UNIT-V

Noise: Undesired electrical signals which are introduced with a message signal during the transmission or processing are called noise. Thus, it is an unwanted signal that corrupts a desired message signal.

In general, noise may be predictable or unpredictable (random) in nature. The predictable noise can be estimated and eliminated by proper engineering design.

Ex: Power supply fluctuations, unwanted oscillations in the amplifiers.

→ The predictable noise is generally a man-made noise and can be reduced or eliminated.

→ Unpredictable noise varies randomly with time and ~~will~~ we have no control over this noise. Identification of the message signal at the receiver depends on the amount of noise added to the message signal during the process of communication. In the absence of noise, identification of the message signal at the receiver is perfect. The amount of noise power present in the received signal decides the minimum power level of the desired message signal at the transmitter.

Sources of noise:

There are two types of sources of noise.

(i) External noise.

(ii) Internal noise.

EXTERNAL NOISE:

External noise is created outside the circuit and it includes

(i) Erratic nature disturbances.

(ii) Man made noise.

(iii) Extra Terrestrial noise.

Erratic Nature Disturbances: This type of noise does not occur generally. It is caused by electrical storms and other atmospheric disturbances.

→ This noise is unpredictable in nature and is also known as atmospheric or static noise.

→ The atmospheric noise is less effective above 30 MHz.

Man made noise (Industrial noise): This noise is because of undesired pick-ups from electrical appliances such as motors, automobiles air crafts etc.

→ This type of noise is under human control and can be eliminated by removing the source of the noise.

→ This noise is effective in the frequency range 1 MHz – 500 MHz.

~~INTERNAL NOISE:~~

~~It is created by~~

Extra Terrestrial noise: It is of two types:

(i) Solar noise (ii) Cosmic noise

Solar noise:

Solar noise is the electrical noise eminating from the sun. There is regular radiation of noise from the sun. This radiation of noise from the sun is due to the fact that sun is a big body at an extremely high temperature and it radiates electric energy in the form of noise over a very wide frequency spectrum including the spectrum which is used for the radio-communications. But, this noise is very high for every eleven years. That is the reason, the satellites will be carefully launched.

Cosmic noise:

This is due to the distant stars emitting the electrical energy in the form of light. These stars also have high temperature and therefore radiation noise is as similar to sun. But this is less effective than solar noise.

INTERNAL NOISE:

This is created inside the circuit or communication system.

It is classified as

✓(i) Short noise.

(ii) Partition noise.

(iii) Low frequency or flicker noise

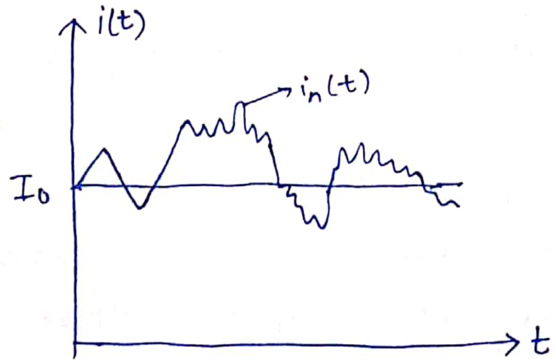
(iv) High frequency or transit noise

✓(v) Thermal noise.

Shot noise:

Shot noise appears in active devices due to the random behaviour of charge carriers (electrons & holes). In electron tubes, short noise is generated due to the random emission of electrons from cathodes. In semiconductors, it is caused due to the random diffussion of minority carriers or random generation and recombination of electron hole pairs.

In general, current is a combination of discrete pulses, everytime a charge carrier moves from one point to another point. Therefore, current appears to be continuous though it is a discrete phenomenon. The nature of current variation with time is shown in figure below.



Here, the current fluctuates above a mean value I_0. This current $i_n(t)$ varies around the mean value (I_0) which is termed as shot noise. These variations of the current are not visualised by normal instruments. So, we assume it as a constant current $= I_0$.

Total current $i(t)$ can be expressed as $\boxed{i(t) = I_0 + i_n(t)}$.

→ Power Density Spectrum of Shot Noise in Diodes :

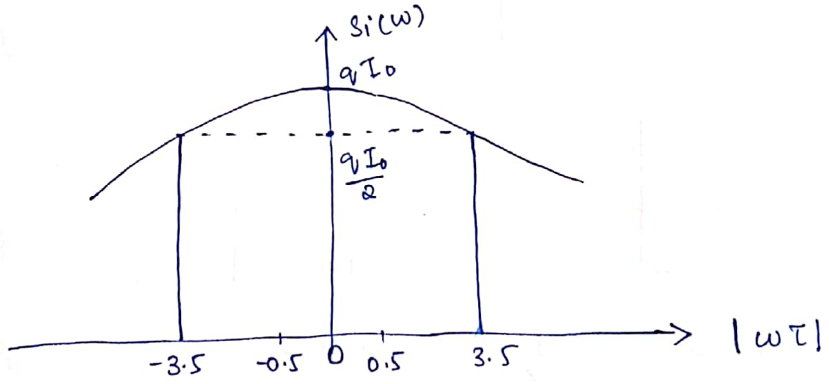
The number of electrons contributing the random stationary current $i_n(t)$ are very large in diodes. Assuming that the electrons do not interact with each other during their movement or emission, so the process may be considered as statistically independent. According to central limit theorem, such a process has a Gaussian distribution. Hence, shot noise is Gaussian distributed with a zero mean. The total diode current is taken as sum of the current pulses, each pulse being formed by the transit of electron from cathode to anode.

For all practical purposes, the power spectral density of such a process is given by $\boxed{S_i(\omega) = q \cdot I_0}$

where q : charge of electron $= 1.6 \times 10^{-19}$ C.

I_0 : mean value of current in Amps.

REPRESENTATION OF POWER SPECTRAL DENSITY OF SHOT NOISE :-



The shot noise is independant of frequency wrt power spectral density. $S_i(\omega) = q \cdot I_0$.

This type of frequency independence is only upto a frequency range decided by transit time of the $\bar{e}$ to reach the anode from cathode. Beyond this frequency the power spectral density varies with the frequency as shown in figure. The transit time of an electron in a diode depends on anode voltage τ

and is given as $\tau = 3.36 \times \frac{d}{\sqrt{V}} \mu$.

where 'd' is the spacing between anode & cathode.

From the figure, it is observed that $|\omega\tau| = 0.5$.

During this time interval, $S_i(\omega)$ is constant (flat).

→ In general, shot noise is independant of frequency below 100 MHz. So most of the applications of communication will be covered in this frequency range except UHF (ultra high frequency) and micro wave frequencies.

SCHOTTKY FORMULA :

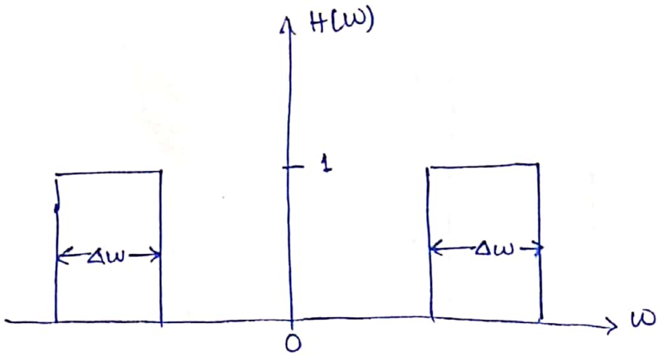
→ Power spectral density of noise current $i_n(t)$ for statistically independant process is given by $S_i(\omega) = q \cdot I_0$.

