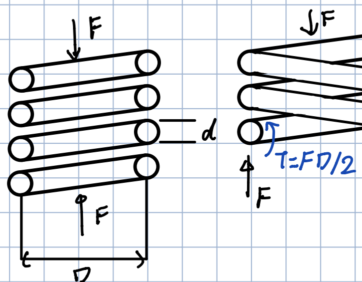
•彈簧應力



$F = V$

⇒ 內側剪應力

$$\tau_{max} = \frac{Tr}{J} + \frac{F}{A}$$

又 $T = \frac{FD}{2}$　$J = \frac{\pi d^4}{32}$

$r = \frac{d}{2}$　$A = \pi d^2/4$

$$\Rightarrow \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

•彈簧指數

$C = \frac{D}{d}$　(常見 $4<C<12$

$K_s = \frac{2C+1}{2C}$　(剪應力修正因數

$$\Rightarrow \tau = K_s \frac{8FD}{\pi d^3}$$　(修正)

•曲率效應

$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$　(Wahl factor

$K_B = \frac{4C+2}{4C-3}$　(Bergsträsser factor

相差不大

$K_c = \frac{K_B}{K_s} = \frac{2C(4C+2)}{(4C-3)(2C+1)}$　(剪應力修正因數

•最終修正

$$\tau = K_B \frac{8FD}{\pi d^3}$$　(應力提升因數,非集中因子)

- 撓曲

- 卡氏定理 Castigliano's theorem

總應變能（扭転分量，剪力分量）

$$U = \frac{T^2 L}{2GJ} + \frac{F^2 L}{2AG} \quad \left(T = FD/2,\ l = \pi D N,\ J = \pi d^4/32,\ A = \pi d^2/4 \right)$$

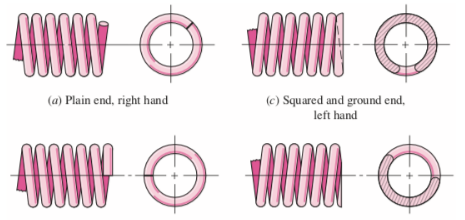
$$\Rightarrow U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 D N}{d^2 G}$$ ($N = N_a$ 有效圈数)

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3N}{d^4 G}\left(1 + \frac{1}{2C^2}\right) \doteq \frac{8FD^3N}{d^4 C^2}$$

彈簧率 scale of spring

$$k = \frac{d^4 G}{8D^3 N}$$

·承压彈簧 压實長度



(a) Plain end, right hand
(b) Squared or closed end, right hand
(c) Squared and ground end, left hand
(d) Plain end, ground, left hand

Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
	Type of Spring Ends			
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

$L_s = (N_t - a)d$ $\quad a \simeq 0.75$

·穩定度

$$y_{cr} = L_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{0.5} \right]$$

