Gabriel GOSMANN
Felipe LORENZON

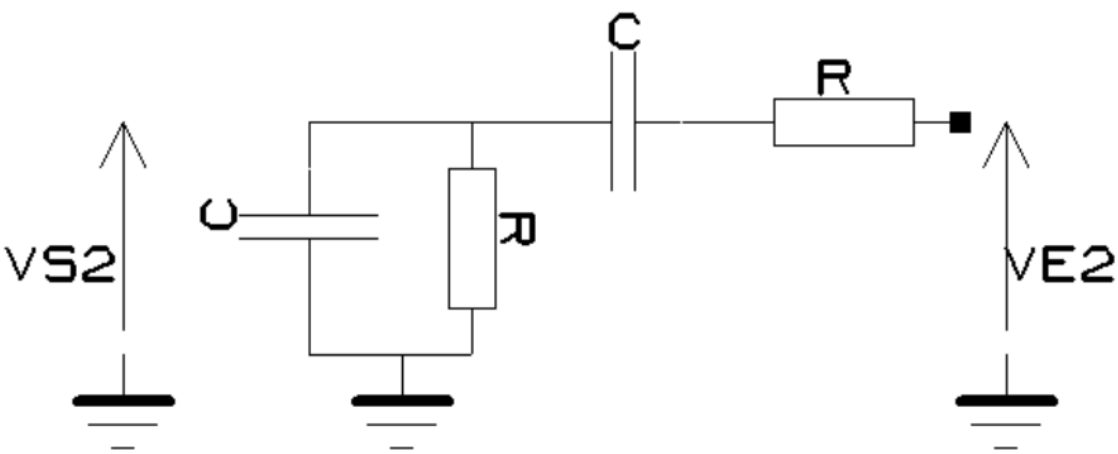
École Centrale de Lille
EBE – Electronics for biomedical engineering

June, 2024.

Oscillators

Part 1 : Theory

1. Give an expression for β(jω).



For the compontents in paralellel from Vs2 to the ground:

$$\frac{1}{Z_1}=\frac{1}{\frac{1}{sC}}+\frac{1}{R}\Rightarrow Z_1=\frac{1}{sC+\frac{1}{R}}=\frac{R}{1+sRC}$$

For the components in series from Ve2 to to Vs2

$$Z_2=R+\frac{1}{sC}$$

The circuit is a voltage divisor:

$$V_{s2}=\frac{Z_1}{Z_1+Z_2}\cdot V_{e2}\Rightarrow\frac{V_{s2}}{V_{e2}}=\frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC+R+\frac{1}{sC}}}\Rightarrow\beta(s)=\frac{sRC}{(sRC)^2+3sRC+1}$$

$$\beta(j\omega)=\frac{j\omega RC}{(1-(\omega RC)^2)+3j\omega RC}$$

2. What is the condition on A_0 and β(jω) for the circuit to oscillate with a stable amplitude ?

The condition is $A_0\cdot\beta(j\omega_0)=1$. It can further broke down into two simpler conditions:

$|A_0||\beta(j\omega_0)|=1$

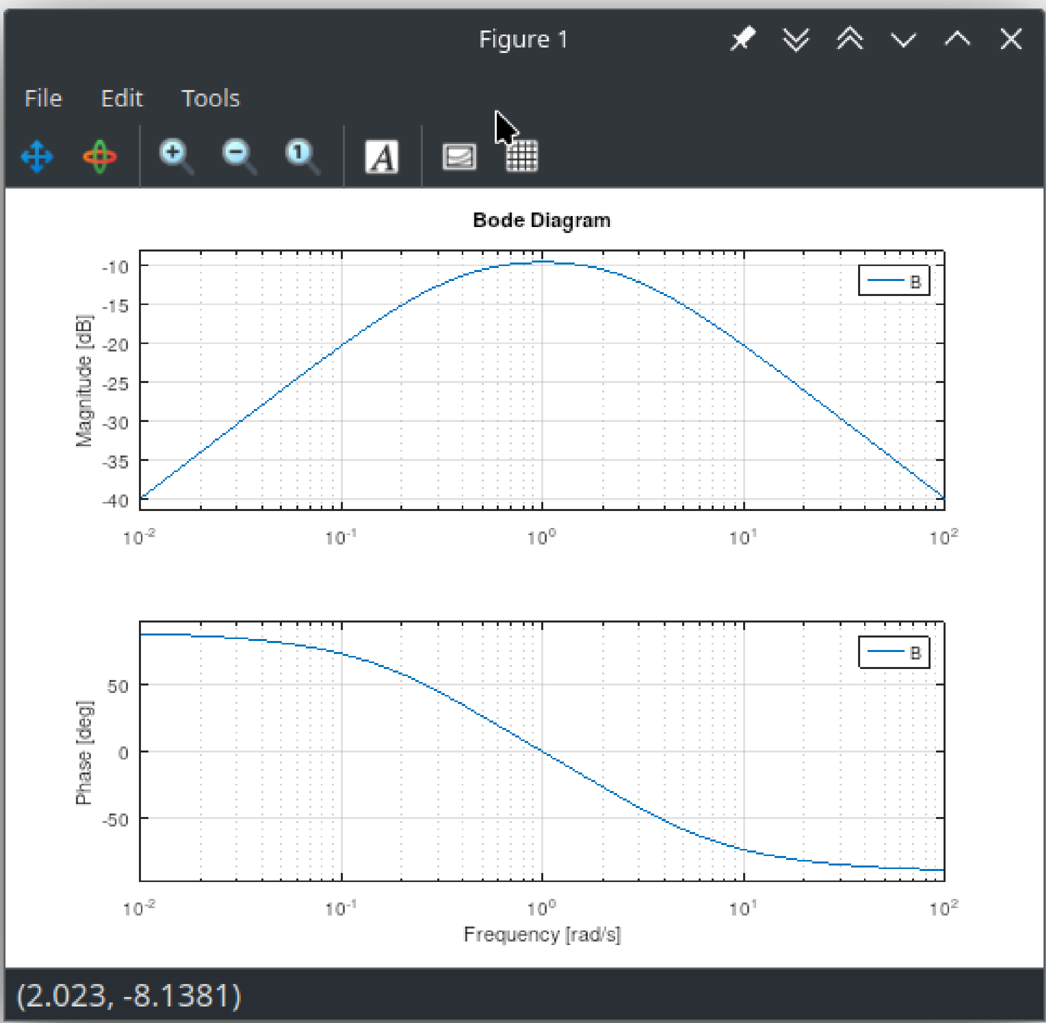
$arg(A)+arg(\beta(j\omega))=2\pi n, n\in\mathbb{N}$

Assuming A is always real, this means that $arg(\beta(j\omega))=\pi n, n\in\mathbb{N}$

3. Derive :

- **An expression for oscillation frequency F_0.**

Plotting the bode diagram for the transferfunction, it is possible to verify that the point of interest is when the phaze is 0, as it never reaches a right angle.



$$arg(\beta(j\omega))=0\Rightarrow arg(j\omega RC)-arg((1-(j\omega RC)^2)+3j\omega RC)=0$$

$$\frac{\pi}{2}-\arctan\left(\frac{3\omega RC}{1-(\omega RC)^2}\right)=0\Rightarrow\frac{3\omega RC}{1-(\omega RC)^2}\to\infty\Rightarrow 1-(\omega RC)^2\to 0\Rightarrow 1=\omega_0 RC\Rightarrow\omega_0=\frac{1}{RC}$$

$$\Rightarrow F_0=\frac{1}{2\pi RC}$$

- **The condition on R_2 et R_1 for a constant amplitude oscillation, without clipping.**

Assuming, as stated, that K is closed, the diodes do not interact wit the circuit. The amplifier is a simple non-inverter, which has a gain as:

$$A_0=1+\frac{R_2}{R_1}$$

In order to know how much gain is needed, it is necessary to calculate the amplitude at F_0.

$$|\beta(j\omega_0)|=\left|\frac{j\omega_0 RC}{(1-(\omega RC)^2)+3j\omega_0 RC}\right|=\left|\frac{j}{(1-(1)^2)+3j}\right|=\left|\frac{1}{3}\right|=\frac{1}{3}$$

$$\Rightarrow A_0\cdot\frac{1}{3}=1\Rightarrow A_0=3\Rightarrow 1+\frac{R_2}{R_1}=3\Rightarrow R_2=2R_1$$

4. Calculate values for C et R_2 so as to obtain a 50 kHz, constant amplitude oscillation.

It is given that

$$R_1=100\,\Omega\Rightarrow R_2=200\,\Omega$$

$$\omega_0=\frac{1}{RC}=50\,kHZ\Rightarrow C=\frac{1}{100\cdot 2\,\%pi50\,000}=3{,}18\,e=8\,F=31.83\,nF$$

5. What is its peak-to-peak amplitude. In practice, how can it be varied ?

Peak-to-peak voltage is the difference between the maximum and the minimum voltages in a signal. It can be varied with another amplification stage after the signal generation, or by changing the supply voltage in the opamp.

