Advances Towards Practical Implementations of Isogeny Based Signatures

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Concerns of Cryptography

There are five rudimentary concerns in information security:

- *Confidentiality*: information must be kept private from unauthorized individuals.
- *Integrity*: information must not be altered by unauthorized individuals.
- *Availability*: information must be available for authorized individuals.
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Public-key Cryptography

The goal of cryptography is to define mathematically precise means of ensuring these information security goals. Proofs of cryptographic security depend, in many cases, on assumptions about the difficulty of solving some problem.

Cryptographic protocols can be either *private-key* or *public-key* systems.

Public-key systems require that every party takes ownership of both a public key (pk), the value of which is known by everyone on the network, and a private key (sk), known only to the owner of the keypair.

Quantum Cryptanalysis

A large-scale quantum computer will have the capability of breaking most modern public-key cryptosystems.

This has lead to the development of the field known as post-quantum cryptography – the aim of which is to develop cryptosystems resistant to quantum cryptanalysis.

Performance of Post-quantum Cryptosystems

Common approaches to post-quantum cryptography include

- Lattice-based cryptography,
- Hash-based cryptography,
- Multivariate-based cryptography,
- Code-based cryptography, and
- Isogeny-based cryptography.

Post-quantum Cryptography

	Key Gen	Sign	Verify
SIDH	84,499,270	4,950,023,141.65	3,466,703,991.09
Sphincs	17,535,886.94	653,013,784	27,732,049
qTESLA	1,059,388	460,592	66,491
Picnic	13,272	9,560,749	6,701,701
RSA	12,800,000	1,113,600	32400
ECDSA	1,470,000	128,928	140,869

Contributions

I outline two ways in which performance improvements can be made to a state-of-the-art isogeny-based signature scheme. We show how

1. certain operations in the Yoo signature scheme can be batched together, improving runtime by roughly 8%, and that
2. by adopting public key compression techniques from the literature, Yoo signatures can be compressed by 144λ bytes.

