A SIMPLE GRAPH TYPE FOR JULIA

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1. Fundamentals

This is documentation for a SimpleGraph data type for Julia. The goal is to make working with graphs as painless as possible. The SimpleGraph data type is for simple graphs (undirected edges, no loops, no multiple edges). Vertices in a SimpleGraph are of a given Julia data type, which might be Any.

To begin, get the repository with this Julia command:

```
Pkg.clone("https://github.com/scheinerman/SimpleGraphs.jl.git")
```

This only has to be done once. To use the SimpleGraph type, give the command using SimpleGraphs.

The key constructor is SimpleGraph which creates a graph whose vertices may be any Julia type. Alternatively, G=SimpleGraph{T}() sets up G to be a graph in which all vertices must be of type T. Two special cases are built into this module: IntGraph() is a synonym for SimpleGraph{Int}() and StringGraph() is a synonym for SimpleGraph{ASCIIString}.

Vertices and edges are added to a graph with add! and deleted with delete!. Membership is checked with has.

```
julia> using SimpleGraphs

julia> G = IntGraph()
SimpleGraph{Int64} (0 vertices, 0 edges)

julia> add!(G,5)
true

julia> add!(G,1,2)
true

julia> G
SimpleGraph{Int64} (3 vertices, 1 edges)
```

Notice that adding an edge automatically adds its end points to the graph.

The number of vertices and edges can be queried with NV and NE. The vertex and edge sets are returned as arrays by vlist(G) and elist(G).

```
julia> vlist(G)
3-element Array{Int64,1}:
1
2
5

julia> elist(G)
1-element Array{(Int64,Int64),1}:
(1,2)
```

Document version 2017:03:30:14:02. Note: This documentation might not quite be in sync with the functionality found in the SimpleGraphs module. I will likely want to redo this at some point.

1

Use deg (G, v) for the degree of a vertex and deg (G) for the graph's degree sequence.

```
julia> deg(G,1)
1

julia> deg(G)
3-element Array{Int64,1}:
    1
    1
    0
```

The neighbors of a vertex can be sought with neighbors (G, v) or, alternatively, with G[v]. Edges can be queried with G[v, w].

```
julia> G[1]
1-element Array{Int64,1}:
2

julia> G[1,2]
true

julia> G[1,5]
false
```

The Complete method can be used to create complete graphs and complete bipartite graphs with Int vertices.

```
julia> Complete(5)
SimpleGraph{Int64} (5 vertices, 10 edges)
julia> Complete(3,3)
SimpleGraph{Int64} (6 vertices, 9 edges)
```

We also provide Cycle(n) and Path(n) to create the graphs C_n and P_n . An instance of a Erdős-Rényi random graph $G_{n,p}$ is returned by RandomGraph(n,p).

The adjacency, Laplacian, and vertex-edge incidence matrices can be found with adjaceny, Laplacian, and incidence. It is important to note that the indexing of the rows/columns of these matrices might not correspond to the natural order of the underlying vertices (or edges).

The incidence matrix method takes an optional argument as to whether the matrix is signed (a 1 and a -1 in each column) or unsigned (only positive 1s). Also, it is a sparse matrix that can be converted to dense storage with full.

Finally, the vertex_type function can be used to query the data type of the vertices in the graph.

```
julia> G = StringGraph()
SimpleGraph{ASCIIString} (0 vertices, 0 edges)
julia> vertex_type(G)
ASCIIString (constructor with 1 method)
```

2. Look but don't touch

The SimpleGraph objects have three internal fields. It is not safe to change these directly, but there is no problem examining their values.

- :V
 This holds the vertex set of the graph. If G is a SimpleGraph {T} then G.V is of type Set {T}.
- :E

 This holds the edge set of the graph. If G is a SimpleGraph {T} then G.E is a Set { (T, T) }.

• :Nflag

This boolean value indicates whether or not fast neighborhood lookup has been activated for this graph. See the discussion below and the description of the function fastN!.

• : N

This is a Dict that keeps track of the neighborhood of each vertex. It is only active if the Nflag is set to true.

