

AN INTRODUCTION TO PULSE OXIMETERS: EQUATIONS AND THEORY

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ABSTRACT

Pulse oximeters are electrical devices used for measuring blood oxygen saturation and pulse rate. Since their conception they have gained widespread acceptance in the medical community. This paper primarily focuses on the theory behind pulse oximetry. Beginning with the Beer-Lambert's Law, which relates the intensity of light before and after an absorbing medium, manipulations are made so that this law may be applied to a real time electrical system used for measuring blood oxygen saturation. When light is transmitted through a part of the human body the received signal is a small pulsation atop a baseline dc offset. The manipulations to the Beer-Lambert's law will result in an equation capable of using parameters from this received signal to compute the blood oxygen saturation. When this equation is used along with the method described in this paper, the blood oxygen saturation may be computed to within 3% accuracy.

INTRODUCTION TO PULSE OXIMETERS

A pulse oximeter is a noninvasive medical device used for measuring the oxygen saturation and pulse rate of a patient. Oxygenated hemoglobin and reduced hemoglobin, being hemoglobin not fully saturated with oxygen, absorb different wavelengths of light differently. A pulse oximeter thus utilizes two signals: an infrared (IR) signal and a near infrared red signal. A pulse oximeter also utilizes pulsations of arteries to determine heart rate and blood oxygen concentration.

Generally, a pulse oximeter consists of two light sources and a photodetector connected to a signal processing unit. The signal received by the photodetector can be broken down into signals: a Direct Current (dc) signal arising from the amount of absorbed IR through the point of measurement, and an Alternating Current (ac) signal arising from the change in absorbed IR due to the pulsation of arteries. The ratio of these two signals may be computed via signal processing. This ratio, measured at red and IR wavelengths can be used to calculate the oxygen saturation [1]. This process will be discussed in greater detail in later sections.

Pulse oximeters have gained widespread acceptance in the medical community for several reasons. Since the first production of pulse oximeters in the early 1980s, pulse oximeters have been recognized and praised for their low cost and simplistic operation [1]. Most pulse oximeters require a relatively small amount of hardware. Even with such simplicity, pulse oximeters measure not one, but two very important patient parameters: blood oxygen concentration and heart rate. Pulse oximeters measure such parameters to a high degree of accuracy. Generally, for blood oxygen concentrations of greater than 70%, pulse oximeters are accurate to within 3%. This is more than adequate for determining hypoxemia. The information provided by pulse oximeters is not only accurate, but immediate and continuous [2, pg. 13]. This is a critical advantage for monitoring patients during surgery and in the emergency room.

The most praised advantage of pulse oximeters is that they are noninvasive. Most pulse oximeters are simple clips that may be attached to a patient's ear lobe. Other probes include nasal and finger probes [1]. These probes are reusable as well. The entire pulse oximeter unit is generally portable [3]. Pulse oximeters are also easy to repair [1].

MOTIVATION: WHY PULSE OXIMETERS ARE IMPORTANT

Oxygenation of the blood is of great importance. Many tissues and organs in the human body will become irreversibly damaged when not properly supplied with oxygen. It is important to monitor blood oxygenation accurately in real-time because organs such as the heart, liver and kidneys have only minutes before becoming permanently damaged due to lack of oxygen. For even more vital organs such as the cerebral cortex, the window of time is less than a minute [2, pg. 14].

