Beyond spectral gap:
The role of the topology in decentralized learning

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October 2023

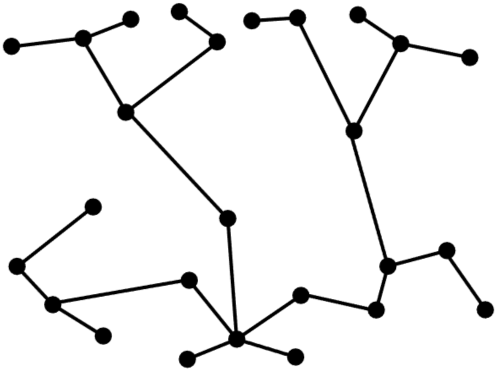
Decentralized learning

The group of nodes is not coordinated by any centralized server. Each node locally holds f_i and exchanges information only with its immediate neighbors.

$$\min_{x_1,\dots,x_m \in \mathbb{R}^d} \sum_{i=1}^{m} f_i(x_i)$$
$$\text{s.t. } x_1 = \dots = x_m.$$

The optimal point in the decentralized sense should be consensual and optimal, i.e.

$$x_1 = \dots = x_m = x^* = \arg\min_{x \in \mathbb{R}^d} \frac{1}{m} \sum_{i=1}^{m} f_i(x).$$



Decentralized Stochastic Gradient Descent (D-SGD)

Algorithm:

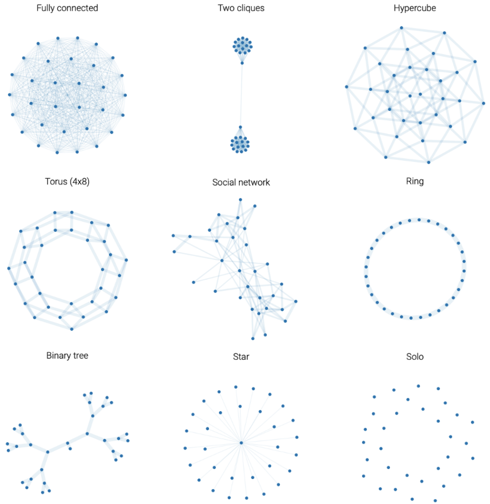
Choose step-size $\alpha > 0$ and pick any $x_i^{(0)} \in \mathbb{R}^n$;

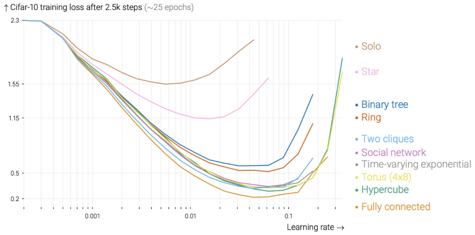
for $k = 0, 1, \ldots$ **do**

$$x_i^{(k+1)} = \sum_{j=1}^n w_{ij} x_j^{(k)} - \alpha \nabla f_i(x_i^{(k)}), \quad i = 1, 2, \ldots, m;$$

end

Article overview





Beyond spectral gap:
The role of the topology in decentralized learning

Thijs Vogels* EPFL

Hadrien Hendrikx* EPFL

Martin Jaggi EPFL

Abstract

In data-parallel optimization of machine learning models, workers collaborate

Spectral Gap vs Effective number of neighbors

The effective number of neighbors measures the ratio of the asymptotic variance of the processes:

$$n_{W}(\gamma) = lim_{t\to\infty} \frac{\sum_{i=1}^{n} Var[\mathbf{y}_i^{(t)}]}{\sum_{i=1}^{n} Var[\mathbf{z}_i^{(t)}]}$$

, where

$$\mathbf{y}^{(t+1)} = \sqrt{\gamma} * y^{(t)} + \xi^{(t)},\ y^{(t)} \in \mathbb{R},\ \xi^{(t)} \sim \mathcal{N}^n(0,1)$$
$$\mathbf{z}^{(t+1)} = W\left(\sqrt{\gamma} * z^{(t)} + \xi^{(t)}\right),\ z^{(t)} \in \mathbb{R},\ \xi^{(t)} \sim \mathcal{N}^n(0,1)$$

We call **y** and **z** **random walks** because workers repeatedly add noise to their state, somewhat like SGD's parameter updates. This should not be confused with a 'random walk' over nodes in the graph.

