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École Centrale de Lille
EBE – Electronics for biomedical engineering

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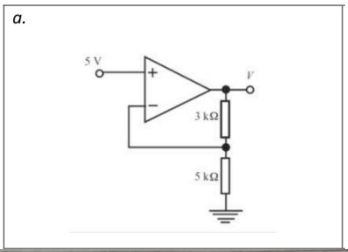
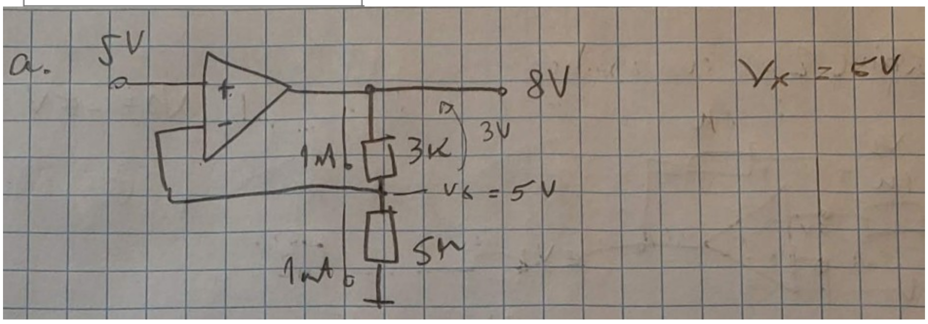
Operational amplifiers (Opamps)

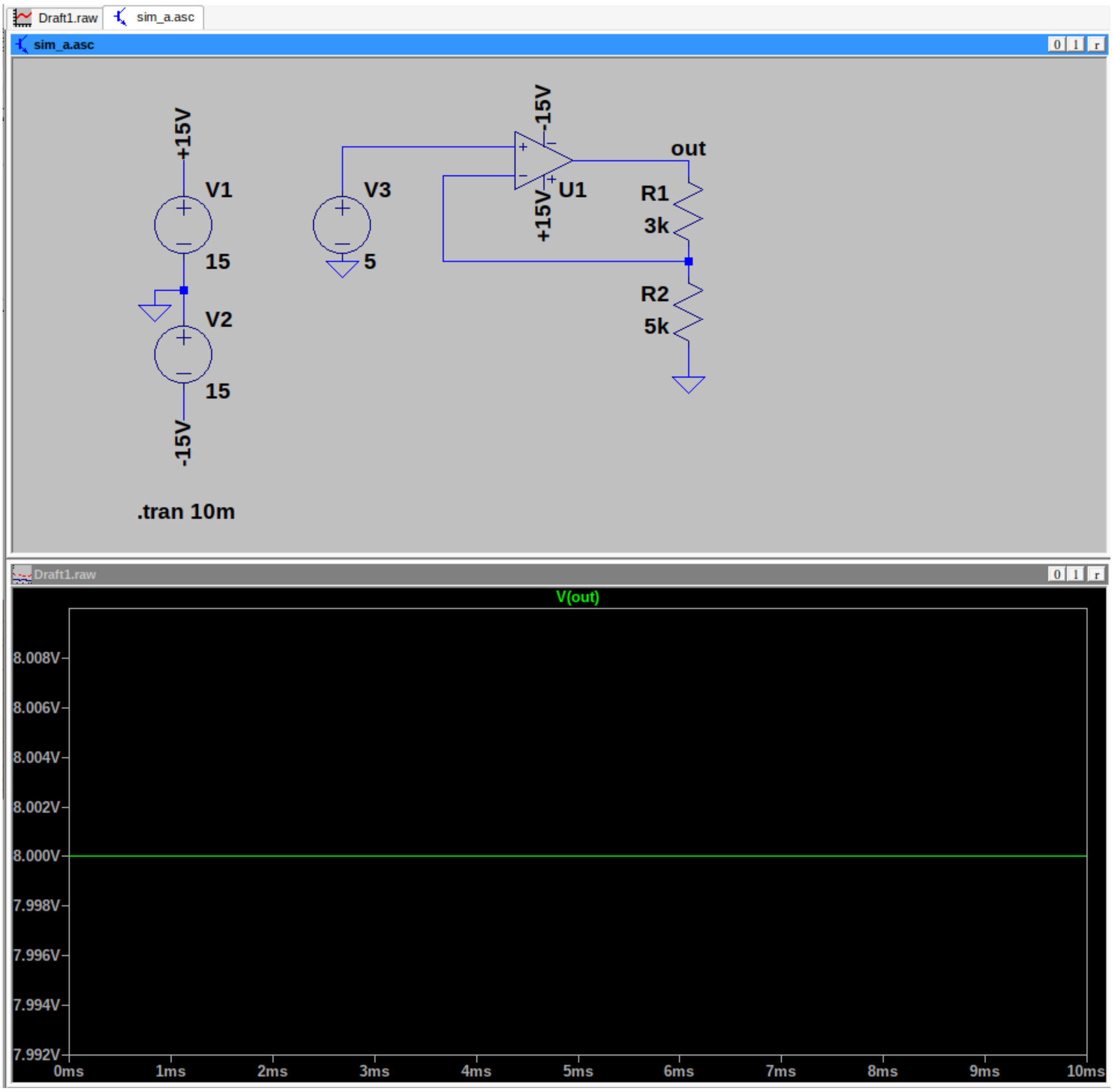
Part 1 : Theory

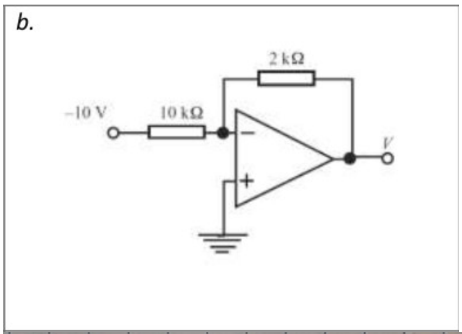
Exercise 1 : In the circuits of figure 1, calculate the value of the voltage V. Use LTSpice to check your results.


a.
5 V
V
3 kΩ
5 kΩ

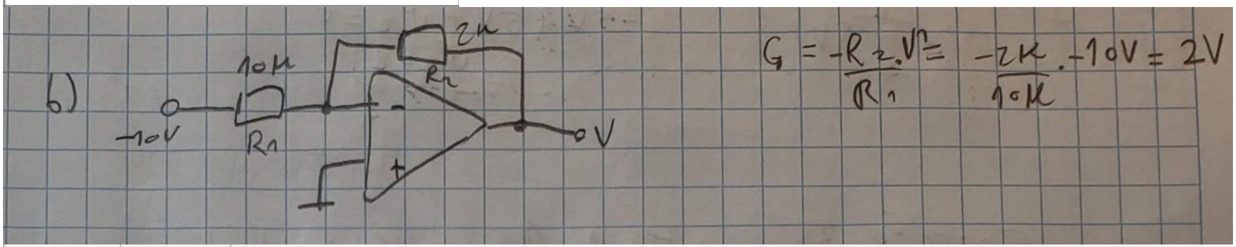
V=8V

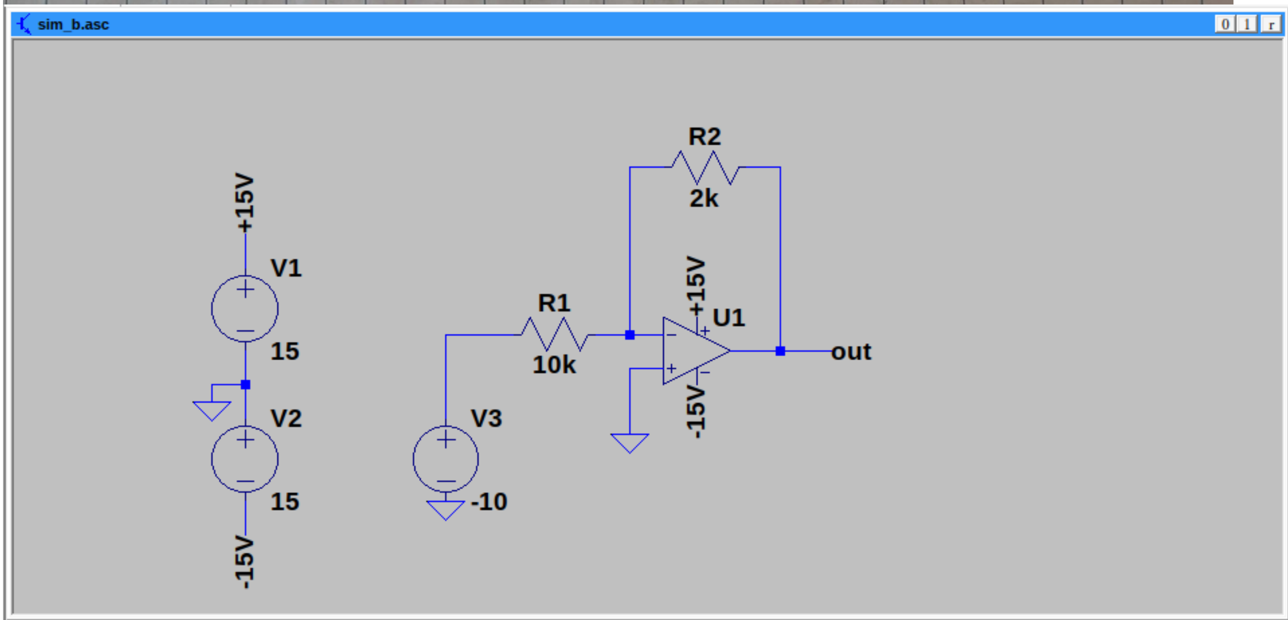

a. 5V
8V
Vx = 5V
3V
1mA
3K
Vx = 5V
1mA
5k

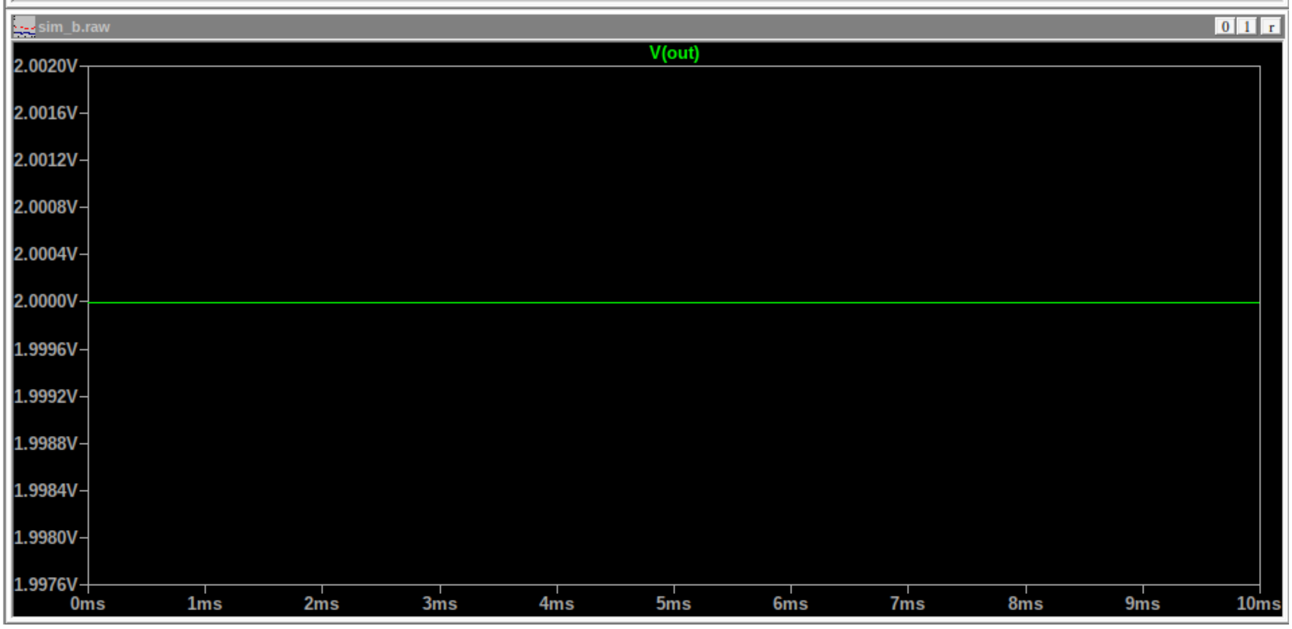

Draft1.raw
sim_a.asc
+15V
V1
15
V2
15
-15V
V3
5
-15V
+15V
U1
out
R1
3k
R2
5k
.tran 10m
V(out)
8.008V
8.006V
8.004V
8.002V
8.000V
7.998V
7.996V
7.994V
7.992V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms

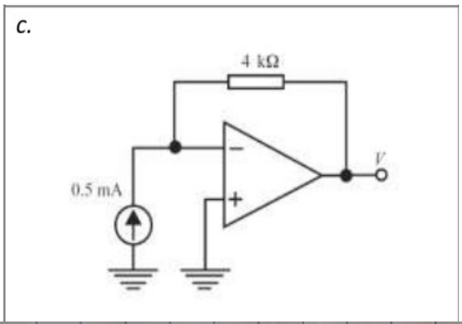

b.
2 kΩ
−10 V
10 kΩ
V

V=2V

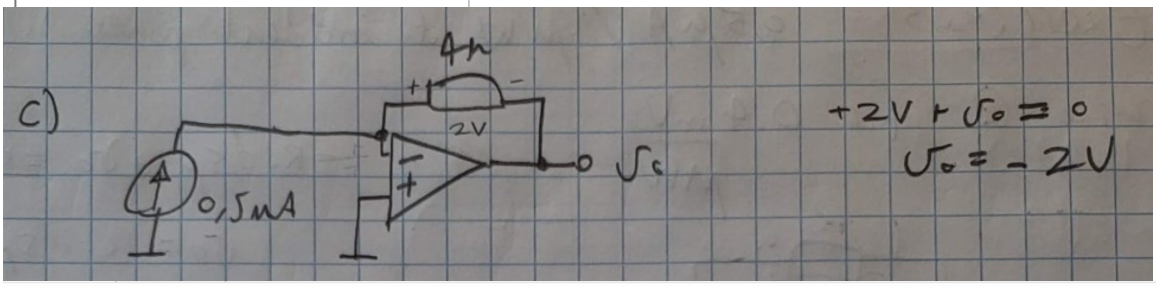

b)
10k
2k
R2
−10V
R1
V
G = -R2·Vi/R1 = -2k/10k · -10V = 2V

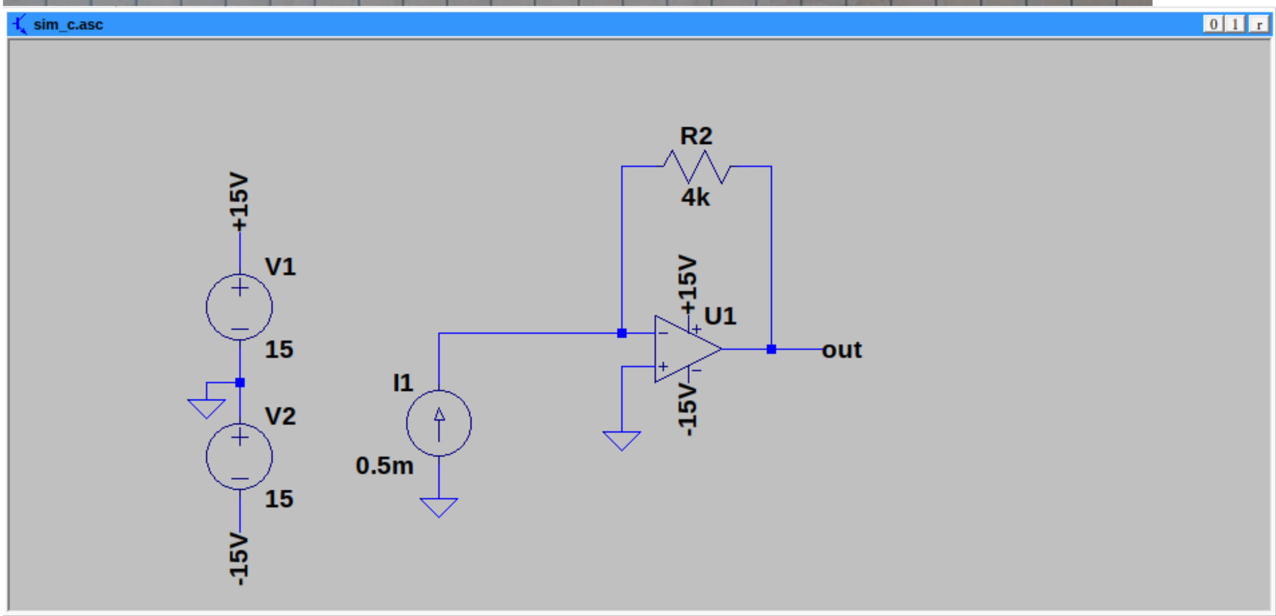

sim_b.asc
+15V
V1
15
V2
15
-15V
R1
10k
V3
-10
R2
2k
+15V
U1
-15V
out

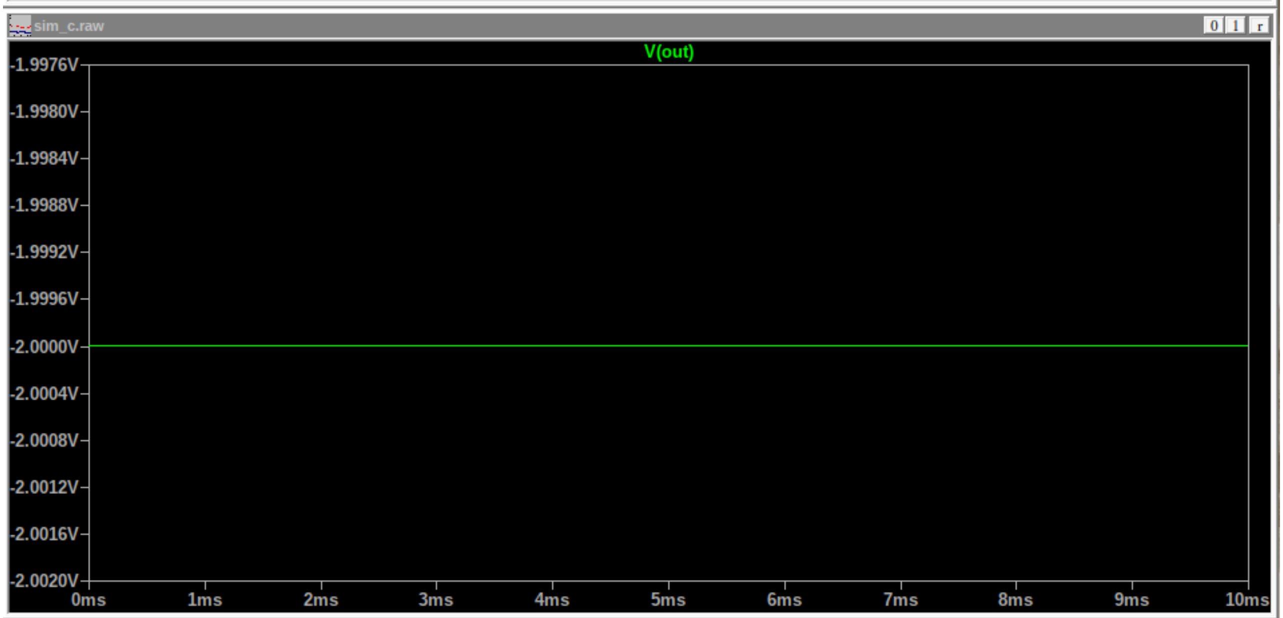

sim_b.raw
V(out)
2.0020V
2.0016V
2.0012V
2.0008V
2.0004V
2.0000V
1.9996V
1.9992V
1.9988V
1.9984V
1.9980V
1.9976V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms

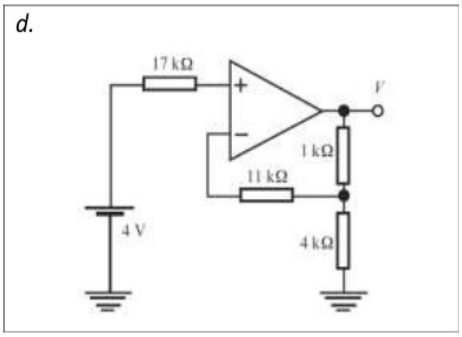

c.
4 kΩ
0.5 mA
V

V = -2V

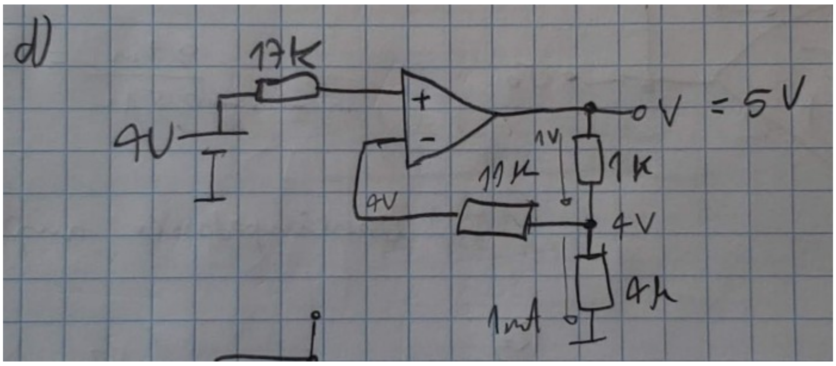

c)
4k
2V
0,5mA
v_o
$+2V + v_o = 0$
$v_o = -2V$

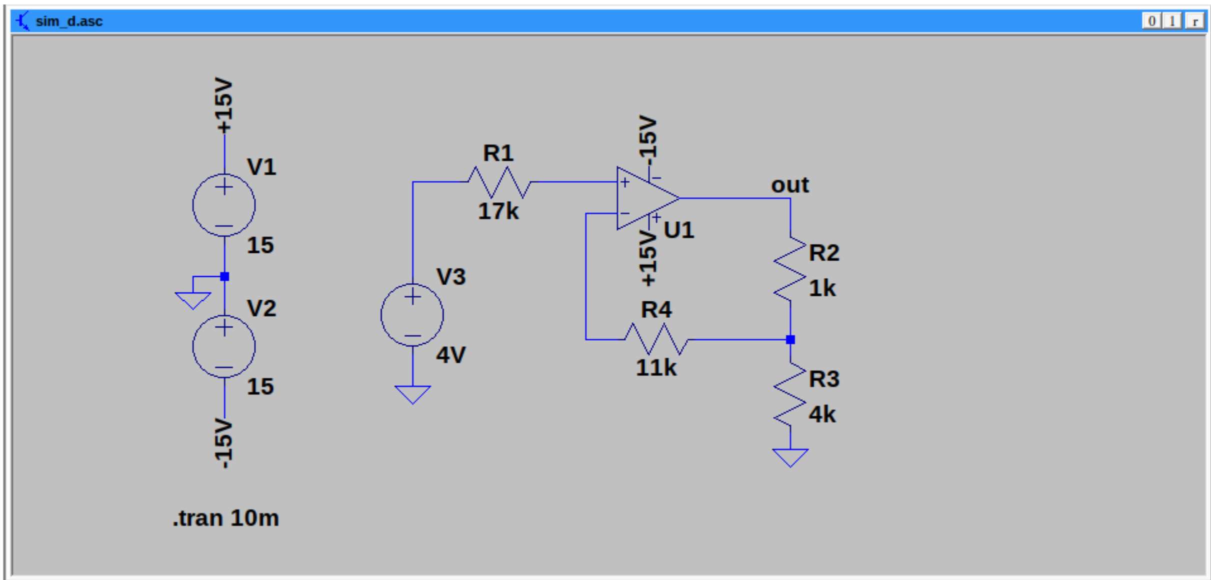

sim_c.asc
R2
4k
+15V
V1
15
V2
15
-15V
I1
0.5m
+15V
U1
-15V
out

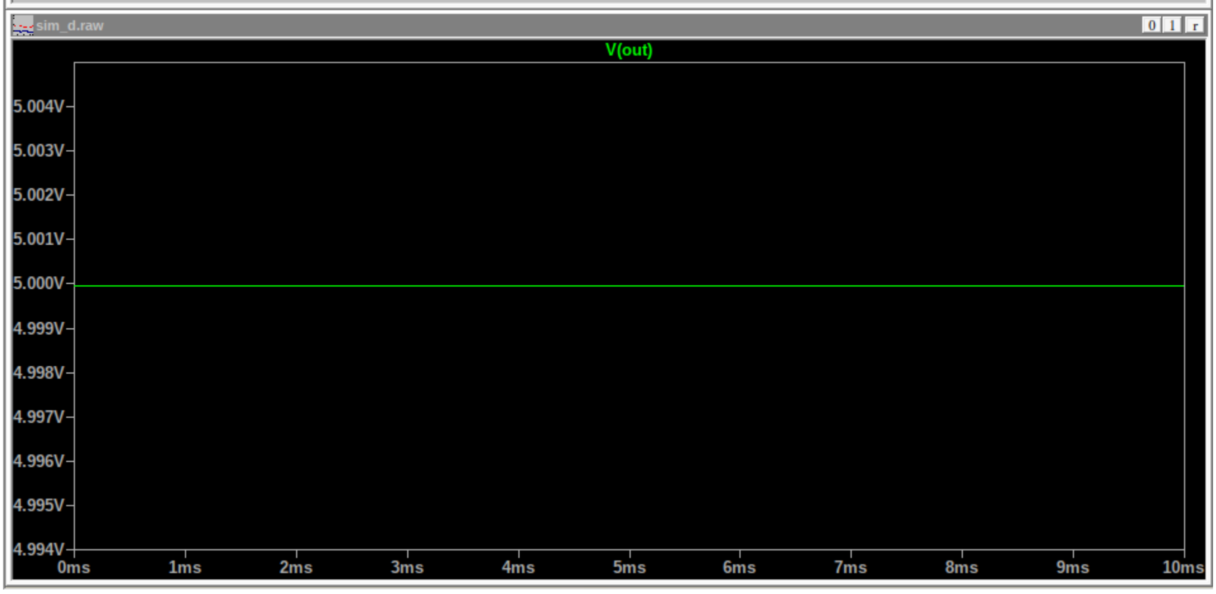

sim_c.raw
V(out)
-1.9976V
-1.9980V
-1.9984V
-1.9988V
-1.9992V
-1.9996V
-2.0000V
-2.0004V
-2.0008V
-2.0012V
-2.0016V
-2.0020V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms


d.
17 kΩ
V
1 kΩ
11 kΩ
4 V
4 kΩ

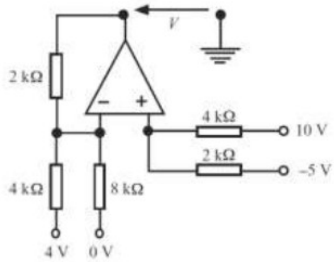
V = 5V


d)
17k
4V
V = 5V
1V
11k
1k
4V
4k
1mA

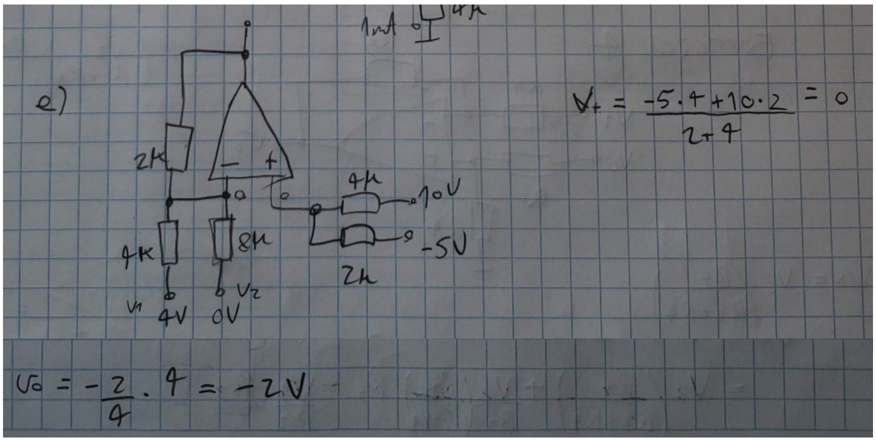

sim_d.asc
+15V
V1
15
V2
15
-15V
R1
17k
V3
4V
-15V
+15V
U1
out
R2
1k
R4
11k
R3
4k
.tran 10m

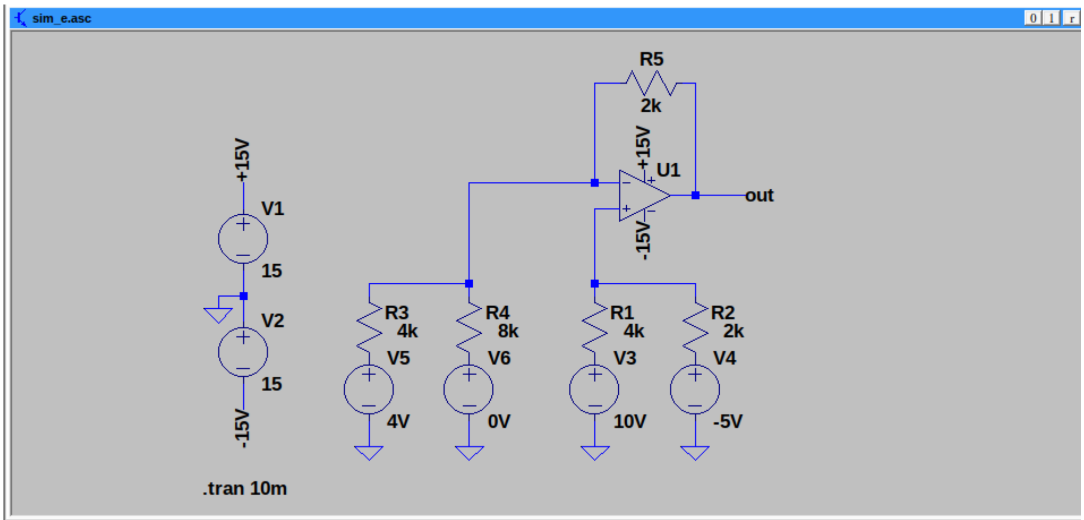

sim_d.raw
V(out)
5.004V
5.003V
5.002V
5.001V
5.000V
4.999V
4.998V
4.997V
4.996V
4.995V
4.994V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms

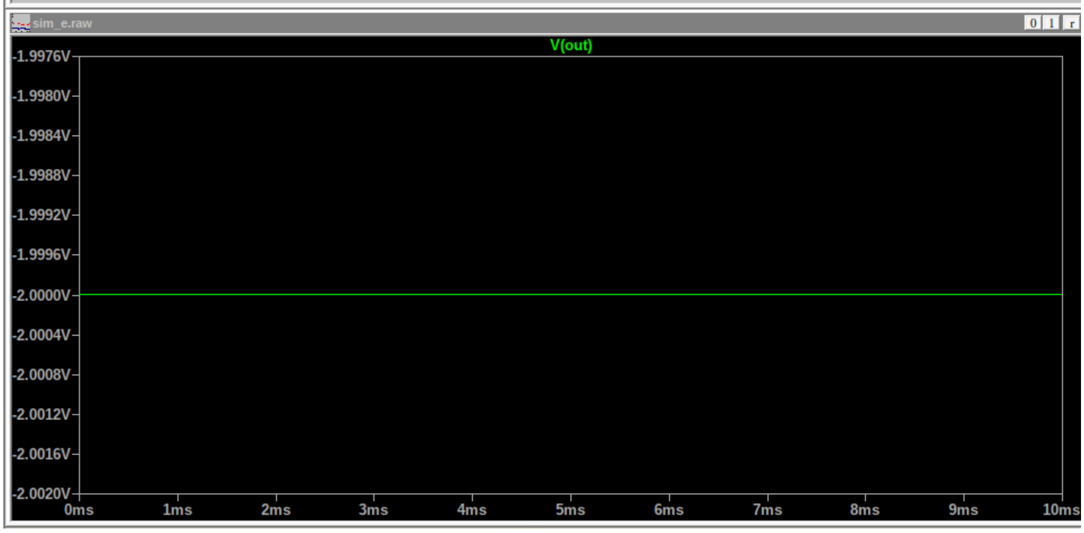
e.


V
2 kΩ
4 kΩ
10 V
2 kΩ
–5 V
4 kΩ
8 kΩ
4 V
0 V

V = -2V


e)
2k
4k
8k
4k
10V
2k
-5V
V1
V2
4V
0V
$V_+ = \frac{-5\cdot4+10\cdot2}{2+4} = 0$
$V_o = -\frac{2}{4}\cdot4 = -2V$


sim_e.asc
R5
2k
+15V
U1
out
-15V
V1
15
V2
15
+15V
-15V
R3
4k
V5
4V
R4
8k
V6
0V
R1
4k
V3
10V
R2
2k
V4
-5V
.tran 10m


sim_e.raw
V(out)
-1.9976V
-1.9980V
-1.9984V
-1.9988V
-1.9992V
-1.9996V
-2.0000V
-2.0004V
-2.0008V
-2.0012V
-2.0016V
-2.0020V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms

Exercise 2 : In the circuit in Figure 2, calculate the values of V1 and V2. Use LTSpice to check your results.

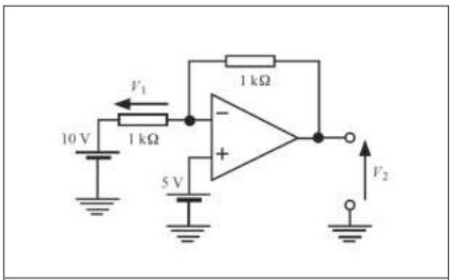
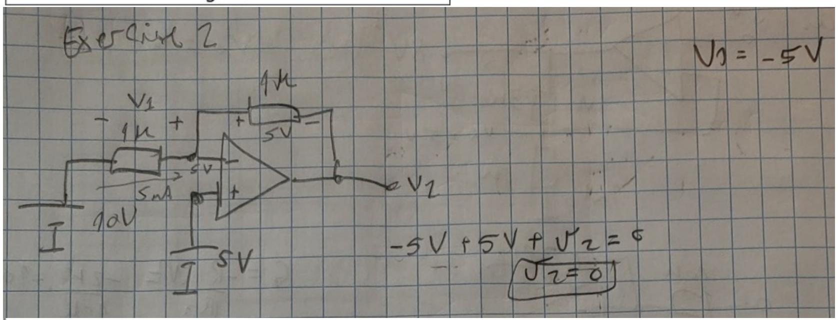
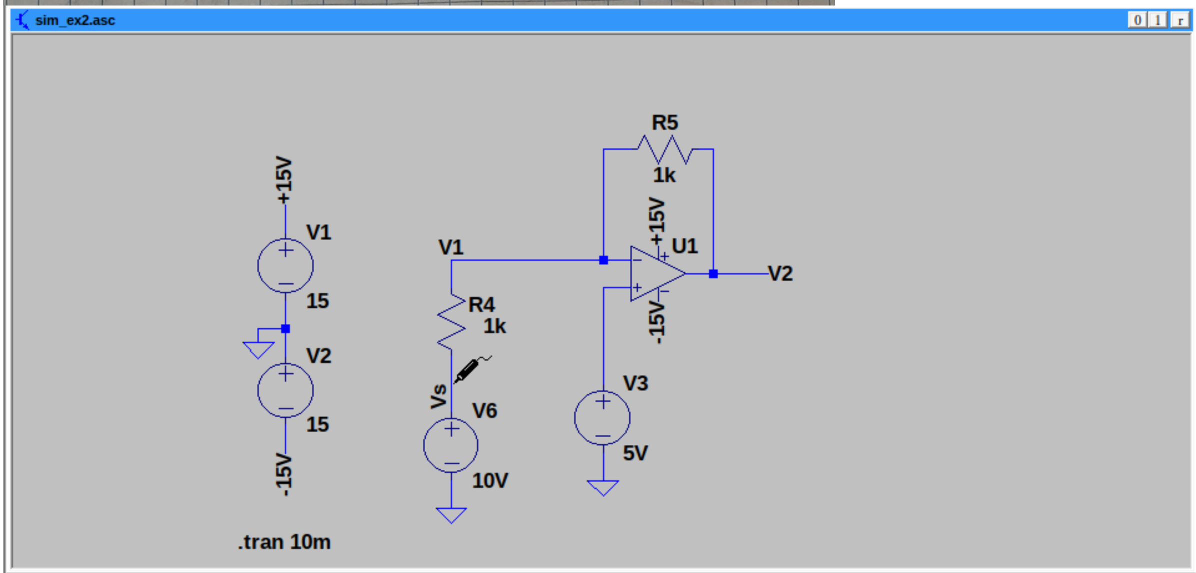
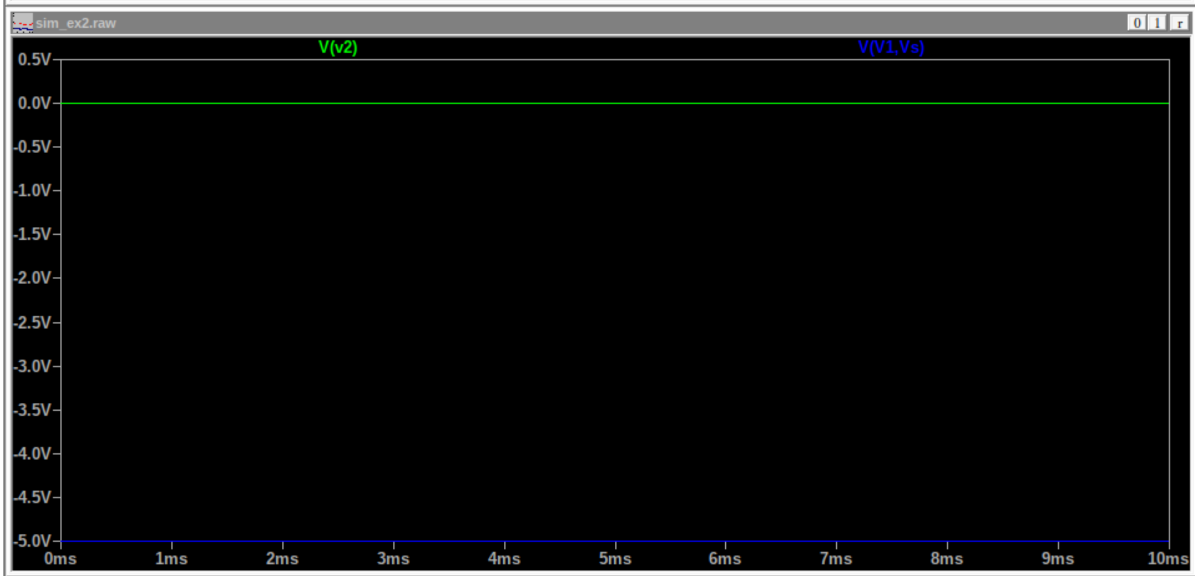


