Post Session 5

Students : Gabriel Gosmann, Gabriel Fontenele, Yann Mello.

In a LaTeX file or Word file give a real-life model (description and equations) described by **linear** ODEs. Then select a measured variable (give the name of the sensor(s) for example "displacement sensor"). Prove that this system is observable and then design an observer (explain carefully the dynamics and gain selection). Provide figures obtained from Simulink showing the effectiveness of your observer.

System : Induction motor (model from the book)

input variables : voltages on the stator on the Clarke-domain (alpha and beta)

$u = [\Gamma_l, V_{s\alpha}, V_{s\beta}]^T$

state varaibles : angular speed, magnetic flux in the rotor and currents on the stator

$x = [\omega, \phi_{r\alpha}, \phi_{r\beta}, i_{s\alpha}, i_{s\beta}]^T$

equations of circular motion :

$$J\frac{d\omega}{dt} = \Gamma_e - \Gamma_l - f_v\omega$$

where:

J is the momment of ineria of the axis

ω is the angular speed of the motor

Γ_e is the electric torque of the motor

Γ_l is the load torque of the motor

f_v is the viscous friction of the motor

equation of electromagnetic torque :

$$\Gamma_e = p\frac{L_m}{L_r}(i_{s\beta}\phi_{r\alpha} - i_{s\alpha}\phi_{r\beta})$$

four electromagnetic equations:

$$\frac{d\phi_{r\alpha}}{dt} = ai_{s\alpha} - b\phi_{r\alpha} - p\omega\phi_{r\beta}$$

$$\frac{d\phi_{r\beta}}{dt} = ai_{s\beta} - b\phi_{r\beta} - p\omega\phi_{r\alpha}$$

$$\frac{di_{s\alpha}}{dt} = -\gamma_1 i_{s\alpha} + \gamma_2\phi_{r\alpha} + \gamma_3\omega\phi_{r\beta} + \gamma_4 V_{s\alpha}$$

$$\frac{di_{s\beta}}{dt} = -\gamma_1 i_{s\beta} + \gamma_2\phi_{r\alpha} - \gamma_3\omega\phi_{r\alpha} + \gamma_4 V_{s\beta}$$

the coefficients $a, b, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \sigma, \alpha_1, \alpha_2, \alpha_3$ are obtained from the parameters of the machine:

R_s is the resistance of the stator in $[\Omega]$

L_s is the inductance of the stator in $[H]$

R_r is the resistance of the rotor in $[\Omega]$

L_r is the inductance of the rotor in $[H]$

p is the number of pole pairs in the machine

L_m is the mutual inductance of the machine in $[H]$

according to the following equations :

$$a = \frac{R_r L_m}{L_r} \qquad b = \frac{R_r}{L_r}$$

$$\gamma_1 = \frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2} \ , \ \gamma_2 = \frac{R_r L_m}{\sigma L_s L_r^2} \ ,$$

$$\gamma_3 = \frac{pL_m}{\sigma L_s L_r} \ , \ \gamma_4 = \frac{1}{\sigma L_s}$$

$$\sigma = 1 - \frac{L_m^2}{\sigma L_s L_r}$$

$$\alpha_1 = \frac{pL_m}{JL_r} \ , \ \alpha_2 = \frac{1}{J} \ , \ \alpha_3 = \frac{f_v}{J}$$

so, we can write the evolution of the state-variable over time according to the following equations :

$$\dot{x_1} = \alpha_1(x_2x_5 - x_3x_4) - \alpha_2\Gamma_l - \alpha_3 x_1$$

$$\dot{x_2} = ax_4 - bx_2 - px_1x_3$$

$\dot{x_3} = ax_5 - bx_3 + px_1x_2$

$\dot{x_4} = -\gamma_1 x_4 + \gamma_2 x_2 + \gamma_3 x_1 x_3 + \gamma_3 u_1$

$\dot{x_5} = -\gamma_1 x_5 + \gamma_2 x_3 + \gamma_3 x_1 x_2 + \gamma_4 u_2$

if written in a state-space model is gives the following equations:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \end{bmatrix} = \begin{bmatrix} -\alpha_3 & \alpha_1 x_5 & -\alpha_1 x_4 & -\alpha_1 x_3 & \alpha_1 x_2 \\ -px_3 & -b & -px_1 & a & 0 \\ px_2 & px_1 & -b & 0 & a \\ \gamma_3 x_3 & \gamma_2 & \gamma_3 x_1 & -\gamma_1 & 0 \\ -\gamma_3 x_2 & -\gamma_3 x_1 & \gamma_2 & 0 & -\gamma_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \gamma_4 & 0 \\ 0 & 0 & \gamma_4 \end{bmatrix} \begin{bmatrix} \Gamma_L \\ V_{s\alpha} \\ V_{s\beta} \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

This space state representation is clearly non-linear as the matrice A has many instances where the evolution of a state depends on the multiplication by another state.

We can linearize the system considering a steady-state operation at $x_1 = \omega_0$,

which would give the following system:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -b & -p\omega_0 & a & 0 \\ 0 & p\omega_0 & -b & 0 & a \\ 0 & \gamma_2 & \gamma_3\omega_0 & -\gamma_1 & 0 \\ 0 & -\gamma_3\omega_0 & \gamma_2 & 0 & -\gamma_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \gamma_4 & 0 \\ 0 & 0 & \gamma_4 \end{bmatrix} \begin{bmatrix} \Gamma_L \\ V_{s\alpha} \\ V_{s\beta} \end{bmatrix}$$

written in matlab:

```
run("params.m");
```

```
parameters = 1×8
    1.6330    0.1420    0.9300    0.0760    2.0000    0.0990    0.0290    0.0038
```

```
A = [ 0 0 0 0 0 ;
      0 -b -p/w0 a 0 ;
      0 p*w0 -b 0 a ;
      0 gamma_2 gamma_3*w0 -gamma_1 0 ;
      0 -gamma_3*w0 gamma_2 0 -gamma_1 ] ;

B = [ 0 0 0 ; 0 0 0 ; 0 0 0 ; 0 gamma_4 0 ; 0 0 gamma_4 ] ;

C = [ 1 0 0 0 0 ; 0 0 0 1 0 ; 0 0 0 0 1 ] ;

Ob = obsv(A,C) ;
rank(Ob)
```

```
ans = 5
```

```
% make Luenberguer observer
z = 0.7
```

```
z = 0.7000
```

```
tr = 2e-3 ;
w = 2.9/tr ;
r1 = roots([1 2*z*w w*w])
```

r1 = 2×1 complex

10^3 ×

```
  -1.0150 + 1.0355i
  -1.0150 - 1.0355i
```

```


z = 0.7
```

```
z = 0.7000
```

```
tr = tr/2 ;
w = 2.9/tr ;
r2 = roots([1 2*z*w w*w])
```

r2 = 2×1 complex

10^3 ×

```
  -2.0300 + 2.0710i
  -2.0300 - 2.0710i
```

```


z = 0.7
```

```
z = 0.7000
```

```
tr = tr/2 ;
w = 2.9/tr ;
r3 = roots([1 2*z*w w*w])
```

r3 = 2×1 complex

```
10^3 ×
  -4.0600 + 4.1420i
  -4.0600 - 4.1420i
```

```
L = place(A', C', [ r1(1) r1(2) r2(1) r2(2) real(r3(1)) ] )' ;

A_obs = A ;
B_obs = [B L] ;
C_obs = C ;
D_obs = zeros(3,6) ;
```