Genetic Tuning on Fuzzy Systems Based on the Linguistic 2-Tuples Representation

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***Abstract*— Linguistic Fuzzy Modeling allows us to deal with the modeling of systems building a linguistic model clearly interpretable by human beings. However, in this kind of modeling the *accuracy* and the *interpretability* of the obtained model are contradictory properties directly depending on the learning process and/or the model structure. Thus, the necessity of improving the linguistic model accuracy arises when complex systems are modeled.**

To solve this problem, one of the research lines of this framework in the last years has leaded up to the objective of giving more accuracy to the Linguistic Fuzzy Modeling, without losing the associated interpretability to a high level.

In this work, a new post-processing method of Fuzzy Rule-Based Systems is proposed by means of an evolutionary lateral tuning of the linguistic variables, with the main aim of obtaining Fuzzy Rule-Based Systems with a better accuracy and maintaining a good interpretability.

To do so, this tuning considers a new rule representation scheme by using the linguistic 2-tuples representation model which allows the lateral variation of the involved labels. As an example of application of these kinds of systems, we analyze this approach considering a real-world problem.

I. Introduction

Fuzzy Modeling (FM)—i.e., system modeling with fuzzy rule-based systems (FRBSs)—may be considered as an approach used to model a system making use of a descriptive language based on fuzzy logic with fuzzy predicates. Several types of modeling can be performed by using different types of FRBSs and depending of the desired degree of interpretability and accuracy of the final model. Unfortunately, both requirements are contradictory properties directly depending on the learning process and/or the model structure.

In this framework, one of the most important areas is *linguistic* FM, where the interpretability of the obtained model is the main requirement. This task is usually developed by means of linguistic FRBSs (also called Mamdani FRBSs [1], [2]), which use fuzzy rules composed of linguistic variables [3] taking values in a term set with a real-world meaning. Thus, the linguistic fuzzy model consists of a set of linguistic descriptions regarding the behavior of the system being modeled.

One of the problems associated to linguistic FM is its lack of accuracy when modeling some complex systems. It is due to the inflexibility of the concept of linguistic variable, which imposes hard restrictions to the fuzzy rule structure [4]. This drawback leads linguistic FM to sometimes move away from the desired trade-off between interpretability and accuracy, thus losing the usefulness of the model.

To overcome this problem, many different possibilities to improve the accuracy of linguistic FM while preserving its intrinsic interpretability have been considered in the specialized literature [5]. A great number of these approaches share the common idea of improving the way in which the linguistic fuzzy model performs the interpolative reasoning by inducing a better cooperation among the rules composing it. This rule cooperation may be encouraged acting on three different model components: the data base (DB) —containing the parameters of the linguistic partitions—, the rule base (RB) —containing the set of rules— and the whole knowledge base (KB) — containing the RB and the DB—. It supposes an extension of the linguistic FM and can be achieved, either by means of learning methods considering the learning of the RB and the DB, either by post-processing mechanisms that are applied to improve the system behavior once the RB and the DB are obtained.

One of the most used approaches to improve the behavior of FRBSs, called as *tuning*, consists of refining the DB from a previous definition once the RB has been obtained. Generally, the tuning is a variation in the shape of the membership functions with the main requirement of improving the linguistic model accuracy.

The tuning methods represent the different parameters that identify the membership functions associated to the labels composing the DB, 3 parameters in the case of triangular membership functions [6]. In the case of problems presenting a large number of variables this leads to tuning models with too many parameters, which could affect to the good performance of the optimization method considered.

In this work, we present a new FRBS post-processing model which performs an evolutionary lateral tuning of the linguistic variables. It is possible thanks to a new rule representation model based on the linguistic 2-tuples representation [7] which allows the lateral variation of the label support considering an unique parameter (slight displacements to the left/right of the original membership functions), with the objective of obtaining FRBSs with a better accuracy and maintaining a high interpretability.

The paper is arranged as follows. The next section describes the proposed lateral tuning and presents the linguistic rule

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representation model based on the linguistic 2-tuples. Section III proposes the evolutionary tuning method considered in this work. Section IV shows an experimental study of the method behavior applied over FRBSs obtained from automatic learning methods (considering an electrical distribution real problem). Finally, Section V points out some concluding remarks.