This design is redundant. One can test if two vertices are adjacent by looking for the edge in E. One can determine the neighbors of a given vertex by iterating over the edge set E. However, this approach is slow. By providing the extra N data structure, these operations are very fast.

By calling fastN! (G, false) the redudant neighborhood structure N is deleted. All functions will still work, but perhaps more slowly. Calling fastN! (G, true) rebuilds the N structure.

We recommend using the default structure unless the graph is so large that it consumes too much memory.

3. List of all functions

Creators. These are functions that create new graphs.

• SimpleGraph

Use G=SimpleGraph() to create a graph whose vertices can be of Any type. To create a graph with vertices of a particular type T use $G=SimpleGraph\{T\}()$.

• IntGraph

This a synonym for SimpleGraph{Int}. Use G=IntGraph() to create a vertex whose vertices are integers (type Int).

Use IntGraph (n) to create a SimpleGraph {Int} graph with vertex set $\{1, 2, ..., n\}$ and no edges.

• StringGraph

This is a synonym for SimpleGraph {ASCIIString}. Use G=StringGraph() to create a graph whose vertices are character strings.

One can also call G=StringGraph (filename) to read in a graph from a file. The file must have the following format:

- Each line should contain one or two tokens (words) that do not contain any whitespace.
- If a line contains one token, that token is added to the graph as a vertex.
- If a line contains two tokens, an edge is added with those tokens as end points. If those two
 tokens are the same, no edge is created (since we do not allow loops) but a vertex is added (if
 not already in the graph).
- If there are three or more tokens on a line, only the first two are read and the rest are ignored.
- If the line is blank, it is ignored.
- If the line begins with a #, the entire line is ignored. (This does put a mild limitation on the names of vertices.)
- Complete

The Complete function can be used to create a complete graph K_n , a complete bipartite graph $K_{n,m}$, or a complete multipartite graph $K(n_1, n_2, \ldots, n_p)$.

- Use Complete (n) to create a complete graph K_n .
- Use Complete (n, m) to create a complete bipartite graph $K_{n,m}$.
- Use Complete ([n1, n2, ..., np]) to create a complete multipartite graph $K(n_1, n_2, ..., n_p)$. Note the last version requires that the part sizes be in an array. In this way we distinguish Complete (n) and Complete ([n]). The first makes K_n and the second an edgeless graph with n vertices, i.e., $\overline{K_n}$. However, Complete (n, m) and Complete ([n, m]) build exactly the same graphs.

```
julia> G = Complete(4,5)
SimpleGraph{Int64} (9 vertices, 20 edges)
julia> H = Complete([4,5])
```

```
SimpleGraph{Int64} (9 vertices, 20 edges)
julia> G == H
true
```

• Cube

Create the cube graph. The 2^n vertices of Cube (n) are ASCIIString objects. For example:

```
julia > G = Cube(3)
SimpleGraph(ASCIIString) (8 vertices, 12 edges)
julia> vlist(G)
8-element Array{ASCIIString,1}:
 "000"
 "001"
 "010"
 "011"
 "100"
 "101"
 "110"
 "111"
julia> elist(G)
12-element Array{(ASCIIString, ASCIIString), 1}:
 ("000", "001")
 ("000","010")
 ("000", "100")
 ("001", "011")
 ("001","101")
 ("010", "011")
 ("010","110")
 ("011", "111")
 ("100","101")
 ("100", "110")
 ("101","111")
 ("110", "111")
```

• Path

Use Path (n) to create a path graph with *n* vertices.

Also, given a list of vertices verts, then Path (verts) creates a path graph with edges (verts[k], verts[k+1]) when k=1:n-1. It's the user's responsibility that there be no repeated entries in verts.

• Grid

Use Grid (n, m) to create an $n \times m$ grid graph.

• Cycle

Use Cycle (n) to create a cycle graph with n vertices. It is required that $n \ge 3$.

Wheel

Use Wheel (n) to create the wheel graph with n vertices. That is, a graph composed of an (n-1)-cycle with one additional vertex adjacent to every vertex on the cycle. This requires $n \ge 4$.