A low blood oxygen concentration is a sign of many bodily ailments, specifically respiratory problems. A low oxygen concentration can be from poor lung compliance due to pneumonia, ADRS, fibrosis, or emphysema. It can be a sign of airway resistance or obstruction as a result of asthma, chronic bronchitis, cystic fibrosis, choking, secretions from intubation, or obstructive sleep apnea. A low blood oxygen concentration can even be used to diagnose lead poisoning and congenital heart disease [2, pg. 15]

Pulse oximeters have gained widespread acceptance as the noninvasive method of measuring blood oxygen concentration [4]. The oxygenation of blood in anesthesiology is important since a patient under anesthesia is unable to breathe on his or her own. It is the anesthesiologist's job to breathe and supply oxygen to the patient. It is very important to determine whether or not the patient is absorbing the necessary amount of oxygen from the ventilator. The reading that the pulse oximeter provides is a key feedback indicator to the anesthesiologist that he or she is successfully respirating the patient. Other variables such as blood pressure and the ECG will yield warnings in a delayed manner, whereas the blood oxygen concentration can be monitored continuously and noninvasively using a pulse oximeter. Because of these factors, pulse oximeters have become the standard in anesthesiology [2, pg. 16-8].

After surgery the effects of the anesthetic drugs do not wear off immediately. This may affect the patient's ability to breathe properly and pulse oximeters are very useful in monitoring a patient during recovery [2, pg. 18].

THEORY BEHIND PULSE OXIMETRY

THE BEER-LAMBERT'S LAW

If a absorbing medium is uniform, light traveling through it will be attenuated at a predictable rate. The intensity of light (I), starting at an intensity of (I_0), can be given as:

$$(1) \quad I = I_0 e^{-\varepsilon(\lambda)cd}$$

where $\varepsilon(\lambda)$ is the absorptivity of the medium (which is a function of the light's wavelength), c is the concentration of the medium, and d is the length of the optical path.

Equation (1) may be manipulated to provide a factor of transmittance (T) as the ratio of transmitted light (I) to the starting intensity (I_0):

$$(2) \quad T = \frac{I}{I_0} = e^{-\varepsilon(\lambda)cd}$$

Naturally transmittance is directly related to absorbance. Equation (2) may be further manipulated by taking the natural log to give what is called the unscattered absorbance (A):

$$(3) \quad A = -\ln(T) = \varepsilon(\lambda)cd$$

For multiple absorbing media, the unscattered absorbance for each of the total n media may be added to give the total absorbance (ΣA):

$$(4) \quad \Sigma A = \sum_{i=1}^n \varepsilon_i(\lambda)c_i d_i$$

Equation (4) is important in that if the total absorbance is measured for n different wavelengths and the absorptivity and optical length of each of the n media are known, a set of n equations can be formed that is solvable for a set of concentrations c_i :

$$(5) \quad \begin{aligned} \varepsilon_1(\lambda_1)c_1 d_1 + \varepsilon_2(\lambda_1)c_2 d_2 + \cdots + \varepsilon_n(\lambda_1)c_n d_n &= \Sigma A(\lambda_1) \\ \varepsilon_1(\lambda_2)c_1 d_1 + \varepsilon_2(\lambda_2)c_2 d_2 + \cdots + \varepsilon_n(\lambda_2)c_n d_n &= \Sigma A(\lambda_2) \\ \vdots \\ \varepsilon_1(\lambda_n)c_1 d_1 + \varepsilon_2(\lambda_n)c_2 d_2 + \cdots + \varepsilon_n(\lambda_n)c_n d_n &= \Sigma A(\lambda_n) \end{aligned}$$

This set of equations may be put into a matrix equation solvable for the concentration vector c :

$$(6) \quad B\bar{c} = \bar{a} \Leftrightarrow \begin{pmatrix} \varepsilon_1(\lambda_1)d_1 & \varepsilon_2(\lambda_1)d_2 & \cdots & \varepsilon_n(\lambda_1)d_n \\ \varepsilon_1(\lambda_2)d_1 & \varepsilon_2(\lambda_2)d_2 & \cdots & \varepsilon_n(\lambda_2)d_n \\ \vdots & \vdots & & \vdots \\ \varepsilon_1(\lambda_n)d_1 & \varepsilon_2(\lambda_n)d_2 & \cdots & \varepsilon_n(\lambda_n)d_n \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} \Sigma A(\lambda_1) \\ \Sigma A(\lambda_2) \\ \vdots \\ \Sigma A(\lambda_n) \end{pmatrix}$$

In other words, the concentrations may be solved for knowing all other parameters [2, pg. 40-2]

PROBLEMS IN USING BEER-LAMBERT'S LAW

For the system of equations represented in Equation (6) to be useful in pulse oximetry, a number of things must be assumed. In order for the absorbance to be linearly related to the concentration, optical distance and absorptivity as in the derivation above, the medium must be homogeneous. This is not a good assumption however, as blood is not homogeneous.

