Procedural Method for Finding Roller Coaster Rails Centerlines Based on Heart-Line Acceleration Criteria

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***Abstract*—Acceleration and jerk on a roller coaster circuit are measured at an imaginary curve passing through the heart of passengers. In this paper, Non Uniform Rational B-Splines are used to represent the heart-line path of roller coasters. Frenet Trihedron is used as the main curve framing, and an additional vector called gravity projection is deducted for generating a framework even at heart-line straights segments and inflection points, when curvature vanishes. A methodology for obtaining track and rails centerlines derived from the heart-line curve is proposed. Using this procedural method, track roll angle is fully determined by local curvature and speed of train. Four cases of study are presented: (1) generating track centerline with Frenet frame, (2) generating track centerline with gravity projection, (3) computing track centerline by the composition of Frenet frame and gravity projection and solving continuity-related problems and (4), generating rails centerlines. It is shown that using composite method all accelerations projected laterally on passengers become null and a continuous, smooth non-jerky track layout is obtained.**

***Index Terms*—nurbs, b-Spline, roller coaster, track, acceleration**

I. Introduction

Roller coasters are the main attractions of amusement parks. Amusement park rides are designed to subject passengers to variations of speed and acceleration, causing the sensation of increase or decrease of weight, in addition to physical reactions such as increased heart rate and the release of adrenaline in the bloodstream. In the case of roller coasters, the acceleration changes in intensity and direction mainly due to the trajectory of the trains.

The dynamic forces produced in human body due to acceleration can be dangerous if they are applied in excess or if they are kept for a longer period than tolerable. These forces can cause problems such as blurred vision, blackout and even complete loss of consciousness. Similarly, the jerk, defined as the derivative of acceleration with respect to time, may cause nausea. It is therefore necessary to limit the acceleration on a roller coaster circuit so as to make it safe and comfortable.

Up until 2002, the Standard DIN 4112 [1], "Temporary Buildings" was the basis of all calculations in the field of amusement rides. The limits of acceleration supported by the human body were defined in 2004, with the publication of European Standard EN 13814:2004 [2], specific to amusement equipment. Due to recent international research and available experiences, acceleration limits could be more precisely defined. With an agreement between American and European normative institutions, it was possible to harmonize the standards internationally [3]. The position of accelerometers may differ; in ASTM F2137 [4] they are positioned at abdomen / heart level.

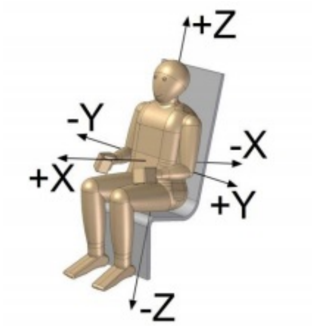

Fig. 1: Body-coordinate system for analyzing accelerations.

The Cartesian coordinate system presented in Fig. 1 is used to analyze accelerations in theme park industry. The expression "eyes down" is used to describe an upwards acceleration in Z direction (this is, the vertical acceleration which is pushing the eyes down). A downwards acceleration in Z direction is referred as "eyes up". Similarly, expressions "eyes back" and "eyes front" are used to describe $+X$ and $-X$ accelerations, respectively.

A roller coaster circuit can be determined by a curve, which represents the path of the circuit, and a roll angle, which represents the bank inclination of track. The curve may pass through the track center or at another position. Modern roller coasters are mainly heart-lined, what means that the coaster path and roll angle are given about the chest level of riders. This reduces overall rolling-related accelerations, by rotating passengers about their center of gravity, as can be seen in Fig. 2.

Tändl and Kecskeméthy [5] use an object-oriented structure to represent the guided spatial movement in multibody systems. Vector decompositions along the path are calculated using three different framings: Frenet Trihedron, Darboux and Bishop frames. It is shown that, for parameterization with Frenet frames, it is possible to avoid singularities at inflection points using L'Hospital limit analysis. The geometry of rails is smoothed down to the fifth order using B-Spline curves,

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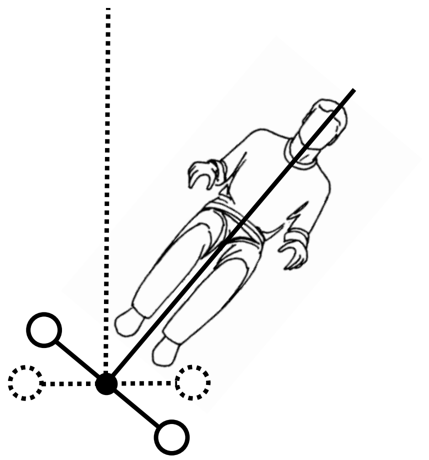
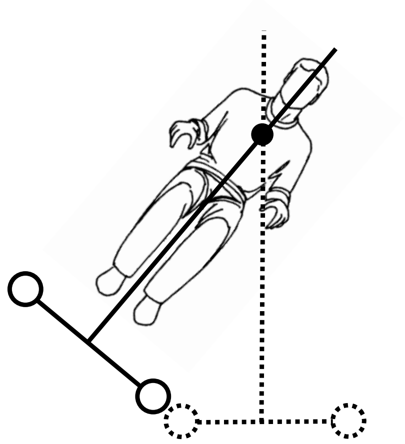


(a) Track centered circuit.



(b) Heart-line centered circuit.

Fig. 2: Different centers of rotation. Positioning riders closer to the center of rotation may reduce rolling-related accelerations.

allowing non-jerky layouts.