Figure 2







Exercise 3 : The circuit in Figure 3 measures light intensity using a photodiode. The latter, polarized in reverse, produces a current of 0.5uA per uW of incident power. Determine the value of R for which the sensitivity of the conditioner is 0.4mV/mW. Verify your result with LTSpice by replacing the{photodiode, voltage source} assembly with a suitable element.

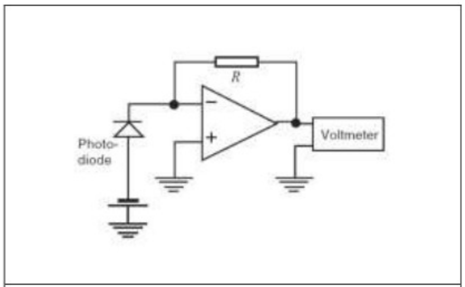
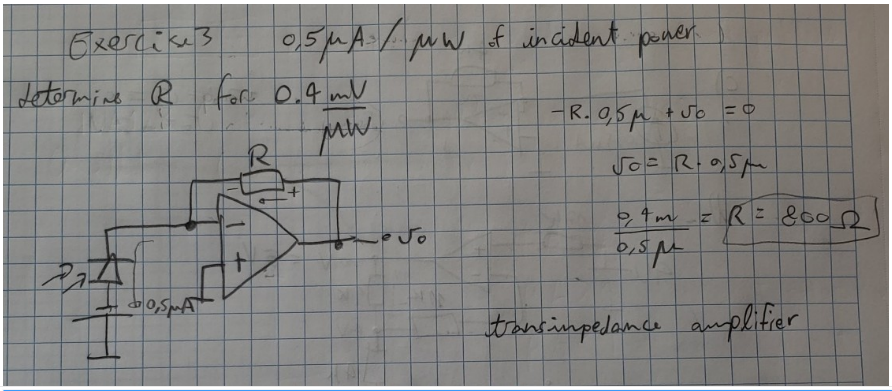
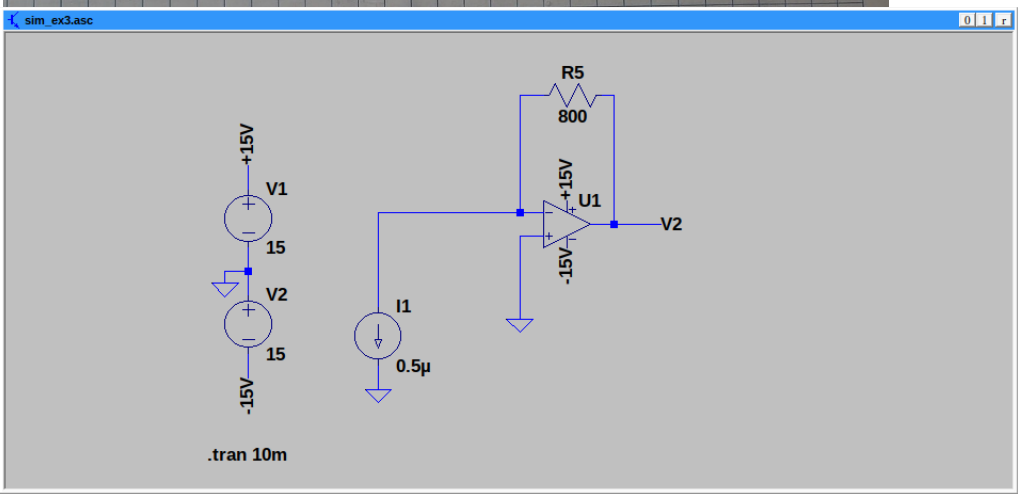


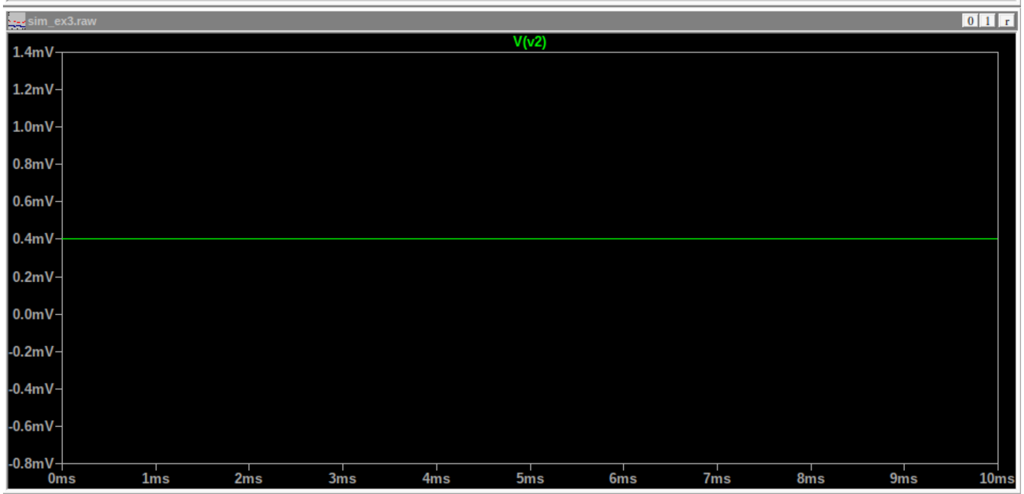
Figure 3

Here I suppose the sensor is submitted to exactly 1mW of power.

R = 800 Ohms







Exercise 4 : In the circuit of figure 4, give the expression of Vo as a function of V1 and V2. Under what condition on resistors R1 to R4 is there a differential amplifier?

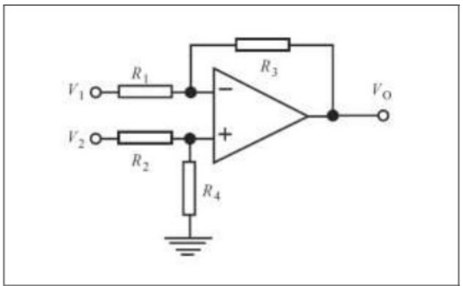
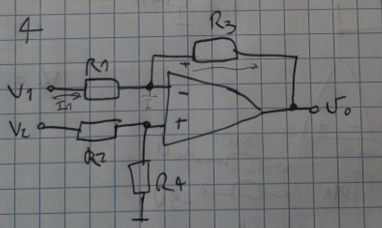


Figure 4

Exercise 4



$\left(\frac{V_2}{R_2+R_4}\right).R_4$

$V_x = V_2 \cdot \left(\frac{R_4}{R_2+R_4}\right)$

$\frac{R_4}{R_2+R_4} = \frac{}{2}$

$I_1 = \frac{V_1 - V_x}{R_1} = \left[\frac{V_1}{R_1} - \frac{V_2}{R_1}\left(\frac{R_4}{R_2+R_4}\right)\right]$

$\frac{}{R_2.R_4}$

$-V_x + I_1.R_3 + V_o = 0$

$V_o = V_x - I_1.R_3$

$= V_2\left(\frac{R_4}{R_2+R_4}\right) + \frac{V_2}{R_1}\left(\frac{R_4}{R_2+R_4}\right).R_3 - \frac{V_1}{R_1}.R_3$

$= V_2\left(\frac{R_4}{R_2+R_4}\right)\left(1+\frac{R_3}{R_1}\right) - V_1.\left(\frac{R_3}{R_1}\right) =$

si $\frac{R_3}{R_1} = \frac{R_4}{R_2} = \alpha$

$= V_2\left(\frac{R_2\alpha}{R_2+R_2\alpha}\right)(1+\alpha) - V_1\alpha$

$= V_2\left(\frac{\alpha}{\cancel{1+\alpha}}\right)\left(\frac{\cancel{1+\alpha}}{1}\right) - V_1\alpha$