$$C_1' = \frac{E}{2(E-G)}$$

$$C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

彈性常數 $\Rightarrow \dfrac{C_2'}{\lambda_{eff}} > 1$

Stable

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

对鋼線 $L_0 < 2.63\frac{D}{\alpha}$ 对plain端、圈端 $\alpha = 0.5$

L_0

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

- S_{ut}

$$S_{ut} = \frac{A}{d^m}$$

粗算

$$\approx S_{sy} = \tau_{all} = 0.56 S_{ut}$$

· 設計彈簧

· 設計條件

$$4 \le C \le 12 \qquad \xi \ge 0.15$$

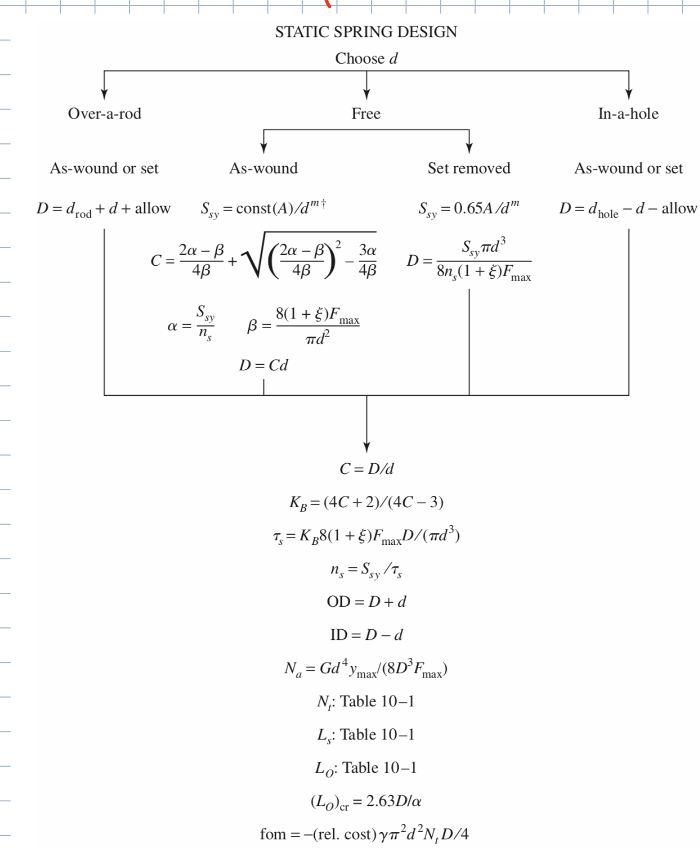
$$3 \le N_a \le 15 \qquad n_s \ge 1.2$$

· 優點數 *the figure of merit*

$$\text{form} = -(\text{相对材料成本})\frac{\gamma\pi^2 d^2 N_t D}{4}$$

$$\underbrace{\frac{S_{sy}}{n_s}}_{\alpha} = K_B\frac{8F_s D}{\pi d^3} = \frac{4C+2}{4C-3}\Big[\underbrace{\boxed{\frac{8(1+\xi)F_{max}}{\pi d^2}}}_{\beta}\,C\Big]$$

Summary



※ C 較大值為
彈簧指數

螺圈彈簧

$$W = AL\gamma = \frac{\pi d^2}{4}(\pi D N_a)(\gamma) = \frac{\pi^2 d^2 D N_a \gamma}{4}$$

$$f = \frac{1}{4}\sqrt{\frac{kg}{W}}$$ （臨界頻率）多15、20倍運動頻率

$\uparrow k \quad \downarrow W$

k=彈簧率

・持久限（Zimmerli：承压彈簧の扭転＋畸变能理論

未經珠擊	珠擊
$S_{sa} = 241\,MPa$	$S_{sa} = 398\,MPa$
$S_{sm} = 379\,MPa$	$S_{sm} = 534\,MPa$

$$S_{se} = \frac{S_{sa}}{1 - \left(\frac{S_{sm}}{S_{su}}\right)^2}$$ （Gerber

$$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}}$$ （Goodman

$$S_{su} = 0.67 S_{ut}$$

$$F_a = \frac{F_{max} - F_{min}}{2}$$

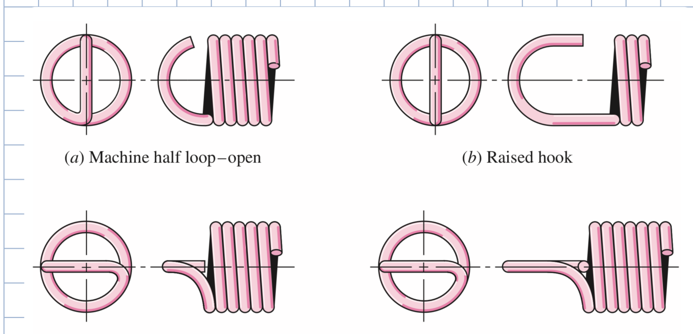
$$\tau_a = K_B \frac{8 F_a D}{\pi d^3}$$

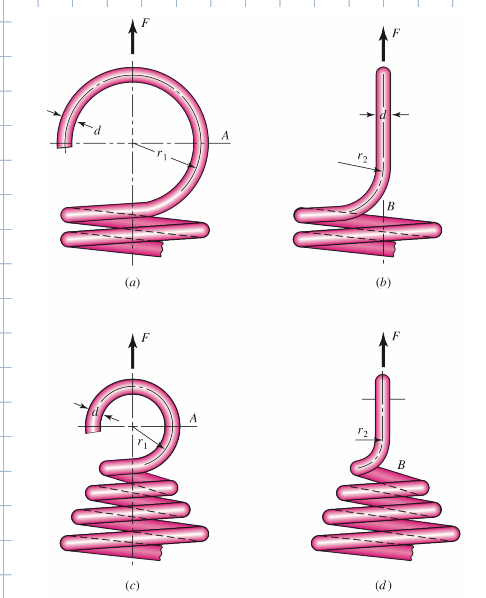
$$F_m = \frac{F_{max} + F_{min}}{2}$$

$$\tau_m = K_B \frac{8 F_m D}{\pi d^3}$$

K_B：应力提升因子

· Hook


(a) Machine half loop–open (b) Raised hook (c) Short twisted loop (d) Full twisted loop



A受最大剪?應力　　B受最大扭轉剪應力

$$\sigma_A = F\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right] \qquad \tau_B = (K)_B \frac{8FD}{\pi d^3}$$

