UNIT-VI

System Noise

Introduction

Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

•*Interference, usually from a human source (man made)*
•*Naturally occurring random noise*

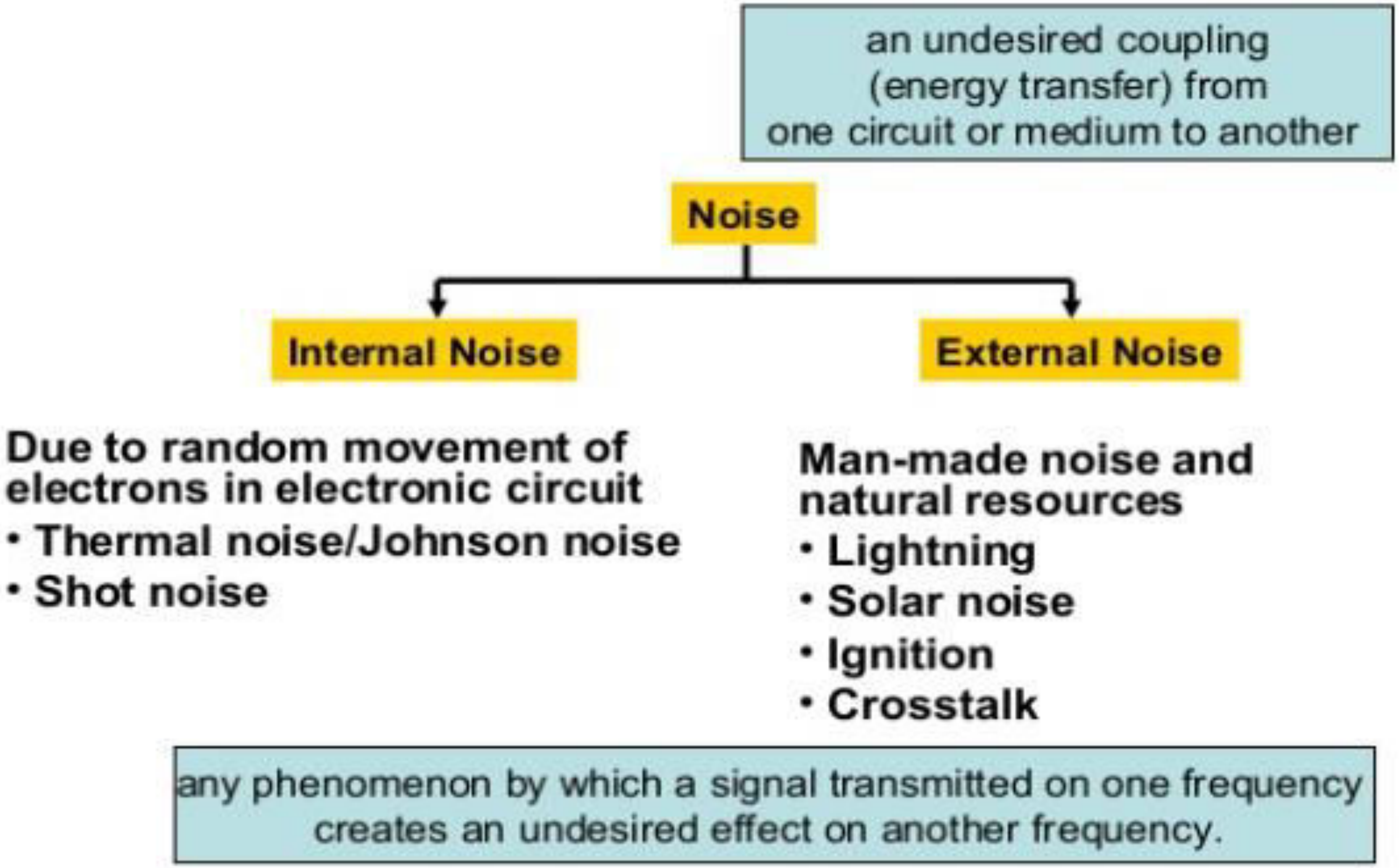
Interference

Interference arises for example, from other communication systems (cross talk), 50 Hz supplies (hum) and harmonics, switched mode power supplies, thyristor circuits, ignition (car spark plugs) motors … etc.