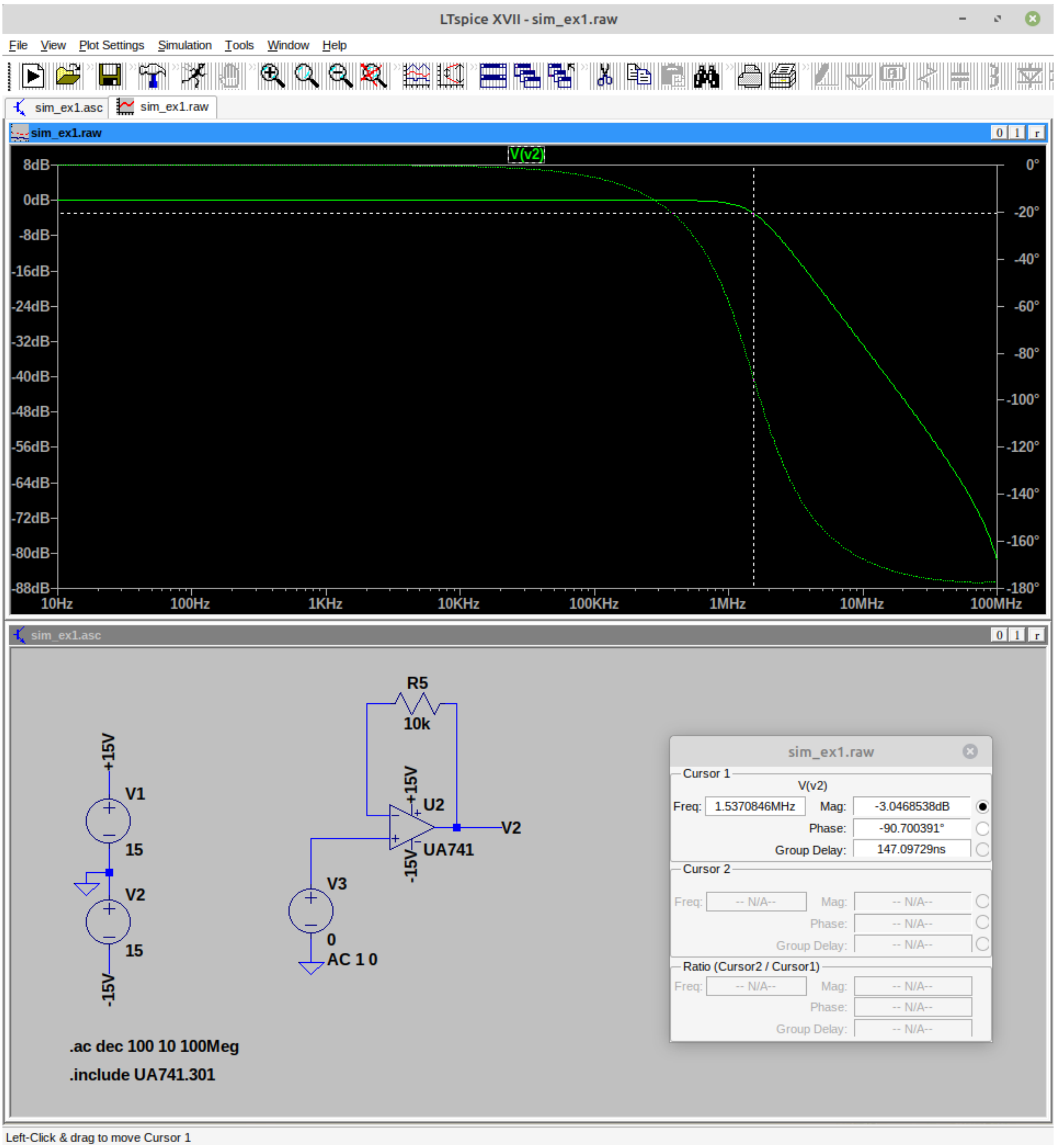
$= \alpha(V_2 - V_1)$

PART2: Simulation

1. Bandwidth : The objective is to verify the invariance of the GBP gain-bandwidth product. To do this we will use the AC simulation mode.

   a. Simulate the opamp as buffer. Determine its cutoff frequency and its GBP product.

The bandwidth of the circuit is 1.53MHz, as defined by the frequency with -3dB attenuation.



b. Simulate the opamp as buffer. Determine its cutoff frequency and its GBP product.

According to the datasheet, the typical value for the bandwidth is of around 1.5MHz, which is very close to the value found in simulation.

TEXAS INSTRUMENTS

LM741

SNOSC25D – MAY 1998 – REVISED OCTOBER 2015 www.ti.com

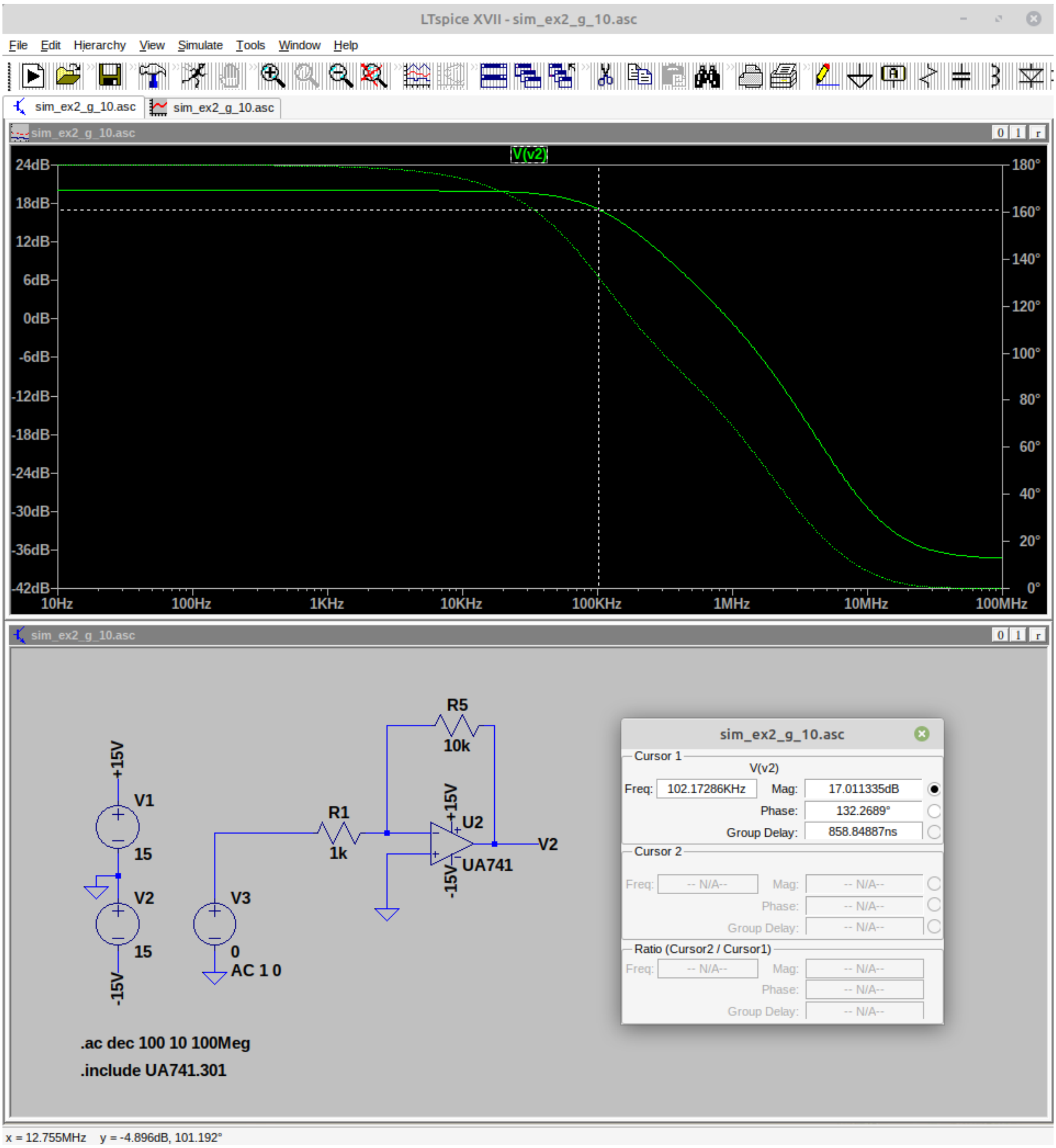
Electrical Characteristics, LM741A(1) (continued)

PARAMETER		TEST CONDITIONS		MIN	TYP	MAX	UNIT
Output voltage swing		$V_S = \pm 20$ V	$R_L \geq 10$ kΩ	±16			V
			$R_L \geq 2$ kΩ	±15			
Output short circuit current		$T_A = 25$°C		10	25	35	mA
		$T_{AMIN} \leq T_A \leq T_{AMAX}$		10		40	
Common-mode rejection ratio		$R_S \leq 50$ Ω, $V_{CM} = \pm 12$ V, $T_{AMIN} \leq T_A \leq T_{AMAX}$		80	95		dB
Supply voltage rejection ratio		$V_S = \pm 20$ V to $V_S = \pm 5$ V, $R_S \leq 50$ Ω, $T_{AMIN} \leq T_A \leq T_{AMAX}$		86	96		dB
Transient response	Rise time	$T_A = 25$°C, unity gain			0.25	0.8	μs
	Overshoot				6%	20%	
Bandwidth (2)		$T_A = 25$°C		0.437	1.5		MHz
Slew rate		$T_A = 25$°C, unity gain		0.3	0.7		V/μs
Power consumption		$V_S = \pm 20$ V	$T_A = 25$°C		80	150	mW
			$T_A = T_{AMIN}$			165	
			$T_A = T_{AMAX}$			135	

(2) Calculated value from: BW (MHz) = 0.35/Rise Time (μs).

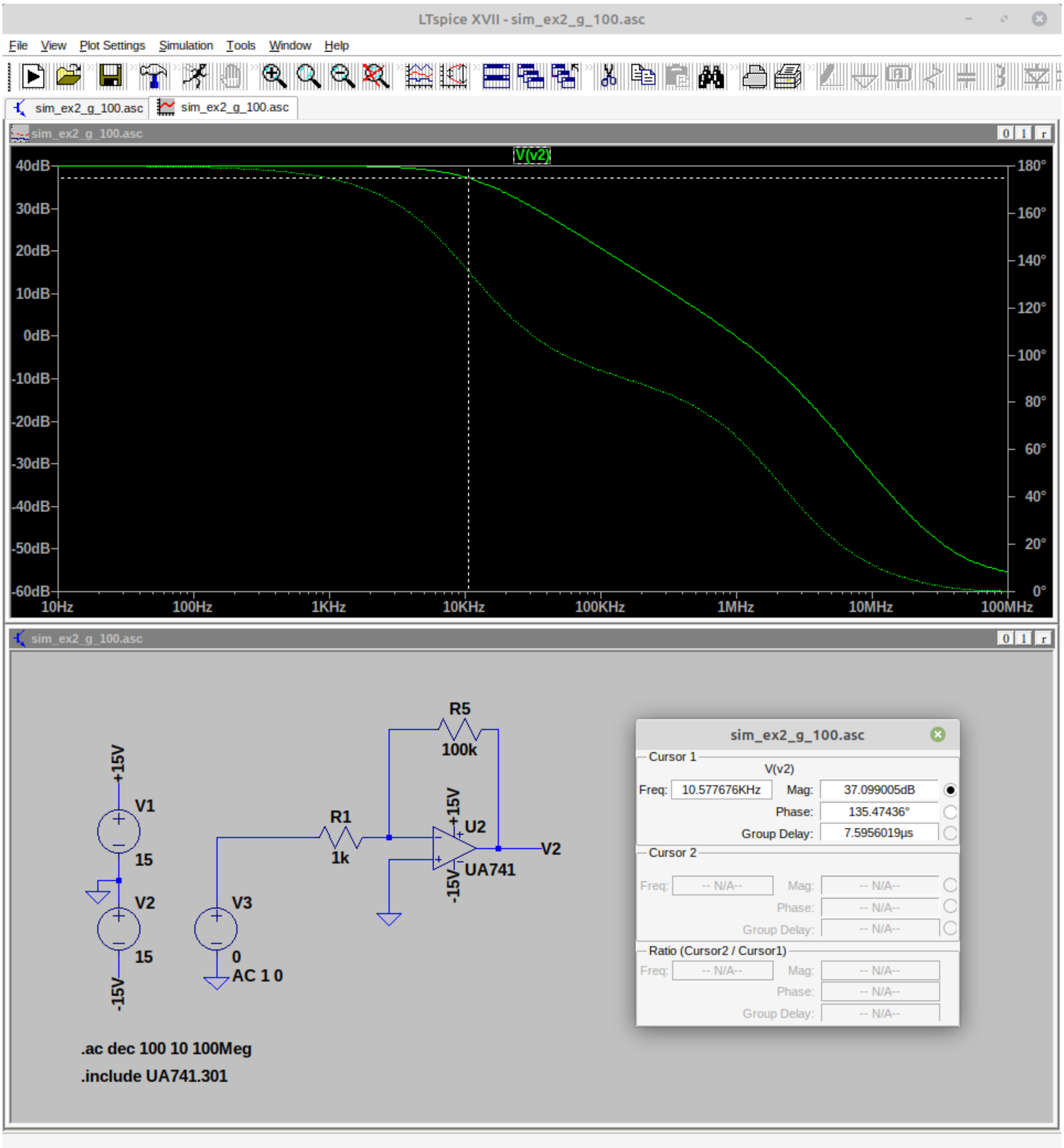
c. Do the same with an inverting amplifier of gain 10, then of gain 100.

gain 10:



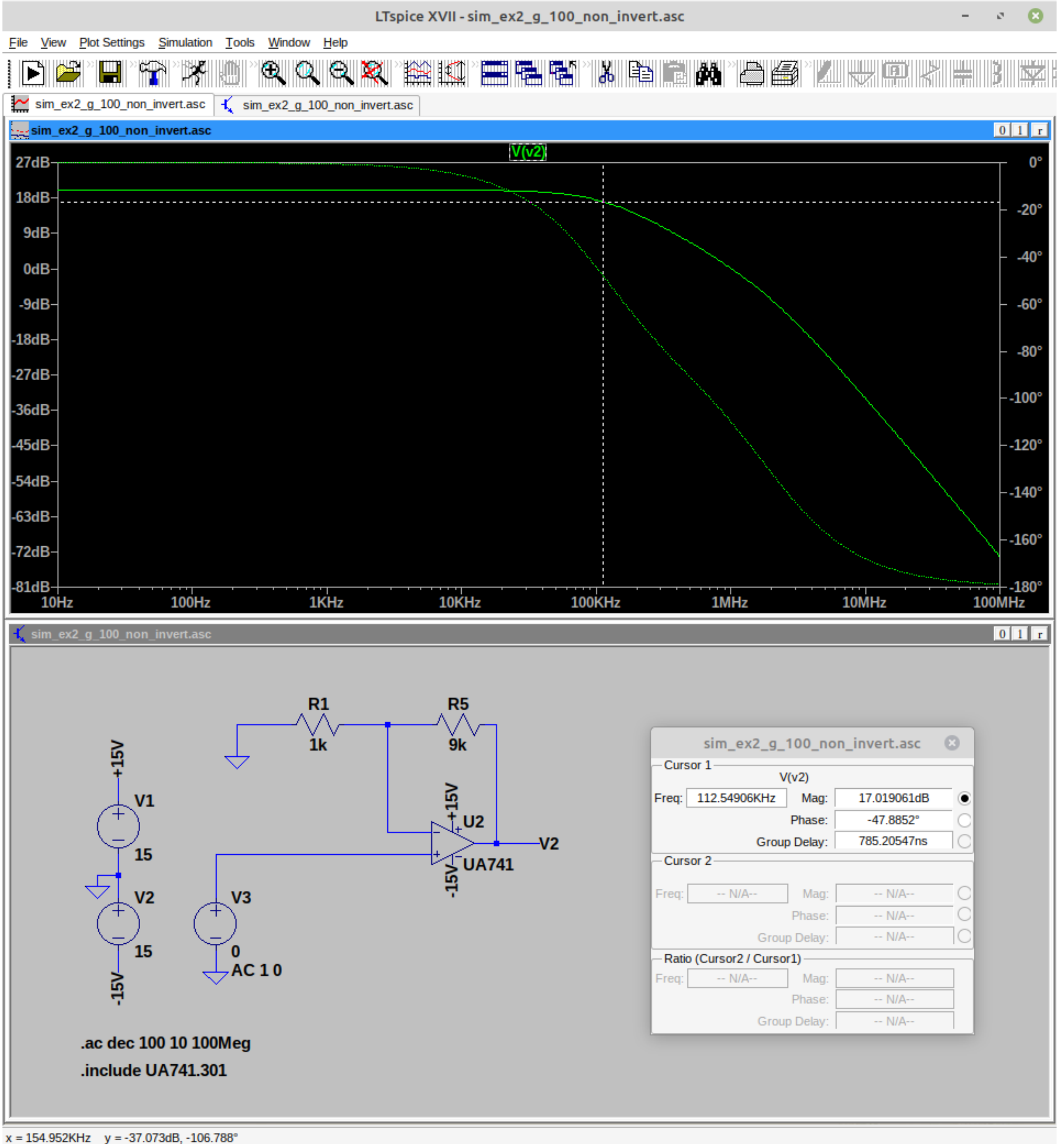
bandwidth : 102 kHz

gain 100:



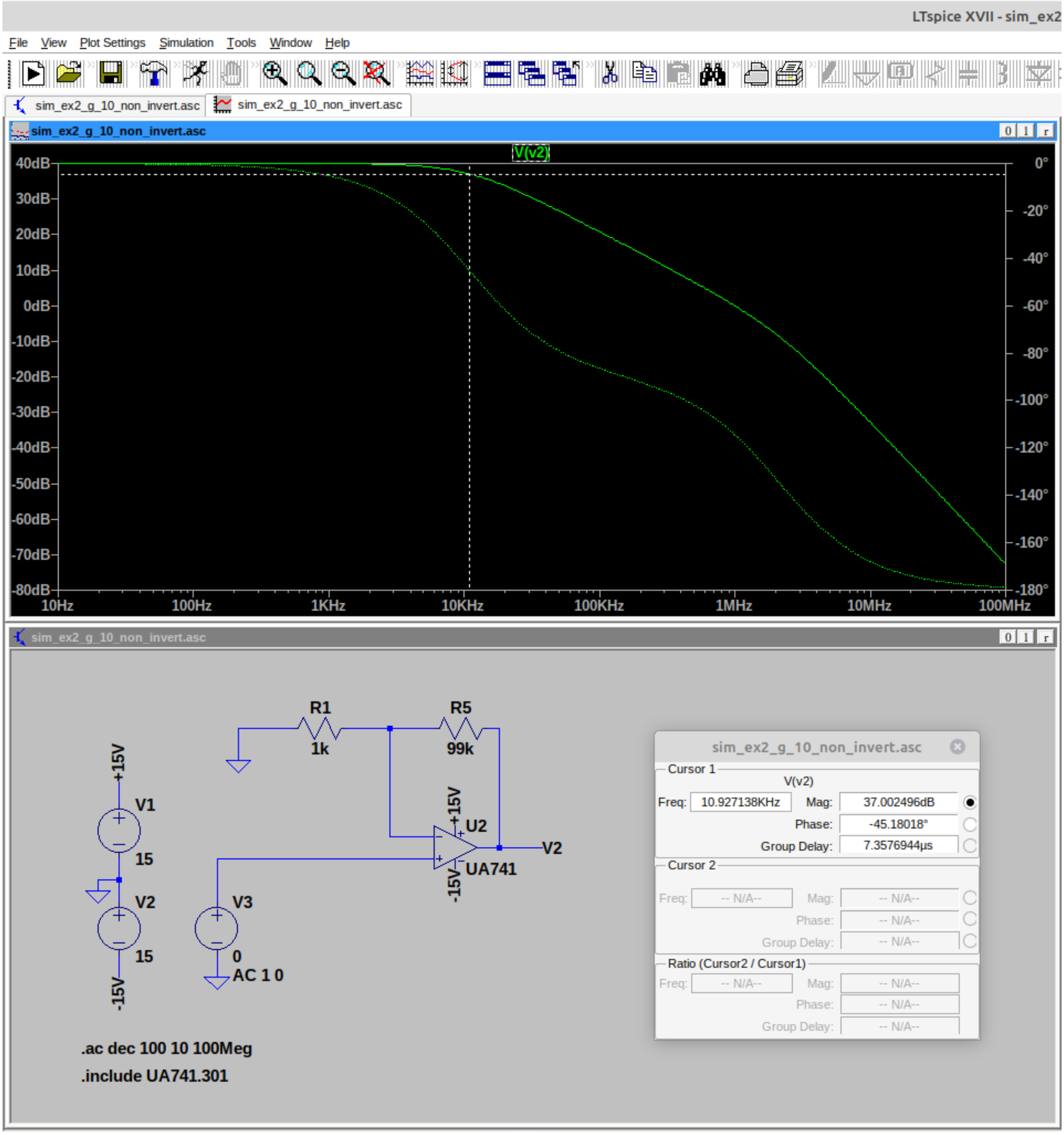
bandwidth : 10 kHz

d. Repeat the previous question with a non-inverting amplifier.
gain 10:



bandwidth : 112 kHz

gain 100:



bandwidth : 11 kHz

e. Conclude

Both in the cases of the inverting and of the non-inverting amplifier the bandwidths were approximately the same. In all cases the product gain and bandwidth is approximately the same.

B. Instrumentation amplifier

PART1: Theory

Exercise 1.a

This is the expression for the output of a differential amplifier:

$s = A_d \cdot (V_p - V_n) + A_{mc} \cdot \frac{V_p + V_n}{2} = A_d U_d + A_{mc} U_{mc}$, where A_d is the differential gain and A_{mc} is the common mode gain.

In practice, a more common datasheet value is the common mode rejection ratio (CMRR), expressed as:

$CMRR = \frac{A_d}{A_{mc}}$, or in dB, $CMRR_{dB} = 20\log(CMRR)$

For this amplifier, $A_{mc} = \frac{A_d}{10^{\frac{CMRR_{dB}}{20}}} = \frac{10}{10^{\frac{80}{20}}} = \frac{10}{10^4} = 10^{-3} = 0.001$

Exercise 1.b

Using the assumption that the amplifier is ideal and has no input current, the input voltages can be calculated from the voltage divisor formula.

$$e1 = U \cdot \frac{r + R1}{r + R1 + R9}$$

$$e2 = U \cdot \frac{R1}{r + R1 + R9}$$

$$\rightarrow U_d = e1 - e2 = U \cdot \frac{r + R1 - R1}{r + R1 + R9} = U \cdot \frac{r}{r + R1 + R9}$$

$$\rightarrow U_{mc} = \frac{e1 + e2}{2} = U \cdot \frac{r + R1 + R1}{2 \cdot (r + R1 + R9)} = U \cdot \frac{\frac{r}{2} + R1}{r + R1 + R9}$$

Exercise 1.c

Substituting all the values:

$$s = A_d U_d + A_{mc} U_{mc} = A_d \cdot U \cdot \frac{r}{r + R1 + R9} + A_{mc} \cdot U \cdot \frac{\frac{r}{2} + R1}{r + R1 + R9} = \frac{A_d U r + A_{mc} U (\frac{r}{2} + R1)}{r + R1 + R9}$$

$$s = \frac{10 \cdot 2 \cdot 1000 + 0.001 \cdot 2 \cdot (\frac{1000}{2} + 1000)}{1000 + 1000 + 9000} = \frac{20003}{11000} = 1.818454545$$

The measurement error is caused by the second term, associated with the common mode.

$$error=\frac{3}{11000}=0.000272727$$

Exercise 1.d

It we swap resistors R1 and R9, we get:

$$\rightarrow U_d=e1-e2=U\cdot\frac{r+R9-R9}{r+R1+R9}=U\cdot\frac{r}{r+R1+R9}$$

$$\rightarrow U_{mc}=\frac{e1+e2}{2}=U\cdot\frac{r+R9+R9}{2\cdot(r+R1+R9)}=U\cdot\frac{\frac{r}{2}+R9}{r+R1+R9}$$

Only the common mode has changed value.

$$s=A_d U_d+A_{mc}U_{mc}=\frac{A_d U r+A_{mc}U\left(\frac{r}{2}+R9\right)}{r+R1+R9}$$

$$s=\frac{10\cdot 2\cdot 1000+0.001\cdot 2\cdot\left(\frac{1000}{2}+9000\right)}{1000+1000+9000}=\frac{20000+19}{11000}=1.819909091$$

$$error=\frac{19}{11000}=0.001727272$$

The error has increased significantly, but remains small compared to the overall value.

Exercise 2.a

$u=e1\cdot\frac{R2}{R1+R2}$(voltage divisor)

$v=e2-R1\cdot i2$

$s=v-R2\cdot i2$

Assuming an ideal opamp:

$$u=v\rightarrow e1\cdot\frac{R2}{R1+R2}=e2-R1\cdot i2\rightarrow R1\cdot i2=e2-e1\cdot\frac{R2}{R1+R2}\rightarrow i2=\frac{e2-e1\cdot\frac{R2}{R1+R2}}{R1}$$

$$s=v-R2\cdot i2=e2-(R1+R2)i2=e2-(R1+R2)\cdot\frac{e2-e1\cdot\frac{R2}{R1+R2}}{R1}=e2+\frac{e1\cdot R2-e2(R1+R2)}{R1}$$

$$s=e1\cdot\frac{R2}{R1}+e2\cdot\frac{R1-R1-R2}{R1}=(e1-e2)\cdot\frac{R2}{R1}$$

The gain is $A_d=\frac{R2}{R1}$

Exercise 2.b

The preliminary analysis holds true:

$u=e1\cdot\frac{R2}{R1+R2}$

$v=e2-R1\cdot i2$

$s=v-R2\cdot i2$

Now assuming the only non-ideal aspect is the gain. Previously it was used to state the "virtual short circuit".