→ The mean square value or average power of the randomly fluctuating noise current will be $\overline{i_n^2} = 2qI_0(\Delta f)$

$$(E[i_n^2] = \overline{i_n^2})$$

where $2\Delta f$ is the bandwidth of the measuring system which is below 100 MHz.

The equation above shown is called Schottky formula.



From the figure, the bandwidth of the signal is obtained as $2\Delta f$.

So, the average power is the product of power spectral density and the magnitude of transfer function $H(\omega)$.

Let us consider the electrons are involving with each other, the process is not being independant, in such a case the power spectral density is given by $S_i(\omega) = 2 \cdot q I_0$.

where 'α' is the smoothing constant varying between 0.01 to 1, the purpose of 'α' is to smoothen the output current under the involvement of $\bar{e}^s$ with each other. If the smoothing is more, 'α' value is greater.

'α' is defined as

$$\alpha = \frac{1.288\, k \cdot T_c \cdot g_d}{q I_0}$$

where T_c : cathode temperature in degrees Kelvin

and 'k' is Boltzmann constant

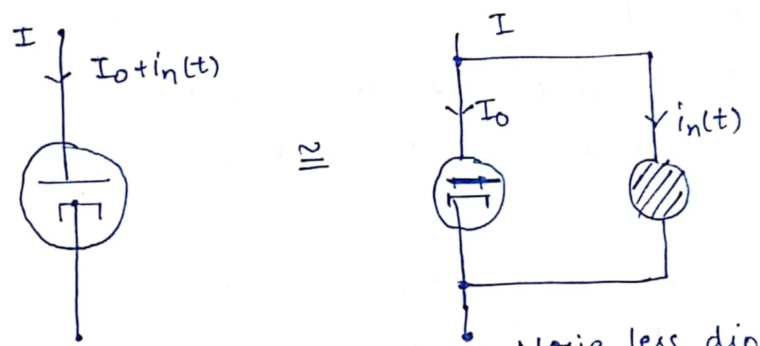
* $k = 1.38 \times 10^{-23}$ J/c

g_d : dynamic conductance of the diode

by substituting this value in $S_i(w)$; we get

$$\boxed{S_i(w) = 1.288\, k\, T_c\, g_d}$$

Equivalent Circuit of Noisy Diode :



Noisy diode

Noise less diode.

*Partition Noise :

When a current has 2 or more paths, the noise generated is called partition noise. Hence the noise generated in diode is less than the noise generated in transistor. The spectrum of this noise is flat.

* Flicker noise (Low Frequency Noise) :

The fluctuations in the carrier density generates fluctuations in the conductivity of the material. This produces a fluctuating voltage drop in the material. This fluctuating voltage drop is called flicker noise. The power spectral density of this noise is inversely proportional to the frequency ie.

$$S(w) \propto \frac{1}{f}.$$

∴ The flicker noise is significant at low frequencies

(Generally below few KHz).

* Transit time noise / High frequency noise:

In semiconductors, when the transit time of charge carriers crossing a junction is comparable with the time period of the signal. Some charge carriers diffuse back to the emitter or source. This process gives rise to conductivity of the material with the frequency.

This type of increased current components are random in nature and are very significant at high frequencies. This type of noise is called transit time noise.

Thermal / Resistor noise:

The thermal / Johnson noise [Resistor noise] is the random noise which is generated in resistor or resistive components due to the random and rapid motion of the molecules, electrons and atoms.

The noise power is proportional to its absolute temperature and also to the bandwidth over which the noise is measured.

$$P_n \propto T B$$

$$\boxed{P_n = K T B}$$

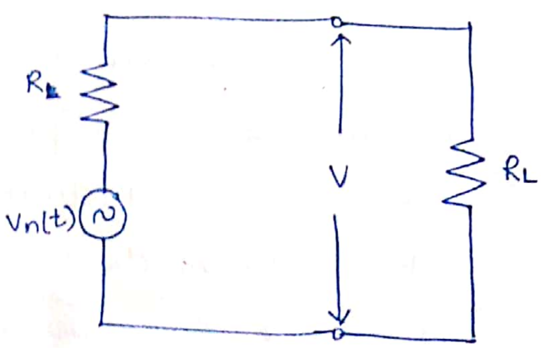
where K : Boltzmann constant

T : Temperature

B : Bandwidth of interest

Voltage and Current Models of a Noisy Resistor:

Voltage Model:



If DC meter is connected across the resistor at room temperature (300K), then no voltage is displayed by the voltmeter. However if a very sensitive electronic voltmeter is used, then it display some voltage. This happens because of the fact that every resistor may be treated as a noise generator. This noise voltage is random in nature.

The figure shown is an equivalent circuit of a resistor as a noise voltage generator. From the equivalent circuit, we can compute the resistor's equivalent noise voltage $V_n(t)$.

$$P_n = \frac{V^2}{R_L}$$

$(\because R_L = R \rightarrow \text{power transfer thm})$.

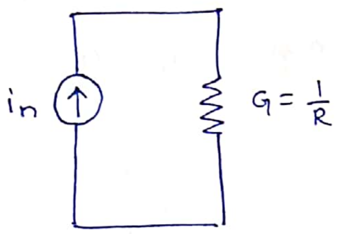
$$P_n = \frac{V^2}{R} \qquad V = \frac{V_n}{2}$$

$$\therefore P_n = \frac{\left(\frac{V_n}{2}\right)^2}{R} = \frac{V_n^2}{4R}.$$

$$\Rightarrow V_n = \sqrt{4RP_n}$$

$$\Rightarrow \boxed{V_n = \sqrt{4KTBR}} \quad (\because P_n = KTB).$$

Current Model:



$$\boxed{i_n = \sqrt{4GKTB}}$$

1. An amplifier operating over the frequency range from 18-20 MHz has a 10 KΩ input resistor. Calculate RMS voltage at the input to this amplifier if ambient temp is 27°C.

Soln:

$$V_n = \sqrt{4KTBR}.$$

$$K = 1.38 \times 10^{-23}$$

$$B = 2 \times 10^{6}\ \text{Hz}.$$

$$R = 10 \times 10^{3}\ \Omega$$

$$T = 300\ \text{K}$$

$$V_n = \sqrt{181.98 \times 10^{-14}}$$

$$= 181.98 \times 10^{-7}\ \text{V}$$

$$= 1.819 \times 10^{-5}\ \text{V}$$

Addition of noise due to several sources in series.

Let us consider several thermal noise sources i.e resistors $R_1, R_2, R_3 \ldots$ etc in series producing noise voltages $V_{n_1}, V_{n_2}, V_{n_3}$ respectively.