II. A Proposal for the Lateral Tuning of Linguistic Labels

In this section, we will briefly discuss about the FRBS tuning parameters to subsequently present the objective of our proposal. Then, the rule structure and two different learning approaches will be stablished.

A. Tuning Parameters of a FRBS

The classical tuning of membership functions works with 3 or 4 parameters per membership function associated to a linguistic label (triangular or trapezoidal shapes respectively). The number of parameters exponentially grows in terms of the number of variables and the number of labels per variable.

Usually, a number of 5 or 7 labels per variable are considered, and never more than 9. However, the number of variables depends on each specific problem, doing the problem complexity exponentially grow when the number of the system variables increases.

This involves a problem for both the tuning methods based on the use of neural networks and those based on the use of Genetic Algorithms (GAs). In the first case, we could highlight the ANFIS method [8] which needs to represent all the possible combinations of rules in the second layer and it provokes a memory overflow when there are more than 4 or 5 input variables. Considering GAs a chromosome with a large number of genes is necessary to represent all the involved parameters and this fact, leads up to convergence problems of the evolutionary methods.

These considerations justifies the design of a tuning method that decrements the complexity of the classical methods for tuning of the parameters of the membership functions. The reduction of the complexity could provoke a better behavior of the tuning method.

B. Objective

Our main objective is the development of a powerful fuzzy partition tuning mechanism to,

1) obtain linguistic labels containing a set of samples with a better covering degree and a better behavior, and
2) to reduce the search space of the tuning approach in order to easily obtain optimal models,

maintaining a good trade-off between accuracy and interpretability.

C. The Lateral Tuning Approach

The main proposal of this work is the design of a new model of tuning of FRBSs considering the linguistic 2-tuples representation to laterally tuning the support of a label, which maintains the interpretability associated to the new representation model. In this way, a new model for rule representation based on the linguistic 2-tuples is introduced. This concept is presented in [7] and allow a lateral displacement of the labels named symbolic translation.

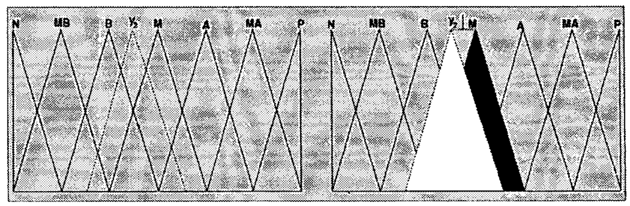

Fig. 1. Lateral Displacement of the Linguistic Label M

Figure 1 shows the lateral displacement of the label M. The new label "y_2" is located between B and M, being enough smaller than M but closer to M.

The symbolic translation of a linguistic term is a number within the interval [-0.5, 0.5) that expresses the domain of a label when it is moving between its two lateral labels. Formally, we have the couple,

$$(s_i, \alpha_i),\ \ s_i \in S,\ \ \alpha_i \in [0.5, -0.5).$$

In [7], both the linguistic 2-tuples representation model and the needed elements for linguistic information comparison and aggregation are presented and applied to the Decision Making framework. In the context of the FRBSs, we are going to see its use in the linguistic rule representation. In the next we present this approach considering a simple control problem.

Let us consider a control problem with two input variables, one output variable and a DB defined from experts determining the membership functions for the following labels:

$$Error \rightarrow \{N, Z, P\}, \qquad \nabla Error \rightarrow \{N, Z, P\},$$
$$Power \rightarrow \{L, M, H\}\ .$$

Classical Rule:
R1: If the **Error** is Zero and the **Error Variation** is Positive then the **Power** is High

Rules with 2-tuples Representation:
R1: If the **Error** is (Zero, 0.3) and the **Error Variation** is (Positive, -0.2) then the **Power** is (High, -0.1)

Fig. 2. Classical Rule and Rule with 2-Tuple Representation

Figure 2 shows the concept of classical rule and linguistic 2-tuples represented rule. Analized from the rule interpretability point of view, we could interpret the tuned rule as:

If the **Error** is "higher than Zero" and
the **Error Variation** is "a little smaller than Positive"
then the **Power** is "a bit smaller than High".