Recalling Equations (2) and (3), the absorption is related to transmittance and transmittance is related to the original intensity (I_0):

$$(7) \quad A = -\ln(T) = -\ln\left(\frac{I}{I_0}\right)$$

Thus, the absorbances in the system of equations can only be found by measuring received intensities and comparing them to the original intensities of transmitted light. There are several problems here. First, the intensity of the original incident light is very difficult to determine accurately. LEDs from the factory do not all emit the same intensity. Further, LEDs are not monochromatic; their intensity is spread over a spectrum of wavelengths. The need to know the value of I_0 is eliminated by taking advantage of the pulsating properties of arteries. This will be discussed in greater detail.

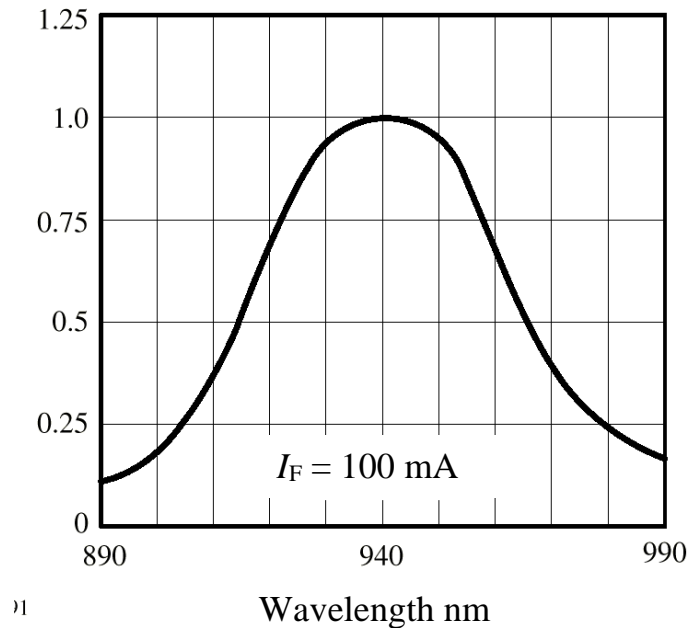


Figure 1 Emittance of Example LED

The Beer-Lambert's Law does not take into account reflection and scattering of light. The difference between the measured intensity I and the original intensity I_0 is not due to just the absorptive property of the media; some intensity is lost due to reflection and scattering. It turns out however that the errors due to this reflection and scattering are acceptable [2, pg. 51-3].

MANIPULATING BEER-LAMBERT'S LAW FOR USE IN PULSE OXIMETRY

As mentioned above, the original intensity of the light is very difficult to determine accurately. By taking advantage of the pulsating properties of arteries, I_0 can become irrelevant. Because arteries pulsate, the physical diameters of the arteries increase and decrease periodically. This can be seen as a change in the optical path distance d in the Beer-Lambert's Law [Equation (1)]. Writing Equation (1) at a pulsation's minimum provides a baseline intensity (I_B):

$$(8) \quad I_B = I_0 \exp(-\varepsilon_B(\lambda)c_B d)$$

In Equation (8), the $\varepsilon(\lambda)$ and c terms are not as simple as they seem here. The human body is not composed of one concentration at one absorptivity. To simplify for now, the $\varepsilon(\lambda)$ term and the c term can be lumped together in one term as a function of wavelength. The absorbance (α) is defined as:

$$(9) \quad \alpha(\lambda) = \varepsilon(\lambda)c$$

So, the baseline in Equation (8) can be rewritten as:

$$(10) \quad I_B = I_0 \exp(-\alpha_B(\lambda)d)$$

At a pulsation's maximum, Equation (1) can be written as a variation of the baseline set in Equation (10):

$$(11) \quad I_P = I_B \exp(-\alpha_P(\lambda)\Delta d) = I_0 \exp(-\alpha_B(\lambda)d - \alpha_P(\lambda)\Delta d)$$