Sequeira et al. [6] present a methodology for the description of geometry from the centerline of roller coaster tracks, railways and trams. Three different parametric modeling are proposed, using cubic Splines, Akima Splines and Splines that preserve shapes. The advantages and disadvantages of each method are discussed through the application of the formulation developed for the dynamic analysis of a roller coaster model with complex spatial geometry. The calculation of the forces reactions imposed on the structure of the roller coaster by moving vehicles are presented.

Given a heart-line curve, the roll angle about the tangent vector $\hat{t}$ at point P could possibly be at any value. In fact, a roller coaster element called *Heart-line Roll* consists in turning the coaster train 360 degrees around a straight heart-line. Different roll angle θ results in different accelerations projections in Y and Z axis. It is apparent that there is an angle θ_0 which minimizes lateral acceleration, this is, acceleration in Y axis. This is done by orienting vector $\hat{v}_{H/T}$ parallel to the resulting acceleration in plane $Y-Z$, $\vec{a}_{Y-Z}$, as shown in Fig. 3.

In this paper, **N**on **U**niform **R**ational **B**-**S**pline (NURBS) is used to describe the heart-line path of roller coasters

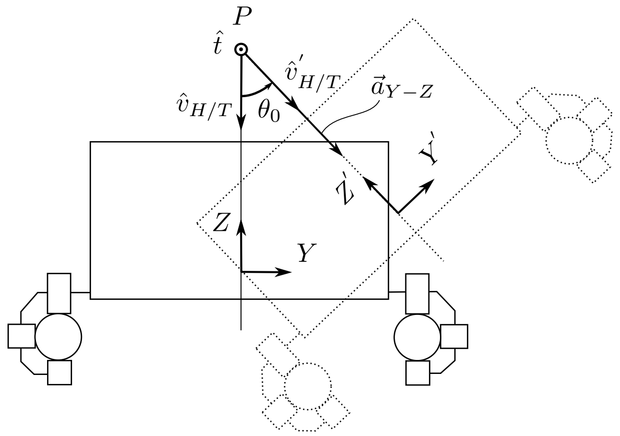


Fig. 3: There is a roll angle θ_0 which minimizes lateral acceleration.

and Frenet Trihedron is used as the main curve framing for orienting roll angle. Although Frenet vectors are easy to compute, they vanish at inflection points and straight curve segments.

There are two motivation for this paper. First, to bypass Frenet frames issues, such as vanishing at inflection points and straight curve segments. This is done by introducing a vector, called gravity projection. Second is to develop a method for obtaining the roll angle based only in curvature and the speed of the train, so that no lateral acceleration will be felt by passengers.

In Section II, NURBS implementation and properties are discussed. Curve derivatives are used to obtain Frenet Trihedron in II-A. In Section III, a front view of a roller coaster car is used to show offset vectors. The first method is proposed in III-A, which uses Frenet frame and normal acceleration for orienting roll angle. The second method is shown in Section III-B, and uses a gravity projection vector to bypass Frenet frames issues. The composition of the two methods results in an smooth, non-jerky track layout presented in III-C.

II. NURBS CURVES

A pth-degree NURBS curve with $n+1$ *control points* is defined as

$$C(t) = \frac{\sum_{i=0}^{n} N_{i,p}(t) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(t) w_i} \tag{1}$$

where P_i are the *control points*, w_i are the *weights* and N_i are the pth-degree B-Spline *basis functions* defined on a nonperiodic and nonuniform knot vector.

Setting

$$R_{i,p}(t) = \frac{N_{i,p}(t) w_i}{\sum_{j=0}^{n} N_{j,p}(t) w_j} \tag{2}$$

and (1) can be rewritten in the form

$$C(t) = \sum_{i=0}^{n} R_{i,p}(t) P_i. \tag{3}$$

The $\{R_{i,p}(t)\}$ are the *rational basis functions*. For any $a \neq 0$, if $w_i = a$ for all i, then $R_{i,p}(t) = N_{i,p}(t)$ for all i. In this case (3) can be rewritten as

$$C(t) = \sum_{i=0}^{n} N_{i,p}(t) P_i \tag{4}$$

which is nonrational B-Spline form.

NURBS curves are usually clamped. The therms clamped/unclamped refers to whether or not the first and last control points are interpolated by the curve. In order to generate a clamped curve, knot values at the beginning and at the end of knot vector must be repeated with multiplicity equal to degree plus one. This causes the B-Spline basis functions to reduce continuity at knot values and interpolate the respective control points. Let M be the sum of degree plus 1

$$M = p + 1 \tag{5}$$

where M represents NURBS *order*, which indicates the number of control points used to generate a single segment of a NURBS curve; e.g, a second degree NURBS curve has at least three control points.

The knot vector for an open, clamped *uniform* B-Spline curve can be defined by

$$U = \{u_0, u_1, \ldots, u_{n+M}\}. \tag{6}$$

By *uniform* it is intended that all *internal* knot spans have equal length and all internal knots have multiplicity of 1. Knot values can be calculated using

$$\begin{aligned} u_i &= a_0 && (i = 0, 1, \ldots, M-1) \\ u_{i+M} &= a_{i+1} && (i = 0, 1, \ldots, n-M) \\ u_{i+n+1} &= a_{n-M+2} && (i = 0, 1, \ldots, M-1) \end{aligned} \tag{7}$$

with

$$a_i = i \qquad (i = 0, 1, ..., n - M + 2). \tag{8}$$

For reference on other B-Spline types and knot vectors reader should view [7] and [8].

