8. 用龙贝格求积方法计算下列积分，使误差不超过 10^{-5}.

(1) $\frac{2}{\sqrt{\pi}}\int_0^1 e^{-x^2}dx$; (1)

(2) $\int_0^{\pi} x\sin x dx$;

(3) $\int_0^1 x\sqrt{1+x^2}dx$.

(1) (1) $T_0^{(0)} = \frac{1}{2}\times(e^{-1}+e^0)\times 1 = 0.77174333$

$f(\frac{1}{2}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{1}{4}} = 2.92228236$

$T_0^{(1)} = \frac{1}{2}T_0^{(0)} + \frac{1}{2}f(\frac{1}{2})$

$= \frac{1}{2}\times 0.77174333 + 0.5\times 2.92228236$

$= 0.72806995$

$T_1^{(0)} = \frac{4}{3}T_0^{(1)} - \frac{1}{3}T_0^{(0)} = 0.71351216$

$f(\frac{1}{4}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{1}{16}} = 0.87078258$

$f(\frac{3}{4}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{9}{16}} = 0.53300858$

$T_0^{(2)} = \frac{1}{2}T_0^{(1)} + \frac{1}{4}[f(\frac{1}{4}) + f(\frac{3}{4})]$

$= 0.71698276$

$\frac{2}{\sqrt{\pi}} = 1.12837917$

$T_1^{(1)} = \frac{4}{3}T_0^{(2)} - \frac{1}{3}T_0^{(1)} = 0.71328704$

$T_2^{(0)} = \frac{16}{15}T_1^{(1)} - \frac{1}{15}T_1^{(0)} = 0.71324203$

$f(\frac{1}{8}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{1}{64}} = 0.99579112$ $f(\frac{3}{8}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{9}{64}} = 0.77552290$

$f(\frac{5}{8}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{25}{64}} = 0.60397785$ $f(\frac{7}{8}) = \frac{2}{\sqrt{\pi}}\times e^{-\frac{49}{64}} = 0.47037842$

$T_0^{(3)} = \frac{1}{2}T_0^{(2)} + \frac{1}{8}[f(\frac{1}{8}) + f(\frac{3}{8}) + f(\frac{5}{8}) + f(\frac{7}{8})]$

$= 0.71420002$

$T_1^{(2)} = \frac{4}{3}T_0^{(3)} - \frac{1}{3}T_0^{(2)}$

$= 0.7132726$

$T_2^{(1)} = \frac{16}{15}T_1^{(2)} - \frac{1}{15}T_1^{(1)} = 0.71327720$

$T_3^{(0)} = \frac{64}{63}T_2^{(1)} - \frac{1}{63}T_2^{(0)}$

$= 0.71327717$

$I \approx 0.71327717$

(2) 类似(1). 有下表

k	h	$T_0^{(k)}$	$T_1^{(k)}$
0	2π	3.14513132×10^{-6}	
1	π	8.62828230×10^{-7}	-4.4469230×10^{-21}

$I \approx 4.4469230\times10^{-21} \approx 0$

(3) 有下表

k	h	$T_0^{(k)}$	$T_1^{(k)}$	$T_2^{(k)}$	$T_3^{(k)}$	$T_4^{(k)}$	$T_5^{(k)}$
0	3	14.2302495					
1	$\frac{3}{2}$	11.1713699	10.1574434				
2	$\frac{3}{4}$	10.4437698	10.2012725	10.2045749			
3	$\frac{3}{8}$	10.2663672	10.2072240	10.2076207	10.2076991		
4	$\frac{3}{16}$	10.2222702	10.2075712	10.2075493	10.2075939	10.2075936	
5	$\frac{3}{32}$	10.2112607	10.2075919	10.2075922	10.2075922	10.2075922	10.2075922

$I \approx 10.2075922$