6. We want the oscillator to operate at frequency Fc, which is not a priori known. What are the values of gain and phase shift induced by the amplifier at this frequency?

$$|G(j\omega)|=\left|\frac{A_0}{1+j\frac{F_c}{F_c}}\right|=\left|\frac{A_0}{1+j}\right|=\frac{A_0}{\sqrt{2}}=A_0\cdot\frac{\sqrt{2}}{2}$$

$$arg(G(j\omega))=arg\left(\frac{A_0}{1+j}\right)=arg(A_0)-arg(1+j)=0-\arctan\left(\frac{1}{1}\right)=-\frac{\pi}{4}$$

7. Derive conditions on R, C and A0 for the circuit to oscillate at constant amplitude, without clipping.

We need to go back to the basic conditions

$$G(j\omega)\beta(j\omega)=\frac{A_0}{1+j\frac{\omega}{\omega_c}}\cdot\frac{j\omega RC}{(1-(\omega RC)^2)+3j\omega RC}$$

Phase condition: it must be null

$$arg(G\beta(j\omega_0))=0\Rightarrow arg\left(\frac{A_0\,j\omega RC}{(1+j\frac{\omega}{\omega_c})((1-(\omega RC)^2)+3j\omega RC)}\right)=0$$

$$\Rightarrow 0=\frac{\pi}{2}-arg\left((1+j\frac{\omega}{\omega_c})((1-(\omega RC)^2)+3j\omega RC)\right)$$

$$\Rightarrow\frac{\pi}{2}=arg\left((1-(\omega RC)^2-3\omega RC\frac{\omega}{\omega_c})+j(3\omega RC+\frac{\omega}{\omega_c}(1-(\omega RC)^2))\right)$$

$$\Rightarrow \frac{\pi}{2} = \arctan\left(\frac{3\omega RC + \frac{\omega}{\omega_c}(1-(\omega RC)^2)}{1-(\omega RC)^2 - 3\omega RC\frac{\omega}{\omega_c}}\right) \Rightarrow \tan\left(\frac{\pi}{2}\right) = \frac{3\omega_c \omega RC + \omega - \omega^3 R^2 C^2}{\omega_c - \omega_c \omega^2 R^2 C^2 - 3\omega^2 RC}$$

$\tan\left(\frac{\pi}{2}\right) \rightarrow \infty \Rightarrow$ *the denominator must equal zero, since the numerator is finite*

$$(\omega_c - \omega_c \omega^2 R^2 C^2 - 3\omega^2 RC) = 0 \Rightarrow \omega_c = (\omega_c R^2 C^2 + 3RC)\omega^2 \Rightarrow \omega = \sqrt{\frac{\omega_c}{\omega_c R^2 C^2 + 3RC}}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{(RC)^2 + 3\frac{RC}{\omega_c}}}$$

Note that this formula is equivalent to what was calculated previously, if we assume the opamp is ideal:

$$\omega_c \rightarrow \infty \Rightarrow \omega_0 = \frac{1}{\sqrt{((RC^2)+0)}} = \frac{1}{RC}$$

now, we still need to use amplitude condition, in order to calculate the necessary gain.

$$|G\beta(j\omega_0)| = \left|\frac{A_0}{1+j\frac{\omega_0}{\omega_c}} \cdot \frac{j\omega_0 RC}{(1-(\omega_0 RC)^2) + 3j\omega_0 RC}\right|$$

$$1 = \frac{A_0}{\left|1+j\frac{\omega_0}{\omega_c}\right|} \cdot \frac{|j\omega_0 RC|}{|(1-(\omega_0 RC)^2) + 3j\omega_0 RC|} \Rightarrow 1 = \frac{A_0}{\sqrt{1+\frac{\omega_0^2}{\omega_c^2}}} \cdot \frac{\omega_0 RC}{\sqrt{(1-(\omega_0 RC)^2)^2 + 9\omega_0^2 R^2 C^2}}$$

$$\Rightarrow A_0 = \frac{\sqrt{1+\frac{\omega_0^2}{\omega_c^2}}\sqrt{(1-(\omega_0 RC)^2)^2 + 9\omega_0^2 R^2 C^2}}{\omega_0 RC}$$

Again, if you assume an ideal opamp, $\omega_c \gg \omega_0 \Rightarrow A_0 = 3$

$$\Rightarrow A_0 = \frac{\sqrt{1+\frac{1}{(\omega_c RC)^2 + 3RC\,\omega_c}}\sqrt{\left(1-\frac{R^2C^2}{(RC)^2+3\frac{RC}{\omega_c}}\right)^2 + 9\frac{1}{(RC)^2+3\frac{RC}{\omega_c}}R^2C^2}}{\frac{RC}{\sqrt{(RC)^2+3\frac{RC}{\omega_c}}}}$$

$$\Rightarrow A_0 = \frac{\sqrt{\frac{(\omega_c RC)^2 + 3RC\,\omega_c + 1}{(\omega_c RC)^2 + 3RC\,\omega_c}}\sqrt{\left(1-\frac{\omega_c^2 R^2 C^2}{(\omega_c RC)^2 + 3\omega_c RC}\right)^2 + \frac{9\omega_c^2 R^2 C^2}{(\omega_c RC)^2 + 3\omega_c RC}}}{\frac{RC\,\omega_c}{\sqrt{(\omega_c RC)^2 + 3\omega_c RC}}}$$

$$\Rightarrow A_0=\frac{\sqrt{\frac{(\omega_c RC)^2+3RC\,\omega_c+1}{(\omega_c RC)^2+3RC\,\omega_c}}\sqrt{\frac{(3\,\omega_c RC)^2}{((\omega_c RC)^2+3\,\omega_c RC)^2}+\frac{(3\,\omega_c RC)^2}{(\omega_c RC)^2+3\,\omega_c RC}}}{\frac{RC\,\omega_c}{\sqrt{(\omega_c RC)^2+3\,\omega_c RC}}}$$

$$\Rightarrow A_0=\frac{3\,\omega_c RC\sqrt{(\omega_c RC)^2+3RC\,\omega_c+1}\sqrt{\frac{1}{((\omega_c RC)^2+3\,\omega_c RC)^2}+\frac{1}{(\omega_c RC)^2+3\,\omega_c RC}}}{RC\,\omega_c}$$

$$\Rightarrow A_0=3\sqrt{(\omega_c RC)^2+3RC\,\omega_c+1}\,\frac{\sqrt{1+(\omega_c RC)^2+3\,\omega_c RC}}{(\omega_c RC)^2+3\,\omega_c RC}$$

$$\Rightarrow A_0=3\,\frac{(\omega_c RC)^2+3\,\omega_c RC+1}{(\omega_c RC)^2+3\,\omega_c RC}=3\left(1+\frac{1}{(\omega_c RC)^2+3\,\omega_c RC}\right)=3\left(1+\frac{1}{\omega_c RC}\,\frac{1}{\omega_c RC+3}\right)$$

8. The opamp has a GBP of 3 MHz. Calculate values of Fc and C. Now, switch K is open, and the oscillation frequency is the same as in question 4.

The easiest way I've found to solve the problem was by using recursion.

$GBP=3\,MHz\Rightarrow A_0F_c=k=3\cdot10^6$

Using the desired frequency:

$$\omega_0=\frac{1}{\sqrt{(RC)^2+3\frac{RC}{\omega_c}}}\Rightarrow 3\,\omega_0^2RC=\omega_c-\omega_c\,\omega_0^2R^2C^2\Rightarrow C=\frac{-3\,\omega_0^2R\pm\sqrt{9\,\omega_0^4R^2+4\cdot\omega_c^2\,\omega_0^2R^2}}{2\,\omega_c\,\omega_0^2R^{2}}$$

$\Rightarrow C=\dfrac{-3\,\omega_0+\sqrt{9\,\omega_0^2+4\cdot\omega_c^2}}{2\,\omega_c\,\omega_0R}$, the square root is always bigger than the other term, so the only valid solution has the plus sign.