Paper's results

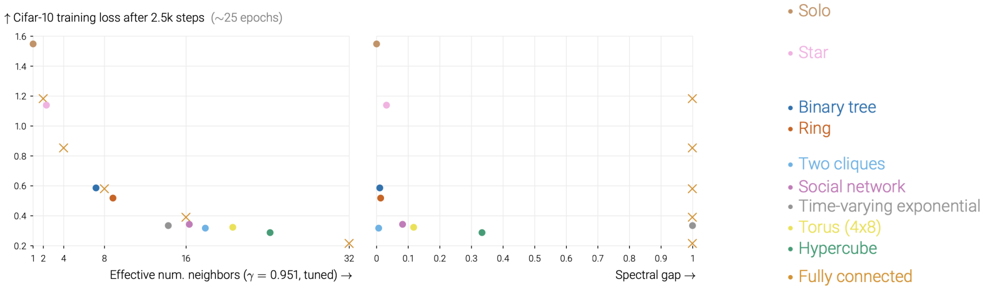


Figure: Cifar-10 training loss after 2.5k steps for all studied topologies with their optimal learning rates.

Our Reproduction of the Experiment

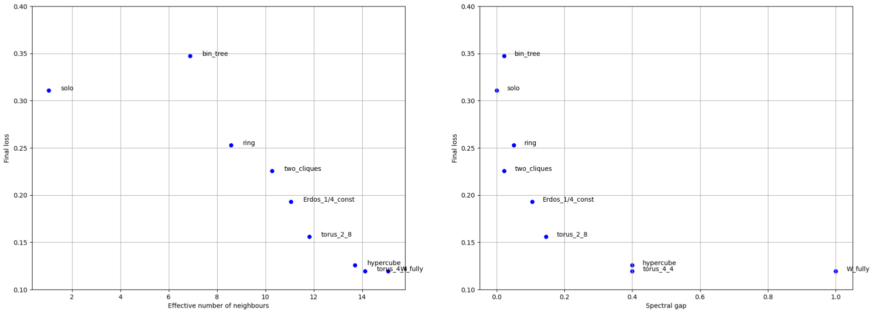


Figure: MNIST training loss after 200 steps for studied **static** topologies with their optimal learning rates.

Time Varying Topologies

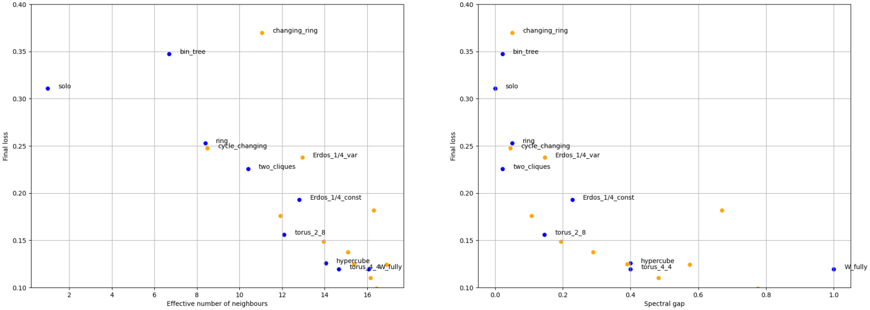


Figure: MNIST training loss after 200 steps for **all** studied topologies with their optimal learning rates.

Regular and Irregular Graphs

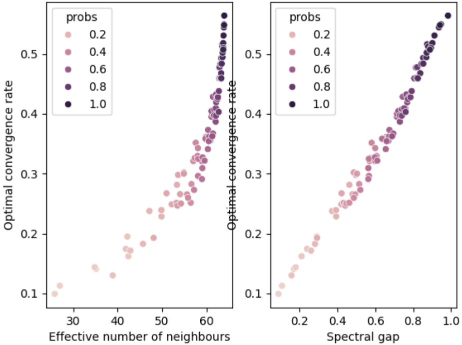


Figure: Optimal convergence rate dependence from ENN and SG for **Erdös-Rényi** random graphs.

