Chapter 16

16-1 Given: $r = 300/2 = 150$ mm, $a = R = 125$ mm, $b = 40$ mm, $f = 0.28$, $F = 2.2$ kN, $\theta_1 = 0°$, $\theta_2 = 120°$, and $\theta_a = 90°$. From which, $\sin\theta_a = \sin 90° = 1$.

Eq. (16-2):

$$M_f = \frac{0.28 p_a (0.040)(0.150)}{1} \int_{0°}^{120°} \sin\theta (0.150 - 0.125\cos\theta)\, d\theta$$
$$= 2.993\left(10^{-4}\right) p_a \ \text{N}\cdot\text{m}$$

Eq. (16-3): $$M_N = \frac{p_a(0.040)(0.150)(0.125)}{1} \int_{0°}^{120°} \sin^2\theta\, d\theta = 9.478\left(10^{-4}\right) p_a \ \text{N}\cdot\text{m}$$

$$c = 2(0.125 \cos 30°) = 0.2165 \text{ m}$$

Eq. (16-4): $$F = \frac{9.478\left(10^{-4}\right) p_a - 2.993\left(10^{-4}\right) p_a}{0.2165} = 2.995\left(10^{-3}\right) p_a$$

$p_a = F/[2.995(10^{-3})] = 2200/[2.995(10^{-3})]$
$= 734.5(10^3)$ Pa for cw rotation

Eq. (16-7): $$2200 = \frac{9.478\left(10^{-4}\right) p_a + 2.993\left(10^{-4}\right) p_a}{0.2165}$$

$p_a = 381.9(10^3)$ Pa for ccw rotation

A maximum pressure of 734.5 kPa occurs on the RH shoe for cw rotation. *Ans.*

(b) *RH shoe*:

Eq. (16-6):

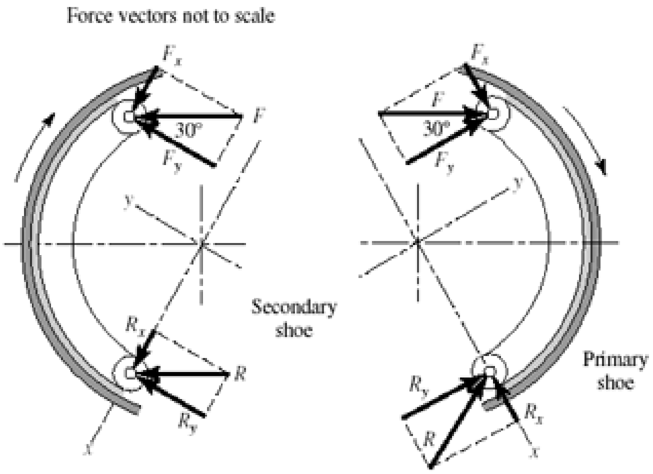
$$T_R = \frac{0.28(734.5)10^3(0.040)0.150^2(\cos 0° - \cos 120°)}{1} = 277.6 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

LH shoe:

$$T_L = 277.6\frac{381.9}{734.5} = 144.4 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$T_{\text{total}} = 277.6 + 144.4 = 422$ N · m *Ans.*

(c)



RH shoe: $F_x = 2200 \sin 30° = 1100$ N, $F_y = 2200 \cos 30° = 1905$ N

Eqs. (16-8): $A = \left(\dfrac{1}{2}\sin^2\theta\right)_{0°}^{120°} = 0.375, \quad B = \left(\dfrac{\theta}{2} - \dfrac{1}{4}\sin 2\theta\right)_{0}^{2\pi/3 \text{ rad}} = 1.264$

Eqs. (16-9):
$$R_x = \frac{734.5\left(10^3\right)0.040(0.150)}{1}[0.375 - 0.28(1.264)] - 1100 = -1007 \text{ N}$$
$$R_y = \frac{734.5\left(10^3\right)0.04(0.150)}{1}[1.264 + 0.28(0.375)] - 1905 = 4128 \text{ N}$$
$$R = [(-1007)^2 + 4128^2]^{1/2} = 4249 \text{ N} \quad \textit{Ans.}$$

LH shoe: $F_x = 1100$ N, $F_y = 1905$ N

Eqs. (16-10):
$$R_x = \frac{381.9\left(10^3\right)0.040(0.150)}{1}[0.375 + 0.28(1.264)] - 1100 = 570 \text{ N}$$
$$R_y = \frac{381.9\left(10^3\right)0.040(0.150)}{1}[1.264 - 0.28(0.375)] - 1905 = 751 \text{ N}$$
$$R = \left(597^2 + 751^2\right)^{1/2} = 959 \text{ N} \quad \textit{Ans.}$$

16-2 Given: $r = 300/2 = 150$ mm, $a = R = 125$ mm, $b = 40$ mm, $f = 0.28$, $F = 2.2$ kN, $\theta_1 = 15°$, $\theta_2 = 105°$, and $\theta_a = 90°$. From which, $\sin\theta_a = \sin 90° = 1$.

Eq. (16-2):
$$M_f = \frac{0.28 p_a (0.040)(0.150)}{1}\int_{15°}^{105°} \sin\theta(0.150 - 0.125\cos\theta)\, d\theta = 2.177\left(10^{-4}\right) p_a$$

Eq. (16-3): $M_N = \dfrac{p_a(0.040)(0.150)(0.125)}{1}\int_{15°}^{105°} \sin^2\theta\, d\theta = 7.765\left(10^{-4}\right) p_a$

$c = 2(0.125) \cos 30° = 0.2165$ m

Eq. (16-4): $F = \dfrac{7.765\left(10^{-4}\right) p_a - 2.177\left(10^{-4}\right) p_a}{0.2165} = 2.581\left(10^{-3}\right) p_a$

RH shoe: $p_a = 2200/\ [2.581(10^{-3})] = 852.4\ (10^3)$ Pa
= 852.4 kPa on RH shoe for cw rotation *Ans.*

Eq. (16-6): $T_R = \dfrac{0.28(852.4)10^3(0.040)(0.150^2)(\cos 15° - \cos 105°)}{1} = 263\ \text{N} \cdot \text{m}$

LH shoe:

$$2200 = \frac{7.765\left(10^{-4}\right) p_a + 2.177\left(10^{-4}\right) p_a}{0.2165}$$

$p_a = 479.1\left(10^3\right)$ Pa $= 479.1$ kPa on LH shoe for cw rotation *Ans.*

$$T_L = \frac{0.28(479.1)10^3(0.040)(0.150^2)(\cos 15° - \cos 105°)}{1} = 148\ \text{N} \cdot \text{m}$$

$T_{\text{total}} = 263 + 148 = 411\ \text{N} \cdot \text{m}$ *Ans.*

Comparing this result with that of Prob. 16-1, a 2.6% reduction in torque is obtained by using 25% less braking material.

16-3 Given: $\theta_1 = 0°$, $\theta_2 = 120°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = R = 3.5$ in, $b = 1.25$ in, $f = 0.30$, $F = 225$ lbf, $r = 11/2 = 5.5$ in, counter-clockwise rotation.