Overview

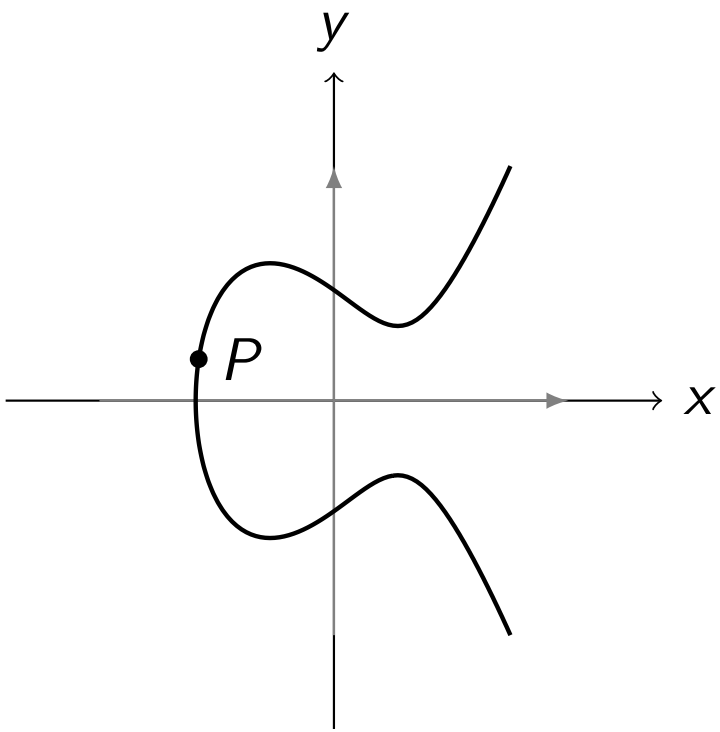
Overview

Elliptic Curves as a Group

Elliptic curves are a class of algebraic curves satisfying

$$E : y^2 = x^3 + ax + b.$$

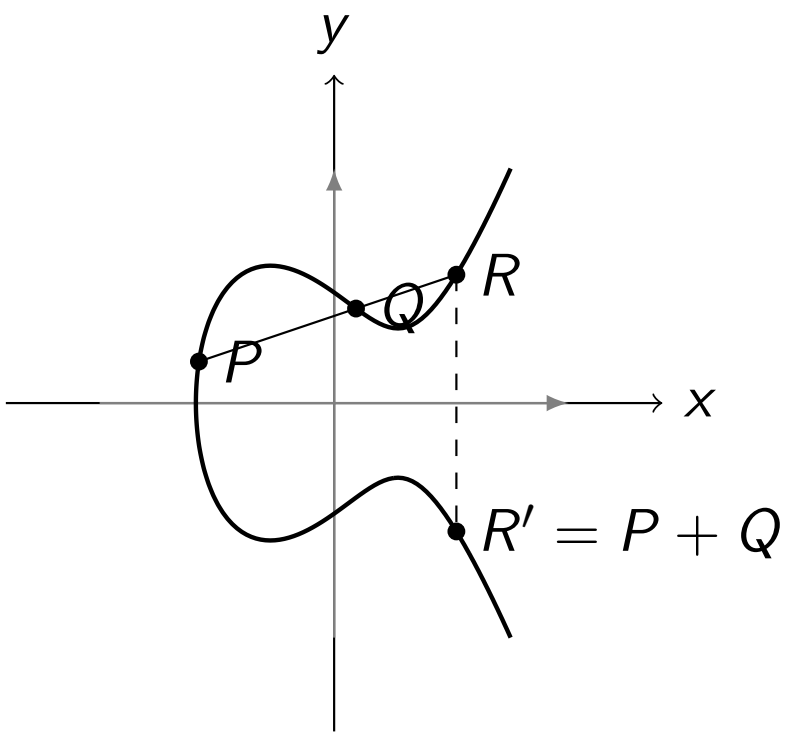
We can define a *group* of elements composed of all the points $P = (x_P, y_P)$ satisfying E.



Elliptic Curves as a Group

We can define the group operation $+$ of an elliptic curve group by computing $P + Q$ for $P, Q \in E$ as shown to the right.

We write $[m]P$ to denote m repeated applications of this operation to the point P.

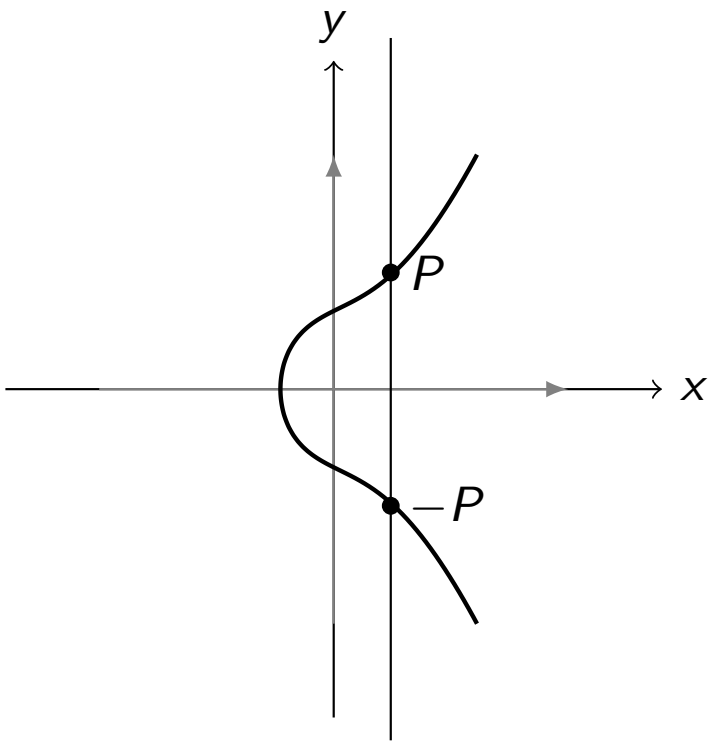


Elliptic Curves as a Group

We Write $\mathcal{O}$ to denote the identity of this group, which is conceptualized as a point residing at positive and negative inifity.

Note to the right how $R + -(R) = \mathcal{O}$ on this elliptic curve.

This example also illustrates the existence of an inverse element, which for $P = (x, y)$ will always be $-P = (x, -y)$.



