POWER SPECTRAL DENSITIES OF VARIOUS LINE ENCODING FORMATS

① Line encoding schemes like NRZ unipolar, NRZ polar etc

② may be described as different realizations (sample functions)

③ of a random process X(t) defined by

$$\boxed{X(t) = \sum_{k=-\infty}^{\infty} A_k \, v(t-kT)} \rightarrow (1)$$

④ where A_k coefficient → discrete random variable

$v(t)$ → basic pulse shape

T → symbol duration

⑤ Basic pulse $v(t)$ is centered at origin, $t=0$ & normalized such that $\boxed{v(0)=1.} \rightarrow (2)$

⑥ Table below summarizes the sample values of coefficient A_k & the basic pulse $v(t)$ for various line encoding schemes.

NRZ FORMAT	COFFFICIENT A_K	BASIC PULSE V(t)
① Unipolar	$A_K = \begin{cases} a, & \text{symbol } 1 \\ 0, & \text{symbol } 0 \end{cases}$	V(t) consists of rectangular pulse of unit amplitude & duration T_b
② Polar	$A_K = \begin{cases} a, & \text{symbol } 1 \\ -a, & \text{symbol } 0 \end{cases}$	
③ Bipolar	$A_K = \begin{cases} a, -a, & \text{alternating 1's} \\ 0, & \text{symbol } 0 \end{cases}$	
④ Manchester	$A_K = \begin{cases} a, & \text{symbol } 1 \\ -a, & \text{symbol } 0 \end{cases}$	V(t) consists of doublet pulse of heights ±1 and total duration T_b

To Evaluate power spectra of various line coding formats

① Able to make a more complete assessment of their individual spectral characteristics

② Mechanism responsible for generation of sequence $\{A_K\}$, defining coefficients in equ①,

③ is modeled as a 'discrete stationary random source'

AUTOCORRELATION FUNCTION

① source is characterized as having ensemble - averaged autocorrelation function

② $$\boxed{R_A(n) = E[A_k A_{k-n}]} \rightarrow (3)$$

$E \rightarrow$ expectation operator

$A_k, A_{k-n} \rightarrow$ Amplitude of k^{th} & $(k-n)^{th}$ symbol position.

③ PSD of signal $x(t)$ is given by

$$\boxed{S_x(f) = \frac{1}{T}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi n f T_b)} \rightarrow (4)$$

④ $V(f) \rightarrow$ Fourier transform of basic pulse $v(t)$

values of $V(f)$ & $R_A(n)$ depends on type of line coding format.

PSD of Unipolar NRZ Format

① Suppose 0's & 1's of a random binary sequence occur with equal probability.

② Then for a unipolar NRZ format, we have

$$\boxed{P(A_k=0) = P(A_k = a) = \frac{1}{2}} \rightarrow (5)$$

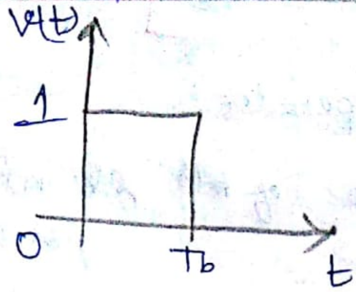
③ Steps to be followed to find out PSD

Step 1: Fourier transform of NRZ pulse $V(f)$

Step 2: Auto correlation of Unipolar $R_A(n)$

Step 3: Calculate PSD based on $V(f)$ & $R_A(n)$

→ Unipolar NRZ pulse



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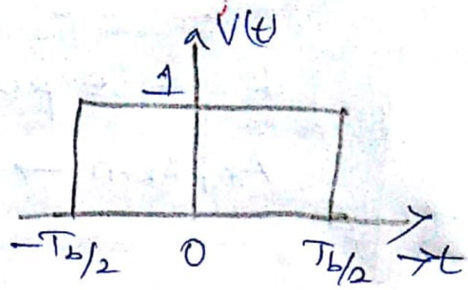

① For a basic pulse $v(t)$, we have a rectangular pulse of unit amplitude & duration T_b.

② Fourier transform of $v(t)$ equals

$$\boxed{V(f) = T_b \operatorname{sinc}(fT_b)} \rightarrow (6)$$

Step 1

③ Derivation is :-

Apply F.T.