Noise

- Practically, we cannot avoid the existence of unwanted signal together with the modulated signal transmitted by the transmitter.
- This unwanted signal is called noise.
- Noise is a random signal that exists in a communication system.
- Random signal cannot be represented with a simple equation.
- The existence of noise will degrade the level of quality of the received signal at the receiver.

Types of Noise



Noise Effect

- Degrade system performance for both analog and digital systems.
- The receiver cannot understand the sender.
- The receiver cannot function as it should be.
- Reduce the efficiency of communication system.

Thermal Noise (Johnson Noise)

- Johnson–Nyquist noise (thermal noise, Johnson noise, or Nyquist noise) is the Electronic noise - generated by the thermal agitation of the charge carriers (the electrons) inside an electrical conductor in equilibrium, which happens regardless of any applied voltage.

- Movement of the electrons will forms kinetic energy in the conductor related to the temperature of the conductor.

- When the temperature increases, the movement of free electrons will increase and the current flows through the conductor.

- Current flows due to the free electrons will create noise voltage, $n(t)$.

- Noise voltage, $n(t)$ is influenced by the temperature and therefore it is called thermal noise.

- Also known as *Johnson noise or white noise.*

Cont'd…

This type of noise was first measured by John B. Johnson at Bell Labs in 1928. He described his findings to Harry Nyquist, also at Bell Labs, who was able to explain the results.

In 1928, J. B. Johnson have proven that noise power generated is proportional to the temperature and the BW.

$$P_n \propto TB$$

$$P_n = kTB \quad \boxed{Watt}$$

where

P_n = noise power ($Watt$)

k = Boltzmann's constant ($1.38 \times 10^{-23}\ J/K$)

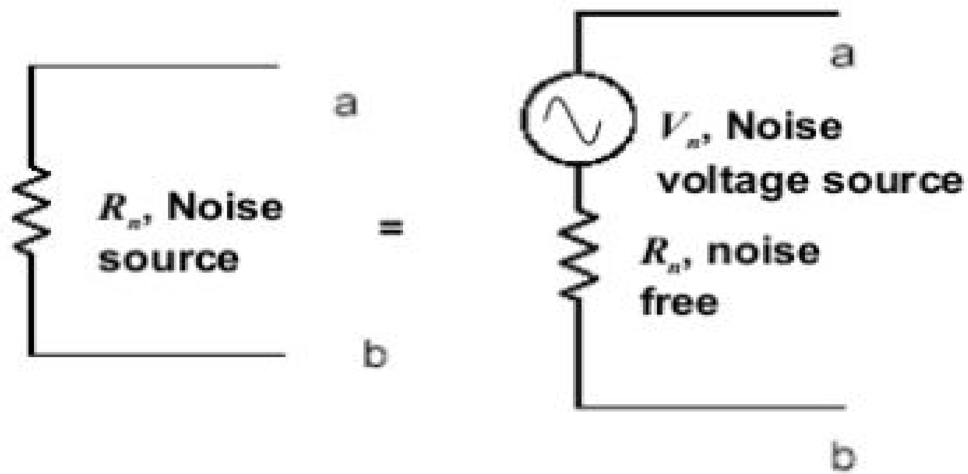
T = Temperature (K)

B = BW spectrum system (Hz)

Noise power can be modeled using voltage equivalent circuit (Thevenin equivalent circuit) or current equivalent circuit (Norton equivalent circuit)

Cont'd…

It can be modeled by a voltage source representing the noise of the non-ideal resistor in series with an ideal noise free resistor.



