathcal{C}_y$) • spike-transmission delays d_{ij} ($\forall i \in y, j \in \mathcal{C}_y$)
<p>Connectivity patterns:</p> <p>¹ <i>random, fixed total number</i> (N_{Syn}): This rule establishes a total number of</p> $K_{yx} = \frac{\ln(1 - C_{yx})}{\ln(1 - (N_x N_y)^{-1})},$ <p>connections between a source population x of size N_x and a target population y of size N_y. C_{yx} denotes the connection probability. Sources and targets are randomly and independently drawn from x and y with replacement. Multiple connections between two neurons and self-connections are permitted (\mathbb{M}, \mathbb{A}).</p> <p>² <i>one-to-one</i> (δ): Each neuron in the source population is connected to one corresponding neuron in the target population (bijection).</p> <p>(see “Network sketch” above and Senk et al., 2022)</p>		

Table 1: Description of the network model (continued).

Neurons	
Cortical neurons	
Type	leaky integrate-and-fire (LIF)
Description	<p>dynamics of membrane potential $V_i(t)$ and spiking activity $s_i(t)$ of neuron $i \in x$ for $x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$:</p> <ul style="list-style-type: none"> emission of kth ($k = 1, 2, \dots$) spike of neuron i at time t_i^k if $V_i(t_i^k) \geq \theta$ <p>with spike threshold θ</p> reset and refractoriness: $\forall k, \forall t \in [t_k^i, t_k^i + \tau_{\text{ref}}] : V_i(t) = V_{\text{reset}}$ <p>with refractory period τ_{ref} and reset potential V_{reset}</p> spike train $s_i(t) = \sum_k \delta(t - t_i^k)$ subthreshold dynamics of membrane potential $V_i(t)$: $\forall k, \forall t \notin [t_i^k, t_i^k + \tau_{\text{ref}}] : \tau_m \frac{dV_i(t)}{dt} = [E_L - V_i(t)] + R_m I_i(t) \quad (1)$ <p>with membrane time constant τ_m, membrane resistance R_m, resting potential E_L, and total synaptic input current $I_i(t)$</p>
Thalamic neurons	
Type	Poisson point process
Description	<p>spike trains $s_i(t)$ ($i \in \mathcal{T}$) modeled as independent realizations of Poisson point process with piece-wise constant rate</p> $\nu_{\mathcal{T}}(t) = \nu_{\mathcal{T}} \cdot (\Theta(t - t_{\text{start}}) - \Theta(t - t_{\text{stop}}))$
Cortico-cortical inputs	
Type	Poisson point process
Description	<p>independent realizations $s_i(t)$ ($i \in \mathcal{C}_x$ for $x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$) of a Poisson point process with constant rate</p> $\nu_{\mathcal{C}_x} = K_{\mathcal{C}_x} \cdot \nu_{\mathcal{C}},$ <p>where $K_{\mathcal{C}_x}$ is the cortico-cortica in-degree and $\nu_{\mathcal{C}}$ a constant rate</p>

Table 2: Description of the network model (continued).