The equation below has been found on question 7:

$$\Rightarrow A_0=\frac{\sqrt{1+\frac{\omega_0^2}{\omega_c^2}}\sqrt{(1-(\omega_0 RC)^2)^2+9\,\omega_0^2R^2C^2}}{\omega_0 RC}$$

The procedure is pretty simple, basically a manual bisection method. I've set everything up in Desmos, but this could be automated using matlab. The gain is the only variable. ω_c is defined by the GBP of the filter, then C is calculated. It then reuses these values to calculate the necessary gain. Note that the calculate gain is not equal to our input. I've then calculated their difference and changed the input digit by digit until the calculated difference gets negligible. I've also made an octave program to verify that the transfer function is, indeed, very close to 1 in the desired frequency.

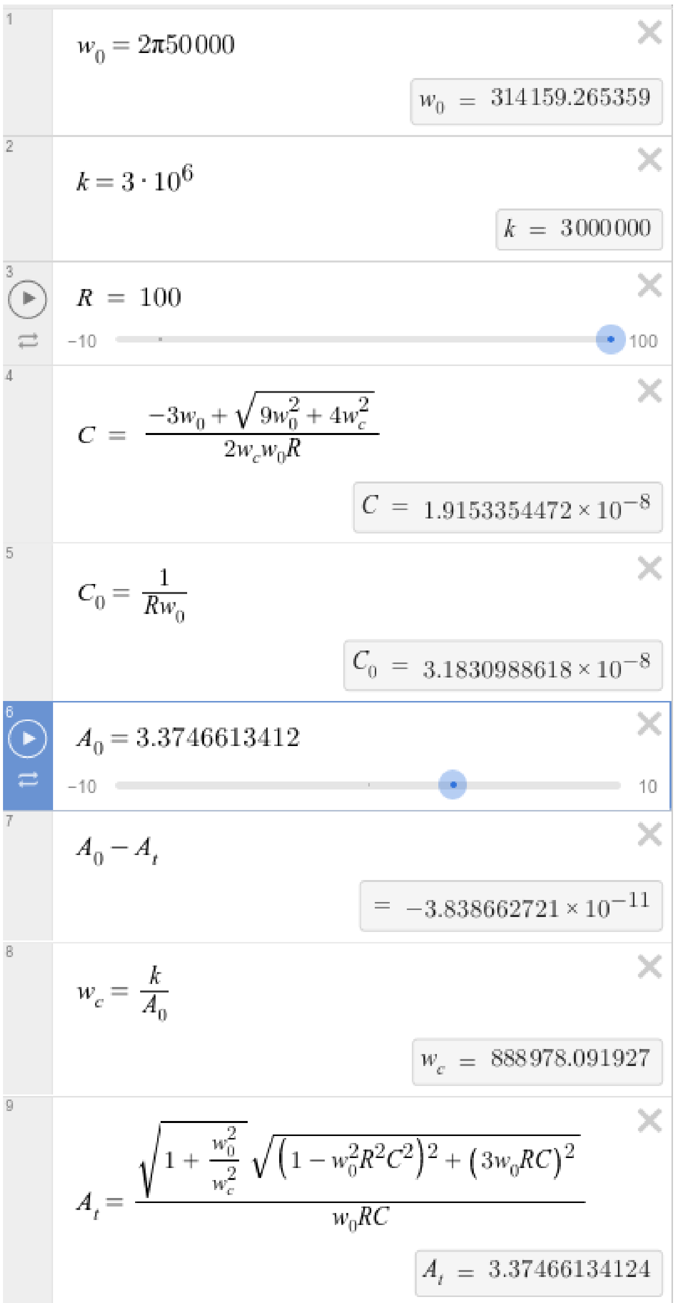


Figure : screenshot from the manual solution

```
pkg load control
s = tf('s')

A0 = 3.3746613412;
wc = 888978.091927;
w0 = 2 * pi * 50000;
A = A0 / ( 1+s/wc)

R=100;
```

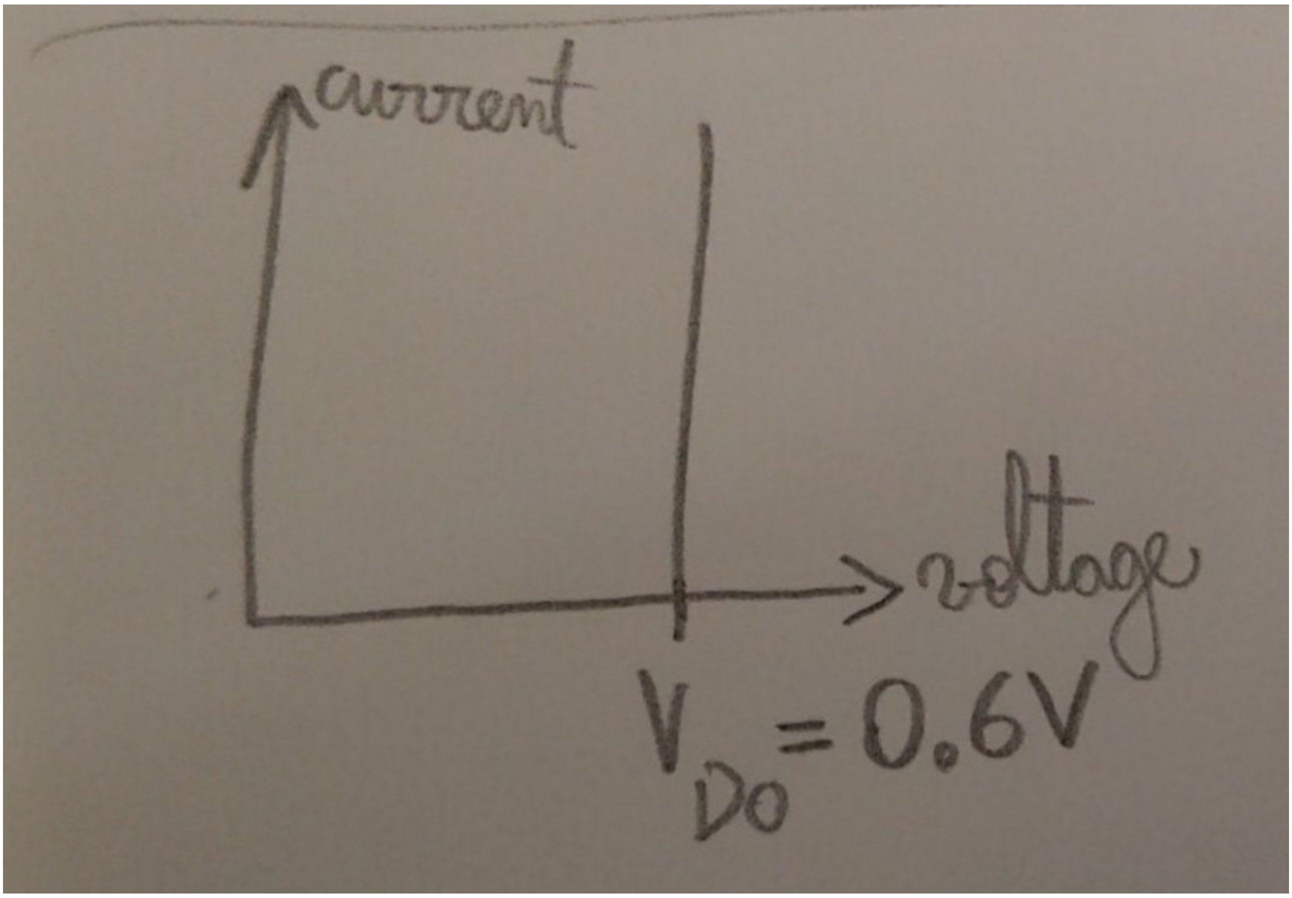
```
C = 1.9153354472e-8;
B = s*R*C/((s*R*C)^2 + 3 * s * R * C + 1)

H = A * B;
bode(H);

sortie= freqresp(H, w0);
"The magnitude should be close to 1"
magnitude = abs(sortie)
"The phase should be close to 0"
phase = arg(sortie)
```

Octave code to verify the solution

Conclusion :

C = 19.1533 nF;
$F_c = 141485\,Hz$

9. Plot the current vs. voltage characteristics of one of the diodes D1 or D2.



Since the series resistance is assumed to be infinite when ON and zero when off, th graph is just a straight line.