Torsion Subgroups

We write $E[r]$ to denote the set of all points on a curve E with order r, e.g;

$$E[r] = \{P \in E : [m]P = \mathcal{O}\}$$

Isogenies

Isogenies are maps that take a point on one elliptic curve to a point on another. For an isogeny ϕ mapping from E_1 to E_2, we can write

$$\phi : E_1 \rightarrow E_2$$

These maps have the following two properties

- $\phi(\mathcal{O}) = \mathcal{O}$
- $\phi(P^{-1}) = (\phi(P)^{-1})$

Isogenies

Lemma (Uniquely identifying isogenies)

Let E be an elliptic curve and let Φ be a finite subgroup of E. There is a unique elliptic curve E' and a seperable isogeny $\phi : E \to E'$ satisfying $ker(\phi) = \Phi$.

Key Exchange Protocols

Key exchange protocols are cryptographic schemes used to establish a shared secret between two party members

These protocols can be defined by a tuple of algorithms (**KeyGen**, **SecAgr**), and then each run **SecAgr** with their own private key and the others public key to generate an equivalent, shared secret key.

Generally, Alice and Bob will both run **KeyGen** to generate their keypairs (pk_A, sk_A) and (pk_B, sk_B), respectively.

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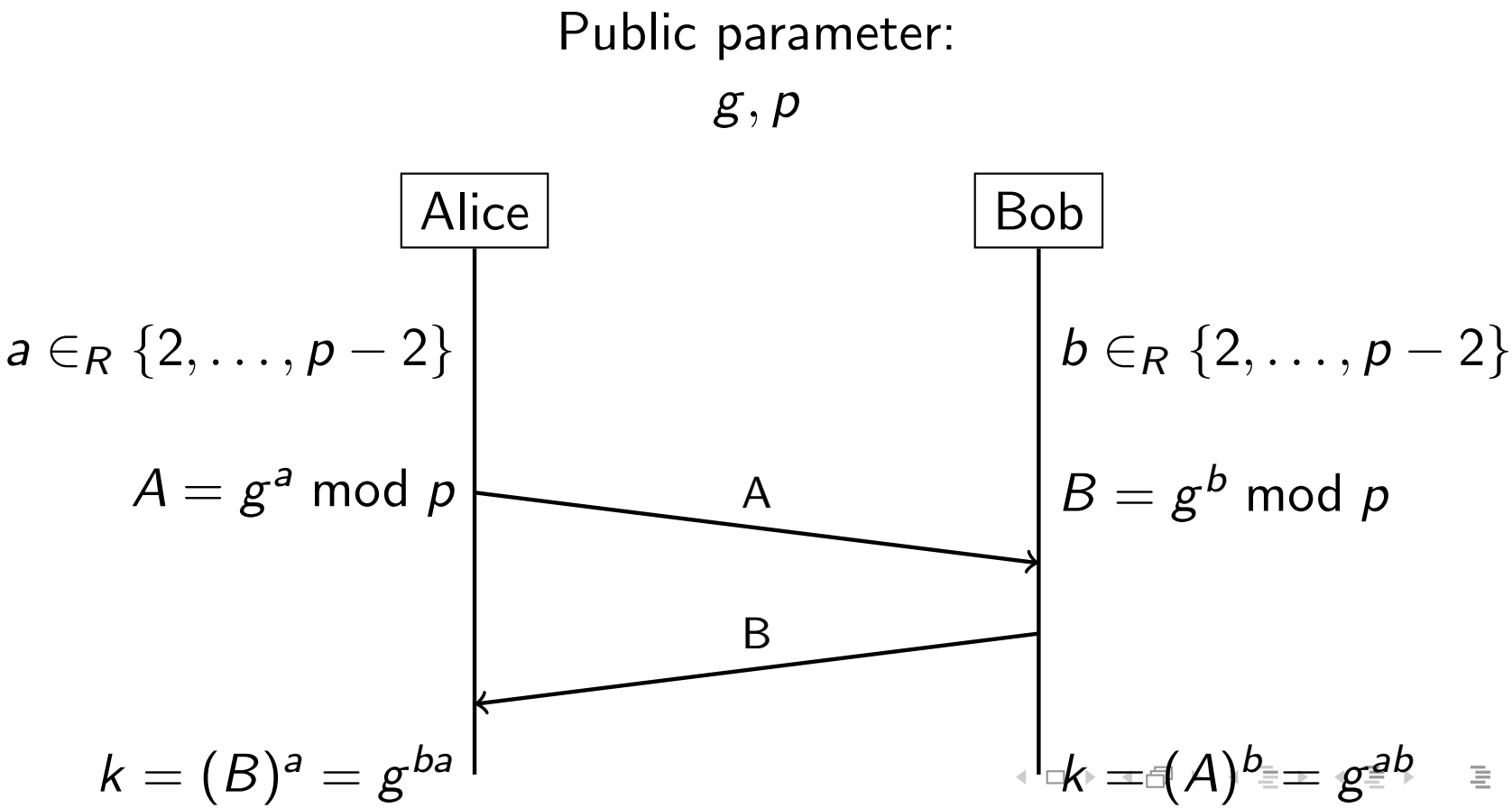
Key Exchange Protocols