• BuckyBall

Create the Buckyball graph with 30 vertices and 90 edges.

• RandomGraph

Use RandomGraph (n, p) to create an Erdős-Rényi random graph with n vertices with edge probability p. If the argument p is omitted, it is assumed $p = \frac{1}{2}$.

• RandomTree

Use RandomTree(n) to create a random tree on vertex set 1:n. All n^{n-2} trees are equally likely.

This works by creating an n-2-long sequence of random values, each in the range 1:n. It then converts that Prüfer code to a tree using code_to_tree. This latter function is exposed for use by the user. It takes as input an array of integers and returns a tree assuming, that is, that the input is valid. No checks are done on the input so user beware.

RandomSBM

Use RandomSBM (bmap, pmat) to create a random stochastic block model graph. Here bmap is an n-long list of integers between 1 and b that assigns the n vertices to blocks. The i, j-entry of the matrix pmat gives the probability that a vertex in block i is adjacent to a vertex in block j.

This can also be invoked by RandomSBM (n, pvec, pmat) where n is the number of vertices, pvec is a probability vector whose j-th entry is the probability a vertex is placed in the j-th block.

• Kneser

Use Kneser (n, k) to create the Kneser graph with those parameters (with $0 \le k \le n$). This is a graph with $\binom{n}{k}$ vertices that are the k-element subsets of $\{1, 2, \dots, n\}$. Two vertices u and v are adjacent iff $u \cap v = \emptyset$.

Part of this implementation is a function subsets (A, k) where A is a Set and k is an Int. This creates the set of all k-element subsets of A.

• Peterser

Use Peteren () to create Petersen's graph. This is created by calculating Kneser (5, 2). To remap the vertex names to $\{1, 2, ..., 10\}$ use relabel (Petersen ()).

Graph operations. These are operations that create new graphs from old.

• line_graph

Use $line_graph(G)$ to create the line graph of G. Note that if G has vertex type T, then this creates a graph with vertex type (T, T).

• complement and complement!

Use complement (G) to create the graph \overline{G} . The original graph is not changed and the vertex type of the new graph is that same the vertex type of G.

We can use G' in lieu of complement (G).

Use complement! (G) to complement a graph in place (i.e., replace G with its own complement).

• сору

Use copy (G) to create an independent copy of G.

• induce

Use induce (G, A) to create the induced subgraph of G on vertex set A.

• spanning_forest

Given a graph, this creates a maximal, acyclic, spanning subgraph. If the original graph is connected, this produces a spanning tree.

• cartesian

Use cartesian (G, H) to compute the Cartesian product $G \times H$ of G and H. For example, to create a grid graph, do this: cartesian (Path (n), Path (m)).

Note that G*H is equivalent to cartesian (G, H).

• relabel

Create a new graph, isomorphic to the old graph, in which the vertices have been renamed. Use relabel (G, label) where G is a simple graph and labels is a dictionary mapping vertices in G to new names. Trouble ensues if two vertices are mapped to the same label (we don't check).

Note that if the vertex type of G is S, then label must be of type $Dict\{S,T\}$. The new graph produced with have type $SimpleGraph\{T\}$.

Calling this with one argument, relabel(G), will produce a relabeled version of G using consecutive integers starting with 1.

• disjoint_union

The disjoint union of two graphs is formed by taking nonoverlapping copies of the two graphs and merging them into a single graph (with no additional edges). In Julia, we do this by appending the intger 1 or 2 to the vertex names. Thus, if a vertex of the first graph is "alpha", then in the disjoint union its name will be ("alpha", 1).

Use $disjoint_union(G, H)$ to form the disjoint union. If the vertex types of the two graphs are both T, then the vertex type of the result is type (T, Int). But if the two graphs have different vertex types, then the result has vertex type Any.

We append a 1 or a 2 to vertex names to ensure that we have two independent copies of the graphs. If the user knows that the two graphs have no vertices in common, then union might be a preferrable choice.

• join

The join of two graphs is formed by taking nonoverlapping copies of the two graphs and then adding all possible edges between the two copies. To ensure the two copies of the given graphs are on distinct vertex sets, we append a 1 or a 2 to the vertex names (see the description for disjoint_union).

