軸計算（手算）

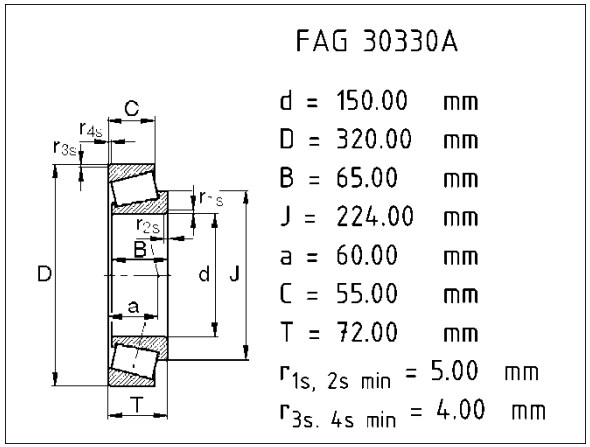
參考料來源：中央大學 機械設計實驗室 自編講義

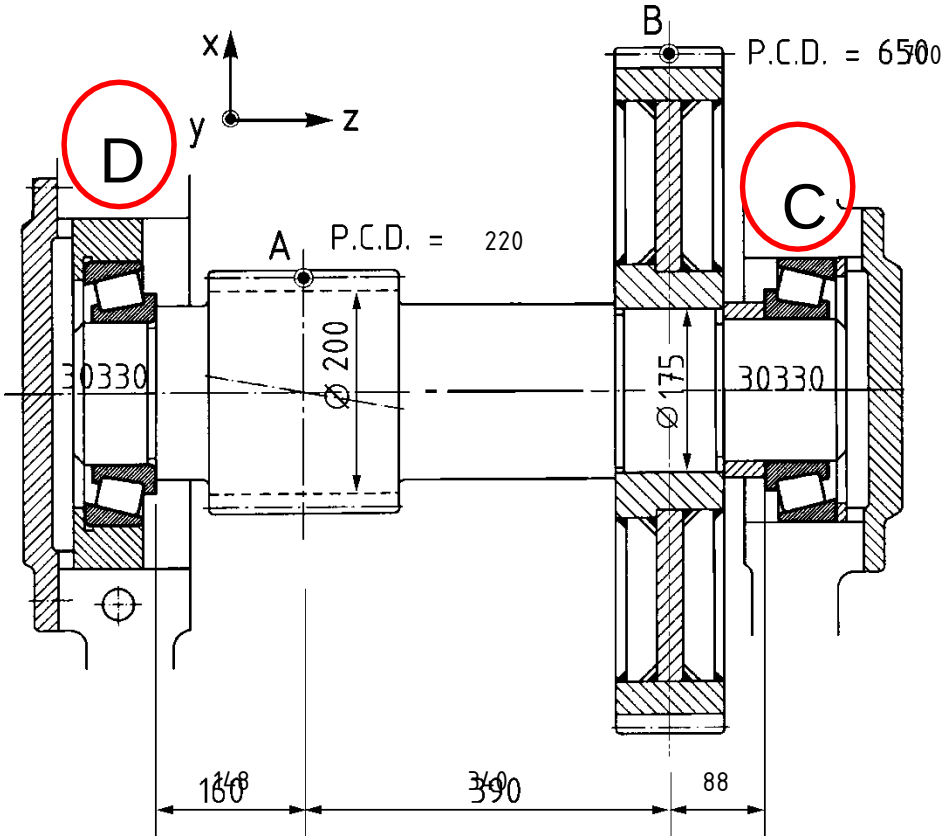
設計問題

圖為減速機之中間軸，請根據所附的軸承與負載數據，求出軸承C與D之受力。

1. y座標正方向為自紙面朝向觀者。
2. ⊙ 表示齒輪負載施加位置
3. P.C.D為節圓直徑
4. 轉速 500 rpm
5. 齒輪齒面間作用負載如右所示

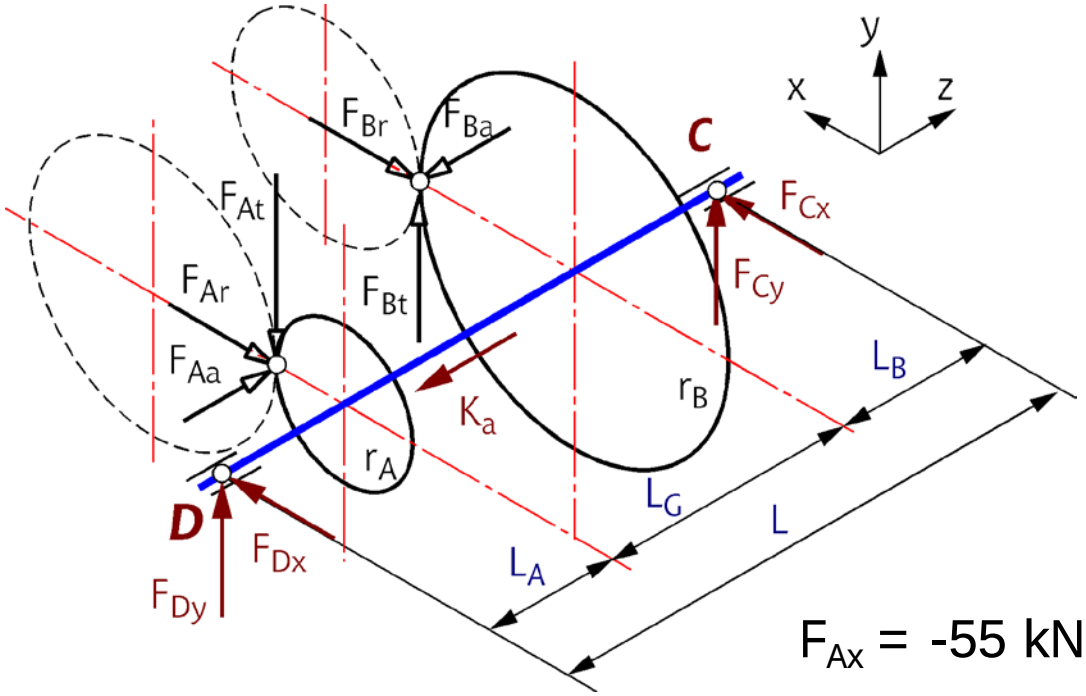
F_{Ax} = -50 kN; F_{Bx} = -20 kN;
F_{Ay} = -160 kN; F_{By} = 50 kN;
F_{Az} = 20 kN; F_{Bz} = -8 kN