The following is an execution of the Diffie-Hellman key exchange protocol between party members Alice and Bob.



Supersingular Isogeny Diffie-Hellman

SIDH is a key-exchange protocol where Alice and Bob use Isogenies as their public keys and points on a curve as their private keys.

Here's how it works...

Supersingular Isogeny Diffie-Hellman

We are concerned with curves over the field $\mathbb{F}_{p^2}$ where

$$p = \ell_A^{e_A} \ell_B^{e_B} \cdot f \pm 1$$

with f chosen such that p is prime.

We then choose a curve E, and bases $\{P_A, Q_A\}$ and $\{P_B, Q_B\}$ generating $E[\ell_A^{e_A}]$ and $E[\ell_B^{e_B}]$.

And so, our set of public parameters is

$$\{p, E, \ell_A, \ell_B, e_A, e_B, P_A, Q_A, P_B, Q_B\}.$$

Interactive Identification Schemes

Identification schemes are used to confirm the identity of a user on a network. These protocols are typically composed by the tuple of algorithms (**KeyGen**, **Commit**, **Prove**, **Verify**).
For Bob to prove his identity to Alice, a protocol of this type would run as follows:

1. Bob runs **KeyGen** (1^{λ}) to generate his keypair (sk, pk).
2. Bob runs **Commit** () to generate *com* and sends it to Alice.
3. Alice sends a randomly generated "challenge" value $ch \in \omega$ and sends it to Bob.
4. Bob runs **Prove** (*sk*, *com*, *ch*) with output *resp*, the response to Alice's challenge.
5. Alice runs **Verify** (*pk*, *com*, *ch*, *resp*) with output $b \in 0, 1$. Bob has successfully proven his identity to Alice if $b = 1$.

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Isogeny-based Proof of Identity

The isogeny-based proof of identity protocol outlined by De Feo et al. consists of the algorithms **KeyGen**, **Commit**, **Prove**, and **Verify**.

In the scheme, Bob can prove to Alice (or vice-versa) that he is the owner of the keypair (pk, sk) by forming many SIDH secret agreements with an arbitrary, random entity Randall, and showing that he can always come to a valid shared secret with Randall.

Isogeny-based Proof of Identity

Commit: Bob generates Randall's random point $R \in E[\ell_A^{e_A}]$ and the corresponding isogeny ψ_R. Bob then performs **SecAgr** with Randall and sets $com = (E/\langle R \rangle, E/\langle B, R \rangle)$.

Challenge: Alice responds with a random challenge bit $ch \in \{0, 1\}$.

Prove: If $ch = 0$ then Bob reveals isogenies ψ_R and $\psi'_R : E/\langle B \rangle \to E/\langle B, R \rangle$. If $ch = 1$ then Bob reveals $\phi'_B : E/\langle R \rangle \to E/\langle B, R \rangle$.

Verify: If $ch = 0$ return 1 iff R and $\phi_B(R)$ have order $\ell_A^{e_A}$ and generate the kernels of isogenies from $E \to E/\langle R \rangle$ and $E \to E/\rangle B, R \langle$ respectively. If $ch = 0$ return 1 iff $\psi_R(B)$ has order $\ell_B^{e_B}$ and generates the kernel of an isogeny over $E/\langle R \rangle \to E/\langle B, R \rangle$.