LH shoe:

Eq. (16-2), with $\theta_1 = 0$:

$$M_f = \frac{f\,p_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin\theta\left(r - a\cos\theta\right) d\theta = \frac{f\,p_a br}{\sin\theta_a}\left[r(1 - \cos\theta_2) - \frac{a}{2}\sin^2\theta_2\right]$$

$$= \frac{0.30\,p_a(1.25)5.5}{1}\left[5.5(1 - \cos 120°) - \frac{3.5}{2}\sin^2 120°\right]$$

$$= 14.31 p_a\ \text{lbf} \cdot \text{in}$$

Eq. (16-3), with $\theta_1 = 0$:

$$M_N = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin^2\theta d\theta = \frac{p_a bra}{\sin\theta_a}\left[\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right]$$

$$= \frac{p_a(1.25)5.5(3.5)}{1}\left[\frac{120°}{2}\left(\frac{\pi}{180°}\right) - \frac{1}{4}\sin 2(120°)\right]$$

$$= 30.41\, p_a\ \text{lbf} \cdot \text{in}$$

$$c = 2r\cos\left(\frac{180^\circ - \theta_2}{2}\right) = 2(5.5)\cos 30^\circ = 9.526 \text{ in}$$

$$F = 225 = \frac{30.41 p_a - 14.31 p_a}{9.526} = 1.690\, p_a$$

$$p_a = 225 / 1.690 = 133.1 \text{ psi}$$

Eq. (16-6):

$$T_L = \frac{f\, p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.30(133.1)1.25(5.5^2)}{1}[1 - (-0.5)]$$

$$= 2265 \text{ lbf}\cdot\text{in} = 2.265 \text{ kip}\cdot\text{in} \quad \textit{Ans}.$$

RH shoe:

$$F = 225 = \frac{30.41 p_a + 14.31 p_a}{9.526} = 4.694\, p_a$$

$$p_a = 225 / 4.694 = 47.93 \text{ psi}$$

$$T_R = \frac{47.93}{133.1} 2265 = 816 \text{ lbf}\cdot\text{in} = 0.816 \text{ kip}\cdot\text{in}$$

$$T_{\text{total}} = 2.27 + 0.82 = 3.09 \text{ kip}\cdot\text{in} \qquad \textit{Ans}.$$

16-4 **(a)** Given: $\theta_1 = 10°$, $\theta_2 = 75°$, $\theta_a = 75°$, $p_a = 10^6$ Pa, $f = 0.24$, $b = 0.075$ m (shoe width), $a = 0.150$ m, $r = 0.200$ m, $d = 0.050$ m, $c = 0.165$ m.

Some of the terms needed are evaluated here:

$$A = \left[r\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta - a\int_{\theta_1}^{\theta_2}\sin\theta\cos\theta\, d\theta\right] = r\left[-\cos\theta\right]_{\theta_1}^{\theta_2} - a\left[\frac{1}{2}\sin^2\theta\right]_{\theta_1}^{\theta_2}$$

$$= 200\left[-\cos\theta\right]_{10°}^{75°} - 150\left[\frac{1}{2}\sin^2\theta\right]_{10°}^{75°} = 77.5 \text{ mm}$$

$$B = \int_{\theta_1}^{\theta_2}\sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_{10\pi/180 \text{ rad}}^{75\pi/180 \text{ rad}} = 0.528$$

$$C = \int_{\theta_1}^{\theta_2}\sin\theta\cos\theta\, d\theta = 0.4514$$

Now converting to Pascals and meters, we have from Eq. (16-2),

$$M_f = \frac{f\, p_a b r}{\sin\theta_a} A = \frac{0.24\left(10^6\right)(0.075)(0.200)}{\sin 75°}(0.0775) = 289 \text{ N}\cdot\text{m}$$

From Eq. (16-3),

$$M_N = \frac{p_a bra}{\sin\theta_a}B = \frac{10^6(0.075)(0.200)(0.150)}{\sin 75°}(0.528) = 1230 \text{ N}\cdot\text{m}$$

Finally, using Eq. (16-4), we have

$$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN} \quad \textit{Ans.}$$

(b) Use Eq. (16-6) for the primary shoe.

$$T = \frac{fp_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$
$$= \frac{0.24\left(10^6\right)(0.075)(0.200)^2(\cos\ 10° - \cos\ 75°)}{\sin 75°} = 541 \text{ N}\cdot\text{m}$$

For the secondary shoe, we must first find p_a. Substituting

$$M_N = \frac{1230}{10^6}p_a \text{ and } M_f = \frac{289}{10^6}p_a \quad \text{into Eq. (16 - 7),}$$
$$5.70 = \frac{(1230\ /\ 10^6)p_a + (289\ /\ 10^6)p_a}{165}, \quad \text{solving gives} \quad p_a = 619\left(10^3\right)\text{ Pa}$$

Then

$$T = \frac{0.24\left[619\left(10^3\right)\right]0.075\left(0.200^2\right)\left(\cos\ 10° - \cos\ 75°\right)}{\sin 75°} = 335 \text{ N}\cdot\text{m}$$

so the braking capacity is $T_{\text{total}} = 2(541) + 2(335) = 1750 \text{ N}\cdot\text{m}$ *Ans.*

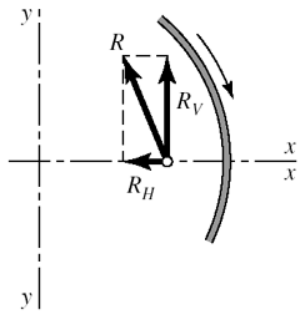
(c) *Primary shoes*:

$$R_x = \frac{p_a br}{\sin\theta_a}\left(C - f\,B\right) - F_x$$
$$= \frac{10^6(0.075)0.200}{\sin 75°}[0.4514 - 0.24(0.528)](10^{-3}) - 5.70 = -0.658 \text{ kN}$$
$$R_y = \frac{p_a br}{\sin\theta_a}(B + f\,C) - F_y$$
$$= \frac{10^6(0.075)0.200}{\sin 75°}[0.528 + 0.24(0.4514)]\left(10^{-3}\right) - 0 = 9.88 \text{ kN}$$

Secondary shoes:

$$\begin{aligned}R_x &= \frac{p_a br}{\sin\theta_a}(C + f\,B) - F_x\\ &= \frac{0.619\left(10^6\right)0.075(0.200)}{\sin 75°}[0.4514 + 0.24(0.528)]\left(10^{-3}\right) - 5.70\\ &= -0.143\text{ kN}\\ R_y &= \frac{p_a br}{\sin\theta_a}(B - f\,C) - F_y\\ &= \frac{0.619\left(10^6\right)0.075(0.200)}{\sin 75°}[0.528 - 0.24(0.4514)]\left(10^{-3}\right) - 0\\ &= 4.03\text{ kN}\end{aligned}$$

Note from figure that $+y$ for secondary shoe is opposite to $+y$ for primary shoe.



Combining horizontal and vertical components,

$$\begin{aligned}R_H &= -0.658 - 0.143 = -0.801\text{ kN}\\ R_V &= 9.88 - 4.03 = 5.85\text{ kN}\\ R &= \sqrt{(-0.801)^2 + 5.85^2}\\ &= 5.90\text{ kN} \quad \textit{Ans.}\end{aligned}$$

16-5 Given: Face width b = 1.25 in, F = 90 lbf, f = 0.25.

Preliminaries: $\theta_1 = 45° - \tan^{-1}(6/8) = 8.13°$, $\theta_2 = 98.13°$, $\theta_a = 90°$, $a = (6^2 + 8^2)^{1/2} = 10$ in

Eq. (16-2):

$$\begin{aligned}M_f &= \frac{f\,p_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta\left(r - a\cos\theta\right)d\theta = \frac{0.25p_a(1.25)6}{1}\int_{8.13°}^{98.13°}\sin\theta\left(6 - 10\cos\theta\right)d\theta\\ &= 3.728p_a\text{ lbf}\cdot\text{in}\end{aligned}$$

Eq. (16-3):

$$\begin{aligned}M_N &= \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta\,d\theta = \frac{p_a(1.25)6(10)}{1}\int_{8.13°}^{98.13°}\sin^2\theta\,d\theta\\ &= 69.405p_a\text{ lbf}\cdot\text{in}\end{aligned}$$

Eq. (16-4): Using $Fc = M_N - M_f$, we obtain

$$90(20) = (69.405 - 3.728)p_a \quad\Rightarrow\quad p_a = 27.4\text{ psi} \quad \textit{Ans.}$$

Eq. (16-6):

$$T = \frac{fp_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.25(27.4)1.25(6^2)(\cos 8.13° - \cos 98.13°)}{1}$$
$$= 348.7 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-6 For $+3\hat{\sigma}_f$:

$$f = \bar{f} + 3\hat{\sigma}_f = 0.25 + 3(0.025) = 0.325$$

From Prob. 16-5, with $f = 0.25$, $M_f = 3.728\, p_a$. Thus, $M_f = (0.325/0.25)\, 3.728\, p_a = 4.846\, p_a$. From Prob. 16-5, $M_N = 69.405\, p_a$.

