Lecture #14: Visual Bag of Words

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1 Introduction

In this lecture, we learn another approach to recognition. To recognize objects in images, we need to first represent them in the form of feature vectors. Feature vectors are mathematical representations of an image's important features. These feature vectors, for example, can be the raw color values of the image or contain information about the position of the pixel in the image as we have seen and implemented in Homework 5. We then create a space representation of the image to view the image values in a lower dimensional space. Every image is then converted into a set of coefficients and projected into the PCA space. The transformed data is classified using a classifier. Some examples of such classifiers include K-means and HAC. This process of going from an image to a useful representation of the image in a lower dimensional space can be achieved in many ways. In this lecture, we discuss another approach entitled Visual Bag of Words.

1.1 Idea of Bag of Words

The idea behind "Bag of Words" is a way to simplify object representation as a collection of their subparts for purposes such as classification. The model originated in natural language processing, where we consider texts such as documents, paragraphs, and sentences as collections of words - effectively "bags" of words. Consider a paragraph - a list of words and their frequencies can be considered a "bag of words" that represents the particular paragraph, which we can then use as a representation of the paragraph for tasks such as sentiment analysis, spam detection, and topic modeling.

Although "Bag of Words" appears to be associated with language, the idea of simplifying complex objects into collections of their subparts can be extended to different types of objects. In Computer Vision, we can consider an **image** to be a **collection of image features**. By incorporating frequency counts of these features, we can apply the "Bag of Words" model towards images and use this for prediction tasks such as image classification and face detection.

There are two main steps for the "Bag of Words" method when applied to computer vision, and these will further be explored in the Outline section below.

1. Build a "dictionary" or "vocabulary" of features across many images - what kinds of common features exist in images? We can consider, for example, color scheme of the room, parts of faces such as eyes, and different types of objects.

2. Given new images, represent them as histograms of the features we had collected - frequencies of the visual "words" in the vocabulary we have built.

1.2 Origins

The origins of applying the "Bag of Words" model to images comes from Texture Recognition and, as previously mentioned, Document Representation.

1. Textures consist of repeated elements, called textons - for example, a net consists of repeated holes and a brick wall consists of repeated brick pieces. If we were to consider each texton a feature, then each image could be represented as a histogram across these features - where the texton in the texture of the image would have high frequency in the histogram. Images with multiple textures, therefore, can be represented by histograms with high values for multiple features.

2. Documents consist of words which can be considered their features. Thus, every document is represented by a histogram across the words in the dictionary - one would expect, for example, the document of George Bush's state of the union address in 2001 to contain high relative frequencies for "economy", "Iraq", "army", etc.

Thus, a "bag of words" can be viewed as a histogram representing frequencies across a vocabulary developed over a set of images or documents - new data then can be represented with this model and used for prediction tasks.

2 Algorithm Summary

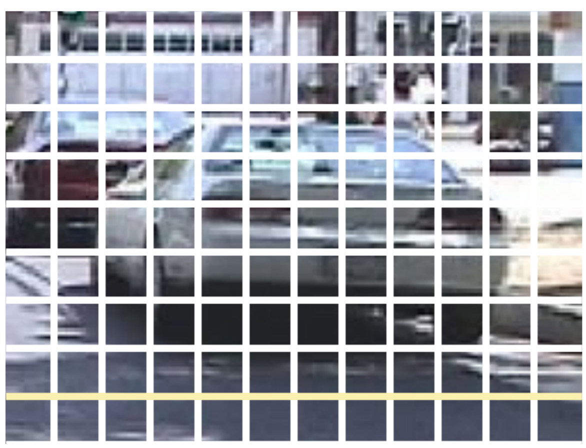
Let's describe in detail how the Bag of Words algorithm can be applied to a large set of images. As noted before, there are two main aspects of the Bag of Words algorithm: building our visual vocabulary, and using this vocabulary to represent new images by the frequencies of their visual "words."

2.1 Building a Visual Vocabulary

To build our visual vocabulary, we will make use of all the images in our dataset.

2.1.1 Extracting Interesting Features

The first step in building our visual vocabulary is to extract the features across all images in our dataset. We can extract features from images using a variety of methods. For example, we can simply split each image into a grid, and grab each of the subimages as features (shown below). Or, we can use corner detection of SIFT features as our features.