This proposal decreases the tuning problem complexity, since the 3 parameters considered per label are reduced to only 1 symbolic translation parameter. As to how perform the lateral tuning there are two possibilities:

- Global Tuning of the Semantics. Tuning is applied to the level of linguistic partition. In this way, the couple (X_i, label) takes the same tuning value in all the rules where it is considered. For example, X_i is (High, 0.3) will present the same value for those rules in which the couple "X_i is High" is initially considered.
- Local Tuning of the Rules. Tuning is applied to the level of rule. The couple (X_i, label) is tuned in a different way in each rule, based on the quality measures associated to the tuning method (usually the system error).

  Rule k: X_i is (High, 0.3) (more than high)
  Rule j: X_i is (High, -0.2) (a little lower than high)

The evolutionary lateral tuning method based on this representation model is shown in the next section.

III. Evolutionary Post-Processing Algorithm

The automatic definition of fuzzy systems can be considered as an optimization or search process and nowadays, Evolutionary Algorithms, particularly GAs, are considered as the more known and used global search technique. Moreover, the genetic coding that they use allow them to include prior knowledge and to use it leading the search up. For this reason, Evolutionary Algorithms have been successfully applied to learn fuzzy systems in the last years, giving way to the appearance of the so called Genetic Fuzzy Systems (GFSs) [6], [9].

Evolutionary Algorithms in general and, GAs in particular, has been widely used in the tuning of FRBSs. In this work, we will consider the use of GAs to design the proposed lateral tuning. A good evolution model is the genetic model of CHC [10].

CHC makes use of a "Population-based Selection" approach. N parents and their corresponding offspring are combined to select the best N individual to take part of the next population.

In the following, the components needed to design the evolutionary tuning process are explained. They are:

- DB codification.
- Chromosome evaluation.
- Genetic operators.

A. DB Codification

Taking into account, that two different types of tuning have been proposed (global tuning of the semantics and local tuning of the rules), there are two different kinds of coding methods. In both cases, a real coding scheme is considered, i.e., the real parameters are the GA representation units (genes).

In the following both schemes are presented:

- Global Tuning of the Semantics: Joint of the parameters of the fuzzy partitions. Let us consider the following number of labels per variable: $(m^1, m^2, \ldots, m^n)$, with n system variables. Then, a chromosome has the following form (where each gene is associated to the tuning value of the corresponding label),

$$(c_{11}, \ldots, c_{1m^1}, c_{21}, \ldots, c_{2m^2}, \ldots, c_{n1}, \ldots, c_{nm^n})$$

- Local Tuning of the Rules: Joint of the rule parameters. Let us condider that the FRBS has M rules: $(R1, R2, \ldots, RM)$, with n system variables. Then, the chromosome structure is,

$$(c_{11}, \ldots, c_{1n}, c_{21}, \ldots, c_{2n}, \ldots, c_{M1}, \ldots, c_{Mn}).$$

Figure 3 graphically depicts an example of correspondence between a chromosome and its associated KB when the global tuning approach is considered.

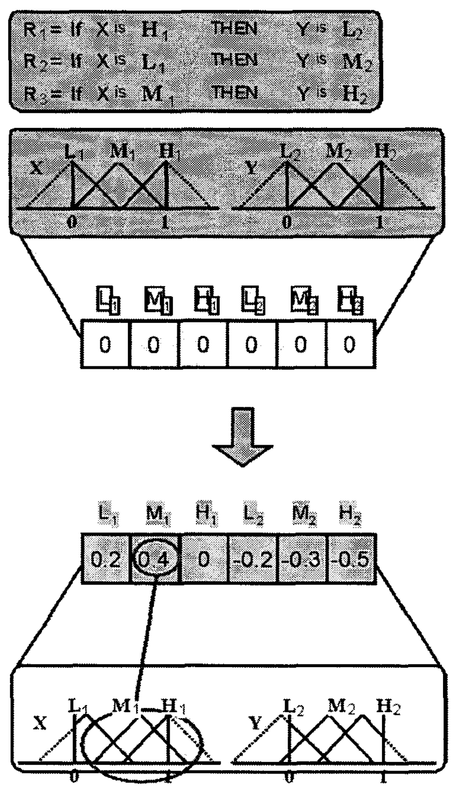

Fig. 3. Example of Coding Scheme Considering the Global Tuning

B. Chromosome Evaluation

To evaluate a determined chromosome we will use the well-known Mean Square Error (MSE):

$$\text{MSE} = \frac{1}{2 \cdot N} \sum_{l=1}^{N} (F(x^l) - y^l)^2,$$

with N being the data set size, $F(x^l)$ being the output obtained from the FRBS decoded from the said chromosome when the l-th example is considered and y^l being the known desired output.