(a) Noise source circuit (b) Thevenin equivalent circuit

- **Noise source will be connected to a system with the input resistance R_L.**
- **Therefore, total noise power is P_n.**
- **With the concept of maximum power transfer ie when $R_n = R_L$, all the power will be transferred to the load.**
- **Also called as impedance matching.**

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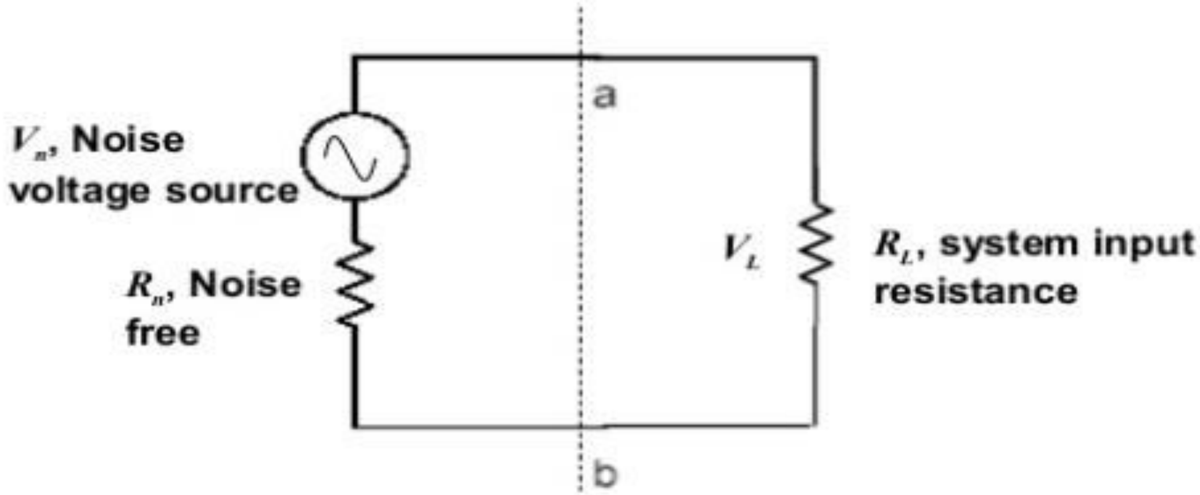


(c) Thevenin equivalent circuit with the load

Given $R_n = R_L = R$

voltage divider rule

Voltage across R_L :

$$V_L = \frac{R_L}{R_n + R_L} V_n$$

$$= \frac{V_n}{2}$$

$$\Rightarrow P_L = \frac{V_L^2}{R} = \frac{\left(V_n/2\right)^2}{R} = \frac{V_n^2}{4R}$$