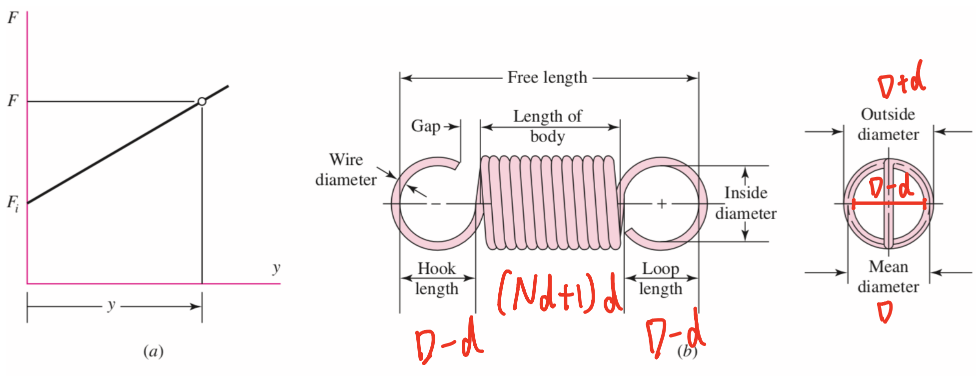
$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1-1)} \quad C_1 = \frac{2r_1}{d} \quad (K)_B = \frac{4C_2-1}{4C_2-4} \quad C_2 = \frac{2r_2}{4}$$

緊密捲繞

一初始拉力

$$F = F_i + ky$$ 彈簧率

(負荷-撓度關係式)



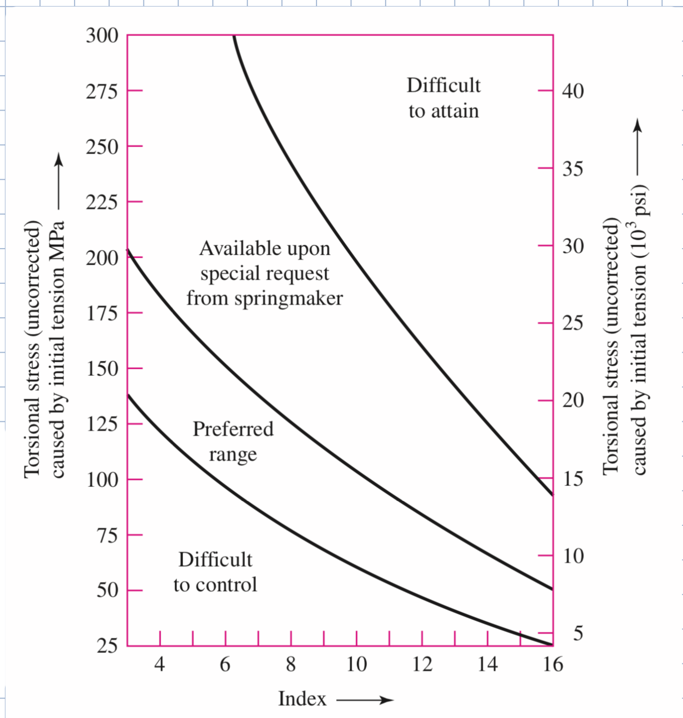
$$L_0 = 2(D-d) + (N_b+1)d$$

Ask 更精確

$$N_a = N_b + \frac{G}{E}$$

本体螺圈数
(含 hook?) ?

等效作用



未修正の扭転剪應力
no K_B, K_W

$$\tau_i = \frac{231}{e^{0.105C}} \pm 6.9\left(4 - \frac{C-3}{6.5}\right)$$

Materials	Percent of Tensile Strength: In Torsion, Body	In Torsion, End	In Bending, End
Patented, cold-drawn or hardened and tempered carbon and low-alloy steels	45–50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

· 持久限 Alternation

S_r：彎曲強度

S_{sr}：扭転強度

$$S_{se} = \frac{S_{sr}/2}{1-\left(\frac{S_{sr}/2}{S_{su}}\right)^2}$$

多に Zimmerli：差 5%

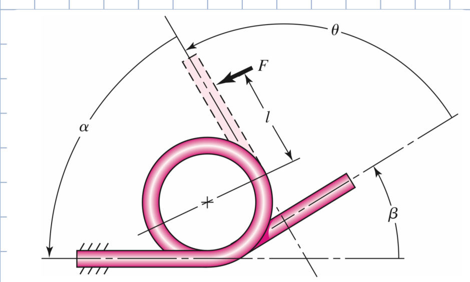
$$S_e = \frac{S_r/2}{1-\left(\frac{S_r/2}{S_{ut}}\right)^2}$$