$s=G(u-v)$

$$\begin{array}{l} v=e2-R1\cdot i2 \\ s=v-R2\cdot i2 \end{array}\rightarrow\begin{array}{l} R2\cdot v=R2\cdot e2-R1\cdot R2\cdot i2 \\ -R1\cdot s=-R1\cdot v+R1\cdot R2\cdot i2 \end{array}\rightarrow R2\cdot v-R1\cdot s=R2\cdot e2-R1\cdot v\rightarrow v=\frac{R1\cdot s+R2\cdot e2}{R1+R2}$$

Substituting in the gain formula:

$$s=G(u-v)=G(e1\cdot\frac{R2}{R1+R2}-\frac{R1\cdot s+R2\cdot e2}{R1+R2})=G(\frac{R2\cdot(e1-e2)-R1\cdot s}{R1+R2})$$

$$\rightarrow s(R1+R2)+G\cdot R1\cdot s=G\cdot R2(e1-e2)\rightarrow s(R1(1+G)+R2)=G\cdot R2(e1-e2)$$

$$s=\frac{G\cdot R2}{R1(1+G)+R2}\cdot(e1-e2)$$

Now, the differential gain is done by: $A_d{}'=\frac{G\cdot R2}{R1(1+G)+R2}$. Note that as G goes to infinity, the limit converges to the previous value of A_d.

Exercise 2.c

If we now consider $G=\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}$, with $A_d=\frac{R2}{R1}$

$$A_d{}'=\frac{\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}\cdot R2}{R1(1+\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}})+R2}=\frac{\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}\cdot R1\cdot A_d}{R1(1+\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}})+R1\cdot A_d}=\frac{\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}\cdot A_d}{1+\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}+A_d}=\frac{\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}\cdot A_d}{1+\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}+A_d}$$

$$A_d{}'=\frac{\frac{G_0}{1+j\frac{\omega}{\omega_c{}'}}\cdot A_d}{\frac{G_0+(1+A_d)(1+j\frac{\omega}{\omega_c{}'})}{1+j\frac{\omega}{\omega_c{}'}}}=\frac{G_0\cdot A_d}{G_0+(1+A_d)(1+j\frac{\omega}{\omega_c{}'})}=\frac{G_0\cdot A_d}{G_0+(1+A_d)(1+j\frac{\omega}{\omega_c{}'})}$$

Exercise 2.d

Exercise 3.a

Exercise 3.b

According to the datasheet, the formula is the following:

$$G=1+\frac{50\,k\Omega}{R_G}\rightarrow A_d=50=1+\frac{50\,k\Omega}{R_G}\rightarrow R_G=\frac{50\,k\Omega}{49}=1020.41\,\Omega$$

Exercise 3.c

According to the datasheet, in the table in the section 7.5, and assuming INA12xP, the CMRR should be between 106 and 125 dB in a typical chip.

Exercise 3.d

The minimum value in the range , by arbitration, is taken to be the actual value. CMRR = 106 dB

Exercise 3.e

$CMRR=\frac{A_d}{A_{mc}}$, or in dB, $CMRR_{dB}=20\log(CMRR)$

$$A_{mc}=\frac{A_d}{10^{\frac{CMRR_{dB}}{20}}}=\frac{50}{10^{\frac{106}{20}}}=250.59\cdot 10^{-6}$$

Considering the input voltages as $p1=301\,mV$ and $p2=300\,mV$, we get:

$U_d=1\,mV\rightarrow s_d=A_d\cdot U_d=50*0.001=50\,mV$

$U_{mc}=300.5\,mV\rightarrow s_{mc}=A_{mc}\cdot U_{mc}=250.59\cdot 10^{-6}\cdot 0.3005=0.075\,mV$

This represents an error of 0.15% in the measurement.

Exercise 3.f

The amplifier must have a high input impedance, in order to avoid drawing current from the sources, and also a very small offset current. According to table in section 7.5 in the datasheet, the input impedance is of $10\,G\Omega$ in differential mode and $100\,G\Omega$ in common-mode. Some of the measurements in the real world present a voltage that drops very significant it current is drawn. It can be modeled as a Thévénin circuit with a very high impedance. Ideally, no current should pass.

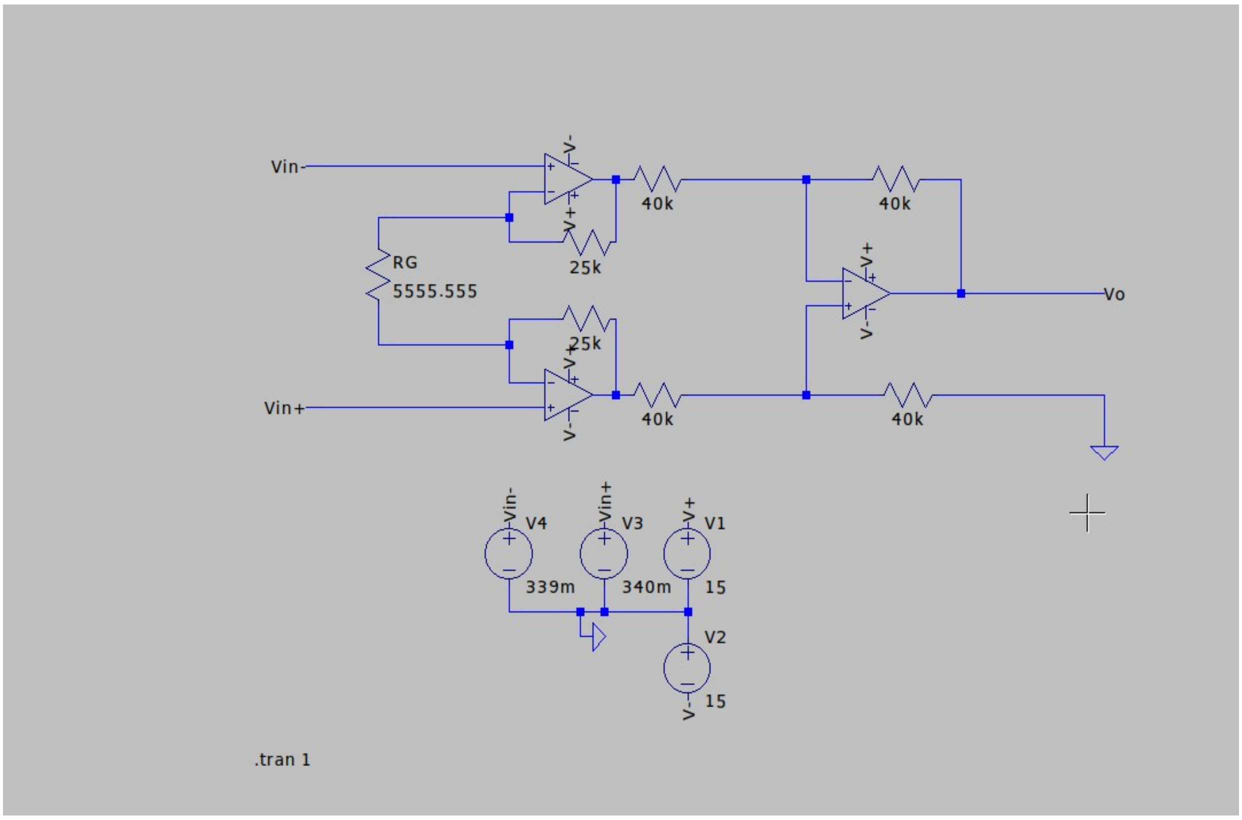
The input bias current, i.e., the average “current source” has $\pm 2\,nA$ and the input offset current (the difference between the currents in each input) is $\pm 1\,nA$.

Exercise 3.g

The bandwidth at -3dB, according again to section 7.5 in the datasheet is between 640 and 200 kHz.

PART2: Simulation

a. Simulate in LTSpice the AI circuit corresponding to the internal schematic of the INA 128, using the "UniversalOpamp2" AO.



b. Set the external resistor RG so as to obtain differential gains of 10, 100 and 1000.