and $P_n = P_L = kTB$

Note: $V_n = V_{rms}$

$$\Rightarrow \frac{V_n^2}{4R} = kTB$$

$$V_n^2 = 4kTBR$$

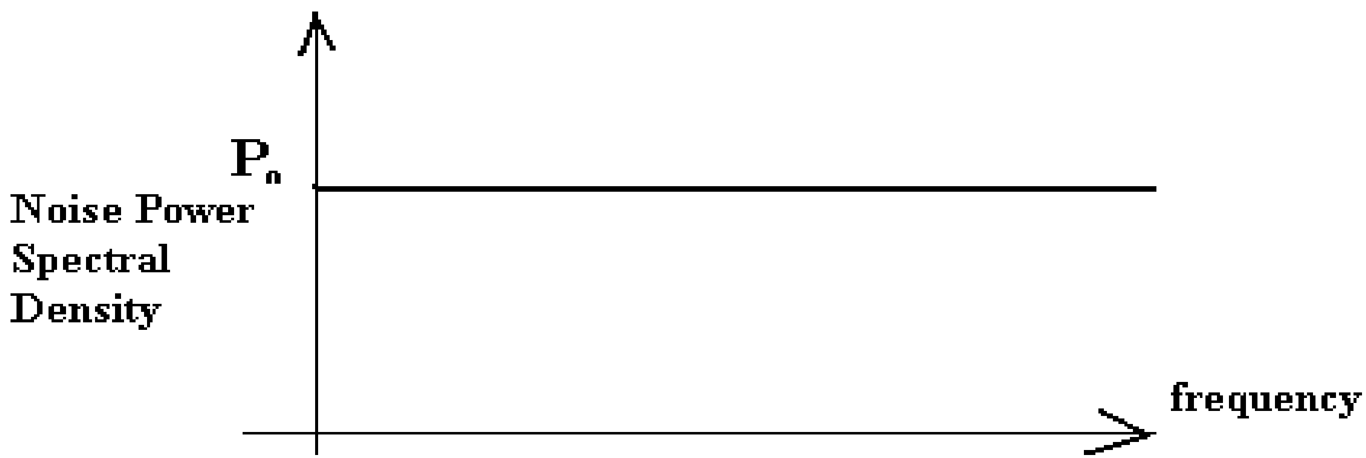
$$V_n = \sqrt{4kTBR}$$

Cont'd…

The law relating noise power, **N** or $\mathbf{P_n}$, to the temperature and bandwidth is

$$\mathbf{N} \text{ or } \mathbf{P_n = k\,TB \text{ watts}}$$

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Uniform Noise Power Spectral Density

Cont'd…

Example 1

One operational amplifier with a frequency range of (18-20) MHz has input resistance 10 kΩ. Calculate noise voltage at the input if the amplifier operate at ambient temperature of 27°C.

BW = fh – fl = (20-18) MHz
= 2 MHz

$V_n^2 = 4KTBR$

$= 4 \times 1.38 \times 10^{-23} \times (273+27) \times 2 \times 10^{6} \times 10^{4}$

$V_n = 18\ \mu\text{volt}$

Shot Noise

- **Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot).**
- **Shot noise also occurs in semiconductors due to the liberation of charge carriers.**
- **For *pn* junctions the mean square shot noise current is**

$$I_n^2 = 2\left(I_{DC} + 2I_o\right)q_e B \qquad (amps)^2$$

Where

I_{DC} is the direct current as the *pn* junction (amps)
I_O is the reverse saturation current (amps)
q_e is the electron charge = 1.6 x 10-19 coulombs
B is the effective noise bandwidth (Hz)

- **Shot noise is found to have a uniform spectral density as for thermal noise.**

Low Frequency or Flicker Noise

Active devices, integrated circuit, diodes, transistors etc also exhibits a low frequency noise, which is frequency dependent (i.e. non uniform) known as flicker noise or ‘one – over – f’ noise(pink noise-1/f).