Eq. (16-4): Using $Fc = M_N - M_f$, we obtain

$$90(20) = (69.405 - 4.846)p_a \quad \Rightarrow \quad p_a = 27.88 \text{ psi} \quad \textit{Ans.}$$

From Prob. 16-5, $p_a = 27.4$ psi and $T = 348.7$ lbf·in. Thus,

$$T = \left(\frac{0.325}{0.25}\right)\left(\frac{27.88}{27.4}\right)348.7 = 461.3 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

Similarly, for $-3\hat{\sigma}_f$:

$$f = \bar{f} - 3\hat{\sigma}_f = 0.25 - 3(0.025) = 0.175$$
$$M_f = (0.175 / 0.25)\, 3.728 p_a = 2.610 p_a$$

$$90(20) = (69.405 - 2.610)\, p_a \quad \Rightarrow \quad p_a = 26.95 \text{ psi}$$
$$T = \left(\frac{0.175}{0.25}\right)\left(\frac{26.95}{27.4}\right)348.7 = 240.1 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-7 Preliminaries: $\theta_2 = 180° - 30° - \tan^{-1}(3/12) = 136°$, $\theta_1 = 20° - \tan^{-1}(3/12) = 6°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = (3^2 + 12^2)^{1/2} = 12.37$ in, $r = 10$ in, $f = 0.30$, $b = 2$ in, $p_a = 150$ psi.

Eq. (16-2): $$M_f = \frac{0.30(150)(2)(10)}{\sin 90°}\int_{6°}^{136°} \sin\theta(10 - 12.37\cos\theta)\, d\theta = 12\,800 \text{ lbf} \cdot \text{in}$$

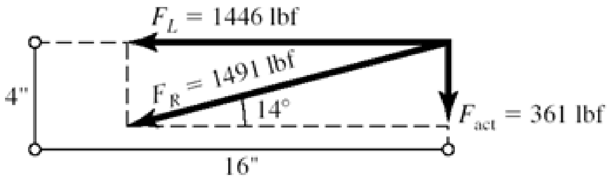
Eq. (16-3): $$M_N = \frac{150(2)(10)(12.37)}{\sin 90°}\int_{6°}^{136°} \sin^2\theta\, d\theta = 53\,300 \text{ lbf} \cdot \text{in}$$

LH shoe:

$$c_L = 12 + 12 + 4 = 28 \text{ in}$$

Now note that M_f is cw and M_N is ccw. Thus,

$$F_L = \frac{53\ 300 - 12\ 800}{28} = 1446 \text{ lbf}$$



Eq. (16-6): $$T_L = \frac{0.30(150)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 15\ 420 \text{ lbf} \cdot \text{in}$$

RH shoe:

$$M_N = 53\ 300 \frac{p_a}{150} = 355.3 p_a, \quad M_f = 12\ 800 \frac{p_a}{150} = 85.3 p_a$$

On this shoe, both M_N and M_f are ccw. Also,

$$c_R = (24 - 2 \tan 14°) \cos 14° = 22.8 \text{ in}$$
$$F_{\text{act}} = F_L \sin 14° = 361 \text{ lbf} \quad \textit{Ans.}$$
$$F_R = F_L / \cos 14° = 1491 \text{ lbf}$$

Thus, $$1491 = \frac{355.3 + 85.3}{22.8} p_a \quad \Rightarrow \quad p_a = 77.2 \text{ psi}$$

Then, $$T_R = \frac{0.30(77.2)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 7940 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 15\ 420 + 7940 = 23\ 400 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-8

$$M_f = 2\int_0^{\theta_2} (f dN)(a' \cos\theta - r) \qquad \text{where } dN = pbr\ d\theta$$
$$= 2 fpbr \int_0^{\theta_2} (a' \cos\theta - r)\ d\theta = 0$$

From which

$$a' \int_0^{\theta_2} \cos\theta\ d\theta = r \int_0^{\theta_2} d\theta$$
$$a' = \frac{r\theta_2}{\sin\theta_2} = \frac{r(60°)(\pi / 180)}{\sin 60°} = 1.209r \quad \textit{Ans.}$$

Eq. (16-15):

$$a = \frac{4r\sin 60°}{2(60)(\pi/180) + \sin[2(60)]} = 1.170r \qquad Ans.$$

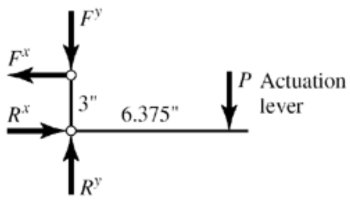
a differs with a' by 100(1.170 –1.209)/1.209 = – 3.23 % *Ans.*

16-9 **(a)** Counter-clockwise rotation, $\theta_2 = \pi/4$ rad, $r = 13.5/2 = 6.75$ in
Eq. (16-15):

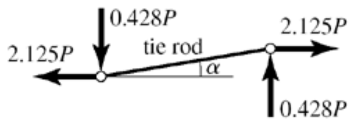
$$a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2} = \frac{4(6.75)\sin(\pi/4)}{2\pi/4 + \sin(2\pi/4)} = 7.426 \text{ in}$$

$$e = 2a = 2(7.426) = 14.85 \text{ in} \qquad Ans.$$

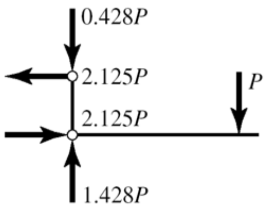
(b)



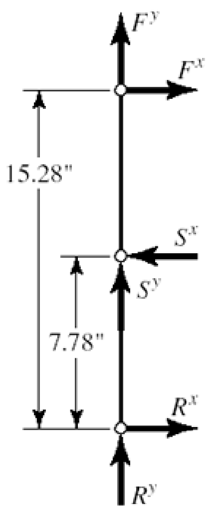
$$\alpha = \tan^{-1}(3/14.85) = 11.4°$$



$$\sum M_R = 0 = 3F^x - 6.375P \quad \Rightarrow \quad F^x = 2.125P$$
$$\sum F_x = 0 = -F^x + R^x \quad \Rightarrow \quad R^x = F^x = 2.125P$$

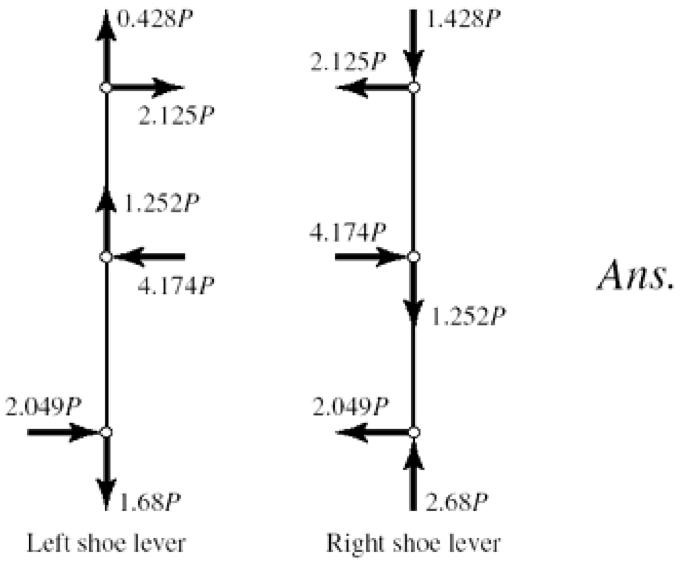


$$F^y = F^x \tan 11.4° = 0.428P$$
$$\sum F_y = -P - F^y + R^y$$
$$R^y = P + 0.428P = 1.428P$$