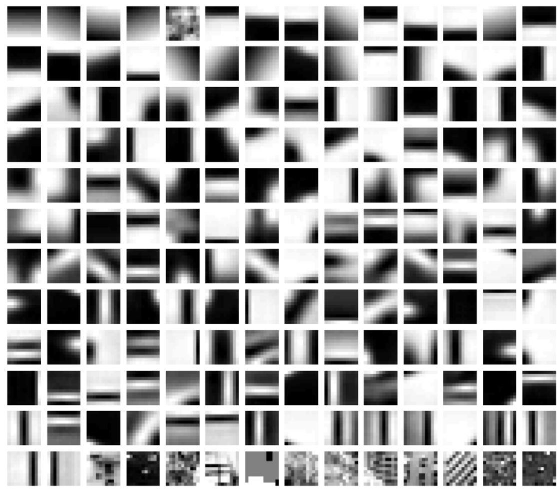
Using grid of subimages as features **?**

2.1.2 Clustering Features for Quantiation

Once we have extracted features from all of the images in the dataset, we must narrow down this large feature set into a smaller, less redundant set of common features, or "words" (to use the Natural Language Processing analogy of the algorithm). As mentioned above, in the Computer Vision application, the "words" are called textons.

To find the common "words" or textons, we simply cluster our full set of features. We can use any clustering technique to accomplish this. K-Means is most common, but Mean Shift or HAC may also work. After clustering, the cluster centers each represent a common "word" or texton in our visual vocabulary. An example of a visual vocabulary is given below.

Note: The word "codevector" is also often used as a synonym for a "word" or cluster center; similarly, "codebook" is the visual vocabulary containing all of our codevectors. This terminology is used to represent the notion of quantizing our features ("words").



Example of a visual vocabulary **?**

2.1.3 Challenges in Building the Visual Vocabulary

Choosing the size of our visual vocabulary (which is exactly equivalent to amount of clusters in our clustering algorithm) is an important hyperparameter. If it's too small, then our words (codevectors) are not representative of the underlying data. Our visual vocabulary will not be descriptive enough to distinguish between classes, and we will underfit. For example, as an analogy, if we wanted our visual vocabulary to differentiate between cats and dogs, a small vocabulary size might place both cat and dog features into the same cluster of all animals. On the other hand, if the vocabulary size is too large, then it will start to overfit the underlying data and be too descriptive. For example, if we wanted a visual word that represented "birds", an unnecessarily large vocabulary size might end up differentiating between heron and egret features. We must be conscious of this when picking the K value for K-Means (if, of course, we decide to use K-Means as our clustering algorithm).

2.2 Representing New Images by Visual Word Frequencies

Once we have built our visual vocabulary (codebook), we can use it to do interesting things. First, we can represent every image in our dataset as a histogram of visual word or codevector frequencies (shown below). We extract features from a new image using the same method we used to extract features while building our visual vocabulary. We then use our codebook to map the new image's features to the indexes of the closest words (codevectors) (feature quantization).

Our type of problem determines what we do next. If we have a supervised learning problem (i.e. our data has labels), we can train a classifier on the histograms. This classifier will be trained on the appearance of the visual words and hence will be a robust way to distinguish between classes. If we have an unsupervised learning problem (i.e. our data does not have labels), we can further cluster the histograms to find visual themes/groups within our dataset.

Representing our images as a histogram of texton frequencies **?**

We can create our visual vocabulary from a different dataset than the dataset that we are interested in classifying/clustering, and so long as our first dataset is representative of the second, this algorithm will be successful. In other words, this visual vocabulary can be universal.

2.3 Large-Scale Image Search

Large-scale image matching is one of the ways that the Bag-of-words model has been useful. Given a large database, which can hold tens of thousands of object instances, how can one match an images to this database?

The Bag-of-words model can help build the database. First, features can be extracted from the database images. Then we can learn a vocabulary using k-means (typical k:100,000). Next we compute the weights for each word. Going back to the word dictionary example, weighting the words can help us decrease the importance of certain words. If we are trying to find the topic of a document, we can give words like "the", "a", and "is" low weights since they are likely to be common between documents and used frequently within a document. With images we can do the same, giving useless features low weights and the more important features higher weights. Once the features have been weighted, we can create an inverted file mapping words to images.

Term Frequency Inverse Document Frequency (TF-IDF) scoring weights each word by it's document frequency.

The inverse document frequency (IDF) of a word j can be found by

$$IDF = \log(\frac{NumDocs}{NumDocs_{jappears}})$$

To compute the value of bin j in image I:

$$Bin_j = frequncy_{j\,in\,I} * IDF$$

We can create an inverted file that holds the mapping of words to documents to quickly compute the similarity between a new image and all of the images in the database. If we have images that have around 1000 features, and a database of around 100,000 visual words, each histogram will be extremely sparse. We would only consider images whose bins overlap with the new image.