Given a knot vector U, basis functions $N_{i,p}(t)$ are defined as follows

$$N_{i,0} = \begin{cases} 1, for\ \langle t_i \leq t < t_{i+1} \rangle \\ 0,\ otherwise \end{cases} \tag{9}$$

for $p = 0$, and

$$\begin{aligned} N_{i,p}(t) =& \frac{t - t_i}{t_{i+p} - t_i} N_{i,p-1}(t) + \\ & \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} N_{i+1,p-1}(t) \end{aligned} \tag{10}$$

for $p > 0$ and $i = 0, 1, \ldots, n$. (10) can yield to quotient $0/0$. This quotient is defined to be zero.

A. Frenet Trihedron

Consider a generic NURBS curve $C(t)$

$$C(t) = [x(t), y(t), z(t)]. \tag{11}$$

The first derivative of curve with respect to curve parameter t is denoted either by $\dot{C}(t)$ or $C(t)^{(1)}$, where

$$\dot{C}(t) \equiv C(t)^{(1)} = \frac{dC(t)}{dt} = [\dot{x}(t), \dot{y}(t), \dot{z}(t)]. \tag{12}$$

The kth derivative of a nonrational B-Spline is obtained by computing the kth derivative of basis function, so that

$$C(t)^{(k)} = \sum_{i=0}^{n} N_{i,p}^{(k)}(t) P_i, \tag{13}$$

where $N_{i,p}^{(k)}$ denotes de kth derivative of basis function $N_{i,p}$. $\{N_{i,p}^{(k)}\}$ is calculated using

$$N_{i,p}^{(k)}(t) = p \left(\frac{N_{i,p-1}^{(k-1)}}{u_{i+p} - u_i} - \frac{N_{i+1,p-1}^{(k-1)}}{u_{i+p+1} - u_{i+1}} \right) \tag{14}$$

$C(t)$ is infinitely differentiable on the interior of knot spans, and is $p-m$ times differentiable at a knot of multiplicity m.

Now, considering the use of curve length s as the derivative parameter, one can write

$$\frac{dC}{ds} \equiv C' = \frac{dC}{dt}\frac{dt}{ds} = \frac{\dot{C}}{\dot{s}}. \tag{15}$$

The symbol $'$ is used in order to distinguish it from derivative with respect to t, and the terms (t) are omitted. Considering the curve parameter t as derivative parameter, one can write

$$\frac{ds}{dt} \equiv \dot{s} = \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2 + \frac{dz}{dt}^2} = \sqrt{\dot{C}^2} \tag{16}$$

then, if $\dot{C} \neq 0$, and

$$C' = \frac{\dot{C}}{\dot{s}} = \frac{\dot{C}}{\sqrt{\dot{C}^2}} \therefore \|C'\| = 1. \tag{17}$$

That is, C' has the same direction of $\dot{C}$ and its norm is equal to 1. C' is the so called *unit tangent vector* and will be represented by $\hat{t}$.

Second derivative of curve with respect to length s is given by

$$C'' = \frac{(\dot{C} \times \ddot{C}) \times \dot{C}}{\|\dot{C}^4\|} = \vec{\kappa} \tag{18}$$

where $\vec{\kappa}$ is curvature vector. C'' can also be expressed in terms of radius of curvature ρ, as in

$$C'' = \kappa\, \hat{n} = \frac{1}{\rho} \hat{n} \tag{19}$$

where κ is the curvature modulus and $\hat{n}$ is the *unit normal vector*.

Unit binormal vector $\hat{b}$ is obtained by taking the cross product of $\hat{t}$ and $\hat{n}$, as in

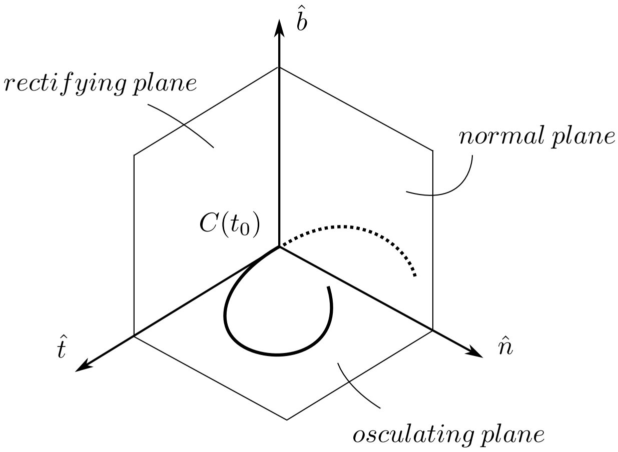


Fig. 4: Frenet trihedron.

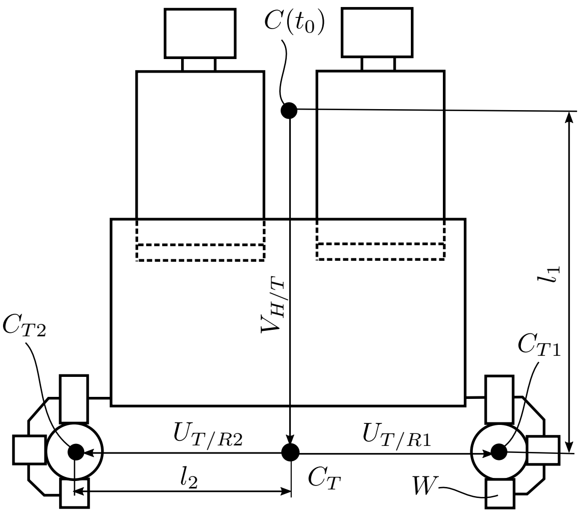


Fig. 5: Offset vectors showed in a front view of a roller coaster car. Wheel assembly is indicated by W.

