TheoryExercise1

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Cylinder formula derivation

We start with the original implicit formula:

$$\|\vec{x} - \vec{x}_0\| = R^2 + ((x - x_0) \cdot \vec{v})^2$$

Where $\vec{x}$ is the ray, $\vec{x_0}$ is the center of the cylinder and $\vec{v}$ is the direction of the cylinder.
We replace $\vec{x} = \vec{o} + t \cdot \vec{d}$ and get

$$(\vec{o} + t \cdot \vec{d} - \vec{x}_0)^T \cdot (\vec{o} + t \cdot \vec{d} - \vec{x}_0) = R^2 + ((\vec{o} + t \cdot \vec{d} - \vec{x}_0) \cdot \vec{v})^2$$

By substituting $\vec{y} = \vec{o} - \vec{x}_0$

$$(t \cdot \vec{d} + \vec{y})^T \cdot (t \cdot \vec{d} + \vec{y}) = R^2 + (t \cdot \vec{d} \cdot \vec{v} + \vec{y} \cdot \vec{v})^2$$

By expanding

$$t^2 \cdot + \left\|\vec{d}\right\|^2 + 2 \cdot t \cdot \vec{d} \cdot \vec{y} + \|\vec{y}\|^2 = R^2 + t^2 \cdot (\vec{d} \cdot \vec{v})^2 + 2t(\vec{d} \cdot v) \cdot (\vec{y} \cdot \vec{v}) + (\vec{y} \cdot \vec{v})^2$$

Giving the second order equation

$$t^2 \cdot (\left\|\vec{d}^2\right\| - (\vec{d} \cdot \vec{v})^2) + 2t \cdot (\vec{d} \cdot \vec{y} - (\vec{d} \cdot \vec{v}) \cdot (\vec{y} \cdot \vec{v})) + \|\vec{y}\|^2 - R^2 - (\vec{y} \cdot \vec{v})^2 = 0$$

1 Normal derivation

We use this formula to project the intersection point on the cylinder's axis:

$$proj = (intersection_{point} - center) \cdot \vec{v}) \cdot \vec{v} + center$$

This is the formula for projecting a point on a line.
Once we have the projection, we use it to check that the intersection point is indeed in the finite cylinder.

Now the normal can be easily found by computing the vector between the projected point and the intersection point ($intersection_{point} - proj$) and divide it by the radius of the cylinder to normalize it. The final normal formula is the following:

$$\vec{normal} = (intersection_{point} - proj)/radius$$