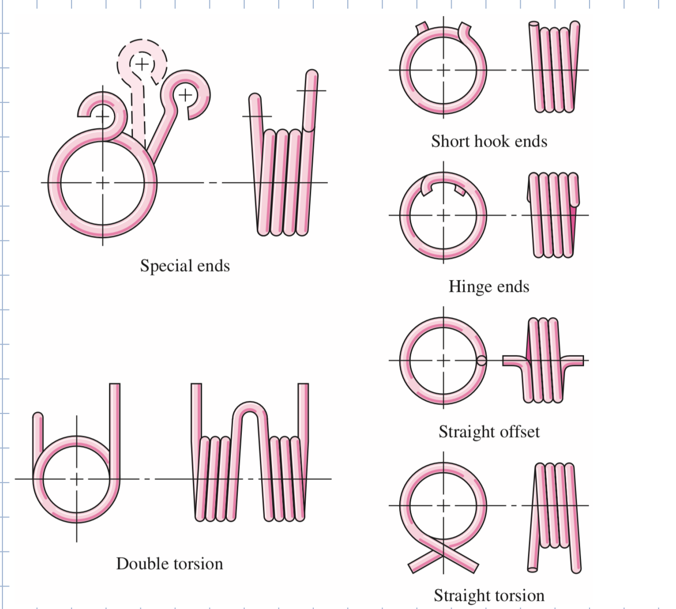
Number of Cycles	Percent of Tensile Strength In Torsion Body	In Torsion End	In Bending End
10^5	36	34	51
10^6	33	30	47
10^7	30	28	45

ex $S_r = 0.45 S_{ut} = 0.45(1819) = 818.6$

• 扭轉彈簧 (torsion spring)

• $N_b = 整数 + \frac{\beta}{360^\circ}$





$\alpha + \theta = const.$

彎應力 $\sigma = K\frac{Mc}{I}$

其中 $K_i = \frac{4C^2-C-1}{4C(C-1)}$ $\cancel{K_o = \frac{4C^2+C-1}{4C(C+1)}}$

inear outer ↳ always < 1

$$\Rightarrow \boxed{\sigma = K_i \frac{32Fr}{\pi d^3}}$$

$M = Fr$ 彎應力

·撓度&彈簧率

角變形 弳度(radian) 捲數(turns)

$$\underline{k'} = \frac{M_1}{\theta_1} = \frac{M_2}{\theta_2'} = \frac{M_2 - M_1}{\theta_2' - \theta_1}$$

(線性)

單位扭矩/圈

$$\theta_e = \frac{y}{l} = \frac{Fl^2}{3EI} = \frac{Fl^2}{3E(\pi d^4/64)} = \frac{64Ml}{3\pi d^4 E}$$

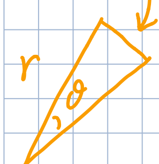
懸臂樑端撓度~所对の角?

Ask

$$r\theta = \frac{\partial U}{\partial F} = \int_0^{\pi D N_b} \frac{\partial}{\partial F}\left(\frac{F^2 r^2 dx}{2EI}\right) = \int_0^{\pi D N_b} \frac{Fr^2 dx}{EI}$$

(F·l 位能

(F 力

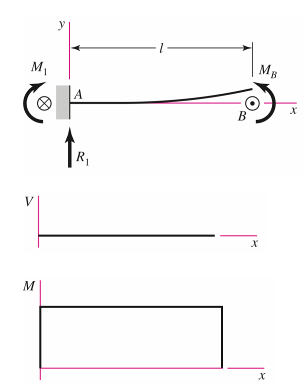


$$M = Fl = Fr$$

2?

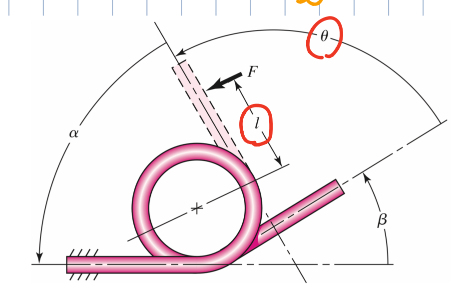
$$\frac{Ml^2}{2EI} = y \qquad y \times F = U \qquad y \times \frac{M}{l}$$

4 Cantilever—moment load



$R_1 = V = 0 \qquad M_1 = M = M_B$

$$y = \frac{M_B x^2}{2EI} \qquad y_{max} = \frac{M_B l^2}{2EI}$$



$I = \pi d^4/64$ 代入

$$\theta = \frac{64 F_r D N_b}{d^4 E} = \frac{64 M D N_b}{d^4 E}$$