Left shoe lever.

$$\sum M_R = 0 = 7.78S^x - 15.28F^x$$
$$S^x = \frac{15.28}{7.78}(2.125P) = 4.174P$$
$$S^y = f\,S^x = 0.30(4.174P) = 1.252P$$
$$\sum F_y = 0 = R^y + S^y + F^y$$
$$R^y = -F^y - S^y = -0.428P - 1.252P = -1.68P$$
$$\sum F_x = 0 = R^x - S^x + F^x$$
$$R^x = S^x - F^x = 4.174P - 2.125P = 2.049P$$



(c) The direction of brake pulley rotation affects the sense of S^y, which has no effect on the brake shoe lever moment and hence, no effect on S^x or the brake torque.

The brake shoe levers carry identical bending moments but the left lever carries a tension while the right carries compression (column loading). The right lever is designed and used as a left lever, producing interchangeable levers (identical levers). But do not infer from these identical loadings.

16-10 $r = 13.5/2 = 6.75$ in, $b = 6$ in, $\theta_2 = 45° = \pi/4$ rad.

From Table 16-3 for a rigid, molded non-asbestos lining use a conservative estimate of $p_a = 100$ psi, $f = 0.33$.

Equation (16-16) gives the horizontal brake hinge pin reaction which corresponds to S^x in Prob. 16-9. Thus,

$$N = S^x = \frac{p_a br}{2}(2\theta_2 + \sin 2\theta_2) = \frac{100(6)6.75}{2}\{2(\pi / 4) + \sin[2(45°)]\}$$
$$= 5206 \text{ lbf}$$

which, from Prob. 6-9 is 4.174 P. Therefore,

$$4.174\,P = 5206 \quad \Rightarrow \quad P = 1250 \text{ lbf} = 1.25 \text{ kip} \qquad Ans.$$

Applying Eq. (16-18) for two shoes, where from Prob. 16-9, $a = 7.426$ in

$$T = 2a\,f\,N = 2(7.426)0.33(5206)$$
$$= 25\,520 \text{ lbf} \cdot \text{in} = 25.52 \text{ kip} \cdot \text{in} \quad Ans.$$

16-11 Given: D = 350 mm, b = 100 mm, p_a = 620 kPa, f = 0.30, ϕ = 270°.
Eq. (16-22):

$$P_1 = \frac{p_a bD}{2} = \frac{620(0.100)0.350}{2} = 10.85 \text{ kN} \quad \textit{Ans.}$$

$$f\phi = 0.30(270°)(\pi / 180°) = 1.414$$

Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 10.85 \exp(-1.414) = 2.64$ kN *Ans.*

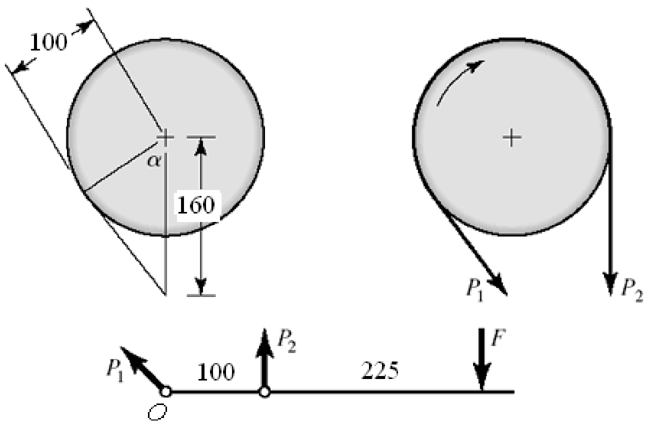
$$T = (P_1 - P_2)(D/2) = (10.85 - 2.64)(0.350/2) = 1.437 \text{ kN} \cdot \text{m} \quad \textit{Ans.}$$

16-12 Given: D = 12 in, f = 0.28, b = 3.25 in, ϕ = 270°, P_1 = 1800 lbf.

Eq. (16-22): $$p_a = \frac{2P_1}{bD} = \frac{2(1800)}{3.25(12)} = 92.3 \text{ psi} \quad \textit{Ans.}$$

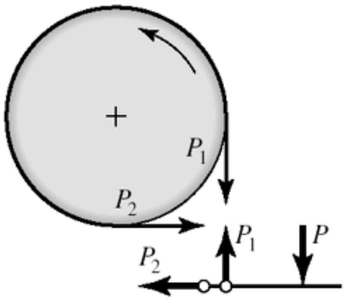
$$f\phi = 0.28(270^\circ)(\pi / 180^\circ) = 1.319$$
$$P_2 = P_1 \exp(-f\phi) = 1800 \exp(-1.319) = 481 \text{ lbf}$$
$$T = (P_1 - P_2)(D/2) = (1800 - 481)(12/2)$$
$$= 7910 \text{ lbf} \cdot \text{in} = 7.91 \text{ kip} \cdot \text{in} \quad \textit{Ans}.$$

16-13



$\Sigma M_O = 0 = 100\, P_2 - 325\, F \Rightarrow P_2 = 325(300)/100 = 975$ N *Ans.*

$$\alpha = \cos^{-1}\left(\frac{100}{160}\right) = 51.32^\circ$$
$$\phi = 270^\circ - 51.32^\circ = 218.7^\circ$$
$$f\phi = 0.30(218.7)\left(\pi / 180^\circ\right) = 1.145$$
$$P_1 = P_2 \exp(f\phi) = 975 \exp(1.145) = 3064 \text{ N} \qquad \textit{Ans.}$$
$$T = \left(P_1 - P_2\right)(D / 2) = (3064 - 975)(200 / 2)$$
$$= 209\left(10^3\right) \text{ N} \cdot \text{mm} = 209 \text{ N} \cdot \text{m} \quad \textit{Ans}.$$

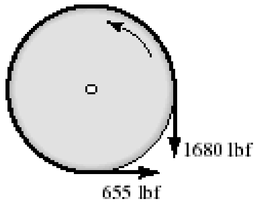


16-14 **(a)** D = 16 in, b = 3 in
n = 200 rev/min
f = 0.20, p_a = 70 psi

Eq. (16-22):
$$P_1 = \frac{p_a bD}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf}$$
$$f\phi = 0.20(3\pi/2) = 0.942$$

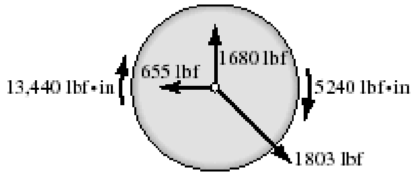
Eq. (16-14):
$$P_2 = P_1 \exp(-f\phi) = 1680 \; \exp(-0.942) = 655 \text{ lbf}$$
$$T = (P_1 - P_2)\frac{D}{2} = (1680 - 655)\frac{16}{2}$$
$$= 8200 \text{ lbf} \cdot \text{in} \quad \textit{Ans}.$$
$$H = \frac{Tn}{63\,025} = \frac{8200(200)}{63\,025} = 26.0 \text{ hp} \quad \textit{Ans.}$$
$$P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504 \text{ lbf} \quad \textit{Ans.}$$

(b) Force of belt on the drum:



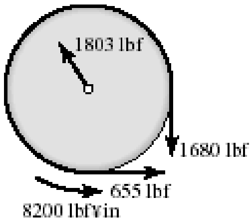
$R = (1680^2 + 655^2)^{1/2} = 1803$ lbf

Force of shaft on the drum: 1680 and 655 lbf



$$T_{P_1} = 1680(8) = 13\ 440 \text{ lbf} \cdot \text{in}$$
$$T_{P_2} = 655(8) = 5240 \text{ lbf} \cdot \text{in}$$

Net torque on drum due to brake band:



$$\begin{aligned} T &= T_{P_1} - T_{P_2} \\ &= 13\ 440 - 5240 \\ &= 8200 \text{ lbf} \cdot \text{in} \end{aligned}$$

The radial load on the bearing pair is 1803 lbf. If the bearing is straddle mounted with the drum at center span, the bearing radial load is 1803/2 = 901 lbf.