$$G=1+\frac{50\,k\Omega}{R_G}\rightarrow R_G=\frac{50\,k\Omega}{G-1}$$

$G=10\rightarrow R_G=5555.56\,\Omega$

$G=100\rightarrow R_G=505.05\,\Omega$

$G=1000\rightarrow R_G=50.05\,\Omega$

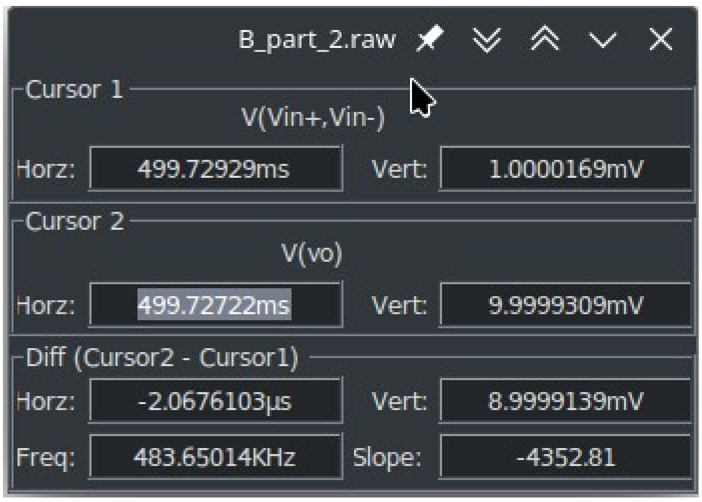


Figure: gain of 10, resistance of 5555 ohms

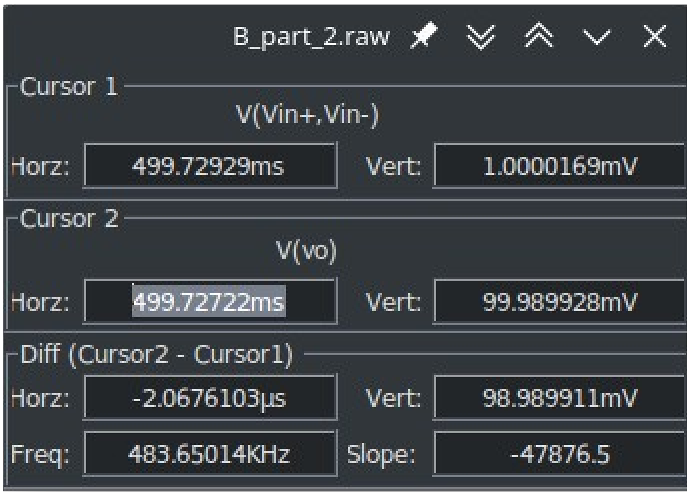


Figure: gain of 100, resistance of 505.05 ohms

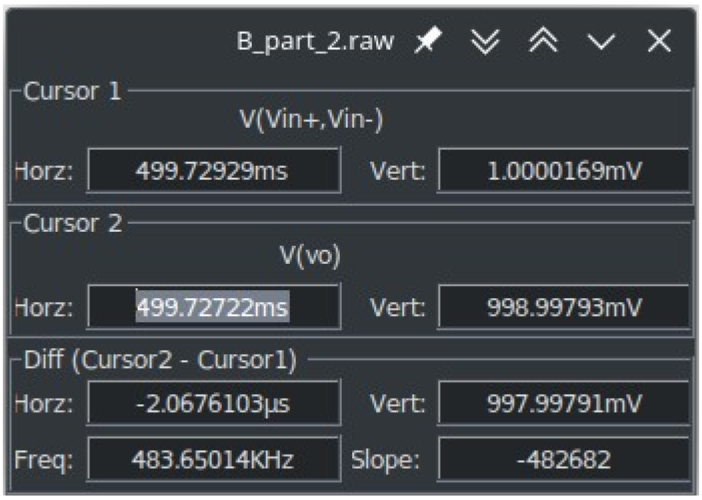
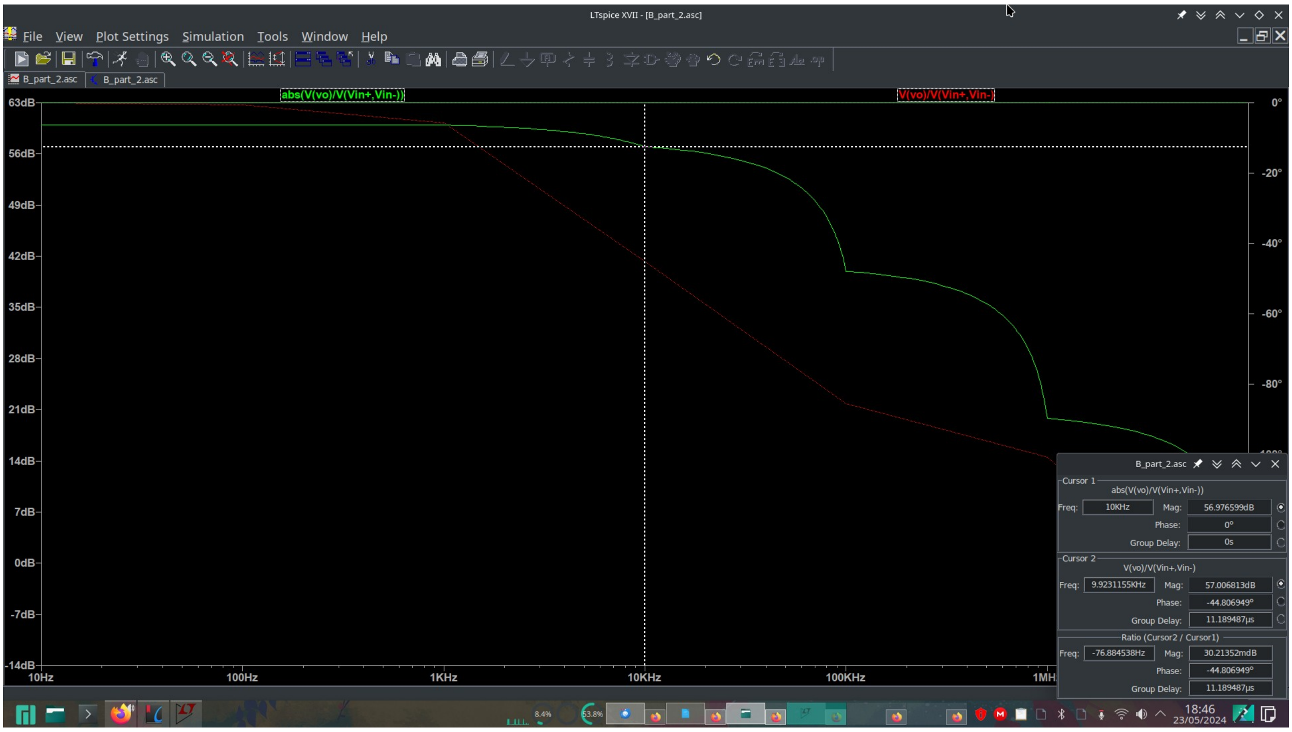


Figure: gain of 1000, resistance of 50.05 ohms

c. Display the differential and common mode gain as a function of frequency, choosing a frequency band wide enough to observe the differential gain cutoff frequency. What do you observe for each of these gains?

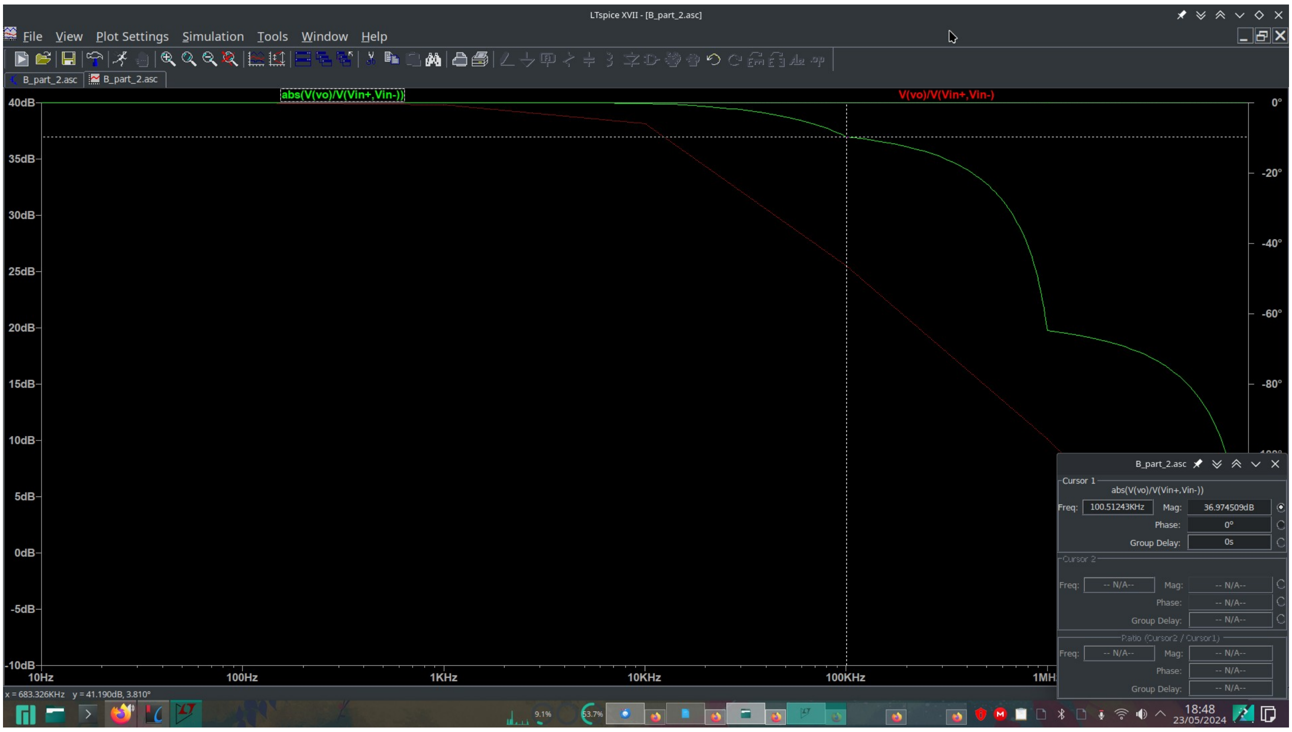
c.1 G = 1000 = 60dB



In the graph, we can see the 60 dB in the pass band, as expected. The graph in green is the magnitude of the differential gain and in red is the phase. At 10kHz, the output is reduced by 3dB.

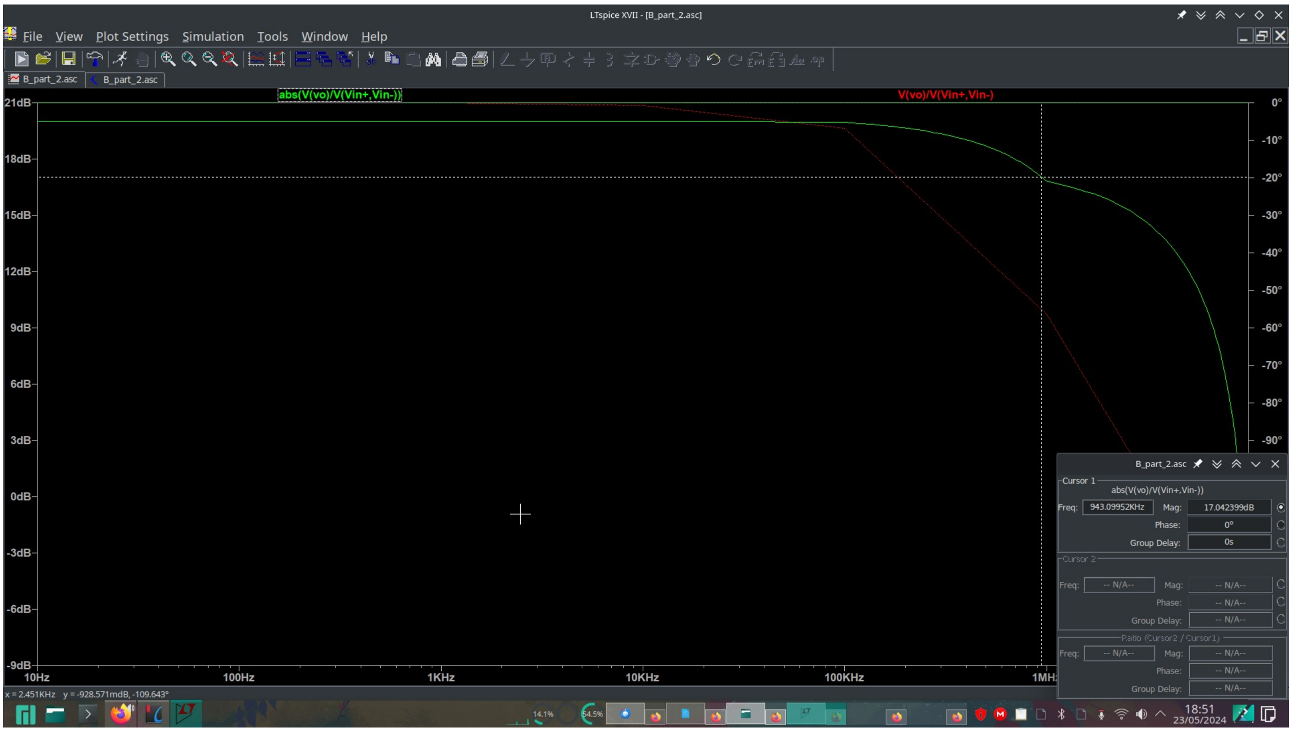
In order to measure the common mode gain, I've modified both voltage inputs to be equal to 5V.

c.2 G = 100 = 40 dB

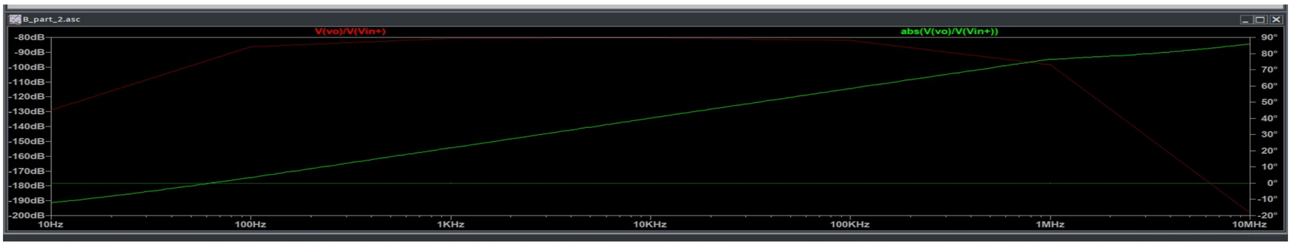


In the pass band, the magnitude is 40 dB as expected. The cutoff frequency is at 100 kHz

c.3 G = 10 = 20 dB



The cutoff frequency is at 943 kHz



This graph shows the common mode gain behaves as a high pass filter. All 3 simulations presented visually identical graphs, so I have omitted the other screenshots for the sake of brevity.

c.conclusion

It is observed that the product of differential gain and bandwidth remains approximately constant. Raising the gain shifts the amplitude graph to the left, but doesn't change much the general form of the graph.

The common mode gain behaves as a high pass filter and is not affected by the increase in differential gain. This is expected if all internal opamps behave ideally, because there would be no voltage across R_G, consequently it has no influence on the output.

This also explains why the Common mode rejection ration improves as the gain increases, the common mode gain gets smaller relative to the differential mode.

d. Compare the values of differential gain, CMRT, and cutoff frequency with the data sheet and comment on the differences.

All the differential gains are very close to values calculated.

The CMRR can be calculated, in dB, as the differential gain minus the common mode gain. I took the value, arbitrarily, at 1 kHz

At G = 10, CMRR = 20 - (-154) = 175 dB

At G = 100, CMRR = 40 - (-154) = 194 dB

At G = 1000, CMRR = 60 - (-154) = 214 dB

These values are significantly better than the datasheet. This is caused by the ideal and very precise resistances and opamps used in the simulation. Changing a resistor by 100 Ohms caused the CMRR to 80 dB at G=10, much closer to the expected value.

The bandwidth at G = 10 was measured as 943 kHz, higher than the 640 kHz from the datasheet.

At G = 100, it was simulated as 100 kHz, lower than the 200 kHz expected.

At G = 1000, it has been found to be 10 kHz, again half of the 20 kHz specification.

e. Concerning the TRMC, propose a solution so that the simulated values are close to those in the datasheet.

We can add a small random deviation in resistance values and non-ideal behaviors to the 3 internal opamps. Simply changing a single resistor by 0.25% decreased the CMRR by around 90 dB