Use join (G, H) to form the join. If the two graphs have the same vertex type T, then the result has vertex type (T, Int). Otherwise, the resulting graph has vertex type Any.

• union

Given graphs G and H, the union has vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$.

• trim

Trimming a graph means to repeatedly remove vertices of a given degree d or less until either all vertices have been removed or the remaining vertices induce a subgraph all of whose vertices have degree greater than d.

Use trim(G, d) to trim the graph, with trim(G) equivalent to trim(G, 0). The latter simply removes all isolated vertices.

Manipulators and inspectors. These are functions that are used to modify a graph and to inspect its structure.

• isequal

Test two graphs to see if they are the same; that is, the graphs must have equal vertex and edge sets. They need not be the same object. While this can be invoked as isequal (G, H) it is more convenient to use G==H.

Note: If the vertex type of either graph cannot be sorted by < then equality testing is slower (unless fast neighborhood lookup is engaged).

• add!

Use this to add vertices or edges to a graph. The syntax add! (G, v) adds a vertex and calling add! (G, v, w) adds an edge.

These return true if the operation succeeded in adding a *new* vertex or edge.

• delete!

Use this to delete vertices or edges from a graph. The syntax delete! (G, v) to delete a vertex (and all edges incident thereon) and delete! (G, v, w) to delete an edge.

Returns true if successful. If the vertex or edge slated for removal was not in the graph, returns false.

• contract!

Mutates a graph by contractin an edge. Calling contract! (G, u, v) adds all vertices in v's neighborhood to u's neighborhood and then deletes vertex v. Typically (u, v) is an edge of the graph but this is not necessary.

This returns true is the operation is successful, but false if either u or v is not a vertex of the graph or if u==v.

• has

Test for the presence of a vertex of edge. Use has (G, v) to test if v is a vertex of the graph and has (G, v, w) to test if the edge is present. Returns true if so and false if not.

Note that G[v, w] is a synonym for has (G, v, w).

• vlist

Use vlist (G) to get the vertex set of the graph as a one-dimensional array. If possible, the vertices are sorted in ascending order.

elist

Use elist (G) to get the edge set of the graph as a one-dimensional array of 2-tuples. If possible, the edges are sorted in ascending lexicographic order.

• neighbors

Use neighbors (G, v) to get the set of neighbors of vertex v as a one-dimensional array. Note that G[v] is a synonym.

dea

Use $\deg(G, v)$ to get the degree of vertex v and $\deg(G)$ to get the degree sequence of G as a one-dimensional array (in decreasing order).

• fastN!

This is explained in §2.

Use this to switch on fastN! (G, true) or to switch off fastN! (G, false) rapid neighborhood lookup. If off, neighborhood lookup can be slow. If on, the data structure supporting the graph is roughly tripled in size.

The difference is especially striking when looking for paths between vertices with find_path.

• NV and NE

Use NV (G) to get the number of vertices and NE (G) to get the number of edges.

• is connected

Use is_connected(G) to determine if the graph is connected.

• num_components

Use num_components (G) to determine the number of connected components in the graph.

components

The function components (G) determines the vertex sets of the connected components of the graph. The return value is a set of sets. That is, if the graph has vertex type T, then this function produces a Partition (requires SimplePartitions).

• find_path

The function $find_path(G, u, v)$ finds a shortest path from u to v if one exists. An empty array is returned if there is no such path. An error is raised if either vertex is absent from the graph.

• dist and dist_matrix

These are used to find distances between vertices in a graph. The distance between vertices u and v is defined to be the number of edges in a shortest (u, v)-path. If there is no such path, one typically says that d(u, v) is undefined of ∞ . However since these functions report distances as Int values, we signal the absence of a (u, v) path by the value -1.

Use dist (G, v, w) to find the distance between the specified vertices in the graph.

Use dist (G, v) to find the distances from vertex v to all vertices in the grpah. This is returned as a Dict.