In the 2 next questions, assume that D1 and D2 are OFF.

10. Give an expression for the gain of the amplifier.

The two resistors can be associated in series.

$$A_0 = 1 + \frac{R_2 + R_3}{R_1}$$

11. Express voltage VR3 (Figure 1.2) vs. Ve.

It can be understood as a voltage divisor.

$$V_E = \frac{R_1}{R_1 + R_2} \cdot V_3 \Rightarrow \frac{V_3}{V_E} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

As shown before, we already have the output voltage. V_{R3} is the difference.

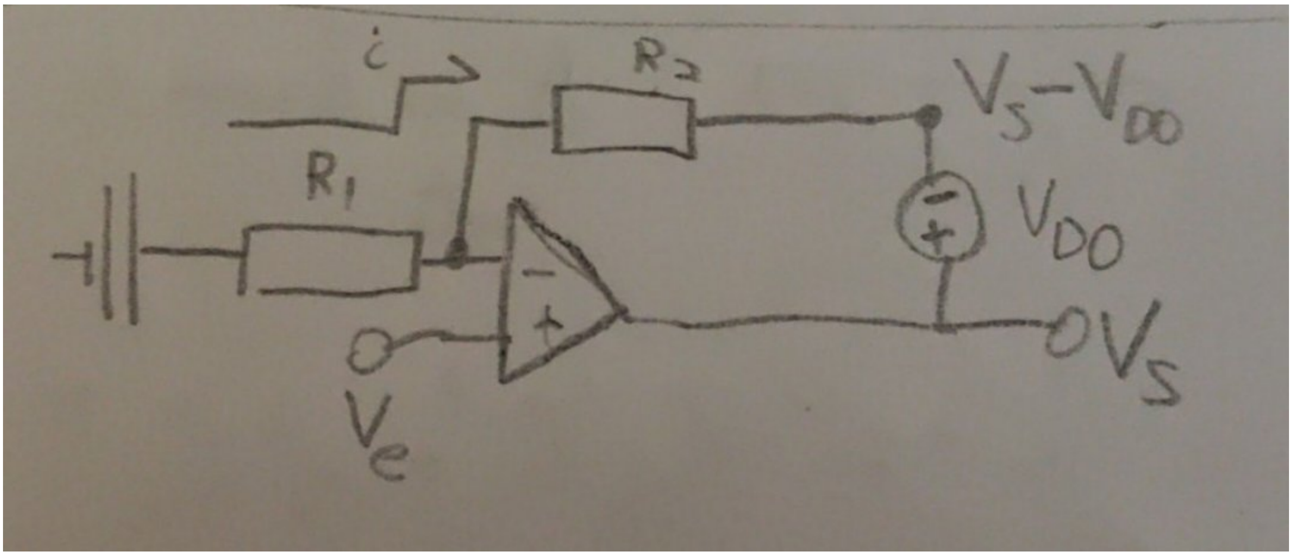
$$V_{R3} = V_3 - V_0 = \left(1 + \frac{R_2}{R_1} - \left(1 + \frac{R_2 + R_3}{R_1}\right)\right) V_E = -\frac{R_3}{R_1} \cdot V_E$$

12. Starting with D1 and D2 OFF, what is the condition on Ve for diode D2 to become ON? What is, then, the state of D1 ?

The voltage in the resistor needs to be greater than V_{DO} in the correct orientation. In the case of D2, it means V_{R3} = -0.6 V. D1 is in reverse polarization and is OFF.

$$V_E = -V_{R3} \frac{R_1}{R_3} = +V_{DO} \frac{R_1}{R_3} = \frac{60}{R_3}$$

13. In that case, redraw the equivalent circuit around the amplifier, and derive a relationship between Vs, Ve and VD0 when D2 in ON and D1 OFF.



Equivalent circuit with D2 ON

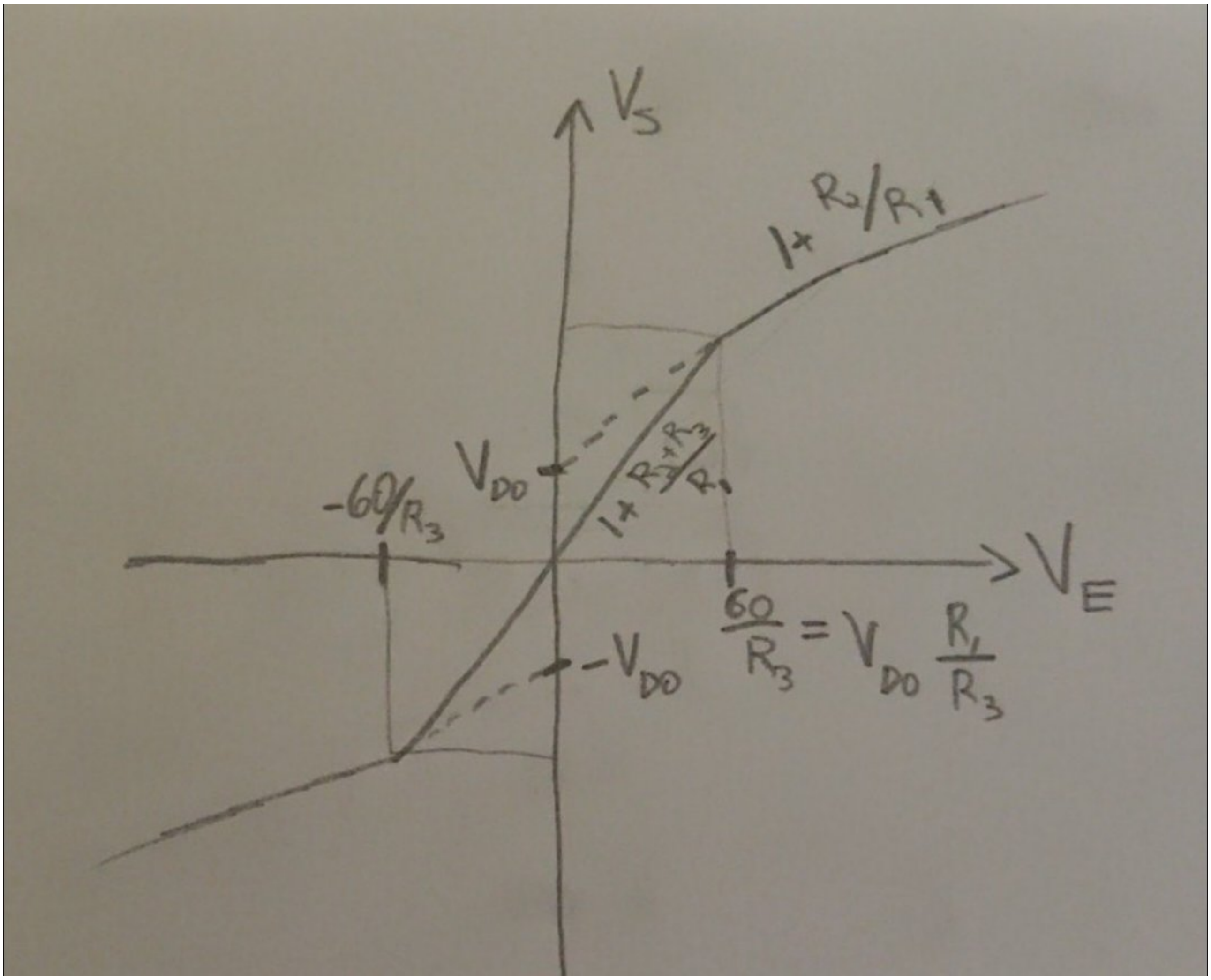
$$i = \frac{0 - V_E}{R_1} = \frac{V_E - (V_S - V_{DO})}{R_2} \Rightarrow -V_E R_2 = V_E R_1 - V_S R_1 + V_{DO} R_1 \Rightarrow V_S = V_E \left(1 + \frac{R_2}{R_1}\right) + V_{DO}$$

Notice that the linear term is just an amplifier without the R3 resistor.