Signature Schemes

Signature schemes are used to prove that a particular party supplied and authenticated a given message m, and that m had not been edited by an unauthorized party. These schemes consist of the algorithms **KeyGen**, **Sign**, and **Verify**.

If Bob wishes to authenticate a message m and send it to Alice, the following will take place:

1. Bob runs **KeyGen** to produce his keypair (pk_B, sk_B).
2. Bob runs **Sign** (m, sk_B) to produce a signature σ, and sends (m, σ) to Alice.
3. Alice runs **Verify** (m, σ, pk_B) returning 1 if Bob's message was successfully verified and 0 otherwise.

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Fiat-Shamir Transform

The Fiat-Shamir transform is a process by which an interactive identification scheme can be turned into a signature scheme.

The transform works by first circumventing the need for challenge input from the verifier, turning it into a non-interactive identification scheme.

Then, the message m is factored into the identification protocol, allowing the prover to verify not just their own identity, but the authenticity of a message as well.

Applying Fiat-Shamir To an IPoI

Returning to our outline for a generic, interactive identification protocol, we would make the following changes:

1. Bob runs **KeyGen** (1^λ) to generate his keypair (sk, pk).
2. Bob runs **Commit** () to generate com and sends it to Alice.
3. ~~Alice sends a randomly generated "challenge" value $ch \in \omega$ and sends it to Bob.~~
3. Bob computes the challenge value as $ch = H(com, m)$ for some cryptographically secure hash function H and for message m.
4. Bob runs **Prove** (sk, com, ch) with output $resp$, the response to Alice's challenge.
5. Alice runs **Verify** $(pk, com, ch, resp)$ with output $b \in 0, 1$. Bob has successfully proven his identity to Alice if $b = 1$.

Yoo Signatures

The signature scheme derived by Yoo et al. applies the Fiat-Shamir transform to the isogeny-based identification scheme to construct the first ever isogeny-based signature scheme.

Recap

SIDH Key Exchange $\rightarrow$ Isogeny-based Proof of Identity $\rightarrow$ Yoo Signatures