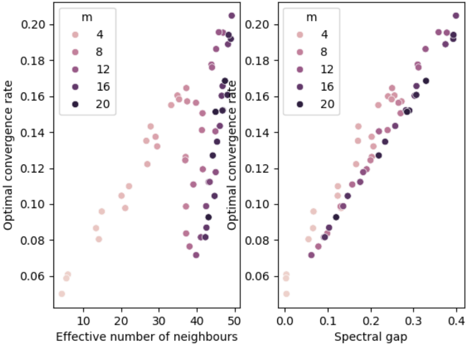
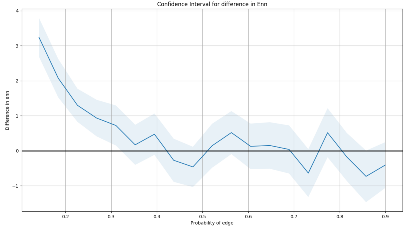
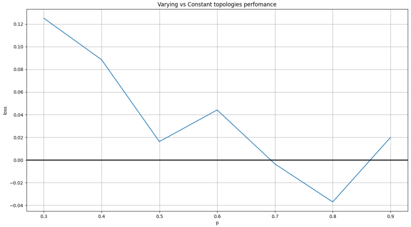


Figure: Optimal convergence rate dependence from ENN and SG for **Barabási-Albert** random graphs.

ENN for random graphs





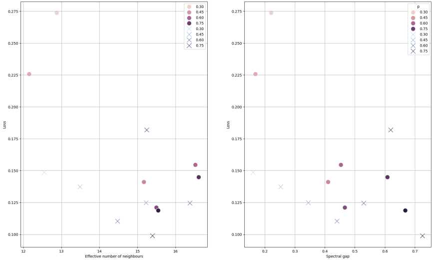


Figure: Time Varying vs Constant random topologies.

Maximization of ENN for a Fixed Graph

G is a fixed graph, W is its weights.

$$\max_W n_W(\gamma) := \frac{\frac{1}{1-\gamma}}{\sum_i \frac{\lambda_i^2}{1-\gamma\lambda_i^2}}$$

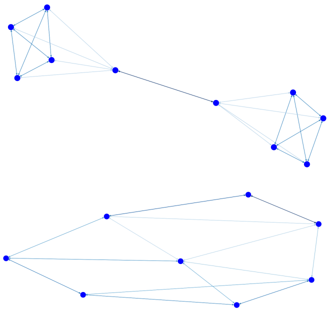
$$\max_W n_W(\gamma) \textit{ is equiv to } \min_W Tr\left(I - \gamma W^2\right)^{-1}$$

$$\text{s.t.}\, W^T = W,\ W\mathbb{1} = \mathbb{1},\ 0 \le W \le G \qquad c$$

$$\min_{W,X,Y} Tr\ X$$

$$\text{s.t.}\, W^T = W,\ W\mathbb{1} = \mathbb{1},\ 0 \le W \le G$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succeq 0, \quad \begin{bmatrix} I - Y & W \\ W & \frac{1}{\gamma} I \end{bmatrix} \succeq 0$$



DIGing Algorithm

Algorithm:

Choose step-size $\alpha > 0$ and pick any $x^{(0)} \in \mathbb{R}^{n \times p}$;

Initialize $y^{(0)} = \nabla f(x^{(0)})$;