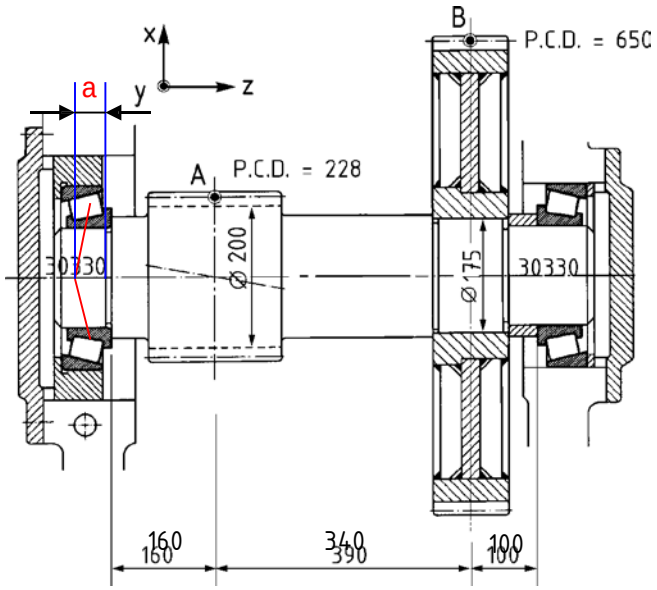




作業檢討

- 將設計問題X型配置改成O型配置



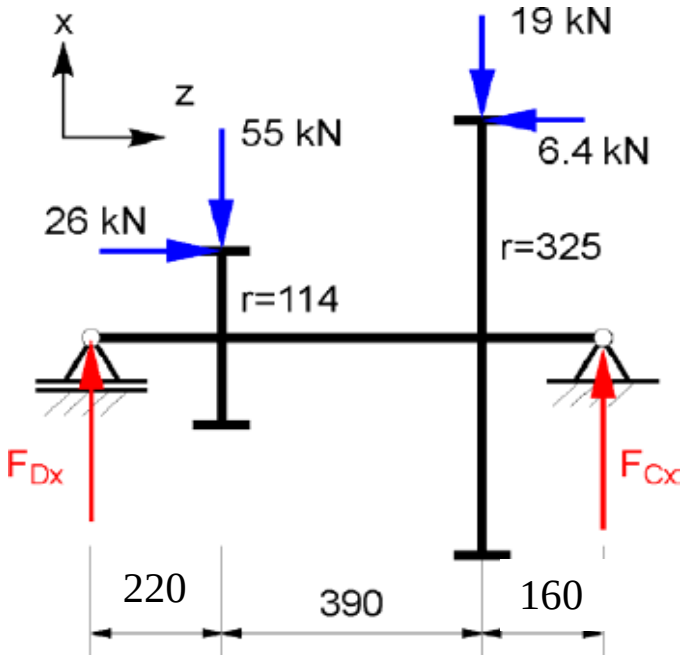


F_{Ax} = -55 kN; F_{Ay} = -148 kN; F_{Az} = 26 kN

F_{Bx} = -19 kN; F_{By} = 52 kN; F_{Bz} = -6.4 kN

L_A = 220 mm; L_G = 340 mm; L_B = 160 mm

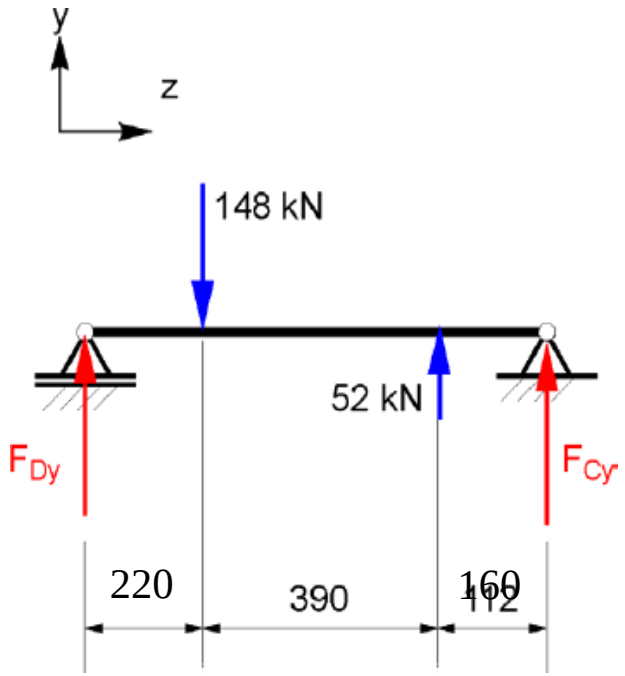
X - Z平面



$$L = 390 + 220 + 160 = 770$$

	FAx	FBx	FAz	FBz	合力
FDx	$55 \cdot \frac{550}{770}$	$19 \cdot \frac{160}{770}$	$-26 \cdot \frac{114}{770}$	$6.4 \cdot \frac{325}{770}$	42.1
FCx	$55 \cdot \frac{220}{770}$	$19 \cdot \frac{610}{770}$	$26 \cdot \frac{114}{770}$	$-6.4 \cdot \frac{325}{770}$	31.9