We know that the RMS value of the noise voltage by a resistor 'R' is given by $V_n = \sqrt{4RKTB}$

$$\Rightarrow V_n^2 = 4RKTB$$

$$\therefore V_{n_1}^2 = 4 \cdot R_1 KTB$$

$$V_{n_2}^2 = 4R_2KTB$$

$$V_{n_3}^2 = 4R_3KTB \ldots$$

Then the resultant noise voltage $V_{nR} = \sqrt{4KTB(R_1+R_2 \ldots}$

$$V_{nr} = \sqrt{4KTB\,R_{eq}}$$

where $R_{eq} = R_1 + R_2 + \cdots + \cdots$ (series)

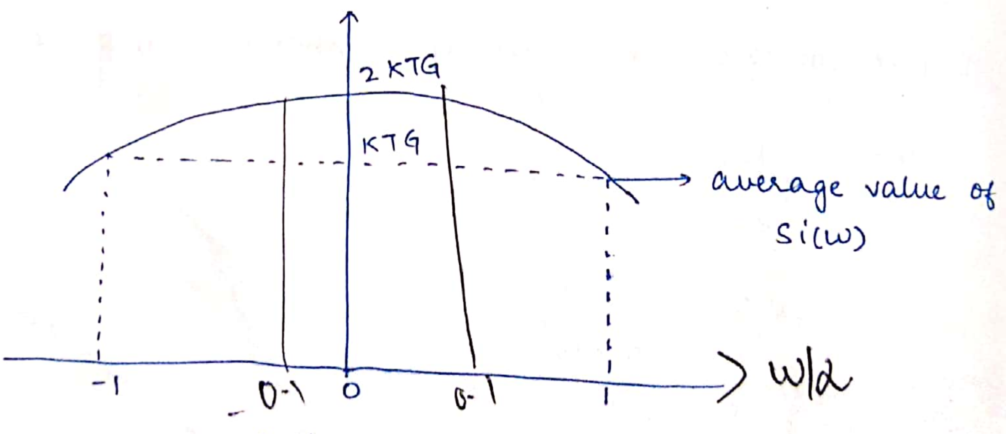
Addition of noise due to several sources in parallel.

Let us consider several thermal noise sources $R_1, R_2 \ldots$ in parallel producing noise voltages $V_{n_1}, V_{n_2}, V_{n_3} \ldots$

$$V_{nr} = \sqrt{4KTB\,R_{eq}}$$

where $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

PSD of Thermal Noise:



The power spectral density of current contributing thermal noise is expressed as

$$S_i(\omega) = \frac{2KTG}{1+\left(\frac{\omega}{\alpha}\right)^2}$$

where T → ambient temperature in kelvin

G → conductance of the resistor

K → Boltzmann's constant

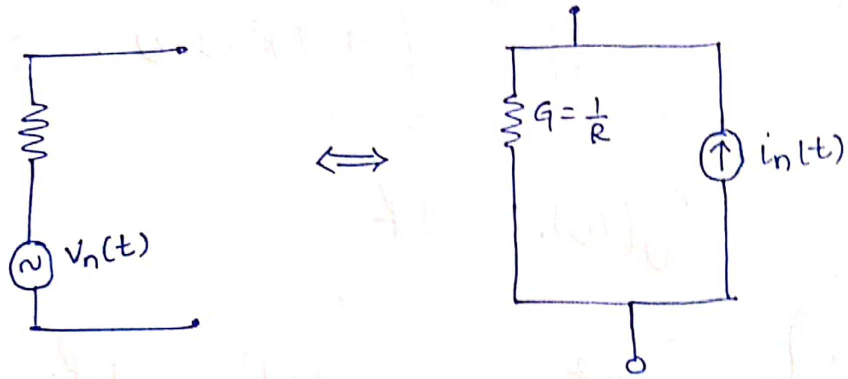
α → Average no. of collisions per second of an electron

$\alpha = 10^{14}$.

The figure shows the PSD of current wrt $\frac{\omega}{\alpha}$. It is observed that $S_i(\omega)$ is constant or flat when $\frac{\omega}{\alpha} \le 0.1$. This means that for this range of frequencies, $S_i(\omega)$ is almost constant and is expressed as $S_i(\omega) = 2KTG$.

Generally, the 'α' value is 10^{14} and so frequency range corresponding to $\frac{\omega}{\alpha} \le 0.1$ is of order 10^{13} Hz. Thus the frequency independant expression $S_i(\omega)$ is valid upto frequency range of 10^{13} Hz. However this range covers all the applications in communication systems. Therefore, it is concluded that thermal noise is considered to contain all frequency components in equal amount.

Derivation of Thermal Noise Expression using P.S.D :-



A noisy resistor is represented as a voltage model / current model. In voltage model of noisy resistor, resistor R is connected with a thermal noise voltage source $V_n(t)$ in series. In current model, the noise resistor R is represented by a conductance 'G' in parallel with thermal noise current source $i_n(t)$. Figure shown above represents both current and voltage model of noisy resistor.

$$v_n(t) = i_n(t) \cdot R.$$

$$\boxed{S_i(\omega) = 2KTG.}$$

$$S_v(\omega) = \cancel{2K} \quad R^2 \cdot S_i(\omega)$$

$$\therefore S_{yy}(\omega) = |H(\omega)|^2 \cdot S_{xx}$$

$$= R^2 \cdot 2KT \cdot \frac{1}{R}$$

$$\boxed{S_v(\omega) = 2KTR}$$

power of thermal noise voltage.

$$P_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_v(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2KTR\, d\omega.$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} KTR \cdot d\omega.$$

$$P_n = E\{v_n^2\}$$

$$= R_{xx}(0)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{v_n}(\omega)\, d\omega$$

$$\boxed{P_n = S_v(\omega) \cdot 2\Delta f}$$

$$E\{v_n^2\} = \overline{v_n^2} = S_v(\omega) \cdot 2\Delta f$$

$$= 2KTR \cdot 2\Delta f$$

Rms $\sqrt{\overline{v_n^2}} = \boxed{\overline{v}_n = 2\sqrt{KTR\Delta f}}$

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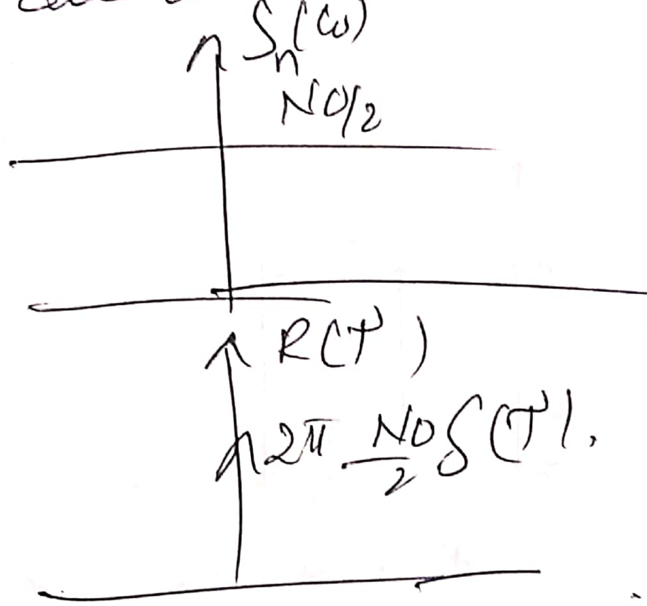
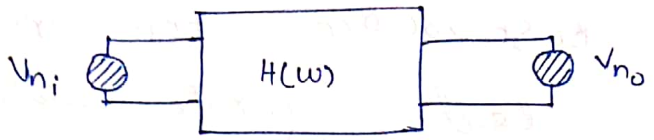
White Noise :- It contains all frequencies in equal amount. The PSD of white noise is independent of frequency which means it contains all frequencies. When probability of occurrence of white noise level is specified by gaussian distribution function then that noise called white gaussian noise.