Partial $\mathbb{F}_{p^2}$ Inversions

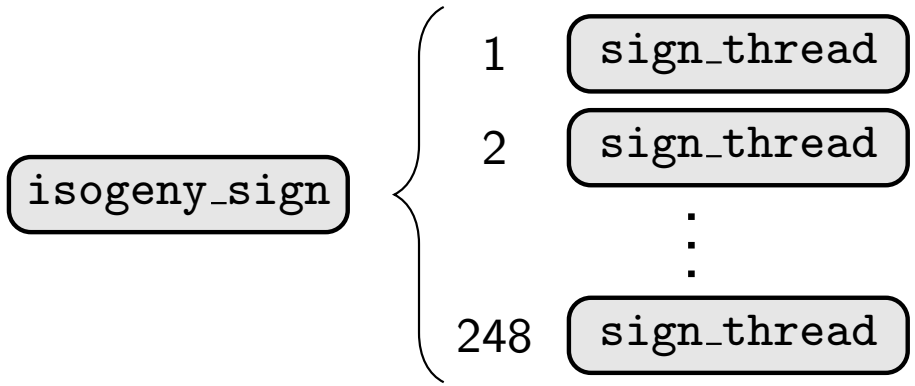
Batching Inversions

Partial Batched Inversions

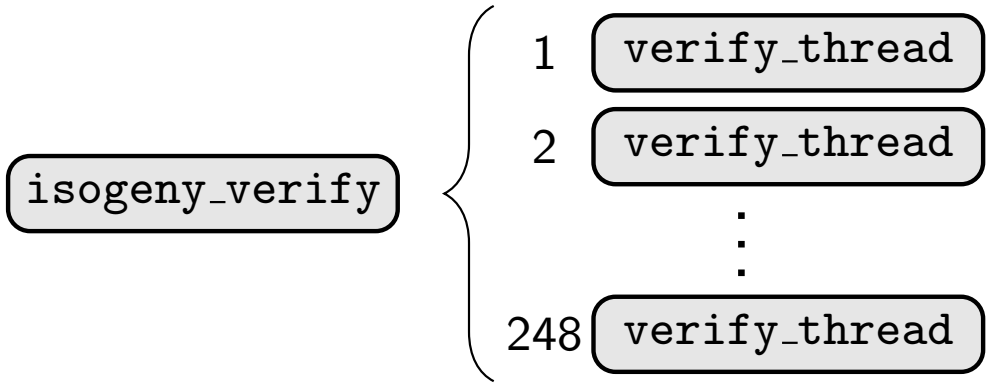
1: **for** `i = 0..(n-1)` **do**
2: $\quad den_i \leftarrow (x_i)_a^2 + (x_i)_b^2 \pmod p$
3: $a_0 \leftarrow den_0$
4: **for** `i = 1..(n-1)` **do**
5: $\quad a_i \leftarrow a_{i-1} \cdot den_i \pmod p$
6: $inv \leftarrow a_{n-1}^{-1} \pmod p$
7: **for** `i = n-1..1` **do**
8: $\quad a_i \leftarrow inv \cdot dest_{i-1} \pmod p$
9: $\quad inv \leftarrow inv \cdot den_i \pmod p$
10: $a_0 \leftarrow a_{inv}$
11: **for** `i = 0..(n-1)` **do**
12: $\quad (xinv_i)_a \leftarrow a_i \cdot (x_i)_a \pmod p$
13: $\quad (xinv_i)_b \leftarrow a_i \cdot -(x_i)_b \pmod p$
14: $\quad x_i^{-1} \leftarrow \{(xinv_i)_a, (xinv_i)_b\}$
15: **return** $\{x_0^{-1}, x_1^{-1}, ..., x_{n-1}^{-1}\}$

Structure of the Yoo Signature Implementation

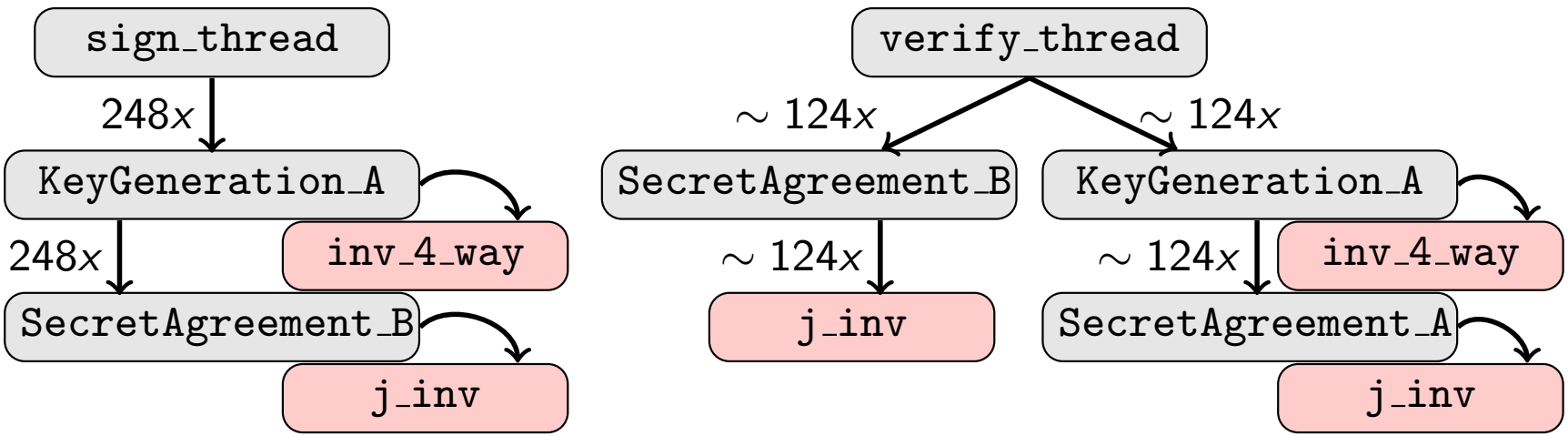
1. The signer executes **Sign** by spawning a thread running `sign_thread` for every iteration of the signing procedure