14. Plot the transfer characteristics Vs = f(Ve) of the amplifier for -15V < Ve < +15V.

Analogously, it can be derived that, when D1 is ON, i.e., the input is negative enough:

$$V_S = V_E\left(1+\frac{R_2}{R_1}\right) - V_{DO}$$



The amplification characteristics (not to scale)

15. What is the purpose of diode D1 and D2 ? Please justify your answer.

They help stabilize the output. They give a non-linear gain to the amplifier that allows small oscillations to get bigger, but once they are big enough the gain decreases to prevent saturation.

16. What is the conditions on R3 for the output oscillation to start?

The graph in the previous image shows the amplification curve. R_2 and R_3 should be chosen son that this curve intersects the straight line from the ideal, calculated gain that matches what is demanded by B(jw). The choice depends on the desired output voltage and where the breakpoint should be located.

The essential condition is that the gain with R2 and R3 must be bigger than the ideal gain, and then only R2 should be smaller than the ideal, to guarantee an intersection. The bigger the difference, i.e., R3, the easier it is to control the output, but the signal will be more distorted.

17. Give an expression for output peak amplitude Vsmax vs. VD0, R1 et R2.

$$V_S = V_E\left(1+\frac{R_2}{R_1}\right)+V_{DO} \Rightarrow 1+\frac{R_2}{R_1}=\frac{V_S-V_{DO}}{V_E} \Rightarrow R_2 = R1\frac{V_S-V_{DO}-V_E}{V_E}=R1\left(A-1-\frac{V_{DO}}{V_E}\right)$$

$R_2=R_1\left(A-1-\frac{V_{DO}A}{V_{Smax}}\right)$, where A is the gain with band correction

Choosing to have an output of 5 volts peak-to-peak, this would yield:

R_2 = 141.8183 Ω

The corresponding Vemax = Vsmax / A = 0.78571 => any value above 60/Vemax = 76.4 Ω should converge. In order to provide a very easy control, I chose R_3 = 152.7 Ω, to be exactly in the middle of the graph.

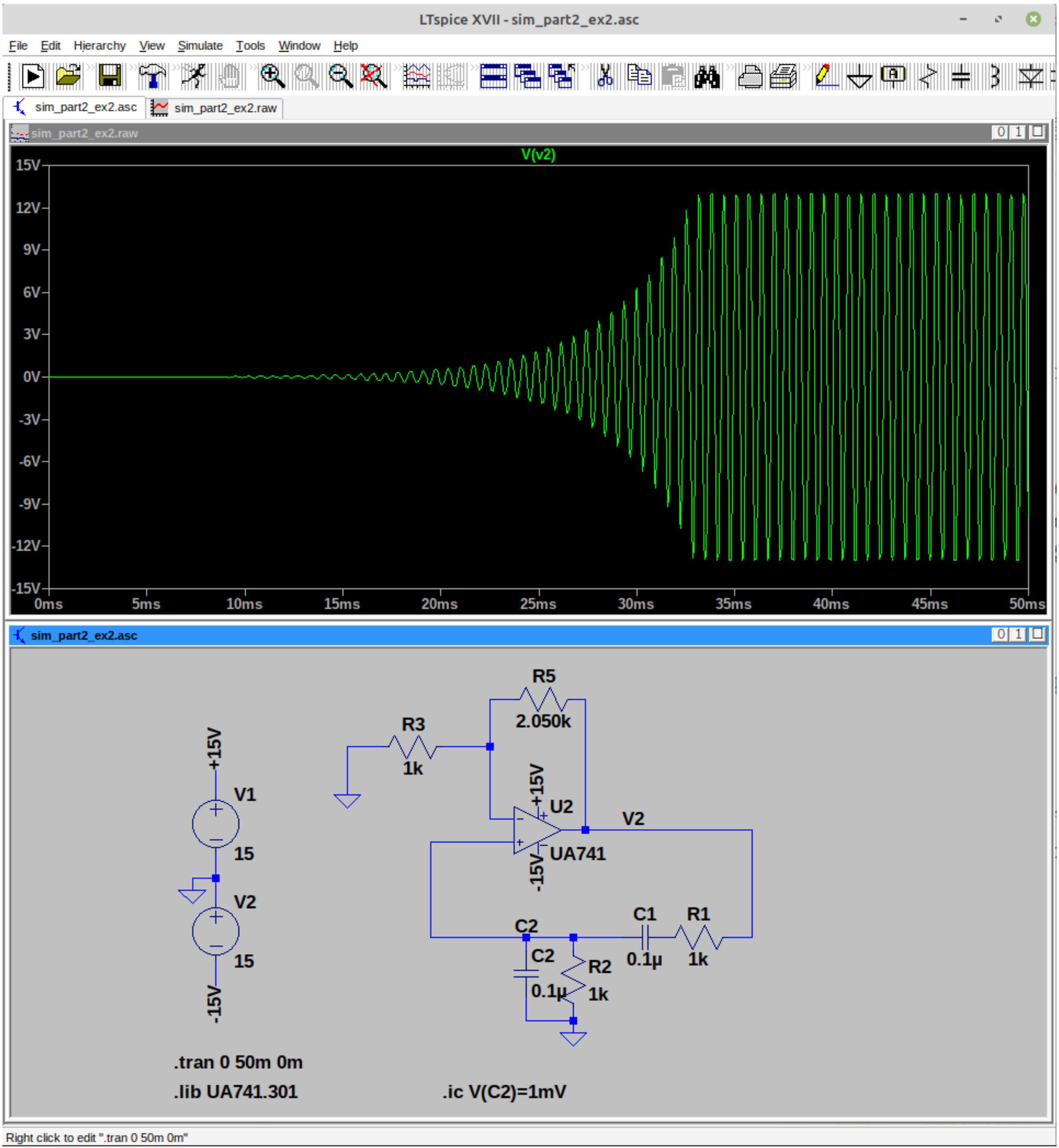
Part 2 : Simulation

Wien Oscillator

1. Calculate the value of oscillation frequency F_0 in Hz and of resistor R2 to maintain a stable oscillation.

$R_2 = R_1 2 = 2k$

2. Simulate the circuit using LTSpice, with UA741 opamp (see homework #1). Take R2 slightly higher than theory. Simulate in 'Transient' mode, and choose a sufficient stop time to observe steady-state. To observe the oscillation startup, select « start external voltage supply at 0V »


LTspice XVII - sim_part2_ex2.asc
File Edit Hierarchy View Simulate Tools Window Help
sim_part2_ex2.asc
sim_part2_ex2.raw
V(v2)
15V
12V
9V
6V
3V
0V
-3V
-6V
-9V
-12V
-15V
0ms 5ms 10ms 15ms 20ms 25ms 30ms 35ms 40ms 45ms 50ms
R5
2.050k
R3
1k
+15V
V1
15
V2
15
-15V
U2
UA741
V2
C1
0.1µ
R1
1k
C2
0.1µ
R2
1k
.tran 0 50m 0m
.lib UA741.301
.ic V(C2)=1mV
Right click to edit ".tran 0 50m 0m"

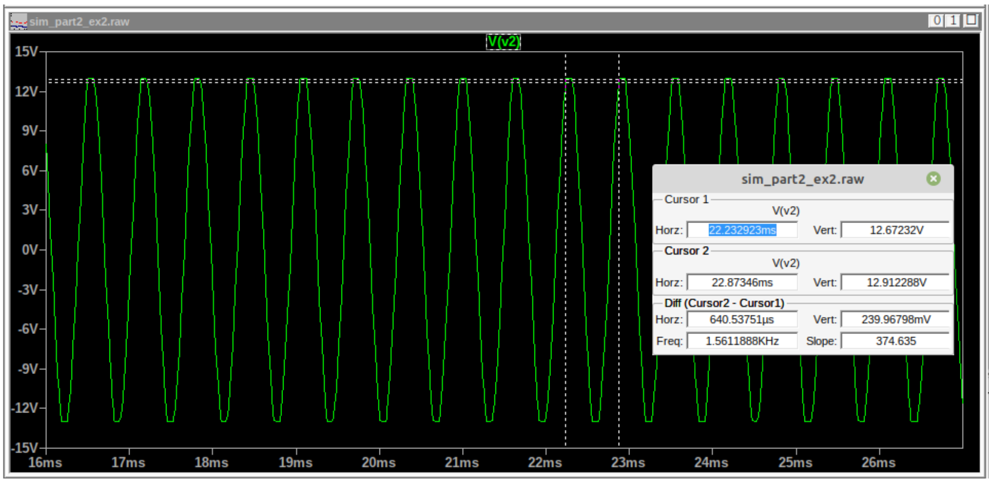
3. Compare theory with simulation and conclude.