Large-scale image search works well for CD covers and movie posters, and real-time performance is possible. The downside for the large scale image search is that the performance of the algorithm degrades as the database grows. Using the Bag-of-Words model for this problem sometimes results in noisy image similarities due to quantization error and imperfect feature detection.**?**

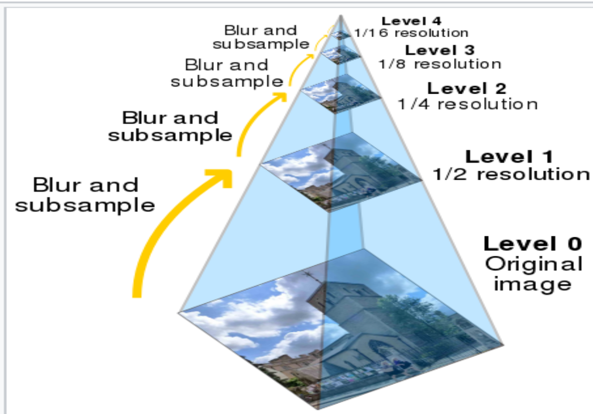
3 Spatial Pyramid Matching

3.1 Motivation

So far, we have not exploited the spatial information. But there is a simple yet smart method to incorporate the spatial information in the model: spatial pyramid matching.

3.2 Pyramids

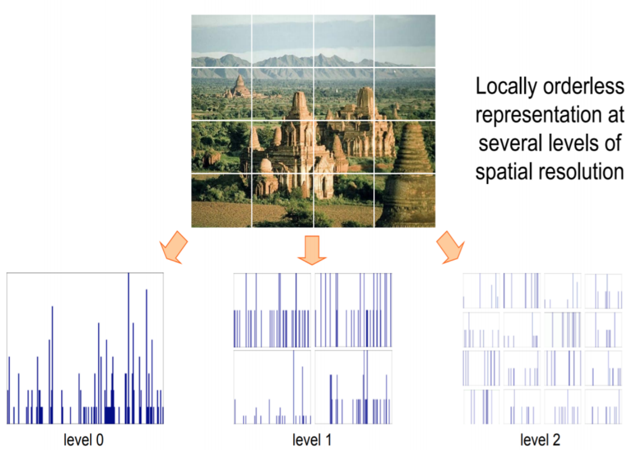
A pyramid is built by using multiple copies of the source image. Each level in the pyramid is $\frac{1}{4}$ of the size of the previous level. The lowest level is of the highest resolution and the highest level is of the lowest resolution. If illustrated graphically, the entire multi-scale representation looks like a pyramid, with the original image on the bottom and each cycle's resulting smaller image stacked one atop the other. **?**



Visual representation of an image pyramid with 5 levels

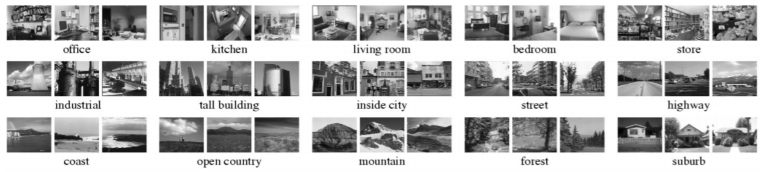
3.3 Bag of Words + Pyramids

Bag of Words alone doesn't discriminate if a patch was obtained from the top, middle or bottom of the image because it doesn't save any spatial information. Spatial pyramid matching partitions the image into increasingly fine sub-regions and allows us to computes histograms (BoW) of local features inside each sub-region. **?**
If the BoWs of the upper part of the image contain "sky visual words", the BoWs in the middle "vegetation and mountains visual words" and the BoWs at the bottom "mountains visual words", then it is very likely that the image scene category is "mountains".



3.4 Some results

Scene category dataset

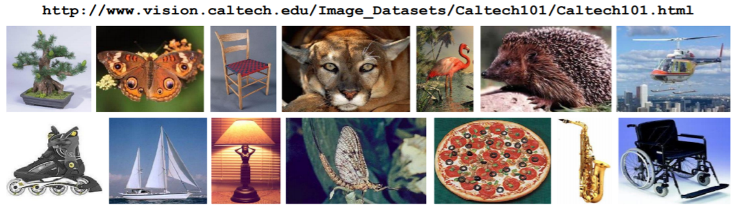


Multi-class classification results
(100 training images per class)

	Weak features (vocabulary size: 16)		Strong features (vocabulary size: 200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0 (1×1)	45.3 $\pm$0.5		72.2 $\pm$0.6	
1 (2×2)	53.6 $\pm$0.3	56.2 $\pm$0.6	77.9 $\pm$0.6	79.0 $\pm$0.5
2 (4×4)	61.7 $\pm$0.6	64.7 $\pm$0.7	79.4 $\pm$0.3	**81.1** $\pm$0.3
3 (8×8)	63.3 $\pm$0.8	**66.8** $\pm$0.6	77.2 $\pm$0.4	80.7 $\pm$0.3

Caltech101 dataset

http://www.vision.caltech.edu/Image_Datasets/Caltech101/Caltech101.html



Multi-class classification results (30 training images per class)