Use dist (G) to find all distances between all vertices in the graph. For example:

```
julia> G = Cycle(10)
SimpleGraph{Int64} (10 vertices, 10 edges)
julia> d = dist(G);
julia> d[(3,9)]
```

The function dist_matrix creates an $n \times n$ -matrix whose i, j-entry is the distance between the ith and jth vertex of the graph where the order is produced by vlist.

• wiener_index

The function wiener_index computes the sum of the distances between distinct vertices in the graph:

$$\frac{1}{2} \sum_{v \in V} \sum_{w \in V} d(v, w).$$

• diam

Compute the diameter of a graph. Note that diam(G) returns -1 if the graph is not connected.

• eccentricty

The *eccentricity* of a vertex v of a graph G is the maximum distance from v to another vertex of G. eccentricity (G, v) returns this value, but if the graph is not connected it returns -1.

radius

The *radius* of a graph is the minimum eccentricty of a vertex. radius(G) returns -1 if the graph is not connected.

• is_ayclic

Determine if the graph is acyclic.

• is_cut_edge

Determine if a given edge is a cut edge. This can be called either as $is_cut_edge(G, u, v)$ where u and v are vertices or as $is_cut_edge(G, e)$ where e is an edge (i.e., a 2-tuple of vertices.

If (u, v) is not an edge of the graph, an error is raised.

• euler

This is used to find Eulerian trails and tours in a graph. Typical call is euler(G, u, v) to find an Eulerian trail starting at u and ending at v. The first element of that arrary is u and the last is v. If a trail is found, the length of the array is NE(G)+1. Otherwise, an empty array is returned.

The graph may have isolated vertices, and these are ignored.

The call euler(G, u) is shorthand for euler(G, u, u). A simple call to euler(G) will attempt to find an Euler tour from some vertex in the graph.

If the graph is edgeless, then euler(G, u) and euler(G, u, u) return the 1-element array [u]. Calling euler(G) will pick u for you. An empty array is returned if the graph has no vertices. (This is mildly unfortunate as an empty array indicates failure to find a trail for nonempty graphs.)

bipartition and two_color

Used to find a bipartition or two-coloring of a graph if the graph is bipartite; otherwise, return an error. The function bipartition returns a Partition with the two color classes. The function two_color returns a map (Dict) from the vertex set to the set {1, 2}.

```
julia> G = Cycle(6)
SimpleGraph{Int64} (6 vertices, 6 edges)
julia> two_color(G)
[5=>1,4=>2,6=>2,2=>2,3=>1,1=>1]
julia> bipartition(G)
{{2,4,6},{1,3,5}}
```

• girth and girth_cycle

Determine the girth of a simple graph and find a shortest cycle in that graph. If the graph is acyclic, girth returns 0 and girth_cycle returns an empty array.

• greedy_color and random_greedy_color

This is a simple graph coloring function. Given an ordering of the vertices of the graph, greedy_color creates a proper, greedy coloring. If the ordering is not provided, then a degree-decreasing ordering is given. Use greedy_color (G, seq) where seq is a permutation of the vertex set (if you wish

to specify the order) or simply greedy_color (G) in which case a degree-decreasing ordering is used.

The second function performs multiple greedy colorings on random orderings of the vertex set. Use random_greedy_color(G, nreps) where nreps is the number of random orders generated.

In all cases a Dict is returned that maps the vertex set to a range of the form [1:k].

Graph matrices. These functions return standard matrices associated with graphs.

- adjacency
 - Use adjacency (G) to return the adjacency matrix of the graph.
- laplace
 - Use laplace (G) to return the Laplacian matrix of the graph.
- incidence

Use incidence (G) to return the signed incidence matrix of the graph. This is equivalent to incidence (G, true). Calling incidence (G, false) returns the unsigned incidence matrix.

Assignment of +1 and -1 in each column tries to put the +1 on the vertex that sorts lower than the vertex that gets a -1. If the vertices are not comparable by < (less than), the assignment is unpredictable.

Note that incidence returns a sparse matrix. Use full (incidence (G)) if a full-storage matrix is desired.

Converting. This is explained in §5.