Y - Z平面



	FAy	FBy	合力
FDy	$148 \cdot \frac{550}{770}$	$-52 \cdot \frac{160}{770}$	94.9
FCy	$148 \cdot \frac{220}{770}$	$-55 \cdot \frac{610}{770}$	-1.3

軸承受力分析

- 合成受力

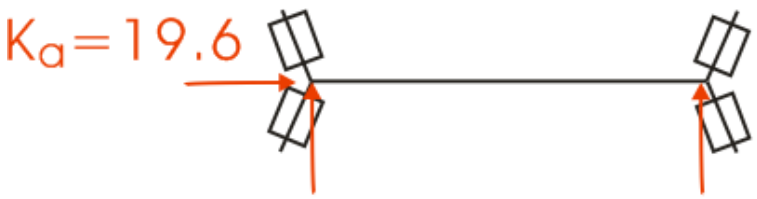
$$F_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = \sqrt{(42.1)^2 + (94.9)^2} = 103.82 \quad [kN]$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(31.9)^2 + (-1.3)^2} = \quad 31.9 \quad [kN]$$

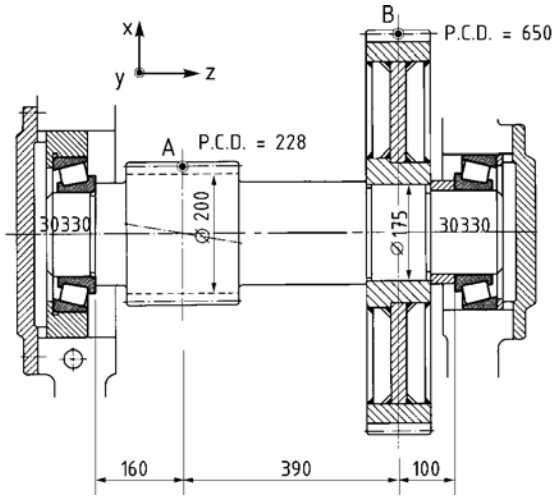
- 軸向受力Ka

Z方向為正 $= 26 - 6.4 = 19.6 \quad [kN]$

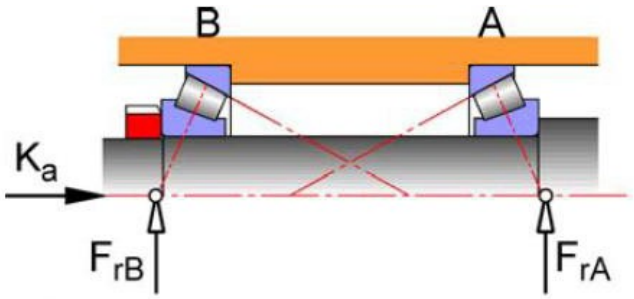
∴Ka為由左至右作用在軸上的力$19.6[kN]$

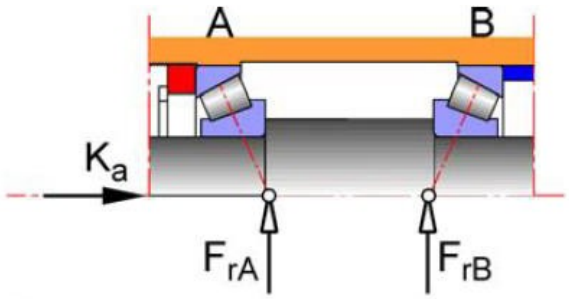


$F_D = 103.82 \quad F_C = 31.9$



錐形滾柱軸承組合之軸向負荷



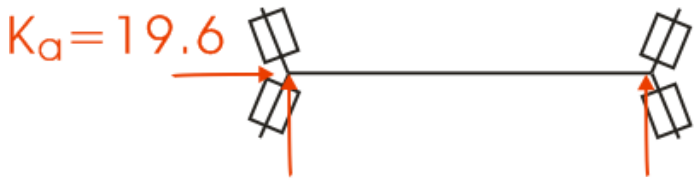


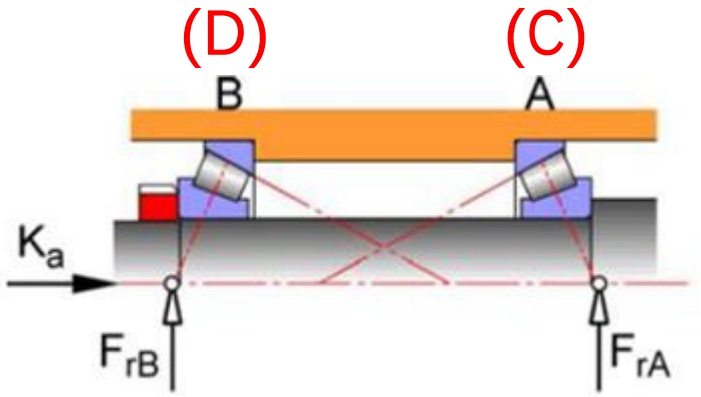
$$F_a = 1.26 \cdot F_r \cdot \tan\alpha$$