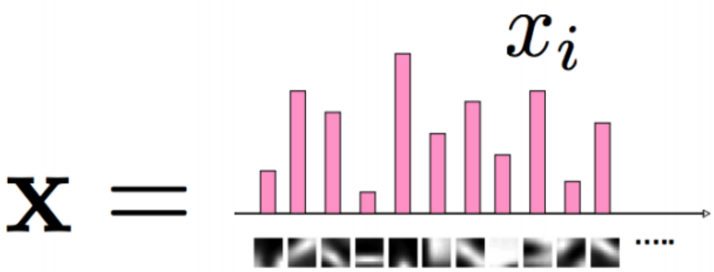
	Weak features (16)		Strong features (200)	
Level	Single-level	Pyramid	Single-level	Pyramid
0	15.5 $\pm$0.9		41.2 $\pm$1.2	
1	31.4 $\pm$1.2	32.8 $\pm$1.3	55.9 $\pm$0.9	57.0 $\pm$0.8
2	47.2 $\pm$1.1	49.3 $\pm$1.4	63.6 $\pm$0.9	**64.6** $\pm$0.8
3	52.2 $\pm$0.8	**54.0** $\pm$1.1	60.3 $\pm$0.9	64.6 $\pm$0.7

Strong features (ie.larger vocabulary size) is better than weaker features (ie. smaller vocabulary size). Notice also that as expected, incorporating pyramid matching always generate better result than single level feature extraction. This is exactly what we expected because under the same circumstance, pyramid approach encodes more information (ie.spacial information) than single-level approach does.

4 Naive Bayes

4.1 Basic Idea

Once we have produced a visual word histogram, we can use Naive Bayes to classify the histogram. To do so, we simply measure whether a given visual word is present or absent, and assume the presence/absence of one visual word to be conditionally independent of each other visual word given an object class.



Consider some visual word histogram *X*, where x_i is the count of visual word *i* in the histogram. We are only interested in the presence or absence of word *i*, we have $x_i \in \{0, 1\}$.

4.2 Prior

$P(c)$ denotes that probability of encountering one object class versus others. For all m object classes, we then have

$$\sum_{i=1}^{m} P(c) = 1$$

For an image represented by histogram x, and some object class c, we can compute

$$P(x|c) = \prod_{i=1}^{m} P(x_i|c)$$

We are able to make the above statement by assuming the mutual independence of each feature x_i when conditioned on c. This assumption is called the Naive Bayes Assumption.

4.3 Posterior

Using the prior equation, we can now calculate the probability than the image represented by histogram x belongs to class category c using **Bayes Theorem**.

$$P(c|x) = \frac{P(c)P(x|c)}{\sum_{c'} P(c')P(x|c')}$$

Note that we express $P(x)$ as the summation of $P(c')P(x|c')$, or equivalently $P(x, c')$ for all c' by using the law of total probability. Expanding the numerator and denominator, we can rewrite the previous equation as

$$P(c|x) = \frac{P(c)\prod_{i=1}^{m} P(x_i|c)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i|c')}$$

by once again using the Naive Bayes Assumption of conditional independence of each x_i on c'.

4.4 Classification

In order to classify the image represented by histogram x, we simply find the class c^* that maximizes the previous equation. That is to say, we look for the label $c = c^*$ such that the label that c has the highest probability of taking on given its feature data is c^*:

$$c^* = argmax_c P(c|x)$$

Since we end up multiplying together a large number of very small probabilities, we will likely run into unstable values as they approach 0. As a result, we use logs to calculate probabilities:

$$c^* = argmax_c log P(c|x)$$

Now consider two classes c_1 and c_2:

$$P(c_1|x) = \frac{P(c_1)\prod_{i=1}^{m} P(x_i|c_1)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i|c')}$$

and

$$P(c_2|x) = \frac{P(c_2)\prod_{i=1}^{m} P(x_i|c_2)}{\sum_{c'} P(c')\prod_{i=1}^{m} P(x_i|c')}$$

Since the denominators are identical, we can ignore it when calculating the maximum. Thus

$$P(c_1|x) \propto P(c_1)\prod_{i=1}^{m} P(x_i|c_1)$$

and

$$P(c_2|x) \propto P(c_2)\prod_{i=1}^{m} P(x_i|c_2)$$

and for the general class c:

$$P(c|x) \propto P(c)\prod_{i=1}^{m} P(x_i|c)$$

and using logs:

$$logP(c|x) \propto logP(c) + \sum_{i=1}^{m} logP(x_i|c)$$

Now, classification becomes

$$c^* = argmax_c P(c|x)$$

$$c^* = argmax_c logP(c|x)$$

$$c^* = argmax_c logP(c) + \sum_{i=1}^{m} logP(x_i|c)$$