for $k = 0, 1, \ldots$ **do**

$$x^{(k+1)} = W^{(k)} x^{(k)} - \alpha y^{(k)};$$

$$y^{(k+1)} = W^{(k)} y^{(k)} + \nabla f(x^{(k+1)}) - \nabla f(x^{(k)});$$

end

DIGing with static topologies

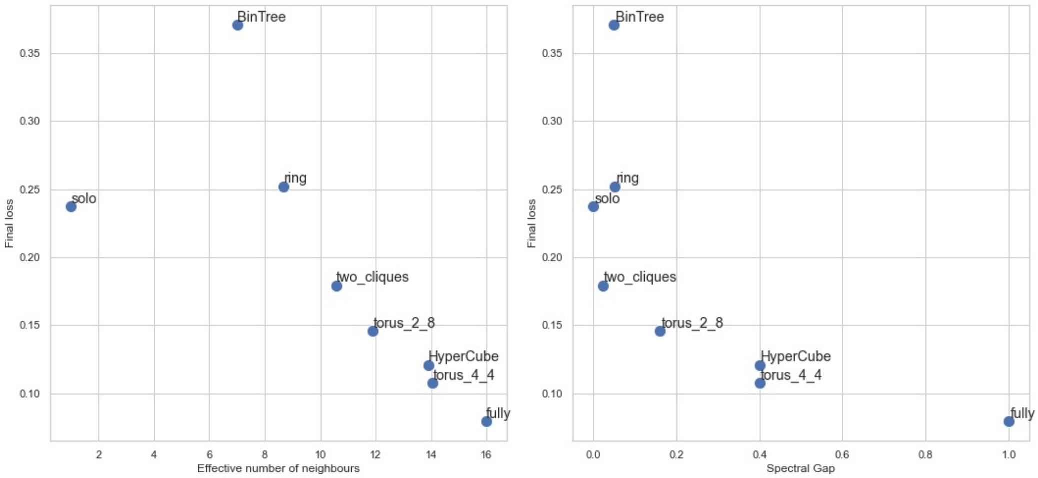


Figure: MNIST training loss after 200 steps for **all** studied topologies with their optimal learning rates.

DIGing with static and varying topologies

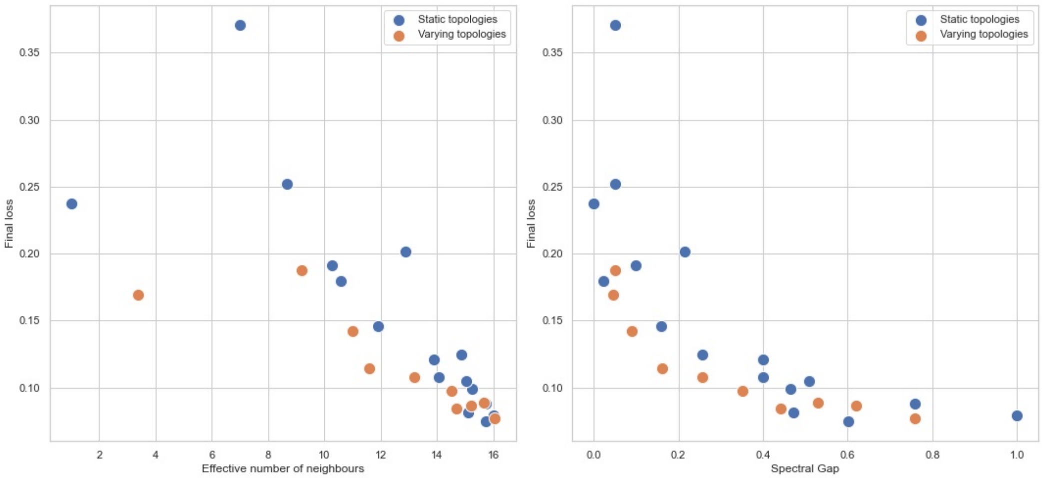


Figure: MNIST training loss after 200 steps for **all** studied topologies with their optimal learning rates.