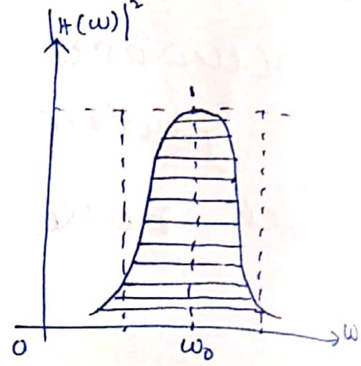
NOISE BANDWIDTH :-

Noise bandwidth is used to specify the output power (noise) at the output of band-pass linear system.

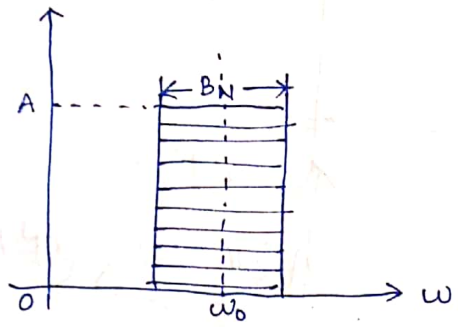
Consider a linear band-pass system as shown in figure. The square of transfer function i.e. $|H(\omega)|^2$ is also plotted in the figure.



Linear band pass system.



$|H(\omega)|^2$ of the actual system.
(A)



$|H(\omega)|^2$ of an ideal system.
(B)

The noise power (Mean square value) at the output of the system is given by

$$P_o = \overline{V_{no}^2}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{S_{ni}(\omega)\,|H(\omega)|^2}_{S_{no}(\omega)}\, d\omega$$

$$\therefore S_{no}(\omega) = S_{ni}(\omega)\,|H(\omega)|^2.$$

$$P_o = \frac{2}{2\pi}\int_0^{\infty} S_{ni}(\omega)\,|H(\omega)|^2\, d\omega$$

$$P_o = \frac{1}{\pi}\int_0^{\infty} S_{ni}(\omega)\,|H(\omega)|^2\, d\omega$$

Let the input power spectral density of the noise $S_{ni}(\omega)$ is constant with frequency.

So, assume $S_{ni}(\omega) = c$ (constant).

$$\therefore P_o = \frac{c}{\pi}\int_0^{\infty} |H(\omega)|^2 d\omega.$$

The integral on the right side is the area under the curve of $|H(\omega)|^2$ as shown in figure 'A'.

$\therefore \quad P_o = \frac{c}{\pi}$ [Area under the curve of $|H(\omega)|^2$].

* let us consider an ideal band pass system with rectangular characteristic of $|H(\omega)|^2$ such that the area under this curve is same as that of an actual system and the amplitude of the ideal curve is equal to 'A', which is the maximum of $|H(\omega)|^2$ in actual system. In this case, the band width of ideal system is called equivalent noise bandwidth (B_N).

The characteristics of ideal system are shown in figure 'B'. The area under this rectangular characteristic specifying the power of the signal is given by $A \times B_N$;

where A = amplitude = $|H(\omega_o)|^2$

Equating the areas of actual & ideal systems.

$$A \times B_N = \int_0^{\infty} |H(\omega)|^2 d\omega .$$

$$\boxed{B_N = \frac{1}{A} \int_0^{\infty} |H(\omega)|^2 d\omega}$$

$$B_N = \frac{\text{Area under the curve of } |H(\omega)|^2}{\text{Amplitude of } |H(\omega)|^2} .$$

The noise power is obtained as $P_o = \overline{V_{no}^2}$

$$P_o = \frac{c}{\pi}\left[\text{Area under the curve of } |H(\omega)|^2 \text{ of actual system}\right].$$

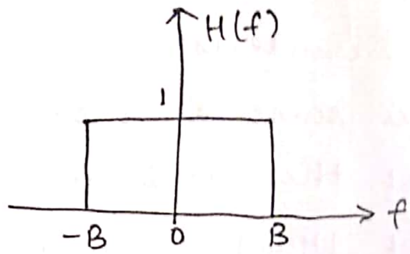
$$= \frac{c}{\pi}\left[\text{Area under the curve of } |H(\omega)|^2 \text{ of ideal system}\right].$$

$$\therefore \boxed{P_o = \frac{c}{\pi} \cdot A B_N}$$

This is the power at output of the system when equivalent noise bandwidth is considered.

* Equivalent noise bandwidth is also defined as the bandwidth of the ideal band pass system which produces the same noise power as the actual system.

* A white noise process $W(t)$ of zero mean & PSD = $\frac{N_0}{2}$ is applied to a low-pass filter of bandwidth and pass band amplitude of 1. Determine PSD of noise $N(t)$ at the output of the filter. Also find auto-correlation fn. of the output noise $N(T)$.



Soln:

~~$S_n(f) =$~~

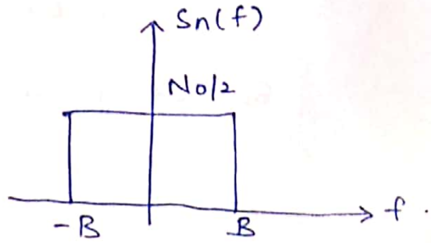
Given $S_w(f) = \frac{N_0}{2}$

$$|H(f)| = 1 \quad ; -B \to B .$$

$$\therefore S_N(f) = |H(f)|^2 \, S_w(f)$$

$$= \frac{N_0}{2} \, , -B \to B .$$

$$S_N(f) = \frac{N_0}{2} \quad ; -B < f < B .$$



$$R_N(\tau) = F^{-1}\left[S_N(f)\right]$$

$$= \frac{1}{2\pi} \int_{-B}^{B} \frac{N_0}{2} . e^{j 2\pi f \tau} \, df \, (2\pi)$$

$$= \frac{N_0}{4\pi} \int_{-B}^{B} e^{j 2\pi \tau f} \, df \, (2\pi)$$

$$= \frac{N_0}{4\pi} \left[\frac{e^{j 2\pi \tau f}}{j 2\pi \tau}\right]_{-B}^{B} (2\pi)$$

$$= \frac{N_0}{4\pi} . \frac{\cancel{2\pi}}{j \cancel{2}\pi \tau} \left[e^{j 2\pi \tau B} - e^{-j 2\pi \tau B}\right]$$

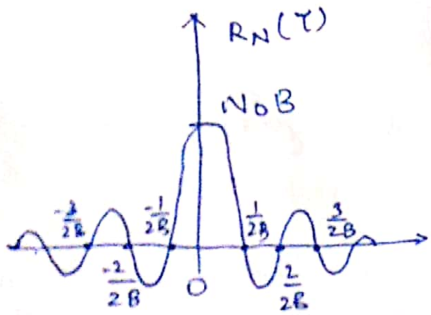
$$= \frac{N_0}{4\pi j \tau} \left[\cos 2\cancel{\pi} \tau B + j \sin 2\pi \tau B - \left[\cos \cancel{2\pi} \tau B - j \sin 2\pi \tau B\right]\right]$$

$$= \frac{N_0}{\cancel{4}_2 \pi \cancel{j} \tau} \, \cancel{2} \cancel{j} \sin 2\pi \tau B .$$