C. Genetic Operators

The genetic operators considered in CHC are crossover and restarting approach (no mutation is considered). A description of these operators is presented in the following:

- Crossover. The crossover operator is based on the the concept of environments. These kinds of operators show a good behavior as said in [11]. Particularly, we consider the PBLX operator (an operator based on the BLX-α). This operator presents a good cooperation when it is introduced within models forcing the convergence by pressure on the offspring.
- Restarting. To get away from local optima, this algorithm uses a restart approach [10]. In this case, the best chromosome is maintained and the remaining are generated at random within the corresponding variation intervals [-0.5, 0.5). It follows the principles of CHC [10], performing the restart procedure when a threshold value is reached.

IV. Main Results

To evaluate the goodness of the two proposed approaches, local and global tuning, several experiments have been carried out considering a real-world problem [12]. The problem consists of estimating the maintenance costs of the medium voltage electrical network in a town from four input variables and therefore, it presents *a large search space*. A short description of this problem can be found in the following subsection.

TABLE I

Methods Considered for Comparison

Ref.	Method	Description
[13]	**WM**	Learning Method
[14]	**T**	Classical Genetic Tuning
[15]	**PAL**	Tuning of Parameters, Domains and Linguistic Modifiers
—	**GL**	Global Lateral Tuning
—	**LL**	Local Lateral Tuning

The methods considered for these experiments are briefly described in Table I. The WM method is considered to obtain the initial RB to be tuned. The tuning methods are applied once this initial RB has been obtained. T is a classical membership function parameter tuning algorithm. The PAL method has been compared with tuning methods of the parameters, domain, linguistic modifiers and with any combination of any two of them obtaining the best results [15]. For this reason, we only consider the PAL method (parameters, domain and linguistic edges) in this study.

The initial linguistic partitions are comprised by *five linguistic terms* with triangular-shaped fuzzy sets giving meaning to them. With respect to the fuzzy reasoning method used, we have selected the *minimum t-norm* playing the role of the implication and conjunctive operators, and the *center of gravity weighted by the matching* strategy acting as the defuzzification operator.

Finally, the following values have been considered for the parameters of each method [1]: 51 individuals, 50,000 evaluations, 0.6 as crossover probability for a PBLX (0.2 as mutation probability per chromosome and 0.35 for the factor a in the max-min-arithmetical crossover operator for T and PAL).

A. Problem Description

Estimating the maintenance costs of the medium voltage electrical network in a town [12] is a complex but interesting problem. Since a direct measure is very difficult to obtain, the consideration of models becomes useful. These estimations allow electrical companies to justify their expenses. Moreover, the model must be able to explain how a specific value is computed for a certain town. Our objective will be to relate the *maintenance costs of medium voltage line* with the following four variables: *sum of the lengths of all streets in the town*, *total area of the town*, *area that is occupied by buildings*, and *energy supply to the town*. We will deal with estimations of minimum maintenance costs based on a model of the optimal electrical network for a town in a sample of 1,059 towns.

To develop the different experiments in this contribution, we consider a *5-folder cross-validation model*, i.e., 5 random partitions of data with a 20%, and the combination of 4 of them (80%) as training and the remaining one as test. In this way, 5 partitions considering an 80% (847) in training and a 20% (212) in test are considered for the experiments.

B. Experimental Results

For each one of the 5 data partitions, the tuning methods has been run 6 times, showing for each problem the averaged results of a total of 30 runs. Moreover, a *t-test* (with 95 percent confidence) was applied in order to ascertain if differences in the performance of the proposed approaches are significant when compared against the one for the other algorithms in the respective table.

The results obtained by the analyzed methods are shown in Table II, where $\#R$ stands for the number of rules, MSE_{tra} and MSE_{tst} respectively for the averaged error obtained over the training and test data, σ for the standard deviation and *t-test* represents the following information:

- $\star$ represents the best averaged result.
- $+$ means that the best result has better behavior that the one in the corresponding row.
- $=$ denotes that the results are equal.

Analyzing the results presented in Table II we can point out the following conclusions:

[1] With these values we have tried to ease the comparisons selecting standard common parameters that work well in most cases instead of searching very specific values for each method. Moreover, we have set a large number of evaluations in order to allow the compared algorithms to achieve an appropriate convergence. No significant changes were achieved by increasing that number of evaluations.