DIGing with Erdos topologies

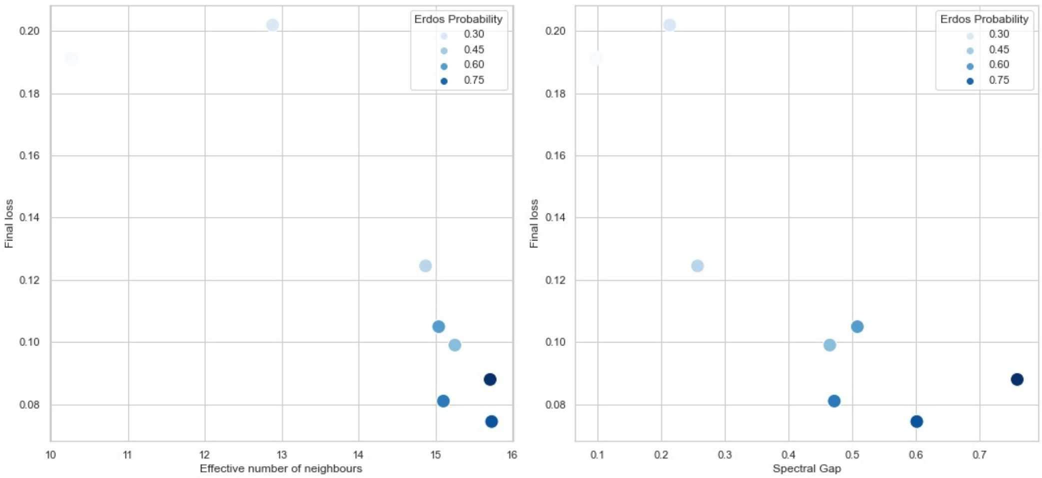


Figure: MNIST training loss after 200 steps for all studied topologies with their optimal learning rates.

DIGing with Varying Erdos topologies

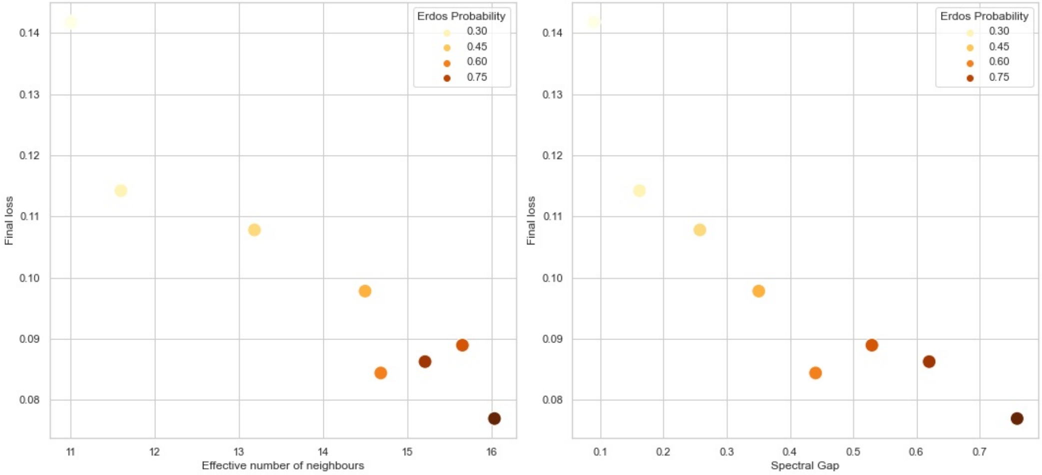


Figure: MNIST training loss after 200 steps for all studied topologies with their optimal learning rates.

Comparison of DSGD and DIGing algorithms

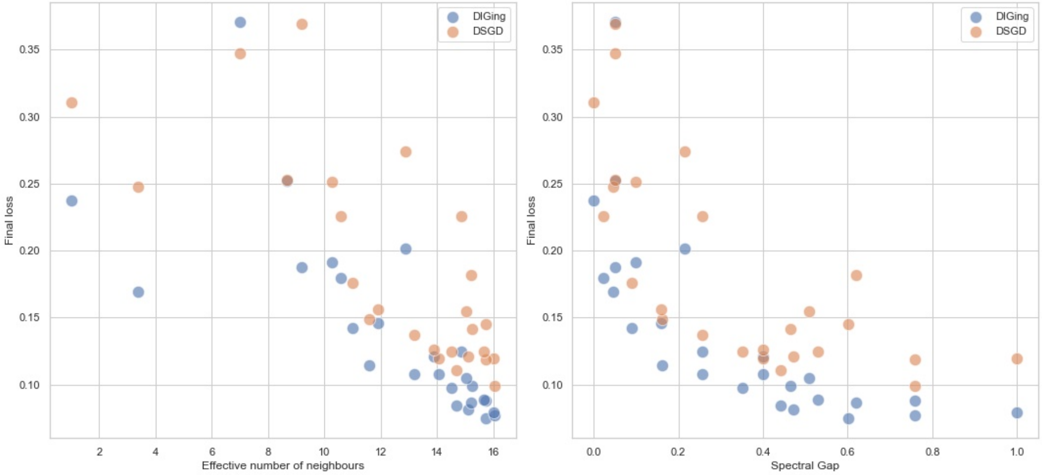


Figure: MNIST training loss after 200 steps for **all** studied topologies with their optimal learning rates.

Thank you for your attention!