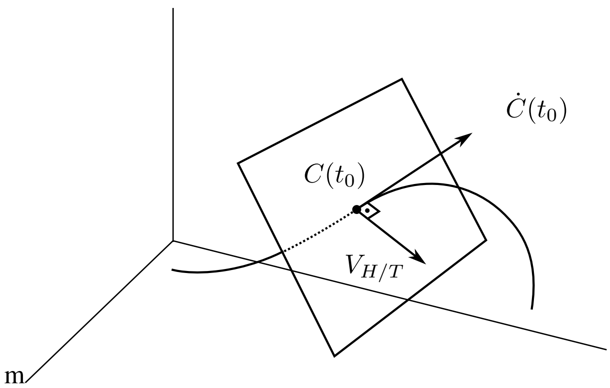


Fig. 6: Normal plane and offset vector $V_{H/T}$ at $C(t_0)$.

$$\hat{b} = \hat{t} \times \hat{n}. \tag{20}$$

Each pair of vectors of Frenet Trihedron defines a plane. The plane in $\mathbb{R}^3$ defined by vectors $\hat{n}$ and $\hat{b}$ is called normal plane, given by

$$\langle P - C(t_0), \hat{t} \rangle = 0 \tag{21}$$

The plane in $\mathbb{R}^3$ defined by vectors $\hat{n}$ and $\hat{t}$ is called osculating plane, given by

$$\langle P - C(t_0), \hat{b} \rangle = 0. \tag{22}$$

Finally, the plane in $\mathbb{R}^3$ defined by vectors $\hat{t}$ and $\hat{b}$ is called osculating plane, written as

$$\langle P - C(t_0), \hat{n} \rangle = 0. \tag{23}$$

Figure 4 shows the Frenet frame and planes.

III. Modeling tracks

Let the vector from passengers heart-line be represented by $V_{H/T}$.

$$V_{H/T} = l_1 \hat{v}_{H/T} \tag{24}$$

where, l_1 is a distance of offset and $\hat{v}_{H/T}$ is unit offset vector. Similarly, to find rails centerlines, one can define offset vectors from track centerline to rails centerlines. Here they are called $U_{T/T1}$ and $U_{T/T2}$,

$$\begin{aligned} U_{T/R1} &= l_2 \hat{u}_{T/R1} \\ U_{T/R2} &= -l_2 \hat{u}_{T/R1} \end{aligned} \tag{25}$$

where l_2 is a distance offset and $\hat{u}_{T/R1}$ is the unit *transverse* offset vector. Vector $\hat{u}_{T/R1}$ is calculated by taking the cross product

$$\hat{u}_{T/T1} = \dot{C}_T \times \hat{v}_{H/T}. \tag{26}$$

where $\dot{C}_T$ is the direction between successive C_T points. Figure 5 shows the offset vectors $V_{H/T}$, $U_{T/R1}$ and $U_{T/R2}$. The vectors are shown in a front view of a roller coaster car. A conventional roller coaster car carries two people side by side. Hence, the heart-line curve is defined to be at the heart level, but in the middle of the distance from each seat.

After the calculation of the offset vectors, the track center C_T and rails centers C_{T1} and C_{T2} are calculated by vector sum. Vector $V_{H/T}$ must be contained in the normal plane at point $C(t_0)$, so that each point of the heart-line curve generates a single point of track centerline. This is shown in Fig. 6.

It is proposed a procedural method for finding the unit vector $\hat{v}_{H/T}$, in order to generate track and rails centerlines. This method is composed by two sub-methods presented in the following.

A. Normal Acceleration Method

Most of the time, the acceleration felt on a roller coaster is the normal acceleration caused by circuit when the direction is changed. In this proposal, the direction of $\hat{v}_{H/T}$ is aligned with the normal acceleration vector. The normal acceleration is projected to the passengers as a vertical acceleration. Hence,

$$\hat{v}_{H/T}(t_0) = \frac{C(t_0) - \vec{a}_n(t_0)}{\|C_0(t_0) - \vec{a}_n(t_0)\|}. \tag{27}$$

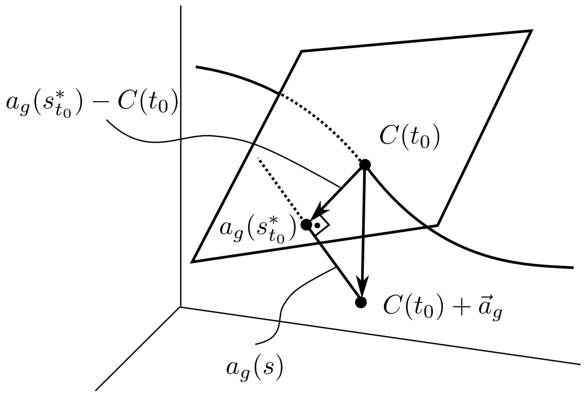


Fig. 7: Projecting gravity acceleration vector into normal plane at point $C(t_0)$.

The negative sign in $\{C(t_0) - \vec{a}_n(t)\}$ is because the offset vector must be in opposition to the normal acceleration, i.e., it is a pseudo centrifugal acceleration. This produces "eyes down" $+Z$ acceleration. Normal acceleration is calculated with

$$\vec{a}_n = \frac{{v_0}^2}{\rho}\,\hat{n} \tag{28}$$

where v_0 is the tangential speed of the coaster train.

B. Gravity Acceleration Method

The gravity acceleration method determines the unit vector $\hat{v}_{H/T}$ considering the gravity. This method consists of projecting the gravity acceleration vector into the normal plane for successive points of the heart-line curve.