TABLE II

RESULTS OBTAINED BY THE STUDIED METHODS

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
WM	65	57605	2841	+	57934	4733	+
WM + T	65	18602	1211	+	22666	3386	+
WM + PAL	65	10545	279	+	13973	1688	+
WM + GL	65	23064	1479	+	25654	2611	+
WM + LL	65	3664	390	⋆	5858	1798	⋆

- Considering these methods, the RB is obtained by means of a generation method to learn few rules (65 from the 625 possible rules). It allow us to obtain a compact and accurate tuned model.
- The local lateral tuning method shows an important reduction of the mean squared error in a problem with a large number of variables and therefore presenting a complex search space. It is due to the use of a unique parameter per label, reducing the search space respect to a classical tuning which usually considers 3 or 4 parameters (in the case of triangular or trapezoidal membership functions). In this way, the local lateral tuning presents a good relationship between the search space complexity and the results obtained which maintains a high trade-off between accuracy and local interpretability in the method. Furthermore, since the lateral displacements are related to the original global labels a global interpretation could be done in these terms.
- In this problem, the global lateral tuning method shows worst results than the classical approaches. However, at present, we are verifying that in other problems, with more complexity and presenting a larger number of variables, this method achieves very good results, even better than the ones obtained by the local approach. It could be again due to the search space reduction, since this approach handles even a lower number of parameters than the local one.

Figures 4 and 5 respectively presents the evolved fuzzy linguistic partitions and the RB obtained by GL from one of the 30 runs performed. The former figure shows that small variations in the membership functions lead to important improvements in the behavior of the obtained FRBSs.

Figure 6 graphically depicts one of the 30 RBs obtained with LL (the local approach) where we can see how the local tuning evolves each label of the different rules in a different way. The difficult trade-off between accuracy and complexity can be observed taking into account both RBs (Figures 5 and 6). The accuracy can be improved but always at the expense of losing some interpretability.

V. CONCLUSIONS

In this work, a new rule representation scheme by using the linguistic 2-tuples representation model has been considered to propose a new post-processing method of FRBSs by means of an evolutionary lateral tuning of the linguistic variables.

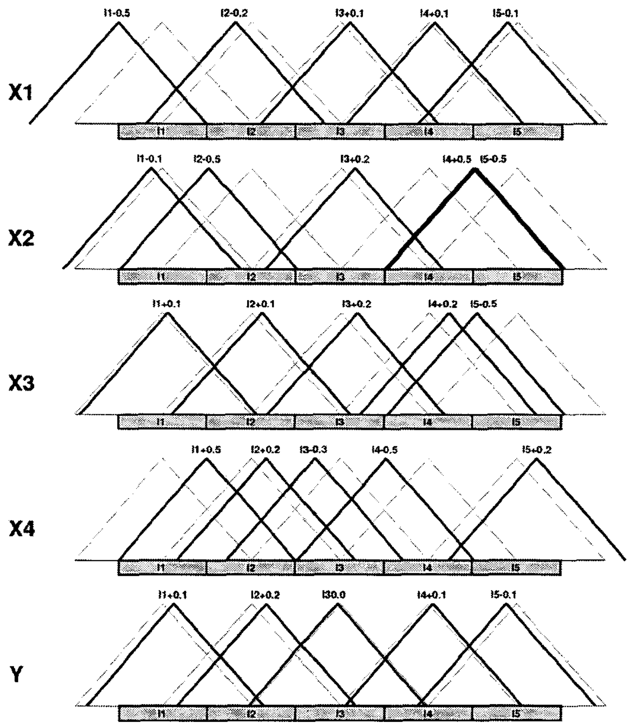

Fig. 4. Initial and Tuned DB of a Model Obtained with GL (Global Approach)

