TheoryExercise1

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Cylinder formula derivation

We start with the original implicit formula:

$$\|\vec{x} - \vec{x}_0\| = R^2 + ((x - x_0) \cdot \vec{v})^2$$

Where $\vec{x}$ is the ray, $\vec{x_0}$ is the center of the cylinder and $\vec{v}$ is the direction of the cylinder.
We replace $\vec{x} = \vec{o} + t \cdot \vec{d}$ and get

$$(\vec{o} + t \cdot \vec{d} - \vec{x}_0)^T \cdot (\vec{o} + t \cdot \vec{d} - \vec{x}_0) = R^2 + ((\vec{o} + t \cdot \vec{d} - \vec{x}_0) \cdot \vec{v})^2$$

By substituting $\vec{y} = \vec{o} - \vec{x}_0$

$$(t \cdot \vec{d} + \vec{y})^T \cdot (t \cdot \vec{d} + \vec{y}) = R^2 + (t \cdot \vec{d} \cdot \vec{v} + \vec{y} \cdot \vec{v})^2$$

By expanding

$$t^2 \cdot + \left\|\vec{d}\right\|^2 + 2 \cdot t \cdot \vec{d} \cdot \vec{y} + \|\vec{y}\|^2 = R^2 + t^2 \cdot (\vec{d} \cdot \vec{v})^2 + 2t(\vec{d} \cdot v) \cdot (\vec{y} \cdot \vec{v}) + (\vec{y} \cdot \vec{v})^2$$

Giving the second order equation

$$t^2 \cdot (\left\|\vec{d}^2\right\| - (\vec{d} \cdot \vec{v})^2) + 2t \cdot (\vec{d} \cdot \vec{y} - (\vec{d} \cdot \vec{v}) \cdot (\vec{y} \cdot \vec{v})) + \|\vec{y}\|^2 - R^2 - (\vec{y} \cdot \vec{v})^2 = 0$$

In the implementation we use this formula to project the intersection point on the cylinder's axis:

$$proj = (intersection_{point} - center) \cdot \vec{v}) \cdot \vec{v} + \vec{center}$$