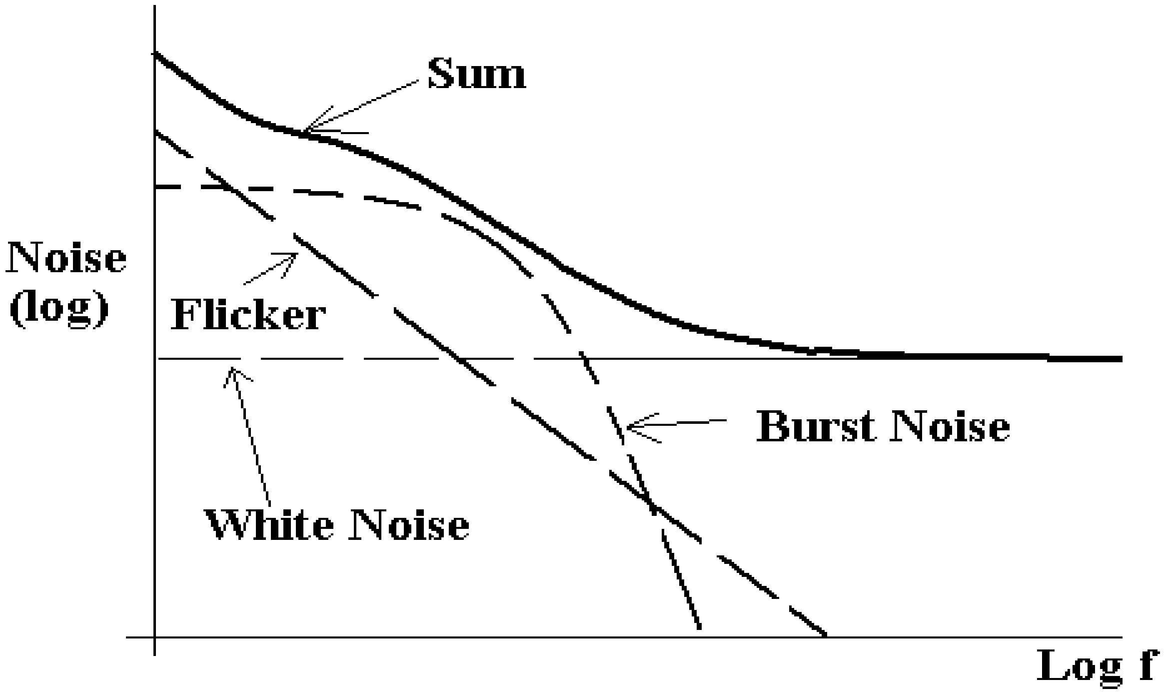
Excess Resistor Noise

Thermal noise in resistors does not vary with frequency, as previously noted, by many resistors also generates as additional frequency dependent noise referred to as excess noise.

Burst Noise or Popcorn Noise

Some semiconductors also produce burst or popcorn noise with a spectral density which is proportional to $\left(\frac{1}{f}\right)^2$

General Comments



For frequencies below a few KHz (low frequency systems), flicker and popcorn noise are the most significant, but these may be ignored at higher frequencies where ‘white’ noise predominates.

Noise Calculation

The essence of calculations and measurements is to determine the signal power to noise power ratio, i.e. the SNR or (S/N) expression in dB.

The signal to noise ratio is given by $SNR = \frac{S}{N} = \frac{Signal\ Power}{Noise\ Power}$

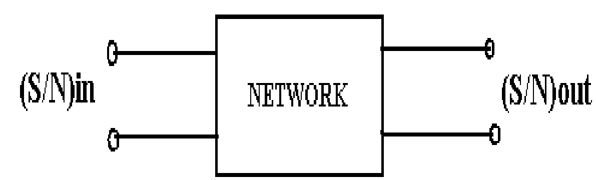
The signal to noise in dB is expressed by

$$SNR_{dB} = \left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

- The amount of noise added by the network is embodied in the Noise Factor F, which is defined by

$$\text{Noise factor F} = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$



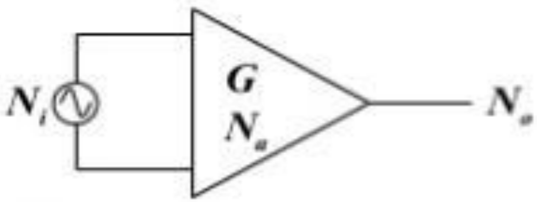
$$\text{Noise Figure} = 10\log_{10} F \qquad F \geq 0\ \text{dB}$$