Synapses	
Type	exponential postsynaptic currents with static weights and delays
Description	<ul style="list-style-type: none"> total synaptic input current $I_i(t)$ to neuron i ($\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$) is governed by: $\left(\frac{d}{dt} + \frac{1}{\tau_s}\right) I_i(t) = f_i(t) \quad (2)$ with superposition from all neurons $j \in x$, $\forall x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}$ $f_i(t) = \sum_x \sum_j f_{ij}(t) = \sum_x \sum_j \hat{I}_{ij} s_j(t - d_{ij})$ of weighted spike trains with static synaptic weights \hat{I}_{ij}, synaptic time constant τ_s, and spike transmission delays d_{ij} solution of (2) for $f_{ij}(t) = \hat{I}_{ij} s_j(t)$ and $I_{ij}(t=0) = 0$: $\text{PSC}_{ij}(t) = \hat{I}_{ij} \exp(-t/\tau_s) \Theta(t)$ with Heaviside function $\Theta(\cdot)$ <p>↪ (exponential decaying) posynaptic current triggered by a single presynaptic spike</p> <ul style="list-style-type: none"> solution of (1) for $I_i(t) = \text{PSC}_{ij}(t)$, $V_i(t=0) = 0$, and $E_L = 0$: $\text{PSP}_{ij}(t) = \hat{I}_{ij} R_m \frac{\tau_s}{\tau_s - \tau_m} \left(e^{-t/\tau_s} - e^{-t/\tau_m} \right) \Theta(t)$ PSC amplitude (synaptic weight): $\hat{I}_{ij} = \frac{J_{ij}}{J_{\text{unit}}(\tau_m, \tau_s, R_m)}$ parameterized by PSP amplitude $J_{ij} = \max_t (\text{PSP}_{ij}(t))$ with unit PSP amplitude (PSP amplitude for $\hat{I}_{ij} = 1$): $J_{\text{unit}}(\tau_m, \tau_s, R_m) = R_m \frac{\tau_s}{\tau_s - \tau_m} \left(\left[\frac{\tau_m}{\tau_s} \right]^{-\tau_m/(\tau_m - \tau_s)} - \left[\frac{\tau_m}{\tau_s} \right]^{-\tau_s/(\tau_m - \tau_s)} \right)$ and time to PSP maximum: $t_{\max} = \frac{\tau_s \tau_m}{\tau_m - \tau_s} \ln \left(\frac{\tau_m}{\tau_s} \right)$

Table 3: Description of the network model (continued).

Synapses (continued)	
Description	<ul style="list-style-type: none"> synaptic weights $\hat{I}_{ij} = \begin{cases} \max(0, z_{yx}), & j \in x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}\} \\ \min(0, z_{yx}), & j \in x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\} \\ \bar{I}_{yx}, & j \in x = \mathcal{C} \end{cases}$ <p>with</p> $z_{yx} \sim \mathcal{N}\{\bar{I}_{yx}, \sigma_{s,yx}^2\}$ <p>drawn from a normal distribution with mean \bar{I}_{yx}, variance $\sigma_{s,yx}^2$</p> <p>note: clipping of synaptic weights leads to a deviation of the total number of synapses with non-zero weights from K_{yx} (see ‘‘Connectivity’’)</p> distributed synaptic delays $d_{ij} = \begin{cases} \max(d_{\min}, z_x), & j \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}\} \\ \bar{d}_x, & j \in x = \mathcal{C} \end{cases}$ <p>with</p> $z_x \sim \mathcal{N}\{\bar{d}_x, \sigma_{d,x}^2\}$ <p>drawn from a normal distribution with mean \bar{d}_x, variance $\sigma_{d,x}^2$, and minimal delay $d_{\min} > 0$</p>
Initial conditions	
Type	random initial membrane potentials and homogeneous initial synaptic currents
Description	<ul style="list-style-type: none"> membrane potentials $V_i(t=0) \sim \mathcal{N}(\bar{V}_{0,x}, \sigma_{v,x}^2)$ <p>randomly and independently drawn from a normal distribution with mean $\bar{V}_{0,x}$ and variance $\sigma_{v,x}^2$ ($\forall i \in x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$)</p> synaptic currents: $I_i(t=0) = 0$ pA ($\forall i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$)

Table 4: Description of the network model (continued).