$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t} \cdot dt$$

$$= \int_{-T_b/2}^{T_b/2} (1)\, e^{-j2\pi f t} \cdot dt = \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-T_b/2}^{T_b/2}$$

$$= \frac{e^{-j2\pi f \frac{T_b}{2}} - e^{-j2\pi f\left(\frac{-T_b}{2}\right)}}{-j2\pi f} = \frac{e^{-j2\pi f \frac{T_b}{2}} - e^{+j2\pi f \frac{T_b}{2}}}{-2j\pi f}$$

$$= \frac{e^{j2\pi f \frac{T_b}{2}} - e^{-j2\pi f \frac{T_b}{2}}}{2j\pi f}$$

$$= \frac{\sin(\pi f T_b)}{\pi f} \times \frac{T_b}{T_b}$$

$$\because \operatorname{sinc}\theta = \frac{\sin \pi\theta}{\pi\theta}$$

$$\boxed{V(f) = \operatorname{sinc}(fT_b) \cdot T_b}$$

$$\therefore \boxed{V(f) = T_b \, \text{sinc}(f T_b)}$$

Step 2:

Autocorrelation of Unipolar scheme

① Coefficient $\boxed{A_k = \begin{cases} a, & \text{symbol } 1 \\ 0, & \text{symbol } 0 \end{cases}}$

② Suppose 0's & 1's of a binary sequence occur with equal probabilities,

③ Then for Unipolar NRZ scheme,

$$\boxed{P(A_k = 0) = P(A_k = a) = \frac{1}{2}}$$

④ Autocorrelation

$$\boxed{R_A(n) = E[A_k A_{k-n}]}$$

(estimation of $A_k A_{k-n}$)

⑤ If $n=0$

$$R_A(n) = E[A_k A_{k-0}] = E[A_k A_k] = E[A_k^2]$$

⑥ $\because n=0$

$$\boxed{R_A(0) = E[A_k^2]}$$

for continuous fry:

$$E[x^2] = \int x^2 f(x) dx$$

for discrete fry:

$$E[x^2] = \sum x^2 p(x)$$

⑦ for binary bit

	A_k	A_k	A_k^2	Prob
0	0	0	0	1/2
1	a	a	a^2	1/2

⑧ $R_A(0) = E[A_k^2] = \sum A_k^2 p(A_k)$

$$= (0)^2 \times p(A_k = 0) + (a)^2 \, p(A_k = a)$$

$$= a^2 \cdot \frac{1}{2}$$

$$\boxed{R_A(0) = \frac{a^2}{2}}$$

① If $n \neq 0$

$$\boxed{R_A(n) = E[A_k A_{k-n}]}$$

② A_k, A_{k-n} will have 4 probabilities

0×0, $0 \times a$, $a \times 0$, $a \times a$ → with probabilities $\frac{1}{4}$ each

③

	A_k	A_{k-n}	$A_k A_{k-n}$	Prob
0 0	0	0	0	1/4
0 1	0	a	0	1/4
1 0	a	0	0	1/4
1 1	a	a	a^2	1/4

④ $R_A(n) = E[A_k A_{k-n}]$

$= \sum A_k A_{k-n} \, p(A_k)$

$= 0 \times \frac{1}{4} + 0 \times \frac{1}{4} + 0 \times \frac{1}{4} + a^2 \cdot \frac{1}{4}$

$$\boxed{R_A(n) = \frac{a^2}{4}}$$

⑤ Auto correlation for $R_A(n)$

$$\boxed{R_A(n) = \begin{cases} \frac{a^2}{2}, & n = 0 \\ \frac{a^2}{4}, & n \neq 0 \end{cases}}$$ 7

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(6) step3:- Calculate PSD based on $V(f)$ & $R_A(n)$

PSD of unipolar NRZ format is

(7) $$S_x(f) = \frac{1}{T}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, exp(-j2\pi f n t)$$

$$= \frac{1}{T_b}\left[T_b^2 \, sinc^2(fT_b)\right] \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$$= T_b\, sinc^2(fT_b)\left[R_A(0) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} R_A(n) e^{-j2\pi f n T_b}\right]$$

$$= T_b\, sinc^2(fT_b)\left[\frac{a^2}{2} + \frac{a^2}{4}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f n T_b}\right]$$

(8) $$\boxed{S_x(f) = \frac{a^2}{2} T_b\, sinc^2(fT_b) + \frac{a^2}{4} T_b\, sinc^2(fT_b) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f n T_b}} \rightarrow (8)$$

(9) Using Poisson's formula

$$\boxed{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)} \rightarrow (9)$$

where $\delta(f) \rightarrow$ Dirac delta fn. at $f=0$.

(10) Substitute (9) in (10)

$$\boxed{\therefore S_x(f) = \frac{a^2}{2} T_b\, sinc^2(fT_b) + \underbrace{\frac{a^2}{4} \cancel{T_b}\, sinc^2(fT_b) \frac{1}{\cancel{T_b}} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)}_{\text{multiplication of sinc fn: with impulse train}}} \rightarrow (10)$$