Noise Calculation in Amplifier

Gain

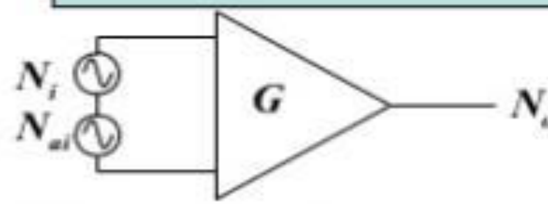
- To simplify the analysis, two types of noise model are used.
- - Amplifier with noise
- - Amplifier without noise

input noise, Ni
artificial noise source, Nai



(a) Amplifier with noise

$$N_o = GN_i + N_a$$

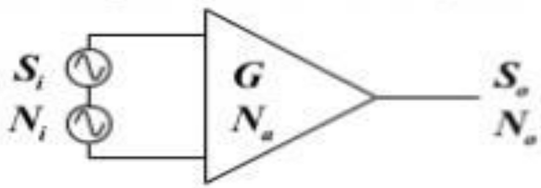


(b) Amplifier without noise

$$N_o = G(N_i + N_{ai})$$

where $N_{ai} = \dfrac{N_a}{G}$ and $P_n = N_i = kT_iB$

Analysis of Amplifier with Noise



Model Amplifier with noise

(1)

$$S_o = GS_i$$

$$N_o = GN_i + N_a$$

$$= G\left(N_i + \frac{N_a}{G}\right)$$

$$= G(N_i + N_{ai})$$

$$SNR_o << SNR_i$$

Noise Figure, F

(2)

$$\frac{SNR_i}{SNR_o} = \frac{\dfrac{S_i}{N_i}}{\dfrac{GS_i}{G(N_i + N_{ai})}}$$

$$= \frac{N_i + N_{ai}}{N_i}$$

$$= 1 + \frac{N_{ai}}{N_i}$$

$$F = 1 + \frac{N_{ai}}{N_i}$$

(3) **We have:**

$N_i = kT_iB$ **and** $N_{ai} = kT_eB$

$$\Rightarrow F = 1 + \frac{kT_eB}{kT_iB}$$

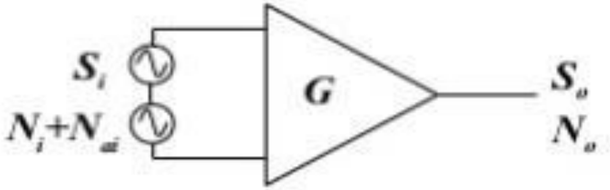
Noise Figure:

$$F = 1 + \frac{T_e}{T_i}$$

Noise Temperature:

$$T_e = (F - 1)T_i$$

Analysis of Amplifier without Noise



Model Amplifier without noise

(1) $S_o = GS_i$

$$N_o = G(N_i + N_{ai})$$

(2)
$$\frac{SNR_i}{SNR_o} = \frac{\dfrac{S_i}{N_i}}{\dfrac{GS_i}{G(N_i + N_{ai})}}$$

$$= \frac{N_i + N_{ai}}{N_i}$$

$$= 1 + \frac{N_{ai}}{N_i}$$

$$F = 1 + \frac{N_{ai}}{N_i}$$

$SNR_o << SNR_i$

(3) **We have:**

$N_i = kT_iB$ **and** $N_{ai} = kT_eB$

$$\Rightarrow F = 1 + \frac{kT_eB}{kT_iB}$$

Noise Figure:

$$F = 1 + \frac{T_e}{T_i}$$

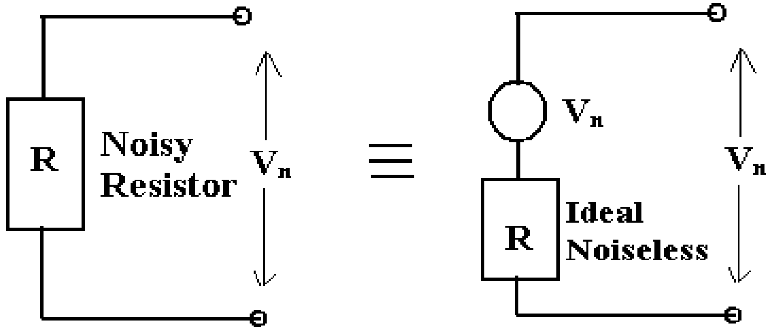
Noise Temperature:

$$T_e = (F-1)T_i$$

Analysis of Noise in Communication Systems

Thermal Noise (Johnson noise)

This thermal noise may be represented by an equivalent circuit as shown below



$$\overline{V^2} = 4kTBR \ (volt^2)$$

(mean square value , power)

then $V_{RMS} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$

i.e. V_n is the RMS noise voltage.