2 Model parameters

Network and connectivity									
Population sizes									
x	\mathcal{E}_{23}	\mathcal{I}_{23}	\mathcal{E}_4	\mathcal{I}_4	\mathcal{E}_5	\mathcal{I}_5	\mathcal{E}_6	\mathcal{I}_6	\mathcal{T}
N_x	20,683	5,834	21,915	5,479	4,850	1,065	14,395	2,948	902
Connection probabilities C_{yx}									
$\begin{matrix} x \\ y \end{matrix}$	\mathcal{E}_{23}	\mathcal{I}_{23}	\mathcal{E}_4	\mathcal{I}_4	\mathcal{E}_5	\mathcal{I}_5	\mathcal{E}_6	\mathcal{I}_6	\mathcal{T}
\mathcal{E}_{23}	0.1009	0.1689	0.0437	0.0818	0.0323	0.0	0.0076	0.0	0.0
\mathcal{I}_{23}	0.1346	0.1371	0.0316	0.0515	0.0755	0.0	0.0042	0.0	0.0
\mathcal{E}_4	0.0077	0.0059	0.0497	0.1350	0.0067	0.0003	0.0453	0.0	0.0983
\mathcal{I}_4	0.0691	0.0029	0.0794	0.1597	0.0033	0.0	0.1057	0.0	0.0619
\mathcal{E}_5	0.1004	0.0622	0.0505	0.0057	0.0831	0.3726	0.0204	0.0	0.0
\mathcal{I}_5	0.0548	0.0269	0.0257	0.0022	0.0600	0.3158	0.0086	0.0	0.0
\mathcal{E}_6	0.0156	0.0066	0.0211	0.0166	0.0572	0.0197	0.0396	0.2252	0.0512
\mathcal{I}_6	0.0364	0.0010	0.0034	0.0005	0.0277	0.0080	0.0658	0.1443	0.0196
Neuron									
Name	Value	Description							
θ	-50 mV	spike threshold							
E_{L}	-65 mV	resting potential							
τ_{m}	10 ms	membrane time constant							
C_{m}	250 pF	membrane capacitance							
R_{m}	$\tau_{\text{m}}/C_{\text{m}} = 40\text{ M}\Omega$	membrane resistance							
V_{reset}	-65 mV	reset potential							
τ_{ref}	2 ms	absolute refractory period							
τ_{s}	0.5 ms	postsynaptic current time constant							
$\nu_{\mathcal{T}}$	120 s^{-1}	rate of thalamic neurons							
t_{start}	700 ms	start time of thalamic input							
$\Delta t_{\mathcal{T}}$	10 ms	duration of thalamic input							
t_{stop}	$t_{\text{start}} + \Delta t_{\mathcal{T}} = 710\text{ ms}$	stop time of thalamic input							
$\nu_{\mathcal{C}}$	8 s^{-1}	rate of cortico-cortical inputs							
Population specific cortico-cortica in-degree K_{C_x}									
C_x	$C_{\mathcal{E}_{23}}$	$C_{\mathcal{I}_{23}}$	$C_{\mathcal{E}_4}$	$C_{\mathcal{I}_4}$	$C_{\mathcal{E}_5}$	$C_{\mathcal{I}_5}$	$C_{\mathcal{E}_6}$	$C_{\mathcal{I}_6}$	$C_{\mathcal{T}}$
K_{C_x}	1600	1500	2100	1900	2000	1900	2900	2100	—

Table 5: Model parameters (continued on next page).

Synapse								
Name	Value	Description						
J	0.15 mV	(mean) weight (PSP amplitude) of excitatory synapses						
\bar{I}_{yx}	$J/J_{\text{unit}} \approx 87.81 \text{ pA}$ $-4J/J_{\text{unit}}$ $2J/J_{\text{unit}}$	synaptic weights: $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$, except for: $(x, y) = (\mathcal{E}_{23}, \mathcal{E}_4)$						
$\sigma_{s, yx}$	$0.1 \cdot \bar{I}_{yx}$	standard deviation of weight distribution						
\bar{d}_x	1.5 ms 0.75 ms	mean spike transmission delays: $x \in \{\mathcal{E}_{23}, \mathcal{E}_4, \mathcal{E}_5, \mathcal{E}_6, \mathcal{T}, \mathcal{C}\}$ $x \in \{\mathcal{I}_{23}, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$						
$\sigma_{d, x}$	$0.5 \cdot \bar{d}_x$	standard deviation of spike transmission delays						
d_{min}	0.1 ms	minimal spike transmission delay						
Initial conditions								
Population specific mean and standard deviation of initial membrane-potential distributions								
population x	\mathcal{E}_{23}	\mathcal{I}_{23}	\mathcal{E}_4	\mathcal{I}_4	\mathcal{E}_5	\mathcal{I}_5	\mathcal{E}_6	\mathcal{I}_6
$\bar{V}_{0, x}$ (mV)	-68.28	-63.16	-63.33	-63.45	-63.11	-61.66	-66.72	-61.45
$\sigma_{v, x}$ (mV)	5.36	4.57	4.74	4.94	4.94	4.55	5.46	4.48

Table 6: Model parameters (continued).