A change in transmittance (ΔT) can be defined by taking the ratio of the pulse intensity over the baseline intensity:

$$(12) \quad \Delta T = \frac{I_P}{I_B} = \frac{I_0 \exp(-\alpha_B(\lambda)d - \alpha_P(\lambda)\Delta d)}{I_0 \exp(-\alpha_B(\lambda)d)} = e^{-\alpha_P(\lambda)\Delta d}$$

Note that Equation (12) in the end is not dependent on I_0 [2, pg. 126-7]. Equation (12) is however a function of Δd which is impossible to measure. The following steps will eliminate the need to know Δd . To dispose of the exponential term, just as in Equation (3) to get the unscattered absorbance, the natural log is taken for both sides of Equation (12):

$$(13) \quad -\ln(\Delta T) = -\ln(e^{-\alpha_P(\lambda)\Delta d}) = \alpha_P(\lambda)\Delta d$$

Equation (13) may be defined as the change in unscattered absorbance (ΔA):

$$(14) \quad \Delta A = \alpha_P(\lambda)\Delta d$$

If the change in unscattered absorbance is measured at two different wavelengths, the term Δd may be dropped. In implementing a pulse oximeter one red and one infrared (IR) wavelengths are selected (λ_R, λ_{IR}):

$$(15) \quad \begin{aligned} \Delta A(\lambda_R) &= \alpha_P(\lambda_R)\Delta d \\ \Delta A(\lambda_{IR}) &= \alpha_P(\lambda_{IR})\Delta d \end{aligned}$$

Assuming that the two wavelengths are supplied at the same distance from the photodetector, the Δd s are the same in Equation (15). Dividing the two equations in Equation (15), Δd may be eliminated:

$$(16) \quad \frac{\Delta A(\lambda_R)}{\Delta A(\lambda_{IR})} = \frac{\alpha_P(\lambda_R)\Delta d}{\alpha_P(\lambda_{IR})\Delta d} = \frac{\alpha_P(\lambda_R)}{\alpha_P(\lambda_{IR})}$$

Equation (16) is defined as the Ratio of Ratios (R_{os}):

$$(17) \quad R_{os} = \frac{\Delta A(\lambda_R)}{\Delta A(\lambda_{IR})} = \frac{\alpha_P(\lambda_R)}{\alpha_P(\lambda_{IR})}$$

This ratio is used in calculating the oxygen saturation level of the patient's blood. Before going into that, it is important to understand what is physically being measured in Equation (17).

In implementing a pulse oximeter, two diodes emit red and infrared signals that are transmitted through a spot in the body and received in their attenuated form. A photodetector picks up two signals, one for the red and the other for the IR. The following is an exaggerated example of what is received by the photodetector.