A) System BW = B Hz

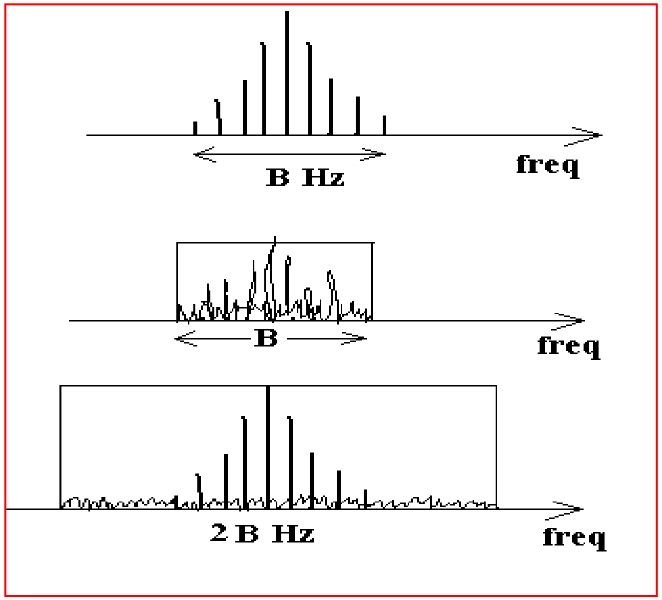
N= Constant B (watts) = KB

B) System BW

N= Constant 2B (watts) = K2B

For A, $\frac{S}{N} = \frac{S}{KB}$

For B, $\frac{S}{N} = \frac{S}{K2B}$



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Resistors in Series

Assume that R_1 at temperature T_1 and R_2 at temperature T_2, then

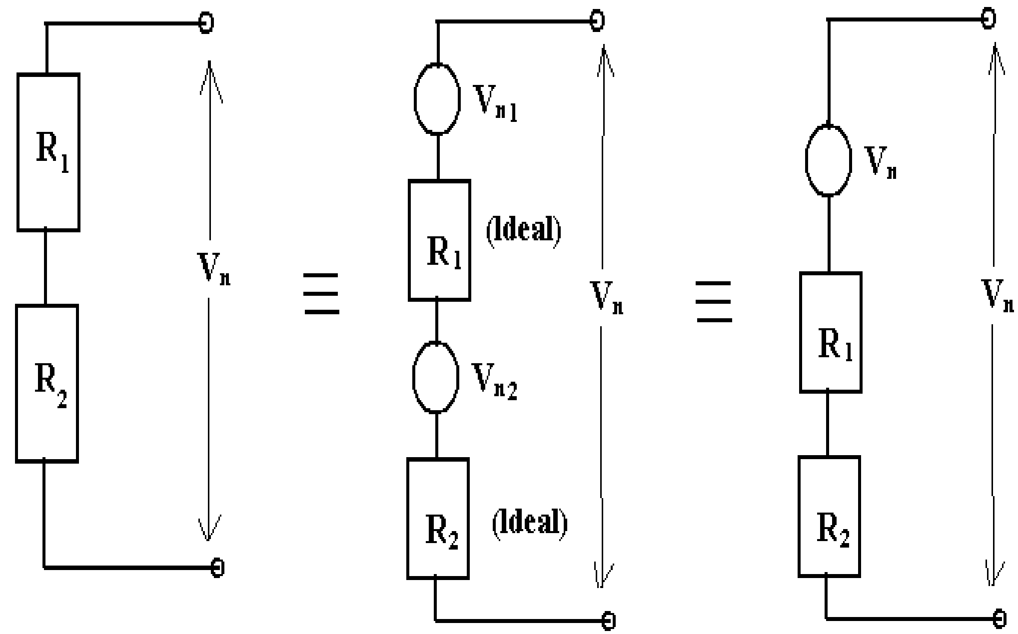
$$\overline{V_n^{\,2}} = \overline{V_{n1}^{\,2}} + \overline{V_{n2}^{\,2}}$$