Load Condition		Axial Load for equivalent dynamic load Bearing A	Bearing B
$\frac{F_{rA}}{Y_A} \geqslant \frac{F_{rB}}{Y_B}$	$K_a \geqslant 0$	$F_{aA} = \frac{F_{rA}}{2\,Y_A}$	$F_{aB} = \frac{F_{rA}}{2\,Y_A} + K_a$
$\frac{F_{rA}}{Y_A} < \frac{F_{rB}}{Y_B}$	$K_a \geqslant 0.5\left(\frac{F_{rB}}{Y_B} - \frac{F_{rA}}{Y_A}\right)$	$F_{aA} = \frac{F_{rA}}{2\,Y_A}$	$F_{aB} = \frac{F_{rA}}{2\,Y_A} + K_a$
	$K_a < 0.5\left(\frac{F_{rB}}{Y_B} - \frac{F_{rA}}{Y_A}\right)$	$F_{aA} = \frac{F_{rB}}{2\,Y_B} - K_a$	$F_{aB} = \frac{F_{rB}}{2\,Y_B}$

Type of Bearing		e	Single Row				Double Row			
			$F_a / F_r > e$		$F_a / F_r \leqq e$		$F_a / F_r > e$		$F_a / F_r \leqq e$	
			X	Y	X	Y	X	Y	X	Y
Deep groove ball bearing										
F_a / C_0 = 0.025		0.22		2.0				2.0		
= 0.04		0.24		1.8				1.8		
= 0.07		0.27	0.56	1.6	1.00	0	0.56	1.6	1.00	0
= 0.13		0.31		1.4				1.4		
= 0.25		0.37		1.2				1.2		
= 0.5		0.44		1.0				1.0		
Angular contact ball bearing	α = 40 °	1.14	0.35	0.57	1.00	0				
	α = 32 °	0.86					0.62	1.17	1.00	0.73
Self-aligning ball bearing		1.5tanα					0.65	0.975/e	1	0.63/e
Spherical roller bearing		1.5tanα					0.67	1/e	1	0.675/e
Taper roller bearing		1.5tanα	0.4	0.6/e	1	0				

計算軸承軸向受力





Y:軸承止推係數=1.74

$\frac{F_{rB}}{Y_B}$ $\frac{F_{rB}}{Y_B} = \frac{F_D}{Y} = \frac{103.82}{1.74} = 60$

$\frac{F_{rA}}{Y_A}$ $\frac{F_{rA}}{Y_A} = \frac{F_C}{Y} = \frac{31.9}{1.74} = 18.33$

$\because \frac{F_D}{Y} > \frac{F_C}{Y}$ $\frac{F_{rA}}{Y_A} < \frac{F_{rB}}{Y_B}$ ➡ $K_a < 0.5\left(\frac{F_{rB}}{Y_B} - \frac{F_{rA}}{Y_A}\right)$ ➡

$K a = 19.6 < 0.5(60 - 18.33) = 20.835$

$F_{aA} = \frac{F_{rB}}{2\,Y_B} - K_a$

$F_{aB} = \frac{F_{rB}}{2\,Y_B}$

計算軸承軸向受力

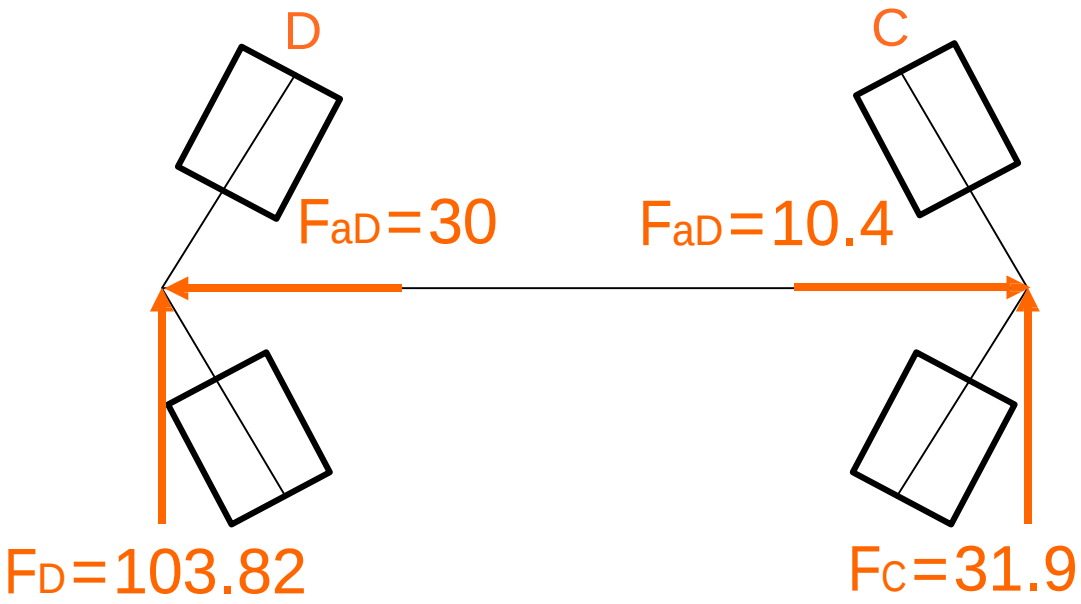
- 軸承軸向受力

$$F_{aB} = \frac{F_{rB}}{2\,Y_B}$$

$$\therefore F_{aD} = \frac{F_D}{2Y} = 30 \quad [kN]$$

$$F_{aA} = \frac{F_{rB}}{2\,Y_B} - K_a$$

$$F_{aC} = \frac{F_D}{2Y} - Ka = 10.4 \quad [kN]$$



計算軸承D的壽命

- Bearing(D)

$$\frac{F_{aD}}{F_D} = \frac{30}{103.82} = 0.289 < e = 0.35$$

$$\therefore X = 1 \quad , \quad Y = 0$$

$$P = X \cdot F_D + Y \cdot F_{aD}$$

$$= 1 \cdot 103.82 + 0 \cdot 30$$

$$= 103.82 \quad [kN]$$

$$\therefore L_{10h_1} = \frac{10^6}{60 \cdot 500}\left(\frac{800}{103.82}\right)^{10/3}$$

