20. 给定数据表如下：

x_j	0.25	0.30	0.39	0.45	0.53
y_j	0.5000	0.5477	0.6245	0.6708	0.7280

试求三次样条插值 $S(x)$，并满足条件：

(1) $S'(0.25)=1.0000$，$S'(0.53)=0.6868$；

(2) $S''(0.25)=S''(0.53)=0$.

(1) $h_0=x_1-x_0=0.05$

$h_1=x_2-x_1=0.09$

$h_2=x_3-x_2=0.06$

$h_4=x_4-x_3=0.08$

$u_j=\frac{h_{j-1}}{h_{j-1}+h_j}$ $u_1=\frac{5}{14}$ $u_2=\frac{3}{5}$ $u_3=\frac{3}{7}$ $u_4=1$

$\lambda_1=\frac{9}{14}$ $\lambda_2=\frac{3}{5}$ $\lambda_3=\frac{4}{7}$ $\lambda_0=1$

$f[x_0,x_1]=\frac{f(x_1)-f(x_0)}{x_1-x_0}=0.9540$

$f[x_1,x_2]=0.8533$ $f[x_3,x_4]=0.7150$

$f[x_2,x_3]=0.7717$

$d_0=\frac{6}{h_0}(f[x_0,x_1]-f_0')=-5.52$

$d_1=6\frac{f[x_1,x_2]-f[x_0,x_1]}{h_0+h_1}=-4.3157$

$d_2=6\frac{f[x_2,x_3]-f[x_1,x_2]}{h_1+h_2}=-3.2640$

$d_3=\frac{6f[x_3,x_4]-f[x_2,x_3]}{h_2+h_3}=-2.4300$

$d_4=\frac{6}{h_3}(f_4'-f[x_3,x_4])=-2.1150$

$$\begin{bmatrix} 2 & 1 & & & \\ \frac{5}{14} & 2 & \frac{9}{14} & & \\ & \frac{3}{5} & 2 & \frac{2}{5} & \\ & & \frac{3}{7} & 2 & \frac{4}{7} \\ & & & 1 & 2 \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} -5.52 \\ -4.3157 \\ -3.2640 \\ -2.4300 \\ -2.1150 \end{bmatrix}$$

解得 $M_0=-2.0278$ $M_1=-1.4643$ $M_2=-1.0313$ $M_3=-0.8012$

$M_4=-0.6539$

$$S(x)=M_j\frac{(x_{j+1}-x)^3}{6h_j}+M_{j+1}\frac{(x-x_j)^3}{6h_j}+\left(y_j-\frac{M_jh_j^2}{6}\right)\frac{x_{j+1}-x}{h_j}+\left(y_{j+1}-\frac{M_{j+1}h_j^2}{6}\right)\frac{x-x_j}{h_j}\quad j=0,1,2$$

$$=\begin{cases} -1.8783x^3-2.4227x^2+1.8591x+0.1573, & x\in[0.25,0.30] \\ 0.8019x^3-1.4538x^2+1.5685x+0.1863, & x\in[0.30,0.39] \\ 0.6225x^3-1.2440x^2+1.4886x+0.1970, & x\in[0.39,0.45] \\ 0.3794x^3-0.8348x^2+1.3085x+0.2496, & x\in[0.45,0.53] \end{cases}$$

(2)

$$\begin{bmatrix} 2 & \frac{9}{14} & 0 \\ \frac{3}{5} & 2 & \frac{2}{5} \\ 0 & \frac{3}{7} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = 6\times \begin{bmatrix} -0.7193 \\ -0.5440 \\ -0.4050 \end{bmatrix}$$

解得 $M_1=-1.8809$ $M_2=-0.8616$ $M_3=-1.0314$

代入三次样条表达式并整理得

$$S(x)=\begin{cases} -6.2697x^3+4.7023x^2-0.2057x+0.2555 & x\in[0.25,0.30] \\ 1.8876x^3-2.6393x^2+1.8966x+0.1353 & x\in[0.30,0.39] \\ -0.4689x^3+0.1178x^2+0.9213x+0.2751 & x\in[0.39,0.45] \\ 2.1487x^3-3.4132x^2+2.5103x+0.0367 & x\in[0.45,0.53] \end{cases}$$