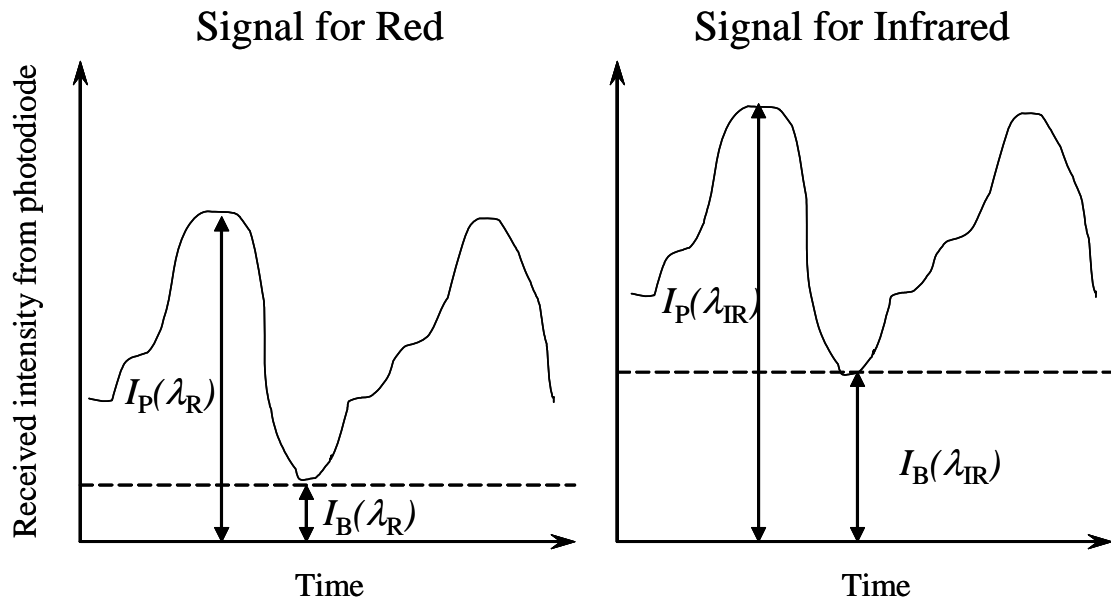


Figure 2 - Received Photodetector Signals

Figure 2 shows that four parameters may be taken from the received signals. Recalling Equation (12), a change in transmittance can be calculated at each of the two wavelengths:

$$(18) \quad \Delta T(\lambda_R) = \frac{I_P(\lambda_R)}{I_B(\lambda_R)} \quad \text{and} \quad \Delta T(\lambda_{IR}) = \frac{I_P(\lambda_{IR})}{I_B(\lambda_{IR})}$$

Recalling Equations (13) and (14), a change in unscattered absorbance can be calculated by taking the natural log for the change in transmittance at each wavelength:

$$(19) \quad \begin{aligned} \Delta A(\lambda_R) &= \ln(\Delta T(\lambda_R)) = \ln\left(\frac{I_P(\lambda_R)}{I_B(\lambda_R)}\right) \\ \Delta A(\lambda_{IR}) &= \ln(\Delta T(\lambda_{IR})) = \ln\left(\frac{I_P(\lambda_{IR})}{I_B(\lambda_{IR})}\right) \end{aligned}$$

Then, from Equation (17), the Ratio of Ratios can be given in terms of the four parameters extracted by the signals provided by the photodetector [2, pg. 126-130]:

$$(20) \quad R_{OS} = \frac{\Delta A(\lambda_R)}{\Delta A(\lambda_{IR})} = \frac{\ln\left(\frac{I_P(\lambda_R)}{I_B(\lambda_R)}\right)}{\ln\left(\frac{I_P(\lambda_{IR})}{I_B(\lambda_{IR})}\right)}$$

USING THE RATIO OF RATIOS TO DETERMINE OXYGEN SATURATION

It is first necessary to describe the oxygen saturation in terms of parameters that have been previously mentioned. For use in pulse oximetry artery blood is modeled as being composed of just two elements: oxyhemoglobin (HbO₂) and reduced hemoglobin (Hb). Oxyhemoglobin is hemoglobin that is fully saturated with oxygen and reduced hemoglobin is not fully saturated with oxygen. The arterial functional oxygen saturation (S_aO₂) is defined as the percentage of oxygenated hemoglobin (HbO₂) divided by the rest of the hemoglobin in the model (HbO₂ + Hb):

$$(21) \quad S_a O_2 = 100\% \times \frac{HbO_2}{HbO_2 + Hb}$$

The arterial functional oxygen saturation can also be defined in terms of concentrations:

$$(22) \quad S_a O_2 = 100\% \times \frac{c_{HbO_2}}{c_{HbO_2} + c_{Hb}} = \frac{c_{HbO_2}}{c_{HbO_2} + c_{Hb}}$$