$$= 30123.72 \quad [hr]$$

Taper roller bearing ➡

$F_a / F_r \leqq e$	
X	Y
1.00	0

P 軸承等效動負載 $= X \cdot F_r + Y \cdot F_a$

X: 軸承徑向係數
Y: 軸承止推係數
F_r: 徑向負荷
F_a: 軸向負荷

基本額定壽命

$$L_{10h} = \frac{10^6}{60 \cdot n}\left(\frac{C}{P}\right)^p \ [hr]$$

L_{10h} 基本額定壽命（小時）

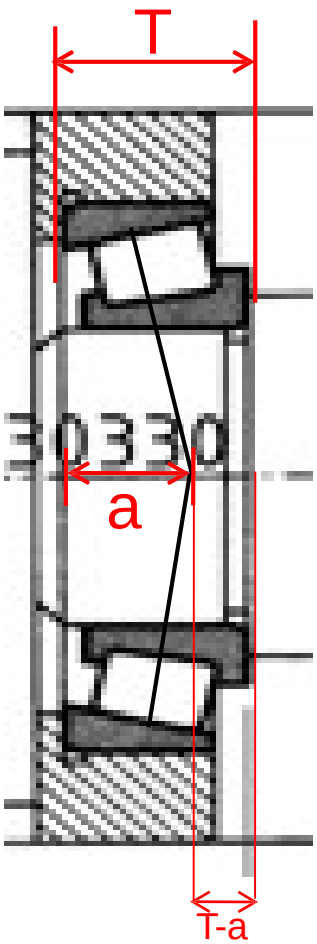
C 基本額定負動負荷

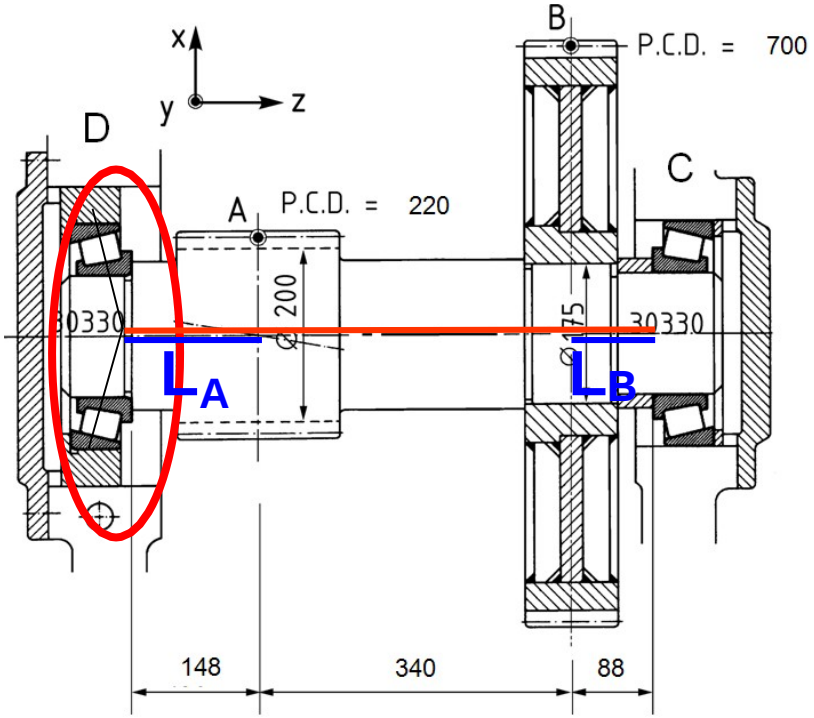
n 軸承轉速 (rpm)