A Single-neuron dynamics in normal form (subthreshold)

- linear, inhomogeneous dynamics of synaptic input currents and (subthreshold) membrane potential for neuron $i \in y \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6\}$ (cf. eqs. (1) and (2)):

$$\begin{aligned} \dot{I}_i + \frac{1}{\tau_s} I_i &= f_i(t) \\ \dot{V}_i + \frac{1}{\tau_m} [V_i - E_L] - \frac{R_m}{\tau_m} I_i &= 0 \end{aligned} \quad (3)$$

with

$$f_i(t) = \sum_x \sum_{j \in x} \hat{I}_{ij} s_j(t - d_{ij}) \quad (x \in \{\mathcal{E}_{23}, \dots, \mathcal{I}_6, \mathcal{T}, \mathcal{C}\}) \quad (4)$$

- rescale membrane potential $v_i(t) = V_i(t) - E_L$ and total current $x_i(t) = \frac{R_m}{\tau_m} I_i(t)$:

$$\begin{aligned} \dot{x}_i + \frac{1}{\tau_s} x_i &= \frac{R_m}{\tau_m} f_i(t) \\ \dot{v}_i + \frac{1}{\tau_m} v_i - x_i &= 0 \end{aligned} \quad (5)$$

- normal form of neuron- i dynamics (5):

$$\frac{d}{dt} \mathbf{y}_i = \mathbf{A} \mathbf{y}_i + \mathbf{f}_i(t) \quad (6)$$

with $D = 2$ dimensional state vector

$$\mathbf{y}_i(t) = \begin{pmatrix} x_i(t), v_i(t) \end{pmatrix}^\top, \quad (7)$$

with constant $(D \times D)$ matrix

$$\mathbf{A} = \begin{bmatrix} -1/\tau_s & 0 \\ 1 & -1/\tau_m \end{bmatrix}, \quad (8)$$

and inhomogeneity vector

$$\mathbf{f}_i(t) = \begin{pmatrix} \frac{R_m}{\tau_m} f_i(t), 0 \end{pmatrix}^\top \quad (9)$$

(see Sec. 3.2.2 in [Rotter & Diesmann, 1999](#))

- see App. B for an efficient, exact integration scheme of (6)
- back-transform to physical quantities:

$$\begin{aligned} V_i(t) &= v_i(t) + E_L \\ I_i(t) &= \frac{\tau_m}{R_m} x_i(t) \end{aligned} \quad (10)$$

B Exact integration of single-neuron dynamics (subthreshold)

- exact integration of (6) for spikes arriving at the target neuron i on a time grid $\mathcal{T}_\Delta = \{t_k = k\Delta | k \in \mathbb{N}, \Delta \in \mathbb{R}^+\}$, i.e., for spike trains $s_j(t) = \sum_l \delta(t - t_j^l)$ with $t_j^l \in \mathcal{T}_\Delta$ (Rotter & Diesmann, 1999):

$$\mathbf{y}_i(t_{k+1}) = \mathbf{P}\mathbf{y}_i(t_k) + \mathbf{f}_i(t_{k+1}) \quad (11)$$

with $(D \times D)$ propagator matrix (matrix exponential)

$$\mathbf{P} = e^{\mathbf{A}\Delta} \quad (12)$$

with components

$$\mathbf{P} = \begin{bmatrix} e^{-\Delta/\tau_s} & 0 \\ \frac{e^{-\Delta/\tau_m} - e^{-\Delta/\tau_s}}{1/\tau_s - 1/\tau_m} & e^{-\Delta/\tau_m} \end{bmatrix} \quad (13)$$

(see Sec. 3.2.2 in Rotter & Diesmann, 1999)

References

- Potjans, T. C., Diesmann, M. (2014). The cell-type specific cortical microcircuit: relating structure and activity in a full-scale spiking network model. *Cerebral cortex* (New York, N.Y. : 1991), 24(3), 785–806. <https://doi.org/10.1093/cercor/bhs358>
- Rotter, S., and Diesmann, M. (1999). Exact digital simulation of time-invariant linear systems with applications to neural modeling. *Biological Cybernetics*, 81:381–402.
- Senk, J., Kriener, B., Djurfeldt, M., Voges, N., Jiang, H.-J., Schüttler, L., Gramelsberger, G., Diesmann, M., Plesser, H.E., van Albada, S.J. (2022). Connectivity concepts in neuronal network modeling. *PLoS Computational Biology*, 18(9):e1010086.