(11) $\delta(f)$ multiplies sinc fn: which has nulls at

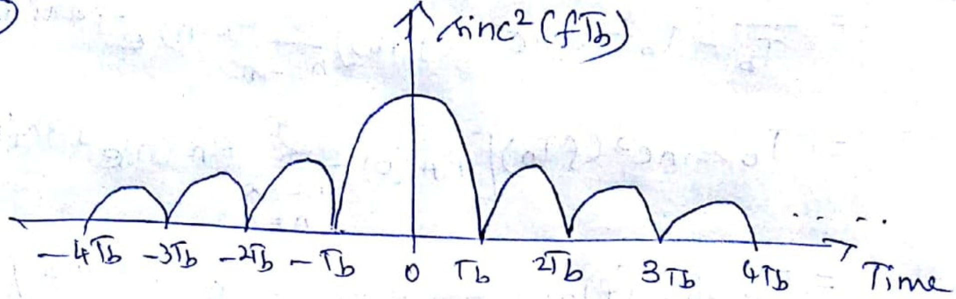
$$f = \pm\frac{1}{T_b}, \pm\frac{2}{T_b} \ldots\ldots$$

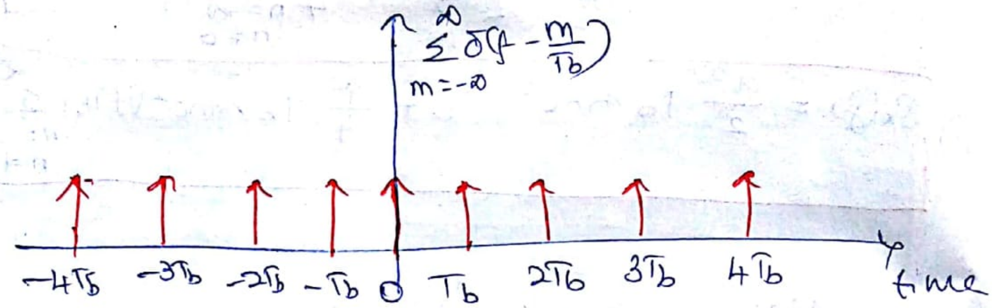
(12) $\therefore$ 2nd term in $= \frac{a^2}{4} sinc^2(fT_b) \cdot \delta(f)$

(1) $\therefore$ $\boxed{S_x(f) = \frac{a^2}{2} T_b \, \text{sinc}^2(f T_b) + \frac{a^2}{4} \delta(f)}$ $\rightarrow$ (ii)

$$\boxed{\because \; \text{sinc}^2(f T_b) \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) = \delta(f)}$$

(2)





(3) @ $T_b \neq 0$, all value of sinc fn: $= 0$

@ $T_b = 0$, $\sin c\, 0 = 1$

$\therefore \delta\left(f - \frac{m}{T_b}\right) = \delta(f)$

(4) Equ (ii) $\Rightarrow$ 2nd term $\frac{a^2}{4}\delta(f)$ is DC Component will lead to distortion.

(5) Normalization in (curve)

$S_x(f)$ is normalized w.r.t. $a^2 T_b$

freq is " " bit rate $1/T_b$

(6) Most of power of NRZ Unipolar format lies b/w dc & bit rate of i/p data

$$S_x(f) = \frac{a^2}{2} T_b \,\mathrm{sinc}^2(fT_b) + \frac{a^2 T_b}{4}\,\mathrm{sinc}^2(fT_b) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi n f T_b}$$

$$= \frac{a^2 T_b}{2}\,\mathrm{sinc}^2(fT_b) + \frac{a^2}{4} T_b \,\mathrm{sinc}^2(fT_b) \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b} - 1\right]$$

to remove $n=0$ term
$e^0 = 1$

$$= \frac{a^2 T_b}{2}\,\mathrm{sinc}^2(fT_b) - \frac{a^2}{4} T_b \,\mathrm{sinc}^2(fT_b) + \frac{a^2}{4} T_b \,\mathrm{sinc}^2(fT_b) \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b}\right]$$

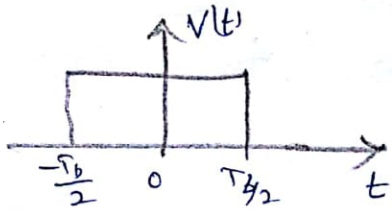
$$\boxed{S_x(f) = \frac{a^2 T_b}{4}\,\mathrm{sinc}^2(fT_b) + \frac{a^2}{4} T_b \,\mathrm{sinc}^2(fT_b) \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b}\right]}$$

PSD of Polar NRZ Format

① $V(f)$
② $R_A(n)$
③ $S_x(f)$

Step 1

① NRZ Polar Pulse



$$\boxed{\therefore\ V(f) = T_b \,\mathrm{sinc}(fT_b)}$$

② for which i/p binary data consists of indept & equally likely symbols.

Step 2

③ Autocorrelation fn: of polar scheme

Coefficient $A_k = \begin{cases} a, & \text{symbol } 1 \\ -a, & \text{symbol } 0 \end{cases}$

④ following similar procedure,

$$\boxed{R_A(n) = \begin{cases} a^2, & n = 0 \\ 0, & n \neq 0 \end{cases}}$$

③ check
PSD of polar scheme is

$$\boxed{S_x(f) = a^2 T_b \operatorname{sinc}^2(f T_b)}$$

PSD of Bipolar NRZ Format

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