(c) Eq. (16-21):

$$p = \frac{2P}{bD}$$

$$p\Big|_{\theta=0^\circ} = \frac{2P_1}{3(16)} = \frac{2(1680)}{3(16)} = 70 \text{ psi} \quad \textit{Ans.}$$

$$p\Big|_{\theta=270^\circ} = \frac{2P_2}{3(16)} = \frac{2(655)}{3(16)} = 27.3 \text{ psi} \quad \textit{Ans.}$$

16-15 Given: $\phi = 270°$, $b = 2.125$ in, $f = 0.20$, $T = 150$ lbf · ft, $D = 8.25$ in, $c_2 = 2.25$ in (see figure). Notice that the pivoting rocker is not located on the vertical centerline of the drum.

(a) To have the band tighten for ccw rotation, it is necessary to have $c_1 < c_2$. When friction is fully developed,

$$P_1 / P_2 = \exp(f\phi) = \exp[0.2(3\pi / 2)] = 2.566$$

If friction is not fully developed,

$$P_1/P_2 \le \exp(f\,\phi)$$

To help visualize what is going on let's add a force W parallel to P_1, at a lever arm of c_3. Now sum moments about the rocker pivot.

$$\sum M = 0 = c_3W + c_1P_1 - c_2P_2$$

From which

$$W = \frac{c_2P_2 - c_1P_1}{c_3}$$

The device is self locking for ccw rotation if W is no longer needed, that is, $W \le 0$. It follows from the equation above

$$\frac{P_1}{P_2} \ge \frac{c_2}{c_1}$$

When friction is fully developed

$$2.566 = 2.25/c_1$$

$$c_1 = \frac{2.25}{2.566} = 0.877 \text{ in}$$

When P_1/P_2 is less than 2.566, friction is not fully developed. Suppose $P_1/P_2 = 2.25$, then

$$c_1 = \frac{2.25}{2.25} = 1 \text{ in}$$

We don't want to be at the point of slip, and we need the band to tighten.

$$\frac{c_2}{P_1 / P_2} \le c_1 \le c_2$$

When the developed friction is very small, $P_1/P_2 \to 1$ and $c_1 \to c_2$ *Ans.*

(b) Rocker has $c_1 = 1$ in

$$\frac{P_1}{P_2} = \frac{c_2}{c_1} = \frac{2.25}{1} = 2.25$$
$$f = \frac{\ln(P_1/P_2)}{\phi} = \frac{\ln\ 2.25}{3\pi/2} = 0.172$$

Friction is not fully developed, no slip.

$$T = (P_1 - P_2)\frac{D}{2} = P_2\left(\frac{P_1}{P_2} - 1\right)\frac{D}{2}$$

Solve for P_2

$$P_2 = \frac{2T}{[(P_1 / P_2) - 1]D} = \frac{2(150)(12)}{(2.25 - 1)(8.25)} = 349 \text{ lbf}$$
$$P_1 = 2.25P_2 = 2.25(349) = 785 \text{ lbf}$$
$$p = \frac{2P_1}{bD} = \frac{2(785)}{2.125(8.25)} = 89.6 \text{ psi} \quad \textit{Ans.}$$

(c) The torque ratio is 150(12)/100 or 18-fold.

$$P_2 = \frac{349}{18} = 19.4 \text{ lbf}$$
$$P_1 = 2.25P_2 = 2.25(19.4) = 43.6 \text{ lbf}$$
$$p = \frac{89.6}{18} = 4.98 \text{ psi} \quad \textit{Ans.}$$

Comment:
As the torque opposed by the locked brake increases, P_2 and P_1 increase (although ratio is still 2.25), then p follows. The brake can self-destruct. Protection could be provided by a shear key.

16-16 Given: OD = 250 mm, ID = 175 mm, $f = 0.30$, $F = 4$ kN.

(a) From Eq. (16-23),

$$p_a = \frac{2F}{\pi d(D-d)} = \frac{2(4000)}{\pi(175)(250-175)} = 0.194\ \text{N/mm}^2 = 194\ \text{kPa} \qquad Ans.$$

Eq. (16-25):

$$T = \frac{Ff}{4}(D+d) = \frac{4000(0.30)}{4}(250+175)10^{-3} = 127.5\ \ \text{N}\cdot\text{m} \qquad Ans.$$

(b) From Eq. (16-26),

$$p_a = \frac{4F}{\pi(D^2-d^2)} = \frac{4(4000)}{\pi(250^2-175^2)} = 0.159\ \text{N/mm}^2 = 159\ \text{kPa} \qquad Ans.$$

Eq. (16-27):

$$T = \frac{\pi}{12} f\, p_a(D^3-d^3) = \frac{\pi}{12}(0.30)159\left(10^3\right)\left(250^3-175^3\right)\left(10^{-3}\right)^3$$
$$= 128\ \text{N}\cdot\text{m} \qquad Ans.$$

16-17 Given: OD = 6.5 in, ID = 4 in, $f = 0.24$, $p_a = 120$ psi.

(a) Eq. (16-23):

$$F = \frac{\pi p_a d}{2}(D-d) = \frac{\pi(120)(4)}{2}(6.5-4) = 1885\ \text{lbf} \qquad Ans.$$

Eq. (16-24) with N sliding planes:

$$T = \frac{\pi f p_a d}{8}(D^2-d^2)N = \frac{\pi(0.24)(120)(4)}{8}(6.5^2-4^2)(6)$$
$$= 7125\ \text{lbf}\cdot\text{in} \qquad Ans.$$

(b) $$T = \frac{\pi(0.24)(120d)}{8}(6.5^2-d^2)(6)$$

d, in	T, lbf · in	
2	5191	
3	6769	
4	7125	*Ans.*
5	5853	
6	2545	

(c) The torque-diameter curve exhibits a stationary point maximum in the range of diameter d. The clutch has nearly optimal proportions.

16-18 **(a)** Eq. (16-24) with N sliding planes:

$$T = \frac{\pi f\, p_a d(D^2 - d^2)N}{8} = \frac{\pi f\, p_a N}{8}\left(D^2 d - d^3\right)$$

Differentiating with respect to d and equating to zero gives

$$\frac{dT}{dd} = \frac{\pi f\, p_a N}{8}\left(D^2 - 3d^2\right) = 0$$

$$d^* = \frac{D}{\sqrt{3}} \qquad Ans.$$

$$\frac{d^2T}{dd^2} = -6\frac{\pi f\, p_a N}{8} d = -\frac{3\pi f\, p_a N}{4} d$$

which is negative for all positive d. We have a stationary point *maximum*.