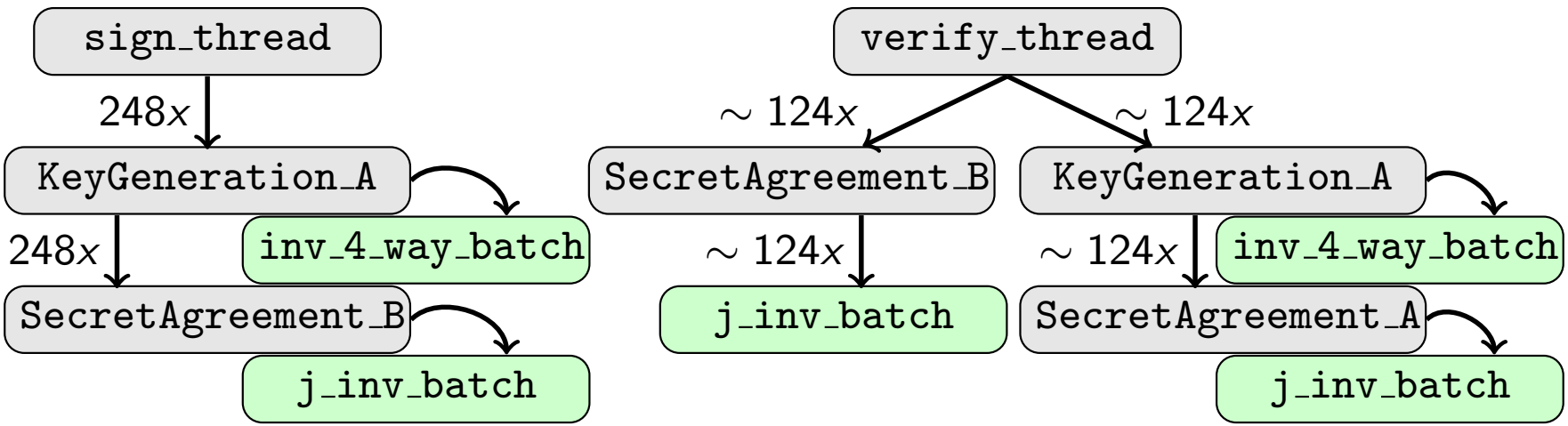
2. The verifier then executes **Verify** in a similar fashion.



Batching Across Threads



Batching Across Threads



Performance Measurements for Partial Batched Inversions

	Without Batching	With Batching
KeyGen	84,499,270	84,499,270
Sign	4,950,023,141.65	4,552,062,482.52
Verify	3,466,703,991.09	3,173,340,239.46

SIDH Public Key Compression

Azerderakhsh et al. showed that SIDH public keys can be compressed in the following way.

Take Alice's public key $pk = (E_A, \phi_A(P_B), \phi_A(Q_B))$.
By generating a basis $\{R_1, R_2\}$ for $E_A[\ell_B^{e_B}]$ we can represent the point components of pk as

$$P_A = \alpha_P R_1 + \beta_P R_2$$

$$Q_A = \alpha_Q R_1 + \beta_Q R_2$$

Giving $pk = (E_A, \alpha_P, \beta_P, \alpha_Q, \beta_Q)$. Public keys in this form are $4 \log p$ bits compared to the usual $6' logp$ bits.

SIDH Public Key Compression

Costello et al. then showed that $(E_A, \alpha_P, \beta_P, \alpha_Q, \beta_Q)$ could be further compressed to $(E_A, b, \zeta, \alpha, \beta)$, where $b \in \{1, 0\}$.

Compressing $\psi(S)$

We use the SIDH public key compression technique developed by Azerderakhsh, Costello, and others to compress every $\psi(S)$ component of a Yoo signature.

Revised Signature Sizes

An example of the `\cite` command to cite within the presentation:

Cost of Compressing Signatures

An example of the `\cite` command to cite within the presentation:

Conclusion of Results

Ay yo.

Next Steps for Implementation

Gotta do sum shiddd

Questions?

References

Reza Azerderalhsh, David Jao, Kassem Kalach, Brian Koziel, and Christopher Leonardi (2016)

Key Compression for Isogeny-Based Cryptosystems

Craig Costello, David Jao, Patrick Longa, Michael Naehrig, Joost Rene, and David Urbanik (2016)

Efficient Compression of SIDH Public Keys

IACR-CRYPTO-2016