Going back to the Beer-Lambert's Law for multiple media in Equation (4), the total absorbance can be written here for two media, oxyhemoglobin and reduced hemoglobin:

$$(23) \quad \Sigma A = \sum_{i=1}^n \varepsilon_i(\lambda) c_i d_i = \varepsilon_{\text{HbO}_2}(\lambda) c_{\text{HbO}_2} d + \varepsilon_{\text{Hb}}(\lambda) c_{\text{Hb}} d$$

Note that the optical path is assumed to be the same for each medium. The concentrations for HbO₂ and Hb can be related to the functional oxygen saturation in Equation (22):

$$(23) \quad \begin{aligned} c_{\text{HbO}_2} &= S_a \text{O}_2 (c_{\text{HbO}_2} + c_{\text{Hb}}) \\ c_{\text{Hb}} &= (1 - S_a \text{O}_2) (c_{\text{HbO}_2} + c_{\text{Hb}}) \end{aligned}$$

Combining the information in Equations (22) and (23) the total absorbance in terms of concentrations and functional oxygen saturation [2, pg. 45]:

$$(24) \quad \Sigma A = [\varepsilon_{\text{HbO}_2}(\lambda) S_a \text{O}_2 + \varepsilon_{\text{Hb}}(\lambda) (1 - S_a \text{O}_2)] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d$$

Recall that in pulse oximetry, Equation (24) will be evaluated at a red and infrared wavelength:

$$(25) \quad \begin{aligned} \Sigma A(\lambda_R) &= [\varepsilon_{\text{HbO}_2}(\lambda_R) S_a \text{O}_2 + \varepsilon_{\text{Hb}}(\lambda_R) (1 - S_a \text{O}_2)] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d \\ \Sigma A(\lambda_{\text{IR}}) &= [\varepsilon_{\text{HbO}_2}(\lambda_{\text{IR}}) S_a \text{O}_2 + \varepsilon_{\text{Hb}}(\lambda_{\text{IR}}) (1 - S_a \text{O}_2)] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d \end{aligned}$$

The Equations in (25) can be put into the form:

$$(26) \quad \begin{aligned} \Sigma A(\lambda_R) &= [\varepsilon_{\text{Hb}}(\lambda_R) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_R) - \varepsilon_{\text{Hb}}(\lambda_R))] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d \\ \Sigma A(\lambda_{\text{IR}}) &= [\varepsilon_{\text{Hb}}(\lambda_{\text{IR}}) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_{\text{IR}}) - \varepsilon_{\text{Hb}}(\lambda_{\text{IR}}))] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d \end{aligned}$$

Just as in computing the Ratio of Ratios in Equation (20), the ratio of absorbance at the red wavelength over the absorbance at the IR wavelength is taken:

$$(27) \quad \begin{aligned} R_{\text{OS}} &= \frac{\Sigma A(\lambda_R)}{\Sigma A(\lambda_{\text{IR}})} = \frac{[\varepsilon_{\text{Hb}}(\lambda_R) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_R) - \varepsilon_{\text{Hb}}(\lambda_R))] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d}{[\varepsilon_{\text{Hb}}(\lambda_{\text{IR}}) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_{\text{IR}}) - \varepsilon_{\text{Hb}}(\lambda_{\text{IR}}))] \times (c_{\text{HbO}_2} + c_{\text{Hb}}) d} \\ R_{\text{OS}} &= \frac{\varepsilon_{\text{Hb}}(\lambda_R) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_R) - \varepsilon_{\text{Hb}}(\lambda_R))}{\varepsilon_{\text{Hb}}(\lambda_{\text{IR}}) + S_a \text{O}_2 (\varepsilon_{\text{HbO}_2}(\lambda_{\text{IR}}) - \varepsilon_{\text{Hb}}(\lambda_{\text{IR}}))} \end{aligned}$$