p 運轉壽命乘冪係數 = 3 滾珠 / = 10/3 滾柱

軸承支撐點的位置

- 注意軸承作用力線與軸線相交，兩交點之間跨距才為軸承在軸上的跨距(P.C.D為齒輪節圓直徑)。



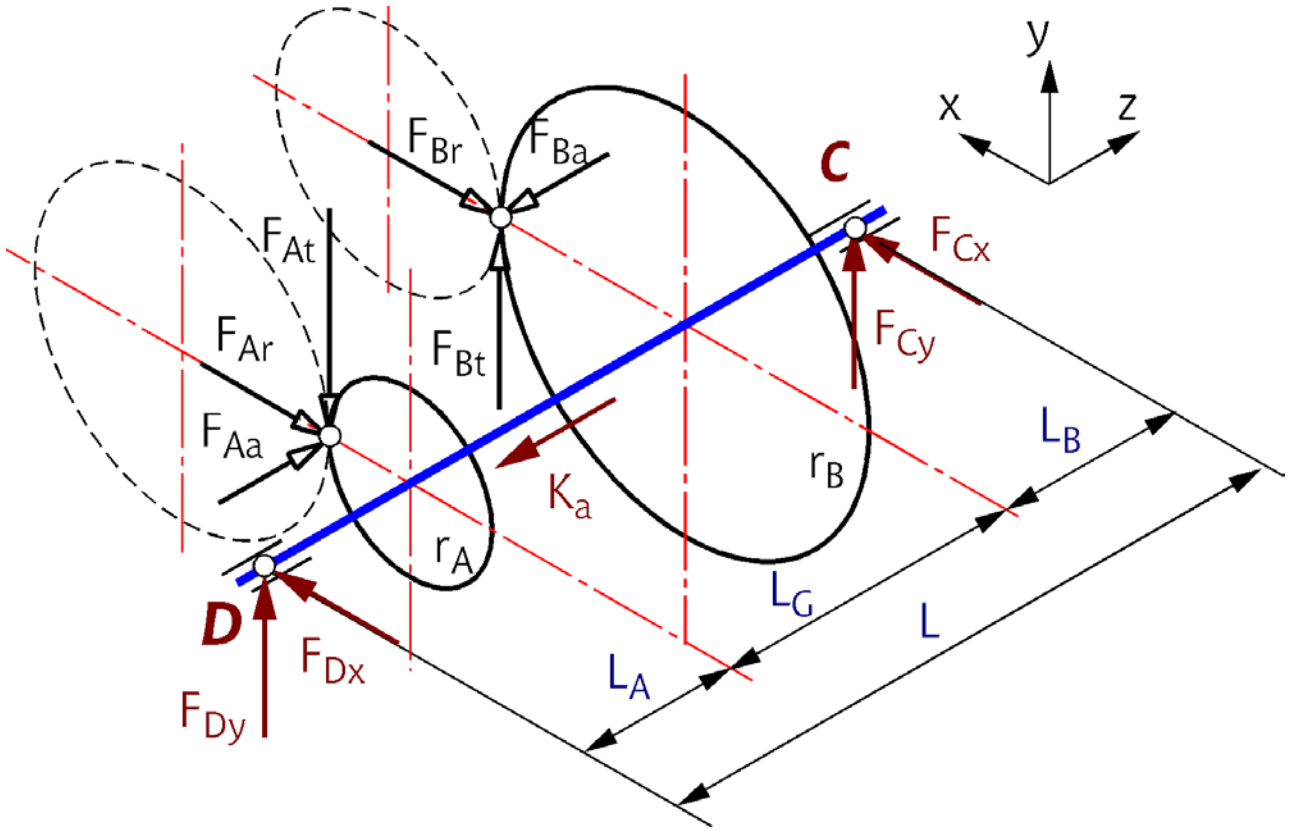


L_A=148+(T-a)=148+12=160 (mm)

L_B=88+(T-a)=88+12=100 (mm)

L=160+100+340=600 (mm)