(b) $d^* = \dfrac{6.5}{\sqrt{3}} = 3.75 \text{ in} \qquad Ans.$

Eq. (16-24):

$$T^* = \frac{\pi(0.24)(120)\left(6.5 / \sqrt{3}\right)}{8}\left[6.5^2 - \left(6.5 / \sqrt{3}\right)^2\right](6) = 7173 \text{ lbf} \cdot \text{in}$$

(c) The table indicates a maximum within the range: $3 \le d \le 5$ in

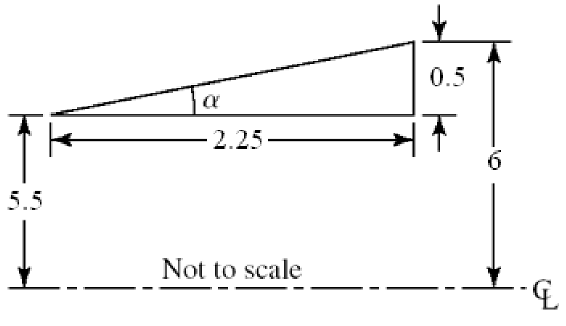
(d) Consider: $0.45 = \dfrac{d}{D} = 0.80$

Multiply through by D,

$$0.45D \le d \le 0.80D$$
$$0.45(6.5) \le d \le 0.80(6.5)$$
$$2.925 \le d \le 5.2 \text{ in}$$
$$\left(\frac{d}{D}\right)^* = d^*/D = \frac{1}{\sqrt{3}} = 0.577$$

which lies within the common range of clutches.
Yes. *Ans.*

16-19 Given: $d = 11$ in, $l = 2.25$ in, $T = 1800$ lbf · in, $D = 12$ in, $f = 0.28$.



$$\alpha = \tan^{-1}\left(\frac{0.5}{2.25}\right) = 12.53°$$

Uniform wear

Eq. (16-45):

$$T = \frac{\pi f\, p_a d}{8\sin\alpha}\left(D^2 - d^2\right)$$

$$1800 = \frac{\pi(0.28)\, p_a(11)}{8\sin 12.53^\circ}\left(12^2 - 11^2\right) = 128.2\, p_a$$

$$p_a = \frac{1800}{128.2} = 14.04 \text{ psi} \quad \textit{Ans.}$$

Eq. (16-44):

$$F = \frac{\pi p_a d}{2}(D - d) = \frac{\pi(14.04)11}{2}(12 - 11) = 243 \text{ lbf} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-48):

$$T = \frac{\pi f\, p_a}{12\sin\alpha}\left(D^3 - d^3\right)$$

$$1800 = \frac{\pi(0.28)\, p_a}{12\sin 12.53^\circ}\left(12^3 - 11^3\right) = 134.1 p_a$$

$$p_a = \frac{1800}{134.1} = 13.42 \text{ psi} \quad \textit{Ans.}$$

Eq. (16-47):

$$F = \frac{\pi p_a}{4}(D^2 - d^2) = \frac{\pi(13.42)}{4}\left(12^2 - 11^2\right) = 242 \text{ lbf} \quad \textit{Ans.}$$

16-20 *Uniform wear*

Eq. (16-34): $T = \frac{1}{2}(\theta_2 - \theta_1) f\, p_a r_i\left(r_o^2 - r_i^2\right)$

Eq. (16-33): $F = (\theta_2 - \theta_1)\, p_a r_i\, (r_o - r_i)$

Thus,

$$\frac{T}{f\,FD} = \frac{(1/2)(\theta_2 - \theta_1) f\, p_a r_i\left(r_o^2 - r_i^2\right)}{f(\theta_2 - \theta_1) p_a r_i (r_o - r_i)(D)}$$

$$= \frac{r_o + r_i}{2D} = \frac{D/2 + d/2}{2D} = \frac{1}{4}\left(1 + \frac{d}{D}\right) \quad \textit{O.K.} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-38): $T = \frac{1}{3}(\theta_2 - \theta_1) f\, p_a\left(r_o^3 - r_i^3\right)$

Eq. (16-37): $$F = \frac{1}{2}(\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right)$$

Thus,

$$\frac{T}{f\ FD} = \frac{(1/3)(\theta_2 - \theta_1)\ f\ p_a\left(r_o^3 - r_i^3\right)}{(1/2)\,f\,(\theta_2 - \theta_1)\,p_a\left(r_o^2 - r_i^2\right)D} = \frac{2}{3}\left\{\frac{(D/2)^3 - (d/2)^3}{\left[(D/2)^2 - (d/2)^2 D\right]}\right\}$$

$$= \frac{2(D/2)^3\left[1-(d/D)^3\right]}{3(D/2)^2\left[1-(d/D)^2\right]D} = \frac{1}{3}\left[\frac{1-(d/D)^3}{1-(d/D)^2}\right]\ O.K. \quad \textit{Ans.}$$

16-21

$$\omega = 2\pi n/60 = 2\pi\ 500/60 = 52.4 \text{ rad/s}$$

$$T = \frac{H}{\omega} = \frac{2(10^3)}{52.4} = 38.2 \text{ N}\cdot\text{m}$$

Key:

$$F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN}$$

Average shear stress in key is

$$\tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \quad \textit{Ans.}$$

Average bearing stress is

$$\sigma_b = -\frac{F}{A_b} = -\frac{3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \quad \textit{Ans.}$$

Let one jaw carry the entire load.

$$r_{av} = \frac{1}{2}\left(\frac{26}{2} + \frac{45}{2}\right) = 17.75 \text{ mm}$$

$$F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN}$$

The bearing and shear stress estimates are

$$\sigma_b = \frac{-2.15\left(10^3\right)}{10(22.5-13)} = -22.6 \text{ MPa} \quad \textit{Ans.}$$

$$\tau = \frac{2.15(10^3)}{10\left[0.25\pi(17.75)^2\right]} = 0.869 \text{ MPa} \quad \textit{Ans.}$$

16-22

$$\omega_1 = 2\pi n / 60 = 2\pi(1600) / 60 = 167.6 \text{ rad/s}$$
$$\omega_2 = 0$$

From Eq. (16-51),

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{Tt_1}{\omega_1 - \omega_2} = \frac{2800(8)}{167.6 - 0} = 133.7 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

Eq. (16-52):

$$E = \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 = \frac{133.7}{2}(167.6 - 0)^2 = 1.877(10^6) \text{ lbf} \cdot \text{in}$$

In Btu, Eq. (16-53): $H = E / 9336 = 1.877(10^6) / 9336 = 201$ Btu

Eq. (16-54):

$$\Delta T = \frac{H}{C_p W} = \frac{201}{0.12(40)} = 41.9°\text{F} \quad \textit{Ans.}$$

16-23

$$n = \frac{n_1 + n_2}{2} = \frac{260 + 240}{2} = 250 \text{ rev/min}$$

Eq. (16-62): $C_s = (\omega_2 - \omega_1) / \omega = (n_2 - n_1) / n = (260 - 240) / 250 = 0.08$ *Ans.*

$$\omega = 2\pi(250) / 60 = 26.18 \text{ rad/s}$$

From Eq. (16-64):

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{6.75(10^3)}{0.08(26.18)^2} = 123.1 \text{ N} \cdot \text{m} \cdot \text{s}^2$$

$$I = \frac{m}{8}\left(d_o^2 + d_i^2\right) \quad \Rightarrow \quad m = \frac{8I}{d_o^2 + d_i^2} = \frac{8(123.1)}{1.5^2 + 1.4^2} = 233.9 \text{ kg}$$

Table A-5, cast iron unit weight = 70.6 kN/m^3 $\Rightarrow$ $\rho = 70.6(10^3) / 9.81 = 7197 \text{ kg / m}^3$.