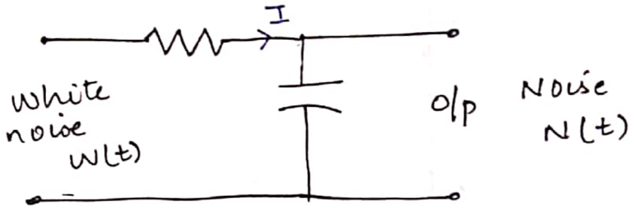
$$= \frac{N_0 B}{2\pi \tau B} \sin 2\pi \tau B$$

$$= N_0 B \, \mathrm{sinc} \, (2\pi \tau B)$$

$$= N_0 B \, \mathrm{sinc} \, (2\pi B \tau)$$

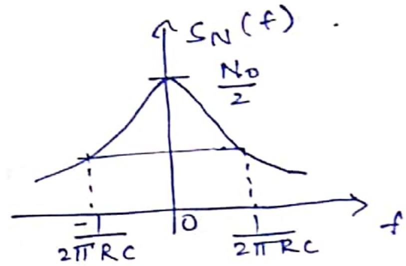


* A white noise process $W(t)$ of zero mean and PSD $= \frac{N_0}{2}$ is applied to a R-C low pass filter shown in figure. Determine the PSD and auto-correlation function of the filtered noise at the output.



soln: $H(f) = \frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$.

$$|H(f)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$



$$S_N(t) = |H(f)|^2 \, S_w(f)$$

$$S_N(t) = \frac{1}{1 + \omega^2 R^2 C^2} \cdot \frac{N_0}{2}$$

$$\underline{\underline{S_N(t) = \frac{1}{1 + (2\pi f RC)^2} \frac{N_0}{2}}}$$

$$R_N(\tau) = F^{-1}[S_N(t)]$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_N(f)\, e^{j\omega\tau} \, d\omega .$$

$$= \frac{N_0}{2} \cdot \frac{1}{\cancel{2\pi}} \int_{-B}^{B} \frac{1}{1 + (2\pi f RC)^2} e^{j2\pi f\tau} \cdot \cancel{2\pi} \, df$$

$$= \frac{N_0}{4} \int_{-B}^{B} \frac{e^{j2\pi f\tau}}{1 + (2\pi f RC)^2} \, df$$

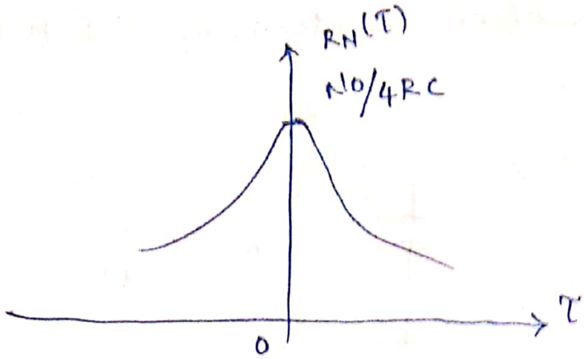
$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2 + \omega^2} .$$

$$= \frac{N_0}{2 \times 2} \int_{-B}^{B} \frac{2}{\left[\left(\frac{1}{2\pi RC}\right)^2 + f^2\right](2\pi RC)^2} e^{j2\pi f\tau} \, df$$

$$= \frac{N_0}{4} \int_{-B}^{B} \frac{2}{1 + (2\pi f)^2 (RC)^2} e^{j2\pi f\tau} \, d\tau$$

$$= \frac{N_0}{4} F^{-1}\left[\frac{2\left(\frac{1}{RC}\right)^2}{\left(\frac{1}{RC}\right)^2 + \omega^2}\right] = \frac{N_0}{4RC} e^{-\frac{1}{RC}|t|}$$

$$R_N(\tau) = \frac{N_0}{4RC} e^{-\frac{|t|}{RC}} = \frac{N_0}{4RC} e^{-\frac{|\tau|}{RC}}$$



* A random process $x(t)$ consists of a sinusoidal wave process and a white noise process $w(t)$ with zero mean and PSD = $\frac{N_0}{2}$ such that the sample function of $x(t)$ can be written as

$$x(t) = A\cos(2\pi f_c + \theta) + w(t) \quad \text{where } -\pi \le \theta \le \pi$$

with equally likely hood. Find PSD & Auto correlation functions of the random process.

Soln:

$$x(t) = A\cos(2\pi f_c + \theta) + w(t)$$

(i) $R_{xx}[A\cos(w_c + \theta)] = \frac{A^2}{2}$ [from previous problem]

$$= \frac{A^2}{2}\cos(w_c\tau)$$

$$= \frac{A^2}{2}\cos(2\pi f_c \tau)$$

$\therefore$ $R_S(\tau) = \frac{A^2}{2}\cos(2\pi f_c \tau)$ [Sinusoidal]

$$S_S(f) = \frac{A^2}{4}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$

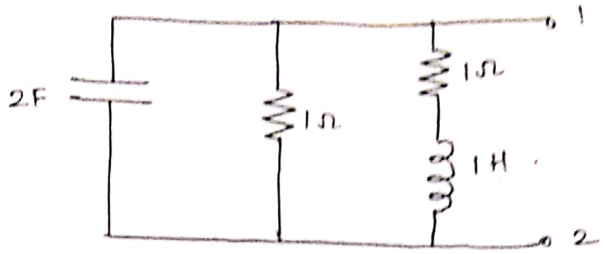
$\longrightarrow$ $PSD[x(t)] = \frac{A^2}{4}\left[\delta(f+f_c) + \delta(f-f_c)\right] + \frac{N_0}{2}$

$$R_N(\tau) = \frac{A^2}{2}\cos(2\pi f_c \tau) + \frac{N_0}{2}\delta(\tau).$$

[Auto correlation fn. of $w(t) = \frac{N_0}{2}\delta(\tau)$

P.S.D of $w(t) = \frac{N_0}{2}$]

* Find the P.S.D of thermal noise voltage across terminals 1 & 2 of a passive network shown in figure.