Consider the line passing through $C(t_0) + \vec{a}_g$ whose direction is given by $\dot{C}(t_0)$. $\vec{a}_g$ is the gravity acceleration, i.e., $\vec{a}_g = \{0, 0, -\|a_g\|\}$. As the direction of this line is $\dot{C}(t_0)$, it is perpendicular to a normal plane at $t = t_0$. Considering the parameter s, the line is given by

$$a_g(s) = C(t_0) + \vec{a}_g + \dot{C}(t_0)s. \tag{29}$$

If $C(t_0) = \{x_0, y_0, z_0\}$, $\dot{C}(t_0) = \{\dot{x}_0, \dot{y}_0, \dot{z}_0\}$ and $a_g(s) = \{x(s), y(s), z(s)\}$, then (29) can be rewritten in function of its coordinates

$$\begin{aligned} x(s) &= x_0 + \dot{x}_0 s \\ y(s) &= y_0 + \dot{y}_0 s \\ z(s) &= z_0 + \vec{a}_g + \dot{z}_0 s \end{aligned} \quad . \tag{30}$$

Let the point that belongs to line $a_g(s)$ and intersects the normal plane at $s = s^*$ be represented by $a_g(s^*)$ (see Fig. 7). Hence, the unit vector $\hat{v}_{H/T}$ is

$$\hat{v}_{H/T}(t_0) = \frac{a_g(s^*_{t_0}) - C(t_0)}{\|a_g(s^*_{t_0}) - C(t_0)\|}. \tag{31}$$

The acceleration projection $a_g(s^*)$ is calculated by finding s which solves

$$\dot{C}(t_0) \cdot [a_g(s) - C(t_0)] = 0 \qquad (\dot{C} \times \vec{a}_g \neq 0). \tag{32}$$

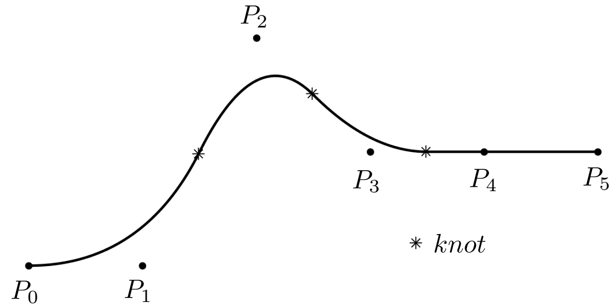


Fig. 8: Curve $C(t)$, control points and knots.

C. Composite Method

Smooth changing direction vector $\hat{v}_{H/T}$ can be calculated by composing the two methods described previously, as

$$\hat{v}_{H/T}(t_0) = \frac{a_g(s^*_{t_0}) - a_n(t_0)}{\|a_g(s^*_{t_0}) - a_n(t_0)\|}, \tag{33}$$

this equation produces a unit vector contained in the normal plane at point $C(t)$, which is obtained by the sum of gravity acceleration projection and pseudo centrifugal acceleration.

All accelerations contained in the normal plane will be felt by the passengers as vertical acceleration. Hence, no lateral forces will be felt by the riders. This is due to the fact that $\hat{v}_{H/T}$, which is vertical oriented (from head to toe), is aligned with the sum of the projected gravity acceleration and the pseudo centrifugal acceleration.

IV. Results

It is shown 4 study cases. Example 1 is a study case of the normal acceleration method. This example has a linear segment where the normal acceleration cannot be calculated. Example 2 is a study case of the gravity acceleration method. Examples 3 and 4 are study cases of the composite method.

A. Example 1

Let a quadratic plane NURBS curve $C(t)$ represents the heart-line path of a roller coaster circuit. The curve is defined by 6 control points

$$\{P\} = \{0\,0,\ 5\,0,\ 10\,10,\ 15\,5,\ 20\,5,\ 25\,5\}$$

and weight vector

$$w = \{1, 1, 1, 1, 1\}.$$

That is, all control points have the same amount of influence over the curve. Knot vector for an open, clamped uniform curve can be calculated using (7) and (8), and it is

$$U = \{u_0, u_1, ..., u_{n+M}\} = \{0, 0, 0, 1, 2, 3, 3, 3\}.$$

(4) is used to obtain the curve points. B-Spline basis functions and derivatives are computed using (9), (10) and (14). Fig. 8 shows the curve $C(t)$, the control points and the knots.

The tangential speed of the train is set to be constant and greater than 0 all over the curve, let it be 5 length units per second. Now, for point $C(0)$, the curvature vector $\vec{k}_0$ is computed using (18). Normal acceleration is computed

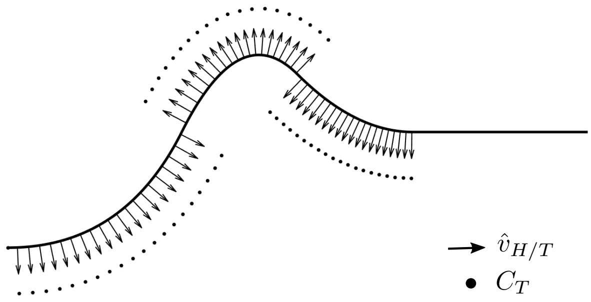


Fig. 9: C(t), $\hat{v}_{H/T}$ (out of scale) and track center points C_T. As the fourth segment P_4P_5 of curve is a line, $\hat{v}_{H/T}$ cannot be computed in this segment.

with (28) and $\hat{v}_{H/T}(0)$ with (27). This procedure is repeated for all the 80 points equally spaced according the t parameter.

Fig. 9 shows $\hat{v}_{H/T}$ and the track center points computed with normal acceleration method.