#R: 65

X1	X2	X3	X4	Y
l1 -0.50	l1 -0.13	l1+0.06	l1+0.50	l1+0.13
l1 -0.50	l1 -0.13	l1+0.06	l2+0.17	l2+0.17
l1 -0.50	l2 -0.48	l1+0.06	l1+0.50	l1+0.13
l1 -0.50	l2 -0.48	l1+0.06	l2+0.17	l2+0.17
l1 -0.50	l2 -0.48	l2+0.11	l1+0.50	l1+0.13
l1 -0.50	l2 -0.48	l2+0.11	l2+0.17	l2+0.17
l2 -0.18	l1 -0.13	l1+0.06	l1+0.50	l1+0.13
l2 -0.18	l1 -0.13	l1+0.06	l2+0.17	l2+0.17
l2 -0.18	l1 -0.13	l2+0.11	l1+0.50	l1+0.13
l2 -0.18	l1 -0.13	l2+0.11	l2+0.17	l2+0.17
l2 -0.18	l2 -0.48	l1+0.06	l1+0.50	l1+0.13
l2 -0.18	l2 -0.48	l1+0.06	l2+0.17	l2+0.17
l2 -0.18	l2 -0.48	l2+0.11	l1+0.50	l2+0.17
l2 -0.18	l2 -0.48	l2+0.11	l2+0.17	l2+0.17
l2 -0.18	l3+0.16	l2+0.11	l1+0.50	l2+0.17
l2 -0.18	l3+0.16	l2+0.11	l2+0.17	l2+0.17
l2 -0.18	l3+0.16	l3+0.18	l1+0.50	l3 -0.03
l2 -0.18	l3+0.16	l3+0.18	l2+0.17	l3 -0.03
l3+0.10	l2 -0.48	l1+0.06	l1+0.50	l1+0.13
l3+0.10	l2 -0.48	l1+0.06	l2+0.17	l2+0.17
l3+0.10	l2 -0.48	l1+0.06	l3 -0.28	l2+0.17
l3+0.10	l2 -0.48	l2+0.11	l1+0.50	l2+0.17
l3+0.10	l2 -0.48	l2+0.11	l2+0.17	l2+0.17
l3+0.10	l2 -0.48	l2+0.11	l3 -0.28	l3 -0.03
l3+0.10	l3+0.16	l2+0.11	l1+0.50	l2+0.17
l3+0.10	l3+0.16	l2+0.11	l2+0.17	l3 -0.03
l3+0.10	l3+0.16	l2+0.11	l3 -0.28	l3 -0.03
l3+0.10	l3+0.16	l3+0.18	l2+0.17	l3 -0.03
l3+0.10	l3+0.16	l3+0.18	l3 -0.28	l3 -0.03
l3+0.10	l4+0.50	l3+0.18	l2+0.17	l3 -0.03
l3+0.10	l4+0.50	l3+0.18	l3 -0.28	l3 -0.03
l3+0.10	l4+0.50	l4+0.22	l2+0.17	l3 -0.03
l3+0.10	l4+0.50	l4+0.22	l3 -0.28	l4+0.05

X1	X2	X3	X4	Y
l4+0.06	l2 -0.48	l2+0.11	l1+0.50	l2+0.17
l4+0.06	l2 -0.48	l2+0.11	l2+0.17	l2+0.17
l4+0.06	l2 -0.48	l2+0.11	l3 -0.28	l3 -0.03
l4+0.06	l2 -0.48	l2+0.11	l4 -0.48	l3 -0.03
l4+0.06	l3+0.16	l2+0.11	l1+0.50	l2+0.17
l4+0.06	l3+0.16	l2+0.11	l2+0.17	l3 -0.03
l4+0.06	l3+0.16	l2+0.11	l3 -0.28	l3 -0.03
l4+0.06	l3+0.16	l2+0.11	l4 -0.48	l3 -0.03
l4+0.06	l3+0.16	l3+0.18	l2+0.17	l3 -0.03
l4+0.06	l3+0.16	l3+0.18	l3 -0.28	l4+0.05
l4+0.06	l3+0.16	l3+0.18	l4 -0.48	l4+0.05
l4+0.06	l4+0.50	l3+0.18	l1+0.50	l3 -0.03
l4+0.06	l4+0.50	l3+0.18	l2+0.17	l3 -0.03
l4+0.06	l4+0.50	l3+0.18	l3 -0.28	l4+0.05
l4+0.06	l4+0.50	l3+0.18	l4 -0.48	l4+0.05
l4+0.06	l4+0.50	l4+0.22	l2+0.17	l4+0.05
l4+0.06	l4+0.50	l4+0.22	l3 -0.28	l4+0.05
l4+0.06	l4+0.50	l4+0.22	l4 -0.48	l5 -0.12
l4+0.06	l5 -0.47	l4+0.22	l2+0.17	l3 -0.03
l4+0.06	l5 -0.47	l4+0.22	l3 -0.28	l4+0.05
l4+0.06	l5 -0.47	l4+0.22	l4 -0.48	l5 -0.12
l4+0.06	l5 -0.47	l5 -0.46	l2+0.17	l5 -0.12
l4+0.06	l5 -0.47	l5 -0.46	l3 -0.28	l5 -0.12
l5 -0.12	l2 -0.48	l2+0.11	l2+0.17	l2+0.17
l5 -0.12	l2 -0.48	l2+0.11	l4 -0.48	l3 -0.03
l5 -0.12	l2 -0.48	l2+0.11	l5+0.23	l4+0.05
l5 -0.12	l2 -0.48	l3+0.18	l2+0.17	l3 -0.03
l5 -0.12	l2 -0.48	l3+0.18	l4 -0.48	l3 -0.03
l5 -0.12	l2 -0.48	l3+0.18	l5+0.23	l4+0.05
l5 -0.12	l4+0.50	l3+0.18	l2+0.17	l3 -0.03
l5 -0.12	l4+0.50	l3+0.18	l4 -0.48	l4+0.05
l5 -0.12	l4+0.50	l3+0.18	l5+0.23	l5 -0.12