soln:

$$S_v(\omega) = 2KTR_{1,2}$$

$$R_{eq} = 1 \parallel (1+j\omega L) \parallel 2$$

$$\cancel{= \frac{1+j\omega L}{2+j\omega L}} \quad \cancel{= \frac{1+j\omega}{2+j\omega}} \quad \cancel{= \frac{(1+j\omega)(2-j\omega)}{4-\omega^2}}$$

$$\therefore \cancel{S_v(\omega) = \frac{2\times 1.38\times 10^{-23}\times 300\times(1+j\omega L)}{2+j\omega L}}$$

$$\cancel{S_v(\omega) = 2\times 1.38\times 10^{-23}\times 300}$$

$$Z_{eq} = \frac{\left(\dfrac{\frac{1}{2j\omega}\cdot 1}{\frac{1}{2j\omega}+1}\right)(1+j\omega)}{\dfrac{\frac{1}{2j\omega}}{\frac{1}{2j\omega}+1} + 1+j\omega}$$

$$Z_{eq} = \frac{\frac{1}{1+2j\omega}(1+j\omega)}{\frac{1}{1+2j\omega}+1+j\omega} = \frac{\frac{1+j\omega}{\cancel{1+2j\omega}}}{\frac{1+(1+j\omega)(1+2j\omega)}{\cancel{1+2j\omega}}}$$

$$Z_{eq} = \frac{1+j\omega}{1+[1+3j\omega-2\omega^2]} = \frac{1+j\omega}{3j\omega-2\omega^2+2}$$

$$Z_{eq} = \frac{1+j\omega}{2(1-\omega^2)+j(3\omega)}$$

$$Z_{eq} = \frac{(1+j\omega)\,[2(1-\omega^2)-3\omega(j)]}{[2(1-\omega^2)]^2+(3\omega)^2}$$

$$\therefore Z_{eq} = \frac{2(1-\omega^2)(1+j\omega) - j3\omega(1+j\omega)}{[2(1-\omega^2)]^2+(3\omega)^2}$$

$$Z_{eq} = \frac{2(1-\omega^2)+2j\omega-3j\omega+3\omega^2}{[2(1-\omega^2)]^2+(3\omega)^2}$$

Req has only real part of Zeq ?

$$\therefore R_{eq} = \frac{2+w^2}{4(1+w^4-2w^2)+9w^2}$$

$$R_{eq} = \frac{2+w^2}{4+4w^4+w^2}$$

$$\therefore S_v(w) = 2KT \cdot \left[\frac{2+w^2}{4+4w^4+w^2}\right]$$

* Evaluate the thermal noise voltage developed across a resistor of 700 Ω. The band-width of the measuring instruments is 7 MHz and the ambient temperature is 27°C.

→ $$V_{rms} = \sqrt{4KTBR}$$

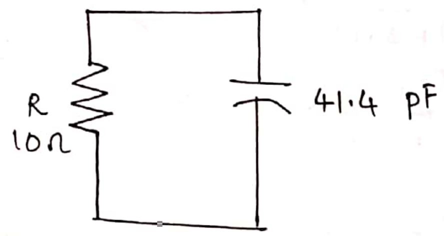
$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 7 \times 10^{6} \times 700}$$

$$= \sqrt{8114400 \times 10^{-17}}$$

$$= \sqrt{81144000 \times 10^{-18}}$$

$$= 9007.99 \times 10^{-9} = 9\,\mu V$$

* Find the RMS value of the noise voltage at 27°C developed across the capacitor of the ckt shown below.



$$V_{rms}^2 \longrightarrow PSD$$

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{n_o}(w)\,dw.$$

$$H(w) = \frac{\frac{1}{jwc}}{\frac{1}{jwc}+R}$$

$$S_{n_o}(w) = |H(w)|^2 \, S_{nv}(w)$$

$$H(w) = \frac{1}{1+jwCR}.$$

$$= |H(w)|^2 \, 2KTR.$$

$$= \frac{1}{1+w^2C^2R^2} \cdot 2KTR.$$

$$\cancel{\frac{1-jwCR}{1+w^2c^2R^2}}$$

$$|H(w)| = \frac{1}{\sqrt{1+w^2c^2R^2}}$$

$$= \cancel{\frac{1}{1+w^2c^2R^2}} \quad \cancel{8280 \times 10^{-23}}$$

$$S_{n_o}(\omega) = \frac{8280\times10^{-23}}{1+\omega^2(1.71\times10^{-19})}$$

$$e^{-a|\tau|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$

$$S_{n_o}(\omega) = \frac{8280\times10^{-23}}{1.71\times10^{-19}\left[\frac{1}{1.71\times10^{-19}}+\omega^2\right]}$$

$$\overline{V_n^2} = P = \frac{1}{\pi}\int_0^\infty \frac{2KTR}{1+\omega^2R^2C^2}\,d\omega$$

$$= \frac{2KTR}{\pi}\int_0^\infty \frac{1}{1+\omega^2R^2C^2}\,d\omega$$

$$= \frac{1}{CR}\frac{2KTR}{\pi}\left[\tan^{-1}(\omega CR)\right]_0^\infty$$

$$= \frac{1}{CR}\frac{2KTR}{\pi}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{1}{C\cancel{R}}\frac{\cancel{2}KT\cancel{R}}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{\cancel{2}}$$

$$= \frac{KT}{C} = \frac{10\times10^{-23}}{10^{-12}} = 10^{-10}$$

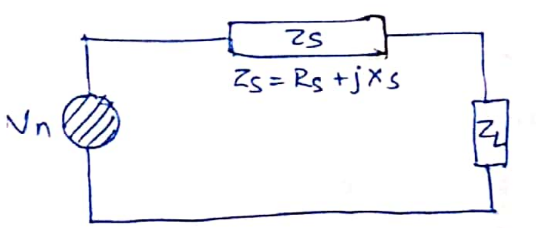
$$\therefore \overline{V_n} = \sqrt{10^{-10}} = 10^{-5}$$

$$= \underline{10\,\mu V}$$

AVAILABLE POWER :-

Available power of source is defined as the maximum power that can be drawn from the source ~~at output load~~. This is occurred only when the impedence of the network is equal to load. Consider a noise voltage source $V_n(t)$ with a source impedence $Z_s = R_s + jX_s$ as shown in figure.

The noise source is driving a load impedence Z_L which is matched with source impedence i.e. $Z_L = Z_s^* = R_s - jX_s$.



The RMS current flowing through the load is given by

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{\sqrt{\overline{V_n^2}}}{Z} = \frac{\sqrt{\overline{V_n^2}}}{Z_L + Z_S} = \frac{\sqrt{\overline{V_n^2}}}{2R_s}$$

Available power $P_a = I_{rms}^2 \cdot R_s$

$$\Rightarrow P_a = \frac{\overline{V_n^2}}{4R_s^{\cancel{2}}} \cdot \cancel{R_s}$$

KTB

$$P_a = \frac{\overline{V_n^2}}{4R_s} = \frac{P_n}{4R_s}\ ?$$ (2√KTRB)

; where P_n is the normalised power and it gives the power delivered by V_n across 1Ω resistor

∴ The power developed across the total ckt resistance $2R_s$ will be $P_n' = \frac{P_n}{2R_s}$.

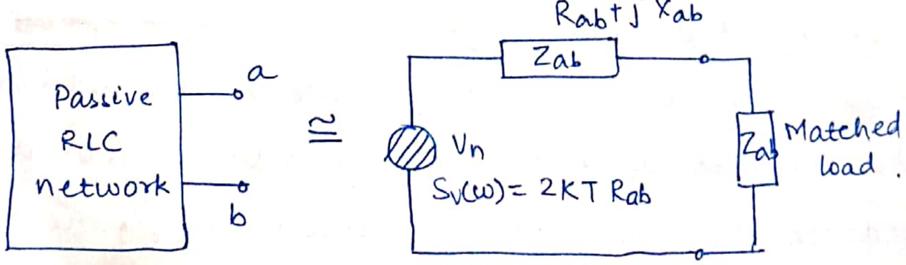
∴ The noise voltage source V_n supplies a noise power equal to P_{n_0} out of which $\frac{P_n}{4R_s}$ is available across the load and the remaining is dissipated across the source resistance R_s.