列出所有受力情形



F_{Ax} = -50 kN; F_{Ay} = -160 kN; F_{Az} = 20 kN

F_{Bx}= -20 kN; F_{By} = 50 kN; F_{Bz} = -8 kN

L_A= 160 mm; L_G = 340 mm; L_B = 100 mm

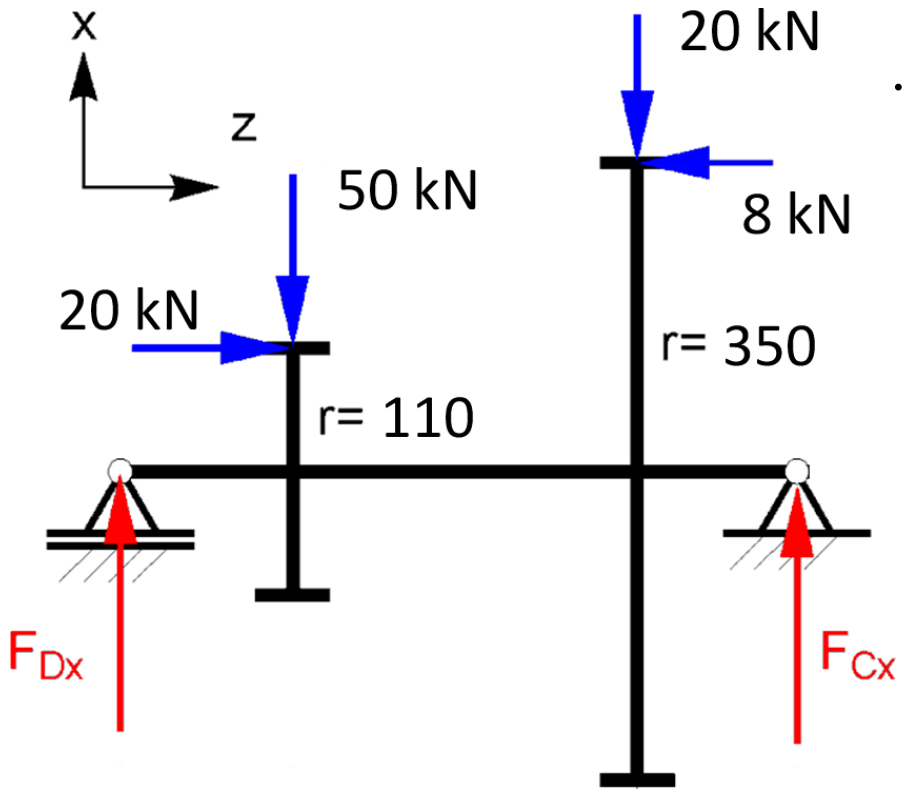
X－Z平面ＦＢＤ與彎曲應力分佈

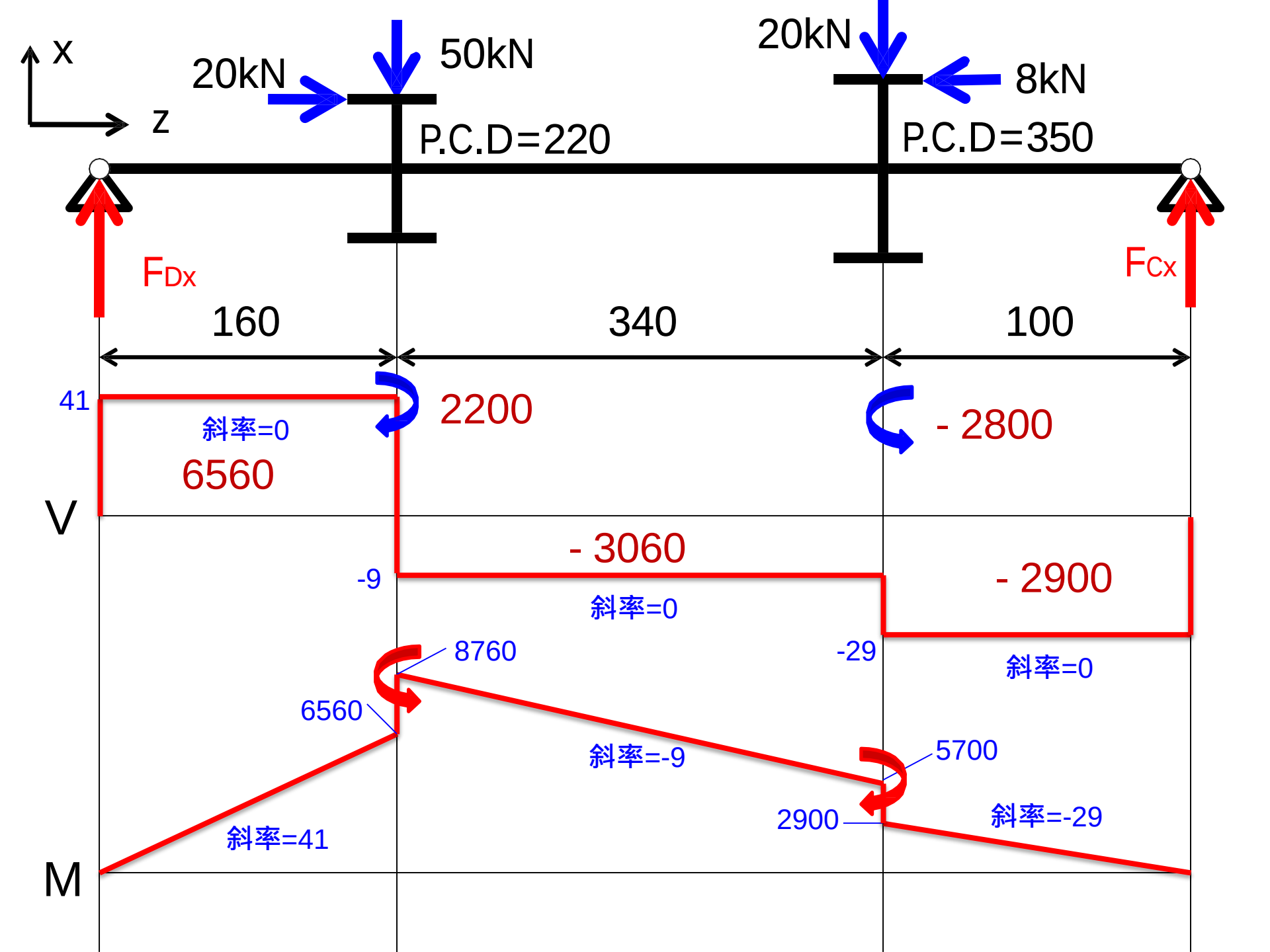
$$\sum F_X = 0 \qquad F_{DX} + F_{CX} - 50 - 20 = 0 \ldots (1)$$

$$\sum M_{CX} = 0 \qquad 8\times 350 + 20\times 100 + 50\times 440 - 20\times 110 - F_{DX}\times 600 = 0$$

$$\therefore F_{DX} = 41 \ [kN] \ldots\ldots \text{代入(1)中得}$$

$$F_{CX} = 29 \ [kN]$$




x
z
20kN
50kN
20kN
8kN
P.C.D=220
P.C.D=350
FDx
FCx
160
340
100
41
斜率=0
6560
2200
- 2800
V
- 3060
-9
斜率=0
- 2900
-29
斜率=0
8760
6560
斜率=-9
5700
2900
斜率=-29
斜率=41
M