So, just as it was achieved in the previous section, here is a relation that is independent of original incident intensity and optical distance. Since the functional oxygen saturation is what needs to be found, Equation (27) is simplified to the form where functional oxygen saturation is a function of the ratio of ratios [2, pg. 50]:

$$(28) \quad S_a \text{O}_2 = \frac{\varepsilon_{\text{Hb}}(\lambda_R) - \varepsilon_{\text{Hb}}(\lambda_{\text{IR}}) R_{\text{OS}}}{\varepsilon_{\text{Hb}}(\lambda_R) - \varepsilon_{\text{HbO}_2}(\lambda_R) + [\varepsilon_{\text{HbO}_2}(\lambda_{\text{IR}}) - \varepsilon_{\text{Hb}}(\lambda_{\text{IR}})] R_{\text{OS}}}$$

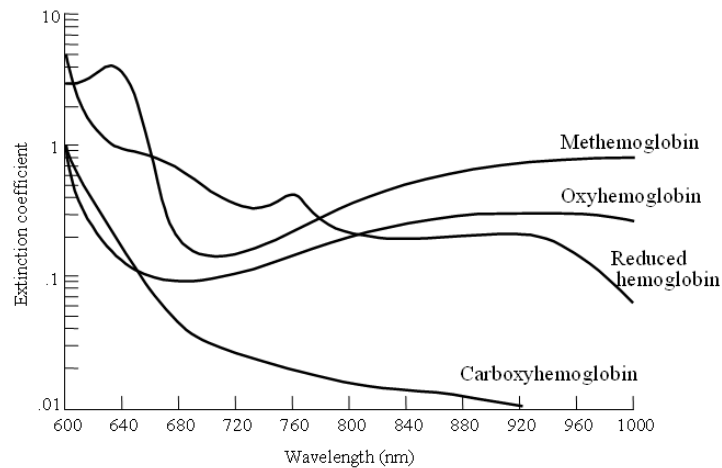


Figure 3 Absorptivity (Extinction) Vs Wavelength [5]

CONCLUSION

An example of a pulse oximeter system is detailed in Figure 4.

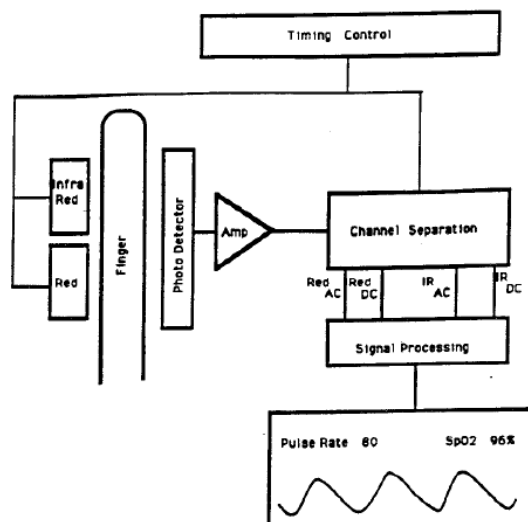


Figure 4 A Sample Pulse Oximeter System [1]

Figure 4 shows two wavelengths of light shining through a body part, namely a finger, and a photodetector measures the attenuated signals. The two wavelengths of light are chosen to be 990 nm for red light and 660 nm for infrared light. These two wavelengths are chosen because the absorptivity of oxyhemoglobin and reduced hemoglobin differ. Figure 3 shows that the absorptivity of oxyhemoglobin is greater than that of reduced hemoglobin at 990 nm and vice versa at 660 nm [2, pg. 45].

The photodetector's signal is amplified and divided into the necessary parameters to compute the ratio of ratios in Equation (20). That value is used in Equation (28) along with the absorptivity values taken from the plot in Figure 3 to compute the functional oxygen saturation.

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