Similarly, the available power density spectrum is given by

$$\boxed{S_a(\omega) = \frac{S_v(\omega)}{4R_s}}$$

where $S_v(\omega)$ is the power spectral density of noise voltage $V_n(t)$.

__Available Power of an RLC network:__



A passive network containing R, L, C elements can be represented by Thevenin's equivalent as shown in figure. The equivalent impedence across the terminals A, B is ~~repres~~ obtained as $Z_{ab} = R_{ab} + jX_{ab}$.

By considering real part of Z_{ab}, the PSD of noise voltage across the resistance R_{ab} is given by $S_v(\omega) = 2KTR_{ab}$

The available power spectrum density $S_a(\omega)$ is obtained as

$$\boxed{S_a(\omega) = \frac{S_V(\omega)}{4R_{ab}}}$$

$$S_a(\omega) = \frac{2KT\cancel{R_{ab}}}{4\cancel{R_{ab}}} = \frac{KT}{2}$$

The available noise power is obtained by using the formula

$$P_a = \frac{\overline{V_n^2}}{4R_{ab}}$$

But $\Rightarrow$ $\overline{V_n^2} \cancel{= 4P_a R_{ab}} = 4KTR\Delta f$

$$\therefore P_a = \frac{4KT\cancel{R_{ab}}\Delta f}{4\cancel{R_{ab}}}$$

$$\boxed{P_a = KT\Delta f}$$

* Noise Temperature :-

In communication systems, signals are processed in a no. of ways. For example, signals may be amplified by an amplifier, a signal may be subjected to a frequency mixer for conversio of intermediate frequency etc.

The processing stages like amplifiers and mixers have two sets of terminals, one for input & other for output. Such systems are called two-port networks. Each port may have resistors as well as active devices as sources of noise within it. The message signal at the input of the port is accompanied by noise sources and while processing through the two-port network, additional noise is added to the signal. Thus at the output of the network, the total noise is due to the contribution of two noise sources. One is internal noise ~~and~~ ~~the~~ generated within the network and second is noise that may be amplified or attenuated by the network. The quantitative analysis of these noise sources is characteri by two parameters :

(i) Equivalent noise temperature.

(ii) Noise figure.

The effective noise temperature is defined as the temperature at which a noisy resistor, has to be maintained such that by connecting this resistor to the input of the noise-less system, it produces the same available noise power at the output of the system as it is produced by all the sources of noise in the actual system.

It is denoted by $T_a = \frac{P_a}{k \Delta f}$ or

$$T_n = \frac{2 S_a(\omega)}{k}$$

* Signal to Noise Ratio :-

The ratio of a signal power to the noise power is referred as signal to noise ratio and is denoted by $\frac{S}{N}$.

If a signal voltage $V_s(t)$ is accompanied by a noise voltage source $V_n(t)$, then the ratio of signal to noise power is

$$\boxed{\frac{S}{N} = \frac{\overline{V_s^2}}{\overline{V_n^2}}}$$

. It can also be expressed in terms of P.S.D.

$$\boxed{\frac{S}{N} = \frac{S_s(\omega)}{S_n(\omega)}}$$

For noise free system $\frac{S}{N}$ ratio is same at the input & output.

But for noisy system, $\frac{S}{N}$ ratio decreases at the output.

Available Power gain:

The ratio of available power density at the output and the input is referred as available power gain and is denoted by

$$G_a(\omega) = \frac{(S_o(\omega))_a}{(S_i(\omega))_a} = \frac{(S_o)_a}{(S_i)_a}$$

$$\Rightarrow (S_o)_a = G_a(\omega)(S_i)_a$$

Here, the power spectral density may belong to a message signal or a noise signal.

Available output power will be

$$P_{ao} = \frac{1}{2\pi}\int_{-\infty}^{\infty}[S_o(\omega)]_a\, d\omega$$

$$\boxed{P_{a_0} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_a(\omega)(S_i)_a \, d\omega}$$

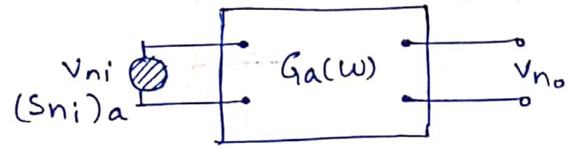
If the input is the white noise with a power spectral density $\frac{KT}{2}$. Therefore, the available power at the output for white noise is

$$(P_{a0})_{\text{white noise}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_a(\omega)\cdot\frac{KT}{2}\, d\omega$$

NOTE: The available power gain of 'n' cascaded stages with available power gains $G_{a_1}, G_{a_2}, G_{a_3} \cdots G_{a_n}$ is given by

$$\boxed{G_a = G_{a_1}\cdot G_{a_2}\cdot G_{a_3}\cdots G_{a_n}}$$

* Effective Input Noise Temperature:



The noise temperature is referred at the input of two port network which accounts for the internal noise produced by the network, and there after the network is considered to be noise-free.

Consider a 2-port network with a white noise source V_{ni} with the available power density $\frac{KT}{2}$ at the input. $G_a(\omega)$ is the available power gain of the network. If the network is noise free, then the available power density at the output will be

$$(S_{no}')_a = G_a\cdot(S_{ni})_a$$

$$\boxed{(S_{no}')_a = \frac{KT}{2}\cdot G_a(\omega)}$$

However, a practical 2-port network introduces its own noise and therefore the available power density at the o/p is higher than $(S_{no}')_a$.

This increase in noise power density referred in terms of noise temperature at the input and the network may be considered as noise free. The input noise temperature of the network which accounts for the internal noise generated by the network is known as effective i/p noise temperature and is denoted by T_e.

∴ The total output power density is given as

$$(S_{n_o})_a = G_a(\omega) \cdot \frac{KT}{2} + G_a(\omega) \cdot \frac{KT_e}{2}$$

$$= G_a(\omega) \frac{K(T+T_e)}{2} \rightarrow (1)$$

where T : noise temperature of the source.

T_e : noise temperature accounting for the network generated noise.

NOISE FIGURE:

It is defined as the ratio of total noise power spectral density S_{n_o} at the output of 2-port network to the noise power spectral density S_{n_o}' at the ~~output~~ input. It is denoted by 'F' and is given as

$$\boxed{F = \frac{(S_{n_o})_a}{(S_{n_o}')_a}}$$

∴ 'F' gives the noise generated in the internal ckt of the network.

where $\boxed{S_{n_o} = S_{n_o}' + S_{n_o}''}$

Here S_{n_o}' is PSD only due to the input noise source

. S_{n_o}'' is PSD contributed by the network

$$\therefore F = \frac{S_{n_o}' + S_{n_o}''}{S_{n_o}'} = 1 + \frac{S_{n_o}''}{S_{n_o}'}$$

Suppose if F=1, the network is completely noise free

if F>1, the network is completely noise.