The image below allows measuring the oscillating frequency of the circuit at around 1.56kHz.

This in inline with the theory as :

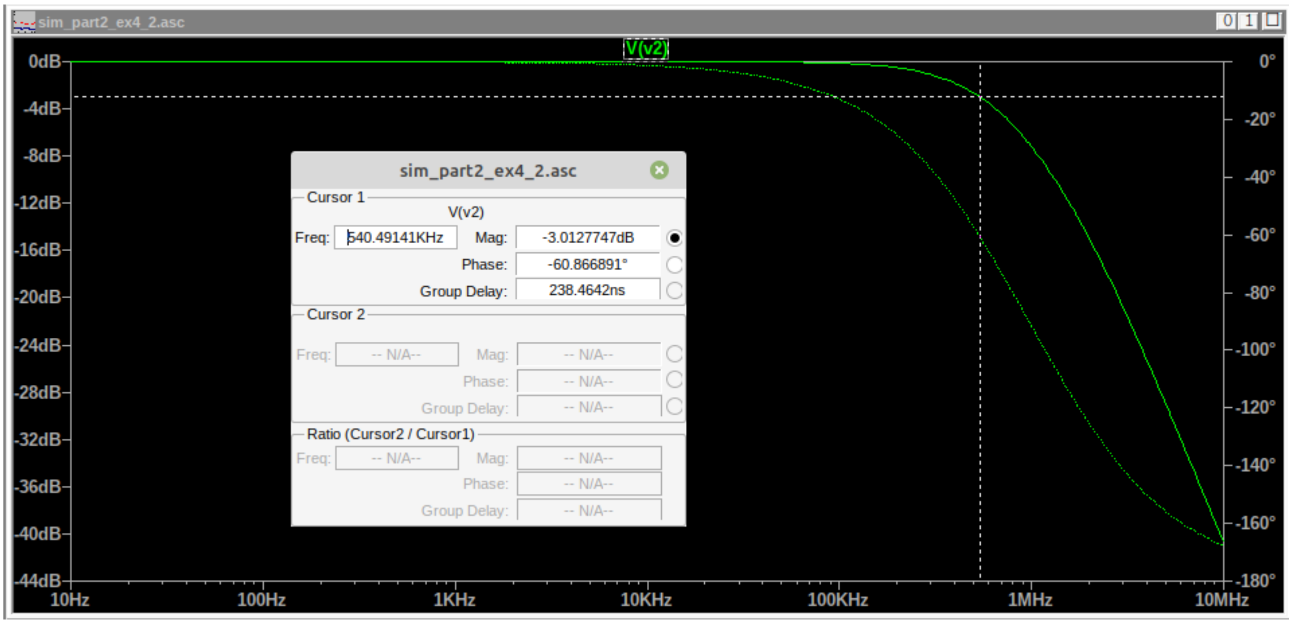
$$w_0 = \frac{1}{RC} = 10000\, rad/s$$

$$w_0 = 2\pi f_0 \rightarrow f_0 = 1591\, Hz$$



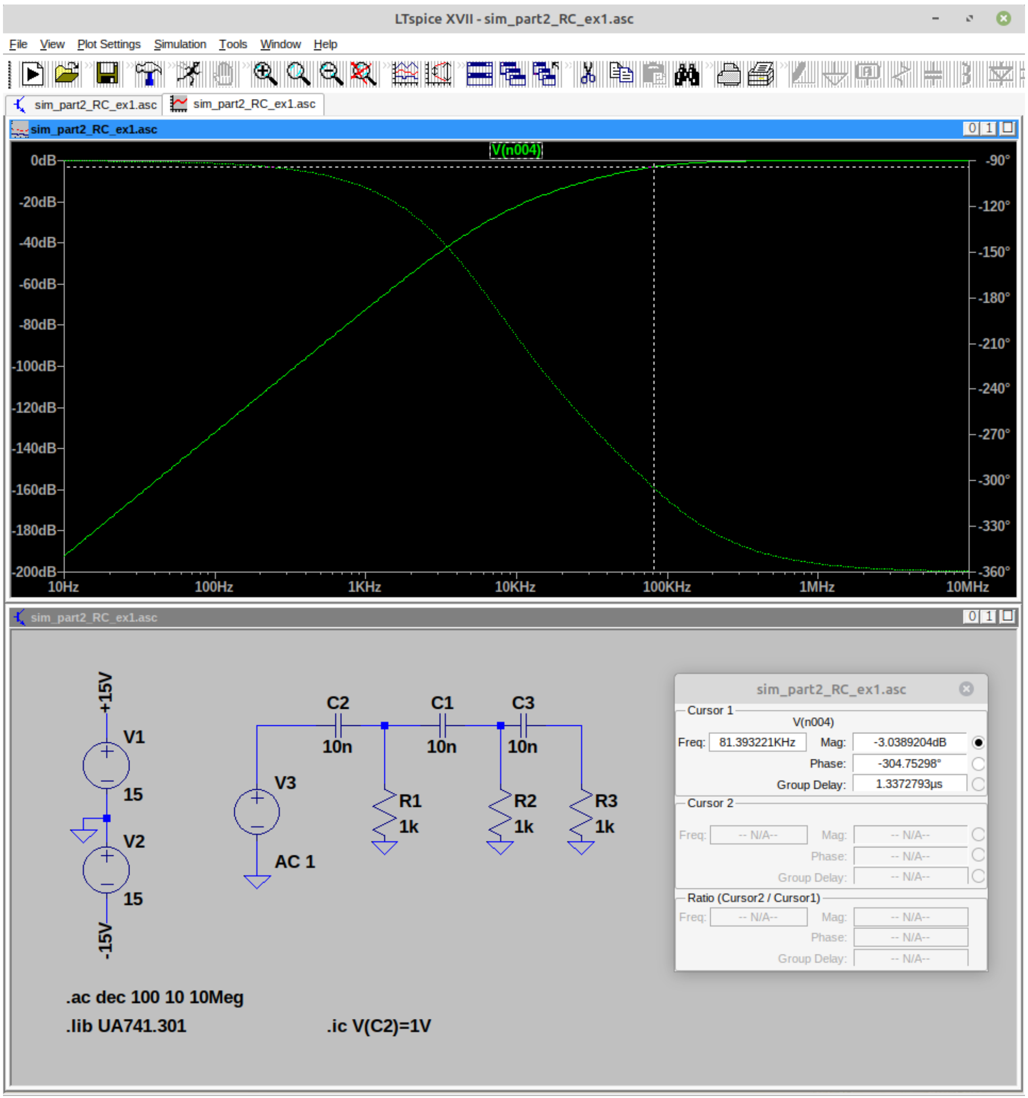
4. We know we want the oscillator to operate at the cutoff frequency of the amplifier. Run a simulation to validate the results from question 6 in part I.

Simulation shows the actual bandwidth of the operational amplifier is around 540kHz:



Oscillator with RC phase shifter

1. Using LTSpice, simulate TF β_{RC} (jω) = V_1 / V_S (amplitude and phase) of the RC phase shifter alone, not connected to the amplifier. Note: the RC phase shifter is loaded by R_1. which should be included in the circuit schematic.



2. Compare with theory (see slides of the course on Oscillators).

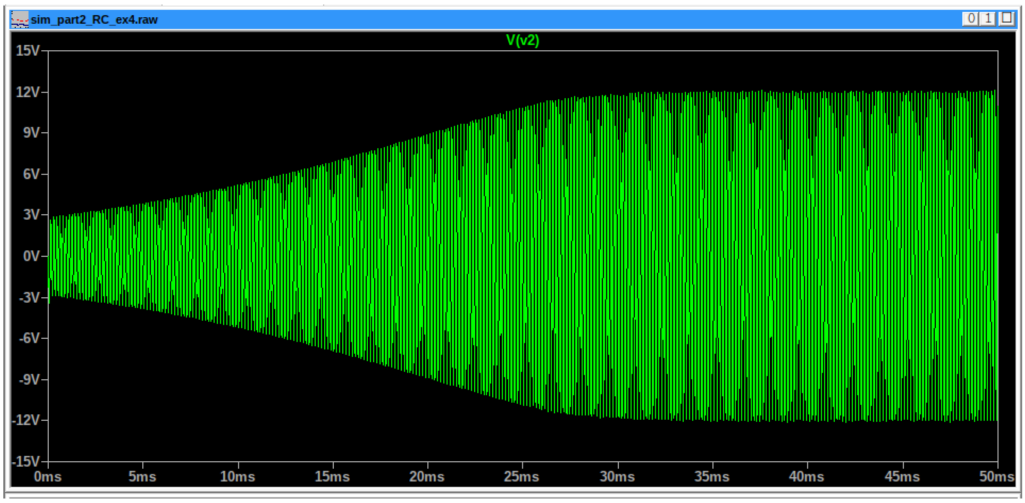
The shape of the graph is adequate, behaving as a high pass filter, and with phase decreasing with frequency.