• convert_simple

Use convert_simple(G) to create a Julia Graphs.simple_graph version of a graph, together with dictionaries to translate between one vertex set and the other.

4. Errors and gotchas

Errors raised. The functions in the SimpleGraphs module generally do not raise errors. Function such as add! and delete! return false if the graph is not changed by the requested modification.

However, there are some instances in which an error might be raised.

• An error is raised if one attempts to add a vertex of a type that is incompatible with the vertex type of the graph. Here's an example:

```
julia> G = StringGraph()
SimpleGraph{ASCIIString} (0 vertices, 0 edges)
julia> add!(G,4)
ERROR: no method convert(Type{ASCIIString},Int64)
```

• An error is raised if one attempts to find the neighborhood or the degree of a vertex that is not in the graph. Here's an example:

```
julia> G = Complete(5)
SimpleGraph{Int64} (5 vertices, 10 edges)
julia> G[6]
ERROR: Graph does not contain requested vertex
```

• An error is raised if one attempts to create a cycle with fewer than three vertices.

Of course, code can be wrapped in a try-catch block to handle these possibilities gracefully.

Please specify the vertex type. Although G=SimpleGraph() allows Any type vertex, we recommend specifying the type of vertex desired. Here's why.

Because we do not allow loops, the code for add! (G, v, w) first checks if v==w and if so, does not add the edge and returns the value false. So far so good.

Internally, the vertices of the graph are held in a Julia Set container. Now Set either does or does not contain a given object; the object cannot be in the Set twice. Where this gets us into a bit of trouble is that the integer value 1 and the floating point value 1.0 are different objects and therefore may cohabit the same Set. Here's an illustration:

```
julia> A = Set()
Set{Any}()

julia> push!(A,1)
Set{Any}(1)

julia> push!(A,1.0)
Set{Any}(1.0,1)
```

The implication of this is that the vertex set of a SimpleGraph might contain both the integer 1 and the floating point number 1.0. However, we cannot add an edge between these two vertices because the test 1==1.0 returns true.

Here's how this plays out:

```
julia> G = SimpleGraph()
SimpleGraph{Any} (0 vertices, 0 edges)
julia> add!(G,1)
true
julia> has(G,1.0)
false
julia> add!(G,1.0)
true
julia> add!(G,1.0)
```

In principle, we could fix this problem by using a more liberal filter in add! (G, v, w) that allows the addition of an edge with v==w provided typeof (v) and typeof (w) are different. But that would not entirely solve the problem.

Consider this example:

```
julia> G = SimpleGraph()
SimpleGraph{Any} (0 vertices, 0 edges)
julia> add!(G,1)
true
julia> add!(G,BigInt(1))
true
julia> vlist(G)
2-element Array{Any,1}:
1
```

1

It appears that the number 1 is in the vertex set twice!

The preferred solution is to avoid using G=SimpleGraph() and, instead, to use $G=SimpleGraph\{T\}()$ where T is the data type of the vertices. The ready-to-use IntGraph and StringGraph are handy for these popular vertex types.

```
julia> G = IntGraph()
SimpleGraph{Int64} (0 vertices, 0 edges)

julia> add!(G,1)
true

julia> add!(G,BigInt(1))
ERROR: 1 is not a valid key for type Int64

julia> add!(G,1.0)
ERROR: 1.0 is not a valid key for type Int64
```

Unsortable vertex types. Edges in a SimpleGraph are held as a tuple. If the end points of the edge can be compared using the < operator, then the smaller end point comes first in the pair. Otherwise, the order is arbitrary. In the latter case, graph equalty checking is slower.

In general, it's best to specify vertex types for graphs, preferring $SimpleGraph\{T\}$ () for some type T that supports < comparison.

5. Convert to Graphs.jl

The Graphs module defined in Graphs.jl is another tool for dealing with graphs. We provide the function convert_simple to convert a graph from a SimpleGraph to a simple_graph type graph from the Graphs module distributed with Julia.

A simple_graph's vertex set is always of the form 1:n, so the output of convert_simple provides two dictionaries for mapping from the vertex set of the SimpleGraph to the vertex set of the simple_graph, and back again.