Volume: $V = m / \rho = 233.9 / 7197 = 0.0325 \text{ m}^3$

$$V = \pi t\left(d_o^2 - d_i^2\right) / 4 = \pi t\left(1.5^2 - 1.4^2\right) / 4 = 0.2278t$$

Equating the expressions for volume and solving for t,

$$t = \frac{0.0325}{0.2278} = 0.143 \text{ m} = 143 \text{ mm} \quad \textit{Ans.}$$

16-24 **(a)** The useful work performed in one revolution of the crank shaft is

$$U = 320\,(10^3)\ 200\,(10^{-3})\ 0.15 = 9.6\,(10^3)\ \text{J}$$

Accounting for friction, the total work done in one revolution is

$$U = 9.6(10^3) / (1 - 0.20) = 12.0(10^3)\ \text{J}$$

Since 15% of the crank shaft stroke accounts for 7.5% of a crank shaft revolution, the energy fluctuation is

$$E_2 - E_1 = 9.6(10^3) - 12.0(10^3)(0.075) = 8.70(10^3)\ \text{J} \qquad \textit{Ans.}$$

(b) For the flywheel,

$$n = 6(90) = 540 \text{ rev/min}$$
$$\omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5 \text{ rad/s}$$

Since $\qquad C_s = 0.10$

Eq. (16-64): $\qquad I = \dfrac{E_2 - E_1}{C_s\omega^2} = \dfrac{8.70(10^3)}{0.10(56.5)^2} = 27.25 \text{ N}\cdot\text{m}\cdot\text{s}^2$

Assuming all the mass is concentrated at the effective diameter, d,

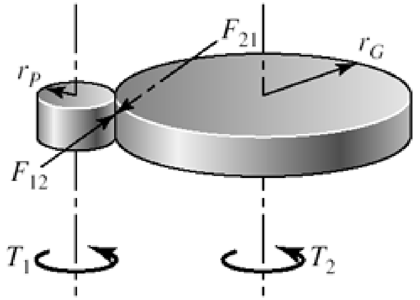
$$I = mr^2 = \frac{md^2}{4}$$
$$m = \frac{4I}{d^2} = \frac{4(27.25)}{1.2^2} = 75.7 \text{ kg} \qquad \textit{Ans.}$$

16-25 Use Ex. 16-6 and Table 16-6 data for one cylinder of a 3-cylinder engine.

$$C_s = 0.30$$
$$n = 2400 \text{ rev/min} \quad \text{or} \quad 251 \text{ rad/s}$$
$$T_m = \frac{3(3368)}{4\pi} = 804 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$
$$E_2 - E_1 = 3(3531) = 10\,590 \text{ in}\cdot\text{lbf}$$
$$I = \frac{E_2 - E_1}{C_s\omega^2} = \frac{10\,590}{0.30(251^2)} = 0.560 \text{ in}\cdot\text{lbf}\cdot\text{s}^2 \qquad \textit{Ans.}$$

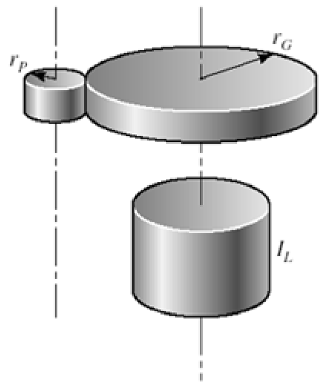
16-26 **(a)**

(1)



$$(T_2)_1 = -F_{21} r_P = -\frac{T_2}{r_G} r_P = \frac{T_2}{-n} \qquad \textit{Ans.}$$

(2)



Equivalent energy

$$(1/2) I_2 \omega_2^2 = (1/2)(I_2)_1 \left(\omega_1^2\right)$$

$$(I_2)_1 = \frac{\omega_2^2}{\omega_1^2} I_2 = \frac{I_2}{n^2} \qquad \textit{Ans.}$$

(3)

$$\frac{I_G}{I_P} = \left(\frac{r_G}{r_P}\right)^2 \left(\frac{m_G}{m_P}\right) = \left(\frac{r_G}{r_P}\right)^2 \left(\frac{r_G}{r_P}\right)^2 = n^4$$

From (2) $\qquad (I_2)_1 = \dfrac{I_G}{n^2} = \dfrac{n^4 I_P}{n^2} = n^2 I_P \qquad$ *Ans.*

(b) $I_e = I_M + I_P + n^2 I_P + \dfrac{I_L}{n^2} \qquad$ *Ans.*

(c) $I_e = 10 + 1 + 10^2(1) + \dfrac{100}{10^2} = 112$

reflected load inertia

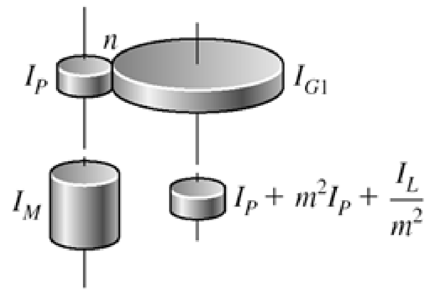
reflected gear inertia

pinion inertia

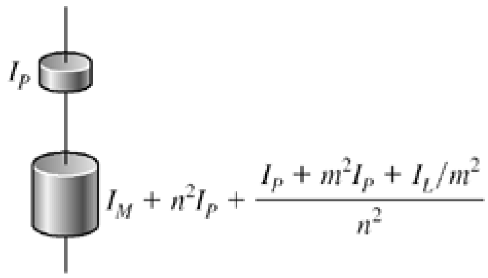
armature inertia

Ans.

16-27 **(a)** Reflect I_L, I_{G2} to the center shaft



Reflect the center shaft to the motor shaft



$$I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2}{n^2} I_P + \frac{I_L}{m^2 n^2} \qquad \textit{Ans.}$$

(b) For R = constant = nm, $\quad I_e = I_M + I_P + n^2 I_P + \dfrac{I_P}{n^2} + \dfrac{R^2 I_P}{n^4} + \dfrac{I_L}{R^2} \qquad$ *Ans.*

(c) For R = 10, $\quad \dfrac{\partial I_e}{\partial n} = 0 + 0 + 2n(1) - \dfrac{2(1)}{n^3} - \dfrac{4(10^2)(1)}{n^5} + 0 = 0$

$$n^6 - n^2 - 200 = 0$$

From which

$$n^* = 2.430 \qquad \textit{Ans.}$$

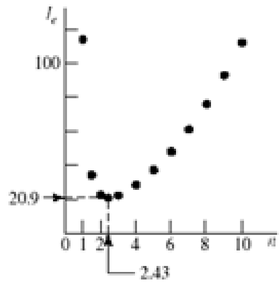
$$m^* = \frac{10}{2.430} = 4.115 \qquad \textit{Ans.}$$

Notice that $n^*$and m^* are independent of I_L.

16-28 From Prob. 16-27,

$$\begin{aligned} I_e &= I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2} \\ &= 10 + 1 + n^2(1) + \frac{1}{n^2} + \frac{100(1)}{n^4} + \frac{100}{10^2} \\ &= 12 + n^2 + \frac{1}{n^2} + \frac{100}{n^4} \end{aligned}$$

n	I_e
1.00	114.00
1.50	34.40
2.00	22.50
2.43	20.90
3.00	22.30
4.00	28.50
5.00	37.20
6.00	48.10
7.00	61.10
8.00	76.00
9.00	93.00
10.00	112.02



Optimizing the partitioning of a double reduction lowered the gear-train inertia to 20.9/112 = 0.187, or to 19% of that of a single reduction. This includes the two additional gears.

