

Name: \_\_\_\_\_

N.º: \_\_\_\_\_

Duration: 120 minutes. (P1 - 60 min + P2 - 60 min).

Mark, on the right, the parts submitted for evaluation.

P1	P2
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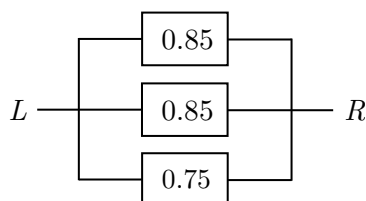
Answer the questions clearly and concisely, showing all necessary calculations.

Bibliographic consultation is allowed.

Internet access is strictly prohibited.

## P1

1. Let  $E_1$  and  $E_2$  be two independent events with probabilities 0.5 and 0.3, respectively. Determine the probability that neither of them occurs using a Venn diagram. (1.5 val)
2. Consider a circuit composed of three devices, as shown in the figure below. The circuit works if, from left ( $L$ ) to right ( $R$ ), there is at least one path with a functional device.



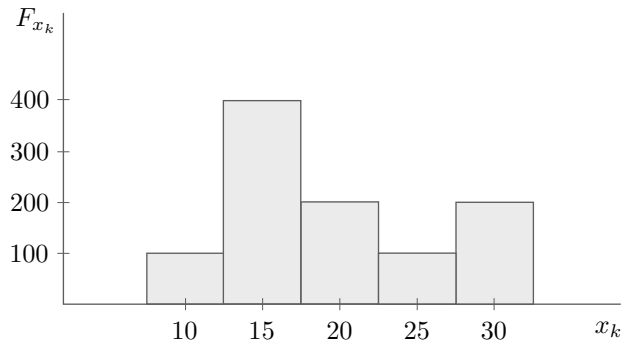
The probability of each device working is shown in the figure and does not depend on the state of the other devices. If the circuit is working, what is the probability that all the devices are working?

(3.0 val)

3. An electronic component manufactured by a well-known technological innovation company has a lifespan characterised by a normal distribution with a mean of 6.5 years and a variance of 3.7 years<sup>2</sup>.
  - a) If the company offers a 3-year warranty, what proportion of components will be replaced under the warranty? (1.5 val)
  - b) Determine the warranty period the manufacturer should offer if they only intend to replace 1% of the components. (2.0 val)
  - c) Out of 10 components produced, what is the probability that at least 2 will last more than 9 years? (2.0 val)

## P2

4. Consider the following histogram of absolute frequencies:



$M_{x_k}$	$F_{x_k}$	$f_{x_k}$
10		
15		
20		
25		
30		

- Draw the corresponding frequency polygon over the histogram and show how to determine the mode graphically. (1.0 val)
- In the table presented, fill in the values of the relative frequencies of the different classes,  $f_{x_k}$ . (1.0 val)
- Determine the mean, the variance, and the 75th percentile ( $p_{75}$ ). (1.5 val)

5. The following statistic was determined on a random sample of 30 elements:

$$\sum_{i=1}^{30} (x_i - \bar{x})^2 = 580$$

- Determine the amplitude of the 95% confidence interval for the population mean. (1.5 val)
- What effect would increasing the sample size have on the amplitude of the confidence interval defined in the previous question? Justify your answer. (0.5 val)
- Calculate the Lower 95% limit for the variance confidence interval. (1.5 val)

6. A company that manufactures light bulbs considers that its production is under control if the proportion of bulbs produced without defects is greater than 90%. In order to check the quality of production, a test was carried out on 100 bulbs, and it was found that 6 were defective.

- Using an appropriate hypothesis test, verify whether the production of defect-free bulbs remains above the stated control limit. Compute the  $p$ -value. (1.5 val)
- Check the same hypothesis using the Confidence Intervals methodology. (1.5 val)

**Good Work**