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Fig. 5. RB and Lateral Displacements of a Model Obtained with GL (Global Approach)

#R: 65

X1	X2	X3	X4	Y
l1+0.24	l1+0.36	l1 -0.44	l1+0.16	l1 -0.34
l1+0.22	l1+0.46	l1+0.49	l2+0.15	l2+0.34
l1+0.12	l2+0.03	l1 -0.13	l1+0.32	l1+0.07
l1+0.44	l2+0.34	l1 -0.44	l2+0.46	l2 -0.48
l1+0.08	l2 -0.42	l2 -0.12	l1+0.08	l1+0.32
l1+0.28	l2 -0.08	l2 -0.32	l2+0.20	l2+0.40
l2 -0.07	l1+0.04	l1 -0.38	l1+0.34	l1+0.10
l2 -0.31	l1+0.26	l1+0.15	l2 -0.25	l2 -0.30
l2+0.02	l1 -0.46	l2+0.46	l1 -0.33	l1+0.29
l2 -0.49	l1 -0.09	l2+0.16	l2+0.00	l2 -0.03
l2+0.15	l2+0.02	l1+0.33	l1 -0.25	l1+0.12
l2 -0.31	l2+0.43	l1 -0.04	l2+0.21	l2 -0.35
l2+0.27	l2+0.25	l2+0.33	l1+0.39	l2+0.16
l2 -0.10	l2 -0.36	l2+0.04	l2+0.13	l2+0.47
l2 -0.25	l3 -0.18	l2 -0.38	l1+0.05	l2+0.02
l2+0.37	l3 -0.33	l2 -0.19	l2+0.26	l2 -0.21
l2 -0.26	l3+0.03	l3+0.35	l1 -0.46	l3 -0.06
l2+0.47	l3 -0.22	l3 -0.20	l2 -0.37	l3+0.16
l3 -0.02	l2 -0.43	l1 -0.20	l1 -0.42	l1 -0.05
l3+0.16	l2+0.14	l1 -0.01	l2+0.02	l2 -0.33
l3+0.13	l2 -0.28	l1+0.08	l3 -0.08	l2 -0.35
l3+0.10	l2+0.45	l2+0.02	l1 -0.29	l2 -0.30
l3+0.03	l2+0.35	l2 -0.34	l2+0.16	l2+0.19
l3+0.15	l2 -0.33	l2+0.04	l3+0.04	l3+0.02
l3+0.11	l3+0.44	l2+0.17	l1+0.39	l2+0.18
l3 0.00	l3+0.06	l2+0.37	l2+0.26	l3 -0.50
l3 -0.50	l3 -0.43	l2+0.37	l3+0.10	l3+0.44
l3 -0.29	l3+0.18	l3+0.40	l2+0.06	l3+0.15
l3 -0.48	l3 -0.41	l3 -0.14	l3 -0.40	l3+0.25
l3+0.37	l4+0.06	l3+0.40	l2 -0.13	l3+0.50
l3+0.48	l4+0.34	l3 -0.31	l3+0.45	l3 -0.15
l3 -0.21	l4 -0.31	l4+0.41	l2+0.26	l3+0.43
l3 -0.15	l4 -0.25	l4 -0.43	l3 -0.16	l4 -0.04