The range of F lies in b/w $1 < F < \infty$

Noise Figure in terms of available power gain Ga :-

$$\boxed{F = \frac{(S_{n_o})_a}{G_a(\omega) \frac{KT}{2}}} \rightarrow (2)$$

Noise figure in terms of transfer fn $H(\omega)$:-

$$\boxed{F = \frac{S_{no}}{S_{ni}\,|H(\omega)|^2}}$$

~~F= Sn(0)~~

In terms of Eq. i/p noise temperature :

$$\boxed{T_e = T(F-1)}$$

$$F = \frac{T+T_e}{T} = 1 + \frac{T_e}{T}$$

$$\Rightarrow T(F-1) = T_e$$

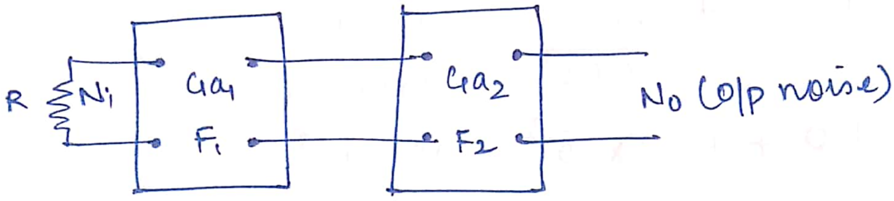
$$\Rightarrow T_e = T(F-1)$$

from eqns 1 and 2

In terms of S/N ratio :

$$F = \frac{\frac{S}{N} \text{ of } i/p}{\frac{S}{N} \text{ of } O/p}$$

Noise figure calculation for cascaded stages:



$$\boxed{F = F_1 + \frac{F_2 - 1}{Ga_1}}$$

For N-stages

* Frii's formula:- $F = F_1 + \frac{F_2 - 1}{Ga_1} + \frac{F_3 - 1}{Ga_1 . Ga_2} + \frac{F_4 - 1}{Ga_1 . Ga_2 . Ga_3} + \cdots$

$$\frac{F_n - 1}{Ga_1 Ga_2 \cdots Ga_3}$$

Let N_i be the noise power generated by resistor (or) R at the input of first stage. The noise power at the final o/p (N_o) due to N_i is given by (assuming that noise contributed by the stages is zero) is equal to $N_{o1} = G_{a1} G_{a2} N_i$.

If first stage introduces it own internal noise, the contribution of this noise at the o/p of the first stage is $G_{a1}(F_1 - 1) N_i$. This noise is amplified at its second stage and appears at the final o/p as $N_{o2} = (G_{a1}(F_1-1)N_i) G_{a2}$. Similarly the second stage also generates ~~the~~ its own internal noise and which is appeared at final output as $N_{o3} = G_{a2}(F_2-1) N_i$.

$\therefore$ The total noise power $N_o = N_{o1} + N_{o2} + N_{o3}$

$$N_o = G_{a1} G_{a2} N_i + G_{a2} G_{a1} (F_1 - 1) N_i + G_{a1} G_{a2} (F_2 - 1) N_i$$

$$= G_{a1} G_{a2} N_i \left[1 + F_1 - 1 + \frac{(F_2 - 1)}{G_{a1}} \right]$$

$$\frac{N_o}{N_i} \times \frac{1}{G_{a1} G_{a2}} = 1 + F_1 - 1 + \frac{F_2 - 1}{G_{a1}}$$

$$= F_1 + \frac{F_2 - 1}{G_{a1}} \qquad G_{a1} G_{a2} = G_a$$

(output noise power)

$$\frac{N_o}{N_i G_a} = F_1 + \frac{F_2 - 1}{G_{a1}} \Rightarrow \boxed{F = F_1 + \frac{F_2 - 1}{G_a}}$$

(o/p noise due to i/p)

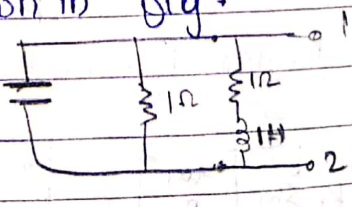
Suppose for N-stages

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}} + \cdots + \frac{F_n - 1}{G_{a1} G_{a2} \cdots G_{an-1}}$$

The formula is called Friis's formula.

Problem:

1) Find PSD of thermal noise voltage across terminals 1 and 2 of the given passive network shown in fig.



Sol: $S_v(\omega) = 2KT R_{12}$

$$Z_{12} = \frac{\left(\frac{1}{2j\omega}\right)(1 + j\omega)}{\left(1 + \frac{1}{2j\omega}\right)} \over {(1 + j\omega + \frac{1}{1 + 2j\omega})} = \frac{1 + j\omega}{3 + 3j\omega - 2\omega^2}$$

$$= \frac{(1+j\omega)(2-2\omega^2-3j\omega)}{(4(1-\omega^2)^2+9\omega^2)}$$

$$= \frac{2-2\omega^2-3j\omega+2j\omega-2j\omega^3-3j^2\omega^2}{4(1-\omega^2)^2+9\omega^2}$$

$$R_{12} = \frac{2-2\omega^2+3\omega^2}{4+4\omega^4+8\omega^2+9\omega^2} = \left(\frac{2+\omega^2}{4\omega^4+4+\omega^2}\right)$$

$$S_v(\omega) = 2KTR$$

$$= 2KT\left(\frac{2+\omega^2}{4\omega^4+\omega^2+4}\right)$$

2) Evaluate Thermal noise voltage developed across 700Ω. The B.W of measuring instrument in 7MHz and Ambient temp in 27°C.

Sol:

$$V_{rms} = \sqrt{\bar{V_n}^2}$$

$$= 2\sqrt{KTRB}$$

$$= 2\sqrt{1.38\times10^{-23}\times300\times700\times7\times10^6}$$

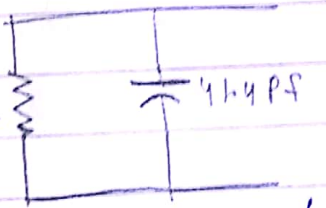
$$= 9\mu V$$

$$P_0 = S_v(\omega)\,(BW)$$

$$= 2KTR(2\Delta f)$$

$$= 4KTR(BW)$$

3) Find the rms value of noise temp at 27°C developed across the capacitor on the circuit



$$P = S_v(\omega)\,B.W$$

$$P = R_{xx}(0)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{no}(\omega)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2\,S_v(\omega)\,d\omega$$

$$|H(\omega)|^2 = \left(\frac{O/P}{I/P}\right)^2 = \left(\frac{1/j\omega C}{\frac{1}{j\omega C}+R}\right)^2 = \frac{1}{1+\omega^2R^2C^2}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{1+\omega^2R^2C^2}(2KTR)\,d\omega$$

$$= \frac{2KTR}{\pi}\int_0^{\infty}\frac{1}{1+\omega^2R^2C^2}\,d\omega$$

$$= \frac{2KRT}{\pi}\left[\frac{\tan^{-1}(\omega RC)}{RC}\right]_0^{\infty}$$

$$= \frac{2KRT}{\pi}\left(\frac{\pi}{2RC}\right)$$

$$= KT/C$$

$$V_{rms} = \sqrt{\bar{v_n}^2} = \sqrt{\frac{KT}{C}} = \sqrt{\frac{1.38\times10^{-23}\times300}{41.4\times10^{-12}}} = 10\mu V$$