3. Derive frequency F0 in Hz et the value of R2 for stable oscillations.

$$f_0 = \frac{1}{2\pi\sqrt{6}RC} = 6498\,Hz$$

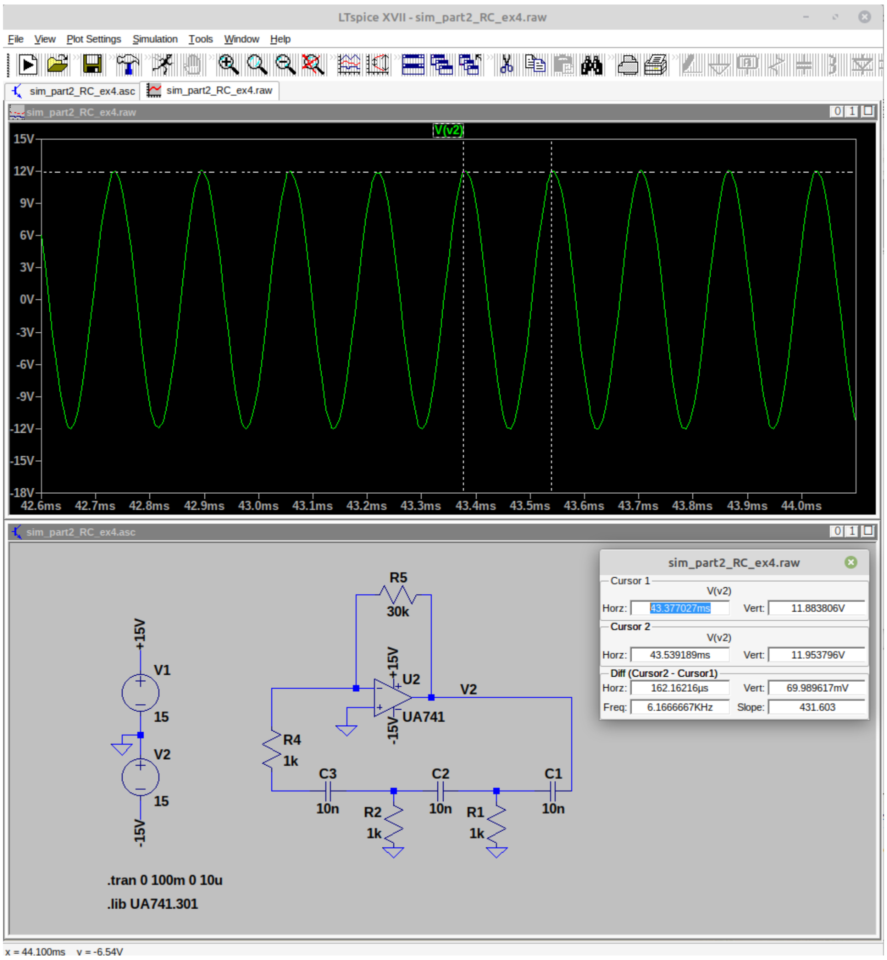
$$A = \frac{-R_2}{R} = -29 \rightarrow R_2 = 29\,k$$

4. Simulate the circuit of Fig. 2.2 using LTSpice, with UA741 opamp (see homework #1). Take R2 slightly higher than theory. Simulate in ‘Transient’ mode, and choose a sufficient stop time to observe steady-state. To observe the oscillation startup, select « start external voltage supply at 0V »



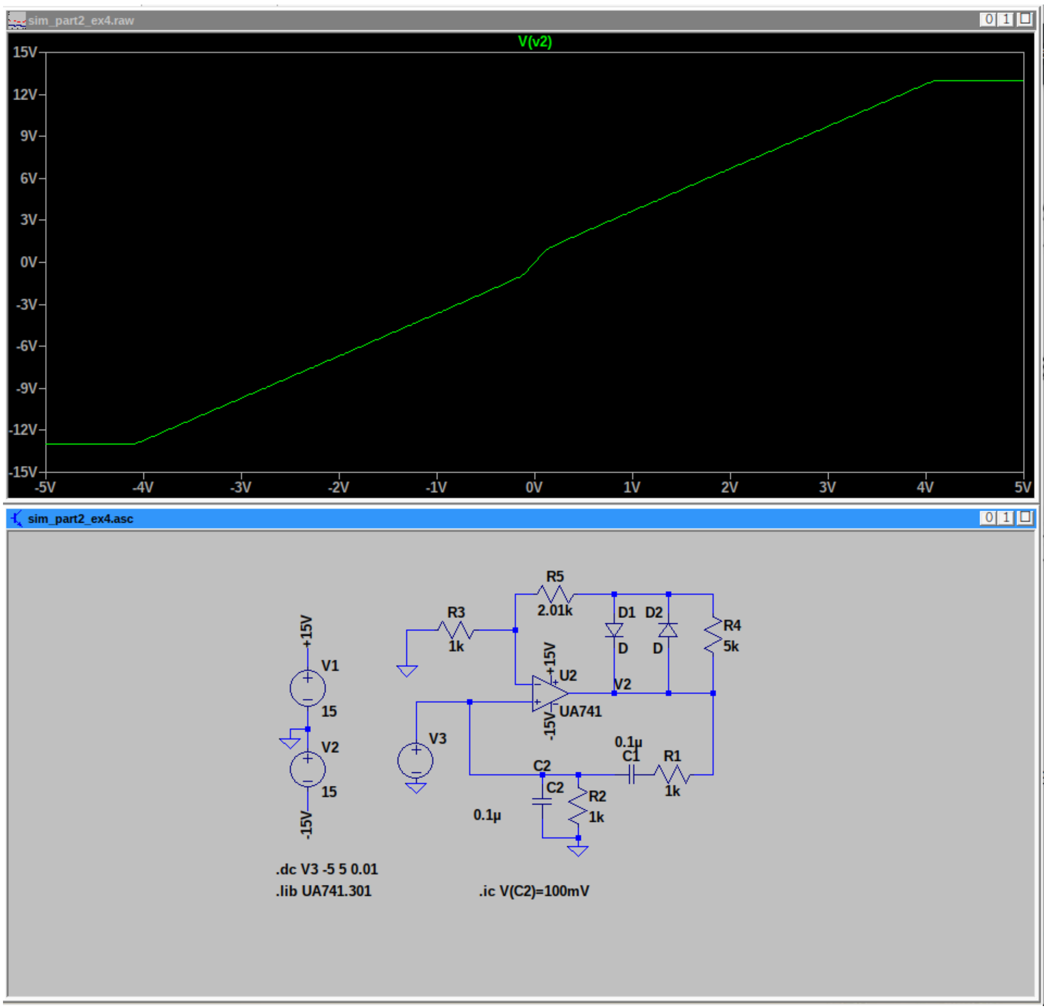
5. Compare theory with simulation and conclude.

The measured frequency corresponds well to the theoretical value of about 6.5kHz.