16-29 Figure 16-29 applies,

$$t_2 = 10\ s, \quad t_1 = 0.5\ s$$
$$\frac{t_2 - t_1}{t_1} = \frac{10 - 0.5}{0.5} = 19$$

The load torque, as seen by the motor shaft (Rule 1, Prob. 16-26), is

$$T_L = \left|\frac{1300(12)}{10}\right| = 1560 \text{ lbf} \cdot \text{in}$$

The rated motor torque T_r is

$$T_r = \frac{63\ 025(3)}{1125} = 168.07 \text{ lbf} \cdot \text{in}$$

For Eqs. (16-65):

$$\omega_r = \frac{2\pi}{60}(1125) = 117.81 \text{ rad/s}$$
$$\omega_s = \frac{2\pi}{60}(1200) = 125.66 \text{ rad/s}$$
$$a = \frac{-T_r}{\omega_s - \omega_r} = -\frac{168.07}{125.66 - 117.81} = -21.41 \text{ lbf} \cdot \text{in} \cdot \text{s/rad}$$
$$b = \frac{T_r \omega_s}{\omega_s - \omega_r} = \frac{168.07(125.66)}{125.66 - 117.81} = 2690.4 \text{ lbf} \cdot \text{in}$$

The linear portion of the squirrel-cage motor characteristic can now be expressed as

$$T_M = -21.41\omega + 2690.4 \text{ lbf} \cdot \text{in}$$

Eq. (16-68):

$$T_2 = 168.07\left(\frac{1560 - 168.07}{1560 - T_2}\right)^{19}$$

One root is 168.07 which is for infinite time. The root for 10 s is desired. Use a successive substitution method

T_2	New T_2
0.00	19.30
19.30	24.40
24.40	26.00
26.00	26.50
26.50	26.67

Continue until convergence to

$$T_2 = 26.771 \text{ lbf} \cdot \text{in}$$

Eq. (16-69):

$$I = \frac{a(t_2 - t_1)}{\ln(T_2 / T_r)} = \frac{-21.41(10 - 0.5)}{\ln(26.771 / 168.07)} = 110.72 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

$$\omega = \frac{T - b}{a}$$

$$\omega_{\max} = \frac{T_2 - b}{a} = \frac{26.771 - 2690.4}{-21.41} = 124.41 \text{ rad/s} \quad \textit{Ans.}$$

$$\omega_{\min} = 117.81 \text{ rad/s} \quad \textit{Ans.}$$

$$\bar{\omega} = \frac{124.41 + 117.81}{2} = 121.11 \text{ rad/s}$$

$$C_s = \frac{\omega_{\max} - \omega_{\min}}{(\omega_{\max} + \omega_{\min})/2} = \frac{124.41 - 117.81}{(124.41 + 117.81)/2} = 0.0545 \quad \textit{Ans.}$$

$$E_1 = \frac{1}{2}I\omega_r^2 = \frac{1}{2}(110.72)(117.81)^2 = 768\,352 \text{ in} \cdot \text{lbf}$$

$$E_2 = \frac{1}{2}I\omega_2^2 = \frac{1}{2}(110.72)(124.41)^2 = 856\,854 \text{ in} \cdot \text{lbf}$$

$$\Delta E = E_2 - E_1 = 856\,854 - 768\,352 = 88\,502 \text{ in} \cdot \text{lbf}$$

Eq. (16-64):

$$\Delta E = C_s I \bar{\omega}^2 = 0.0545(110.72)(121.11)^2$$
$$= 88\,508 \text{ in} \cdot \text{lbf}, \quad \text{close enough} \quad \textit{Ans.}$$

During the punch

$$T = \frac{63\,025H}{n}$$
$$H = \frac{T_L\bar{\omega}(60/2\pi)}{63\,025} = \frac{1560(121.11)(60/2\pi)}{63\,025} = 28.6 \text{ hp}$$

The gear train has to be sized for 28.6 hp under shock conditions since the flywheel is on the motor shaft. From Table A-18,

$$I = \frac{m}{8}\left(d_o^2 + d_i^2\right) = \frac{W}{8g}\left(d_o^2 + d_i^2\right)$$
$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(110.72)}{d_o^2 + d_i^2}$$

If a mean diameter of the flywheel rim of 30 in is acceptable, try a rim thickness of 4 in

$$d_i = 30 - (4\ /\ 2) = 28 \text{ in}$$
$$d_o = 30 + (4\ /\ 2) = 32 \text{ in}$$
$$W = \frac{8(386)(110.72)}{32^2 + 28^2} = 189.1 \text{ lbf}$$

Rim volume V is given by

$$V = \frac{\pi l}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi l}{4}(32^2 - 28^2) = 188.5l$$

where l is the rim width as shown in Table A-18. The specific weight of cast iron is γ= 0.260 lbf / in^3, therefore the volume of cast iron is

$$V = \frac{W}{\gamma} = \frac{189.1}{0.260} = 727.3 \text{ in}^3$$

Equating the volumes,

$$188.5\,l = 727.3$$
$$l = \frac{727.3}{188.5} = 3.86 \text{ in wide}$$

Proportions can be varied.

16-30 Prob. 16-29 solution has I for the motor shaft flywheel as

$$I = 110.72 \ \text{lbf} \cdot \text{in} \cdot \text{s}^2$$

A flywheel located on the crank shaft needs an inertia of 10^2 I (Prob. 16-26, rule 2)

$$I = 10^2(110.72) = 11\ 072 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

A 100-fold inertia increase. On the other hand, the gear train has to transmit 3 hp under shock conditions.

Stating the problem is most of the solution. Satisfy yourself that on the crankshaft:

$$T_L = 1300(12) = 15\ 600 \text{ lbf} \cdot \text{in}$$
$$T_r = 10(168.07) = 1680.7 \text{ lbf} \cdot \text{in}$$
$$\omega_r = 117.81 / 10 = 11.781 \text{ rad/s}$$
$$\omega_s = 125.66 / 10 = 12.566 \text{ rad/s}$$
$$a = -21.41(100) = -2141 \text{ lbf} \cdot \text{in} \cdot \text{s/rad}$$
$$b = 2690.35(10) = 26903.5 \text{ lbf} \cdot \text{in}$$
$$T_M = -2141\omega_c + 26\ 903.5 \text{ lbf} \cdot \text{in}$$
$$T_2 = 1680.6\left(\frac{15\ 600 - 1680.5}{15\ 600 - T_2}\right)^{19}$$

The root is 10(26.67) = 266.7 lbf · in

$$\bar{\omega} = 121.11 / 10 = 12.111 \text{ rad/s}$$
$$C_s = 0.0549 \text{ (same)}$$
$$\omega_{\max} = 121.11 / 10 = 12.111 \text{ rad/s} \quad \textit{Ans.}$$
$$\omega_{\min} = 117.81 / 10 = 11.781 \text{ rad/s} \quad \textit{Ans.}$$

E_1, E_2, ΔE and peak power are the same. From Table A-18

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(11\ 072)}{d_o^2 + d_i^2} = \frac{34.19\left(10^6\right)}{d_o^2 + d_i^2}$$

Scaling will affect d_o and d_i , but the gear ratio changed I. Scale up the flywheel in the Prob. 16-29 solution by a factor of 2.5. Thickness becomes 4(2.5) = 10 in.

$$\bar{d} = 30(2.5) = 75 \text{ in}$$
$$d_o = 75 + (10\ /\ 2) = 80 \text{ in}$$
$$d_i = 75 - (10\ /\ 2) = 70 \text{ in}$$

$$W = \frac{34.19(10^6)}{80^2 + 70^2} = 3026 \text{ lbf}$$

$$V = \frac{W}{\gamma} = \frac{3026}{0.260} = 11\,638 \text{ in}^3$$

$$V = \frac{\pi}{4} l(80^2 - 70^2) = 1178\, l$$

$$l = \frac{11\,638}{1178} = 9.88 \text{ in}$$

Proportions can be varied. The weight has increased 3026/189.1 or about 16-fold while the moment of inertia I increased 100-fold. The gear train transmits a steady 3 hp. But the motor armature has its inertia magnified 100-fold, and during the punch there are deceleration stresses in the train. With no motor armature information, we cannot comment.

16-31 This can be the basis for a class discussion.