X1	X2	X3	X4	Y
l4+0.20	l2+0.22	l2 -0.37	l1+0.24	l2+0.08
l4+0.09	l2+0.48	l2 -0.41	l2+0.15	l2+0.47
l4+0.45	l2 -0.40	l2+0.01	l3 -0.11	l3 -0.18
l4 -0.05	l2+0.46	l2 -0.30	l4+0.11	l3 -0.19
l4 -0.23	l3 -0.12	l2 -0.03	l1+0.29	l2 -0.35
l4+0.01	l3+0.45	l2+0.45	l2+0.19	l3 -0.18
l4 -0.33	l3+0.07	l2+0.15	l3 -0.02	l3 -0.43
l4 -0.48	l3+0.47	l2 -0.03	l4+0.19	l3 -0.31
l4 -0.32	l3 -0.45	l3 -0.04	l2+0.13	l3+0.49
l4 -0.44	l3+0.13	l3+0.15	l3 0.00	l4+0.16
l4+0.10	l3 -0.36	l3+0.03	l4+0.31	l4 -0.09
l4+0.22	l4 -0.07	l3 -0.28	l1+0.36	l3 -0.15
l4 -0.41	l4+0.11	l3 -0.23	l2+0.15	l3 -0.32
l4+0.39	l4+0.18	l3 -0.13	l3 -0.06	l4 -0.45
l4 -0.49	l4 -0.28	l3+0.42	l4 -0.26	l4+0.01
l4+0.14	l4+0.04	l4+0.01	l2+0.26	l4+0.03
l4 -0.11	l4+0.41	l4+0.01	l3+0.06	l4+0.47
l4+0.18	l4 -0.31	l4+0.08	l4 -0.10	l5+0.48
l4 -0.33	l5+0.02	l4 -0.42	l2 -0.43	l3+0.29
l4+0.08	l5+0.15	l4 -0.08	l3+0.01	l4+0.16
l4+0.09	l5+0.14	l4+0.07	l4+0.05	l5 -0.26
l4+0.14	l5 -0.10	l5 -0.09	l2+0.19	l5 -0.47
l4+0.17	l5+0.46	l5+0.28	l3+0.06	l5+0.01
l5 -0.26	l2 -0.38	l2+0.13	l2+0.04	l2 -0.06
l5+0.08	l2+0.29	l2 -0.49	l4+0.27	l3 0.00
l5 -0.08	l2+0.19	l2+0.24	l5+0.14	l4+0.23
l5 -0.29	l2+0.24	l3 -0.06	l2+0.20	l3+0.32
l5 -0.22	l2 -0.30	l3 -0.14	l4 -0.13	l3+0.48
l5 -0.40	l2+0.12	l3 -0.49	l5 -0.17	l4+0.27
l5 -0.16	l4+0.19	l3+0.38	l2+0.48	l3+0.09
l5 -0.41	l4+0.07	l3 -0.12	l4+0.14	l4 -0.39
l5 0.00	l4 -0.34	l3+0.30	l5 -0.41	l5 -0.22

ECM-ent: 3085.859
ECM-pru: 4124.033

Fig. 6. RB and Lateral Displacements of a Model Obtained with LL (Local Approach)

In the following, we present our conclusions to subsequently present some further considerations:

- The lateral tuning of fuzzy partitions to design FRBSs offers a great improvement of the system accuracy when complex problems have to be solved.
- The lateral tuning together with the linguistic 2-tuples representation model offers a good mechanism to obtain an interpretable model, providing a local description of the system behavior to the experts.

An interesting further question is the accomplishment of a wider analysis about the existent relation between the problem complexity and the behavior of the different tuning methods. At this moment, we are working on this issue to present a complete study of these kinds of techniques.

The use of rule selection methods to reduce the number of rules while a lateral tuning is performed could be a good approach to obtain more compact and precise models. As further work, we propose a smart combination of both techniques, rules selection and lateral tuning.

On the other hand, recently, we are working on the determination of some possible measures of quality of the obtained fuzzy rules. The use of multi-objective AGs considering two objectives, accuracy (error measures) and interpretability (quality measures), for the lateral tuning of FRBSs could be an interesting further work.

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