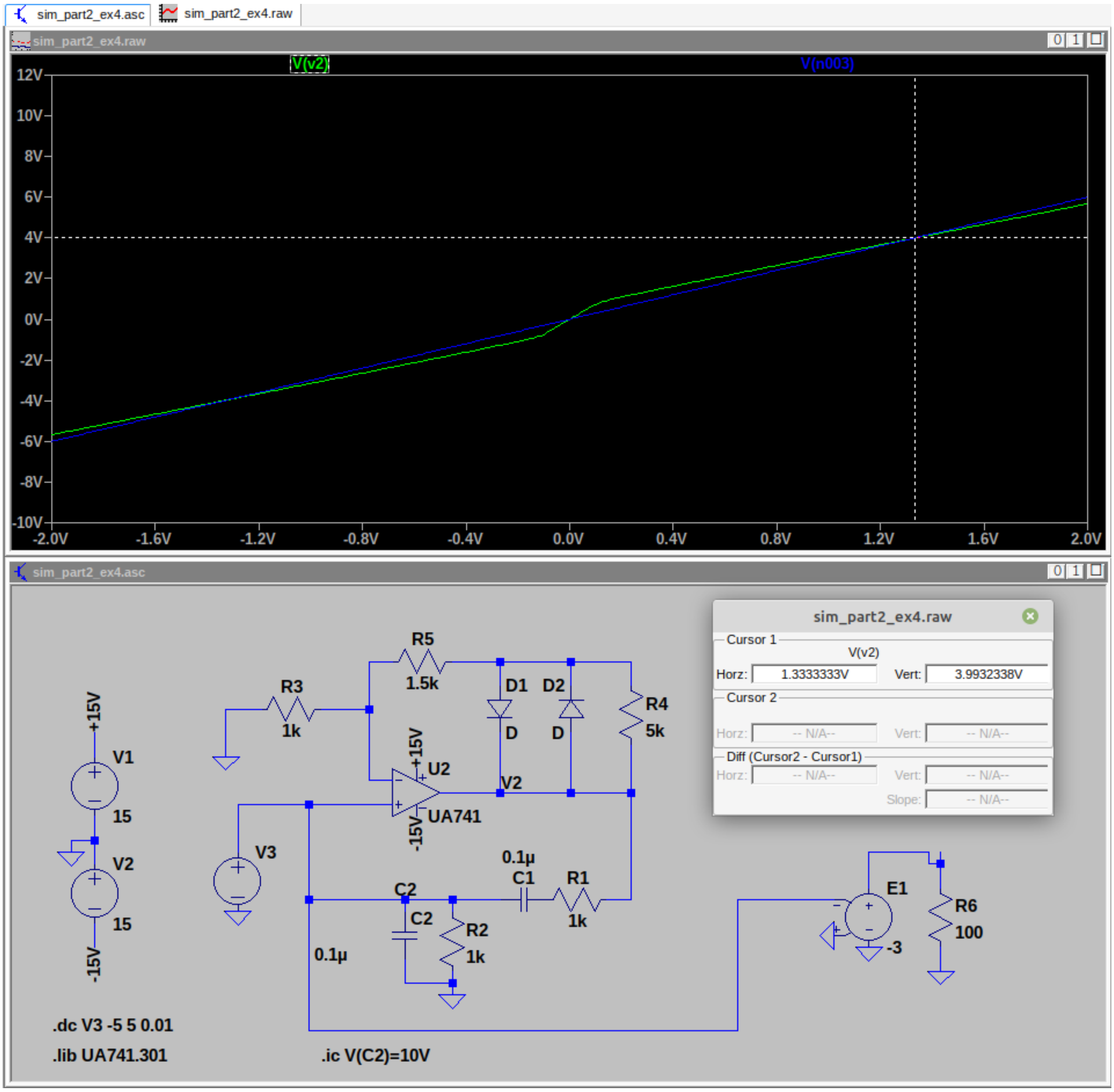
Amplitude control

1. Using LTSpice, simulate the transfer characteristics V_S vs. V_1 of the amplifier alone, including diodes (Fig. 2.3). For the purpose, set simulation mode to « DC sweep » corresponding to an input voltage ramp. For the diodes, use the standard model in the LTSpice library.



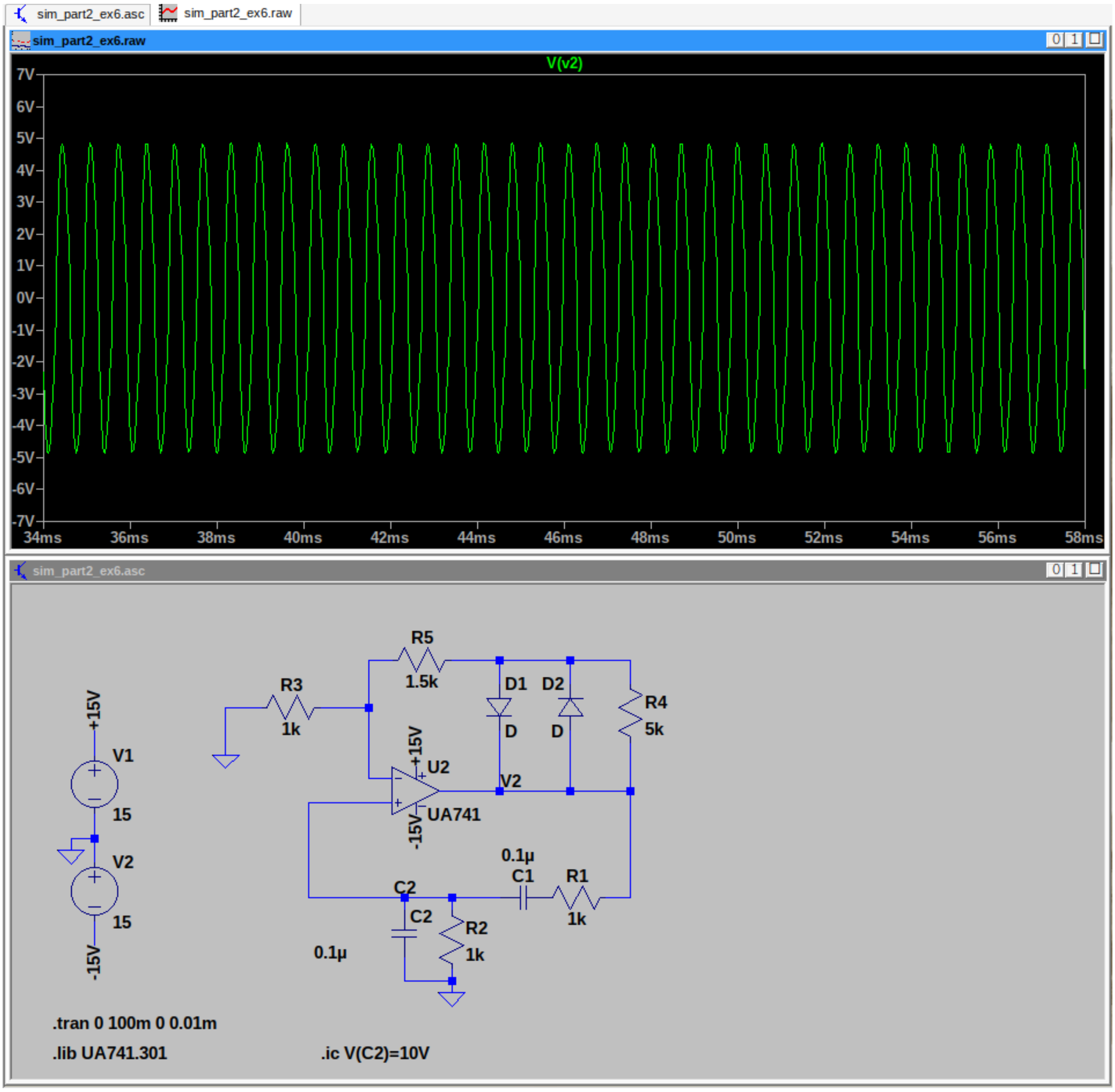
2. For R2 = 1,5 kΩ, show how the amplitude of V_S can be predicted from transfer characteristics.

It is the point in which the new non-linear transfer curve intersects the 1/beta transfer curve. As it can be seen in the image below, the result should be about 4V peak voltage amplitude.



3. For R2 = 1,5 kΩ, simulate the full Wien oscillator (Fig. 2.2), including diodes.

4. Compare amplitude of Vs obtained from simulation and from the previous question.



The resulting curve is well at around 4V peak.

5. Compare with theory (question 17), if you have gone that far.

$$R_2 = R_1\left(A - 1 - \frac{V_{DO} A}{V_{Smax}}\right) \Rightarrow 1500 = 1000\left(3 - 1 - \frac{0.66 \cdot 3}{Vsmax}\right) \Rightarrow 1.5 = 2 - \frac{1.98}{Vsmax} \Rightarrow \frac{1.98}{Vsmax} = 0.5$$

$$Vsmax = \frac{1.98}{0.5} = 3.96\,V$$

This simple calculation does not take into account the low pass characteristics of the amplifier, maybe that is why the 4V are not exactly seen on the output. It is an approximations that gets close to the real value.