At this moment it is apparent that vector $\hat{v}_{H/T}$ changes direction at an inflection point on curve $C(t)$. Hence, track center points position vary abruptly. Moreover, as the three last control points lie on the same line, curvature is zero at any point on the last curve segment and offset vector can not be computed at the last 20 points. These problems are bypassed by introducing a non vanishing vector called gravity projection.

B. Example 2

Let a quadratic 3D NURBS curve $C(t)$ represents the heart-line path of a roller coaster circuit. The curve is defined by 5 control points

$$\{P\} = \{0\,0\,0,\ 10\,0\,0,\ 10\,10\,5,\ 20\,10\,5,\ 20\,0\,5\}$$

and weight vector

$$w = \{1, 1, 2, 1, 1\}.$$

The knot vector for an open, clamped uniform curve is

$$U = \{0, 0, 0, 1, 2, 3, 3, 3\}.$$

(1) is used to compute curve points. The B-Spline rational basis functions are computed using (2), (9) and (10). Fig. 10 shows the rational basis functions for weight vector $w = \{1, 1, 2, 1, 1\}$ in solid lines and for weight vector $w = \{1, 1, 1, 1, 1\}$ in dashed lines. Fig. 11 shows $x - y$ projection of curve $C(t)$.

NURBS curves and surfaces have local control. This is due to the *strong convex hull property*. According to this property, if $t \in [t_i, t_{i+1})$, then $C(t)$ is contained in the convex hull of polygon formed by control points $P_{i-p}, ..., P_i$. The control polygon for the third curve segment and the third curve segment are shown in dashed lines in Fig. 11.

Now, for point $C(0) = \{0\,0\,0\}$, parametric function given by (29) is generated and then (32) is solved. The resulting parameter is s_0^*. Acceleration projection points are computed using (29) or (30) and the unit vector $\hat{v}_{H/T}(0)$ is calculated with (31). Gravity acceleration is set to

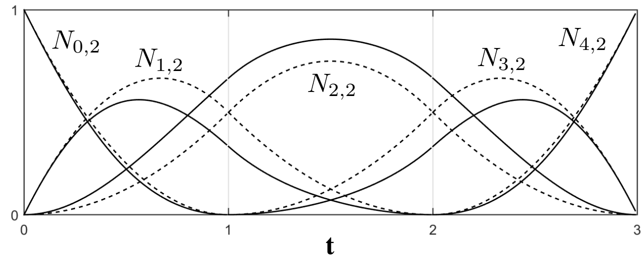


Fig. 10: Rational basis functions. $\{N_{i,p}\}$ for $w_2 = 2$ are showed in solid lines. For comparison, rational basis functions for $w_2 = 1$ are showed in dashed lines.

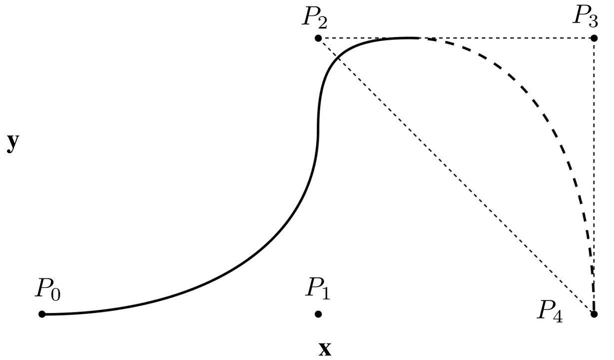


Fig. 11: $x - y$ projection of curve $C(t)$. The strong convex hull property for a quadratic curve: for $t \in [t_4, t_5]$, $C(t)$ is in the triangle $P_2P_3P_4$.

$$\vec{a}_g = \{0, 0, -9.8\}[L/s^2].$$

(32) is solved for all 60 points of curve $C(t)$ equally spaced according to parameter t. Points $a_g(s_t^*)$ are then computed using (29) or (30) and $\hat{v}_{H/T}(t)$ is computed with (31). Fig. 12 shows $\hat{v}_{H/T}$ and the track center points C_T obtained by the gravity acceleration method.

As the third segment of $C(t)$ is parallel to the $x - y$ plane, all normal planes generated on this segment are parallel to the z axis. Hence, the gravity acceleration projection in this segment will be equal to the gravity acceleration itself.

C. Example 3

Let a cubic nonrational B-Spline curve $C(t)$ represents the heart-line of a roller coaster circuit. The curve is defined by 6 control points

$$P = \{0\,0\,0, 4\,10\,0, 8\,0\,0, 12\,10\,0, 16\,0\,5, 20\,10\,5\}.$$

Basis functions, derivatives and curve points are computed as previous examples.

Gravity projection $a_g(s^*)$ is calculated solving (32). Normal acceleration is found with (28). The speed is set to be constant 5 length units per second. Both vectors are combined using (33). Fig. 13 shows a 3D perspective of $C(t)$, $\hat{v}_{H/T}$ and the track center points generated using the composite method. Fig. 14 shows $x - y$ and $x - z$ projections of $C(t)$, $\hat{v}_{H/T}$ and a dashed line between successive C_T points.

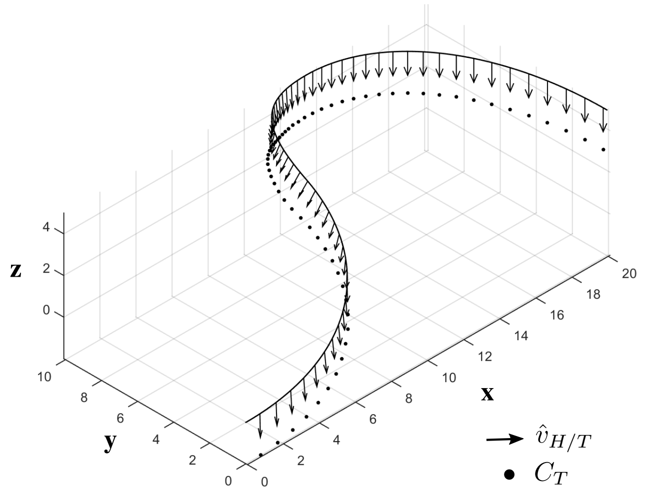


Fig. 12: C(t), $\hat{v}_{H/T}$ (out of scale) and the track center points C_T are shown. As the third segment of curve is parallel to the $x-y$ plane, $\hat{v}_{H/T}$ on this segment is completely aligned to the z axis.