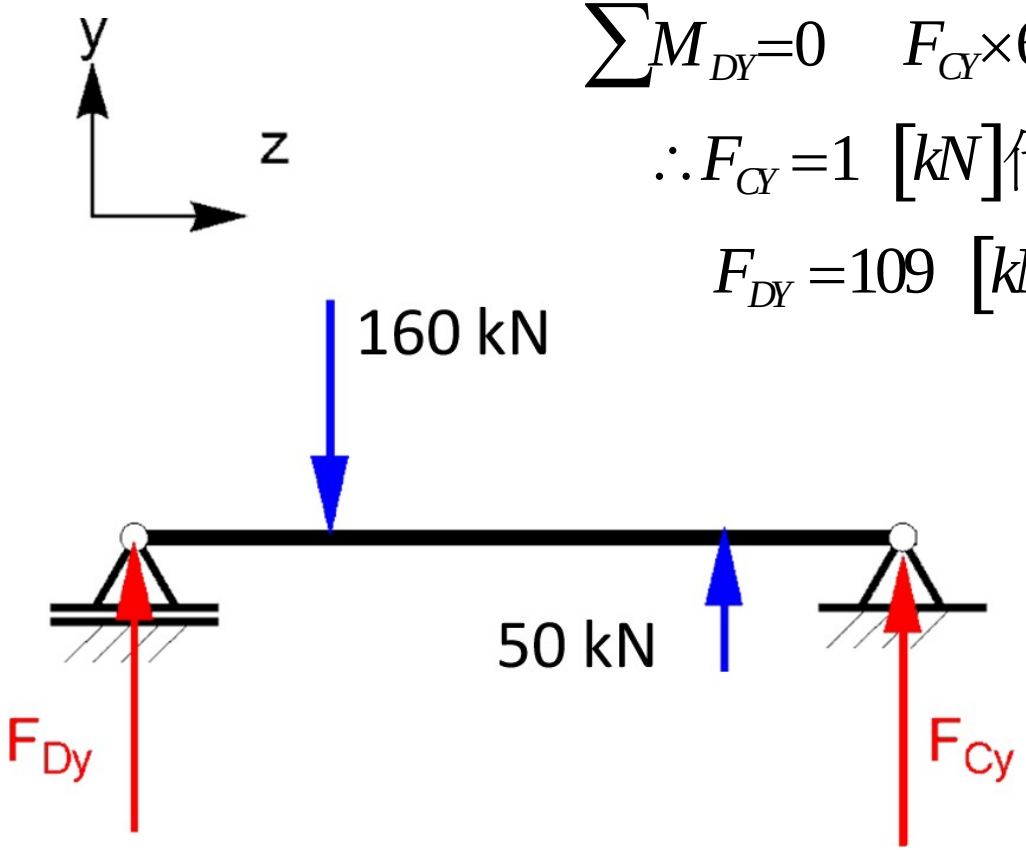
Y－Z平面ＦＢＤ與彎曲應力分佈

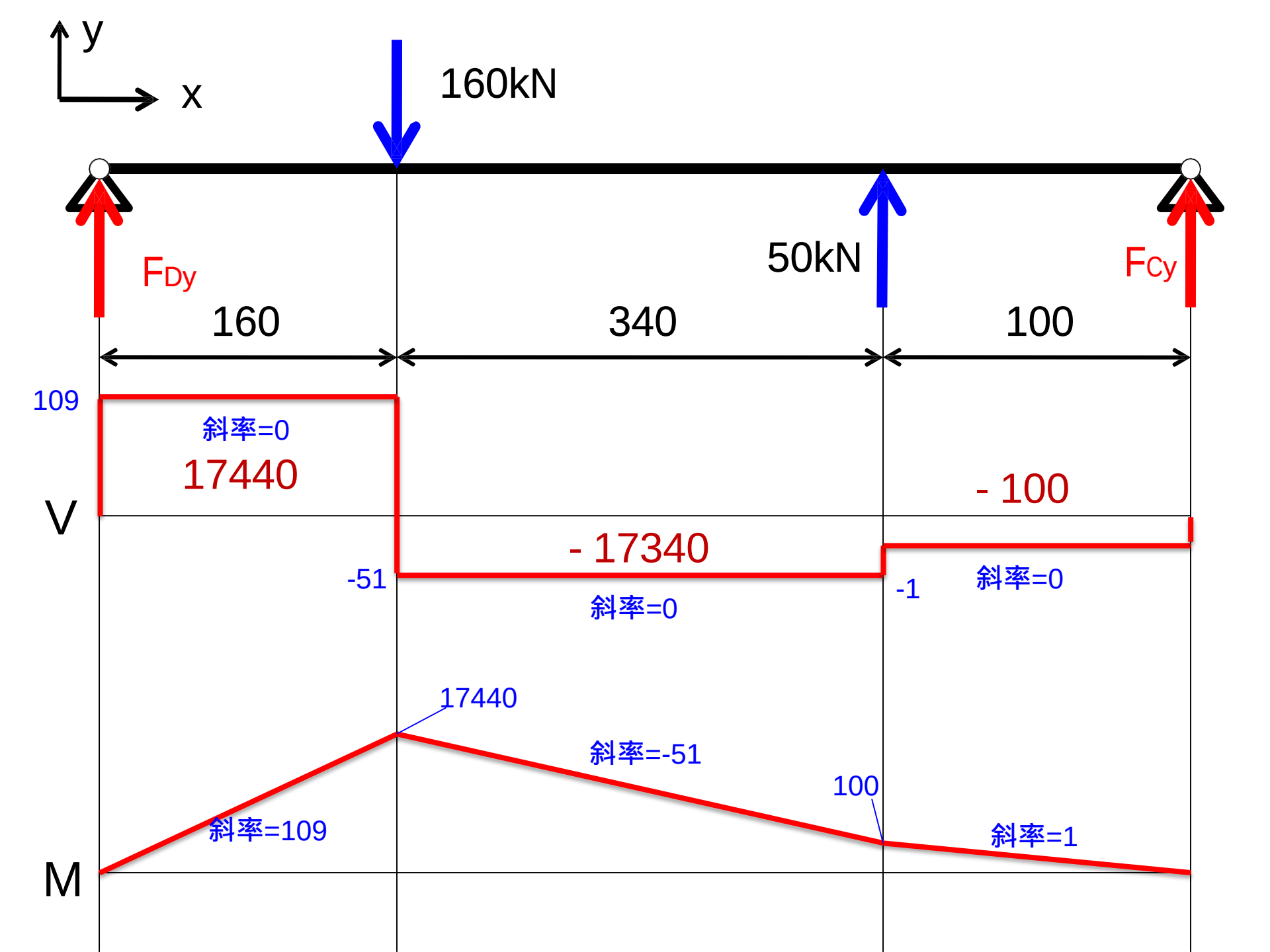
$$\sum F_Y = 0 \qquad F_{DY} - 160 + 50 + F_{CY} = 0 \ldots\ldots (1)$$

$$\sum M_{DY} = 0 \qquad F_{CY} \times 600 - 160 \times 160 + 50 \times 500 = 0$$

$$\therefore F_{CY} = 1 \ [kN] \text{代入(1)中得}$$

$$F_{DY} = 109 \ [kN]$$




y
x
160kN
50kN
FDy
FCy
160
340
100
109
斜率=0
17440
V
- 100
- 17340
-51
斜率=0
-1
斜率=0
17440
斜率=-51
100
斜率=109
斜率=1
M

- 合成受力

$$F_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = \sqrt{(41)^2 + (109)^2} = 116.456 \quad [kN]$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(29)^2 + (1)^2} = \quad 29.02 \quad [kN]$$