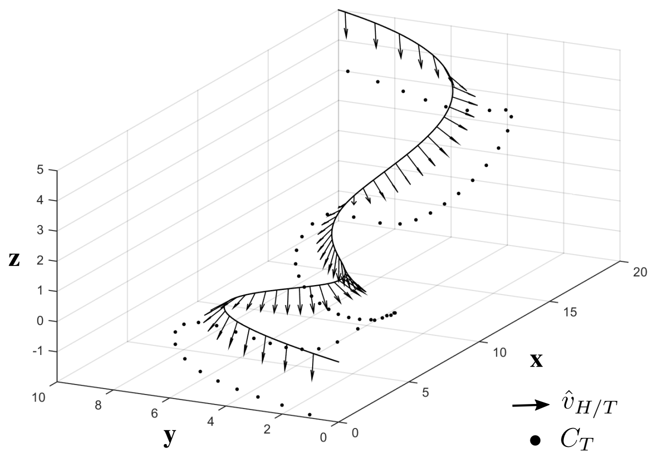


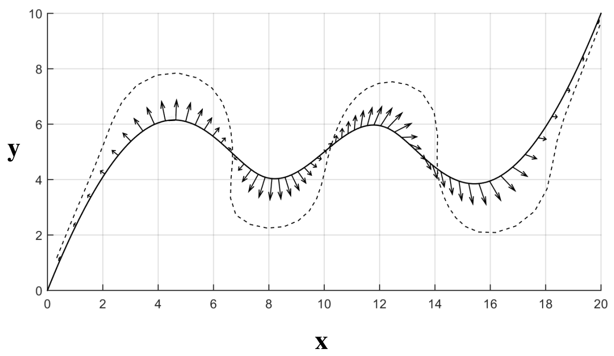
Fig. 13: $C(t)$, $\hat{v}_{H/T}$ (out of scale) and the track center points C_T.

Now let a quadratic nonrational B-Spline curve $C(t)$ represent the heart-line of a roller coaster circuit defined by the same 6 control points. Fig. 15 shows $x-y$ and $x-z$ projections of $C(t)$, $\hat{v}_{H/T}$ and a dashed line between successive points C_T.

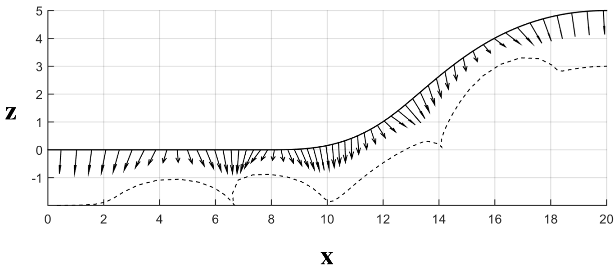
As seen in Fig. 15, track centerline gets discontinuous at $C(t)$ knots. This is due to the fact that $\hat{v}_{H/T}$ is computed from a second degree curve $C(t)$. A second degree B-Spline curve is C^1 class, that is, first derivative is continuous at knots but second derivative is discontinuous. At an inflection point, curvature magnitude and/or direction may change abruptly, as well as normal acceleration vector. This results in a discontinuous $\hat{v}_{H/T}$. Fig. 16 shows $C(t)$ first and second derivatives magnitudes.

D. Example 4

Let a cubic nonrational B-Spline curve $C(t)$ represent the heart-line of a roller coaster circuit. Let the coaster train speed

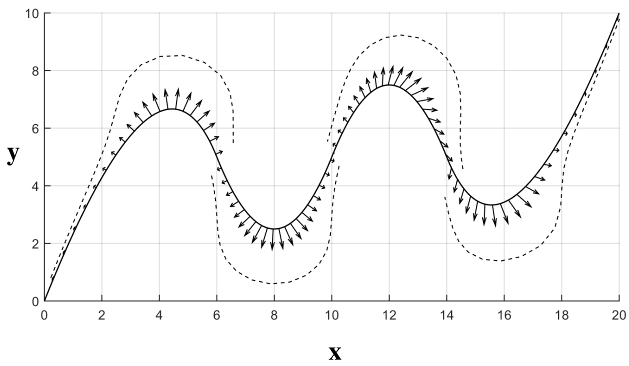


(a) $x-y$ projection.

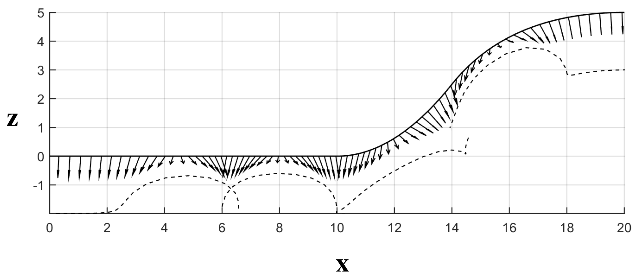


(b) $x-z$ projection.

Fig. 14: $C(t)$ (solid line), $\hat{v}_{H/T}$ (out of scale) and the track centerline (dashed line).



(a) $x-y$ projection.



(b) $x-z$ projection.

Fig. 15: $C(t)$ (solid line), $\hat{v}_{H/T}$ (out of scale) and the track center points C_T. Points C_T vary abruptly at knots because of curvature discontinuity.

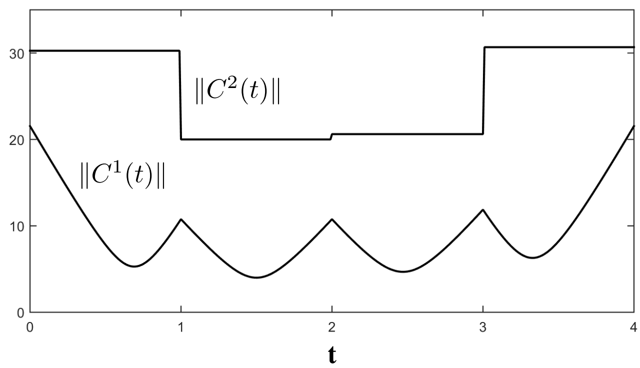


Fig. 16: $C^{(1)}(t)$ and $C^{(2)}(t)$ derivatives magnitudes.

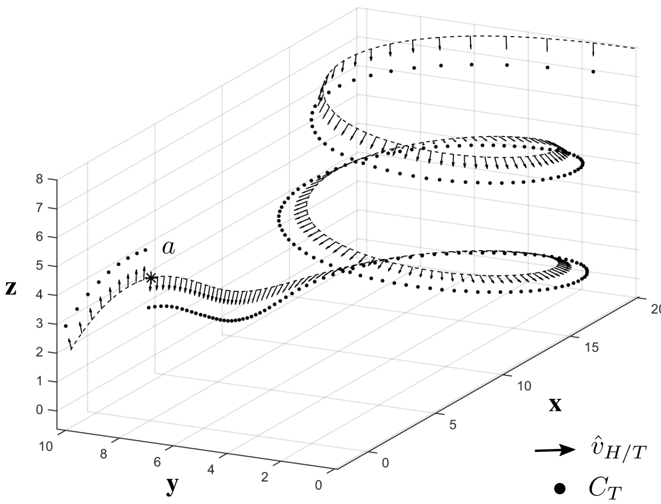


Fig. 17: $C(t)$, $\hat{v}_{H/T}$ (out of scale) and the track center points C_T. Points C_T vary abruptly at knot a because of curvature discontinuity in a vertical plane.

be approximated by

$$v(t) = \sqrt{2gz_{max} - gz(t)} \tag{34}$$

where g is gravity acceleration magnitude and z_{max} is maximum z value of $C(t)$ (roller coaster height). Fig. 17 shows $C(t)$ and track center points obtained using (33).

According to (34), on the first segment of curve $C(t)$ the train is at low speed. On this segment there is low normal acceleration, then $\hat{v}_{H/T}$ is mainly guided by

$$a_g(s^*_{t_0}) - C(t_0).$$

As the train speeds up, normal acceleration increases and pushes $\hat{v}_{H/T}$ to become more aligned to the osculating plane of $C(t)$.

At point a in Fig. 17, the curve is contained in a vertical plane parallel to $x - z$. At this point, the normal acceleration gets larger than the gravity projection, forcing $\hat{v}_{H/T}$ to invert direction. In a curve parallel to the $x - y$ plane, the gravity projection will change $\hat{v}_{H/T}$ direction gradually if the curve degree is equal or greater than 3, but as this curve is in a vertical plane, both $a_g(s^*_{t_0}) - C(t_0)$ and $C_0(t_0) - \vec{a}_n(t_0)$ lie on the same plane. This problem is solved by inverting the direction of $\hat{v}_{H/T}$ when the track center points change

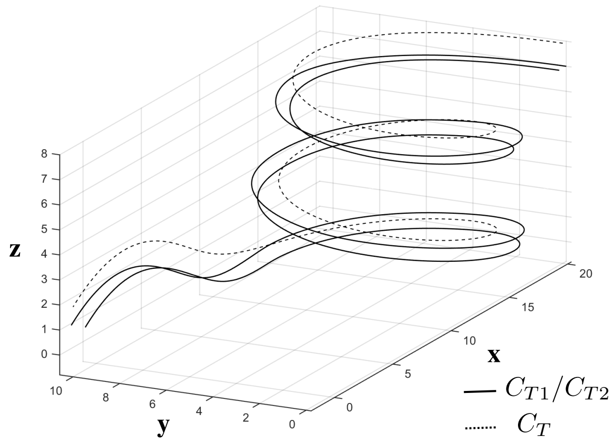


Fig. 18: $C(t)$ and points C_{T1} and C_{T2}. The discontinuity shown in Fig. 17 is removed.

abruptly. In Fig. 18, for each point C_T, transverse unit vector $\hat{u}_{T/R1}$ is computed using (26) and points C_{T1} and C_{T2} are computed using (25).

V. Conclusions

Acceleration can be dangerous to human body if applied in excess. Using the presented method, all acceleration, except tangential acceleration, are projected in passengers as vertical acceleration, whose limits are greater than lateral limits. Problems with curvature vanishing are bypassed by introducing a gravity projection vector, which can be computed even when Frenet frames are unavailable. Because of good local control and intuitive design, NURBS are used to represent the heart-line path of roller coasters. Given a heart-line path, a smooth, non-jerky track layout can be obtained using the proposed methodology. Problems related to track continuity can be bypassed using an appropriate degree for the heart-line curve, as well as observing abrupt variations in vertical planes.

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