$$\overline{V_{n1}^{\,2}} = 4kT_1BR_1$$

$$\overline{V_{n2}^{\,2}} = 4kT_2BR_2$$

$$\therefore\ \overline{V_n^{\,2}} = 4kB(T_1R_1 + T_2R_2)$$

$$\overline{V_n^{\,2}} = 4kTB(R_1 + R_2)$$



i.e. The resistor in series at same temperature behave as a single resistor

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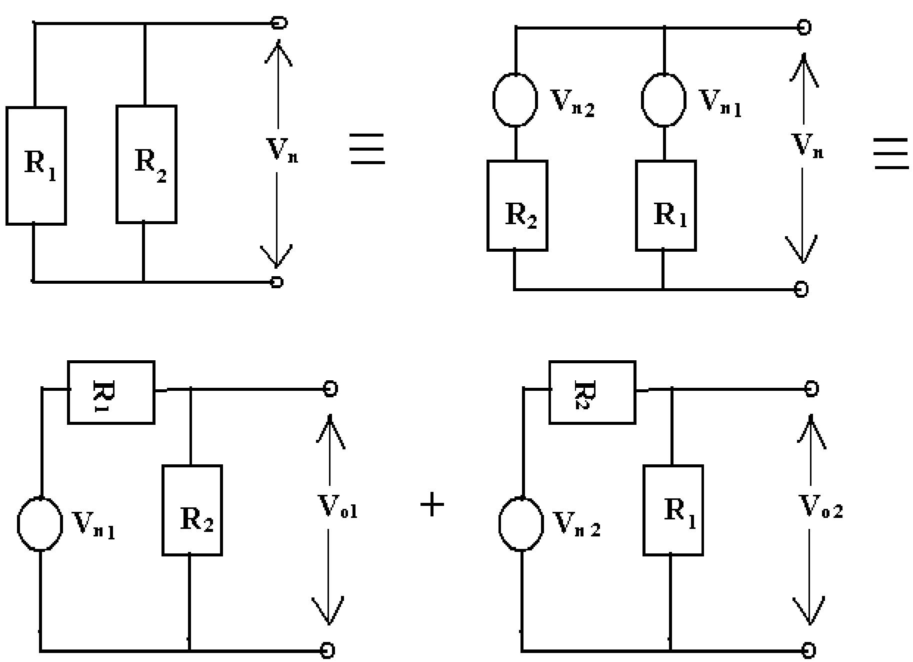
Resistance in Parallel



$$V_{o1} = V_{n1}\frac{R_2}{R_1 + R_2} \qquad V_{o2} = V_{n2}\frac{R_1}{R_1 + R_2}$$

$$\overline{V_n^{\,2}} = \overline{V_{o1}}^{\,2} + \overline{V_{o2}}^{\,2}$$

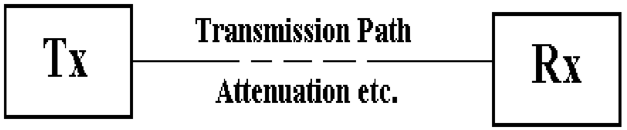
$$\overline{V_n^{\,2}} = \frac{4kB}{(R_1 + R_2)^2}\left[R_2^2\, T_1 R_1 + R_1^2\, T_2 R_2\right] \times \left(\frac{R_1 R_2}{R_1 R_2}\right)$$

$$\overline{V_n^{\,2}} = \frac{4kB\, R_1 R_2 (T_1 R_1 + T_2 R_2)}{(R_1 + R_2)^2}$$

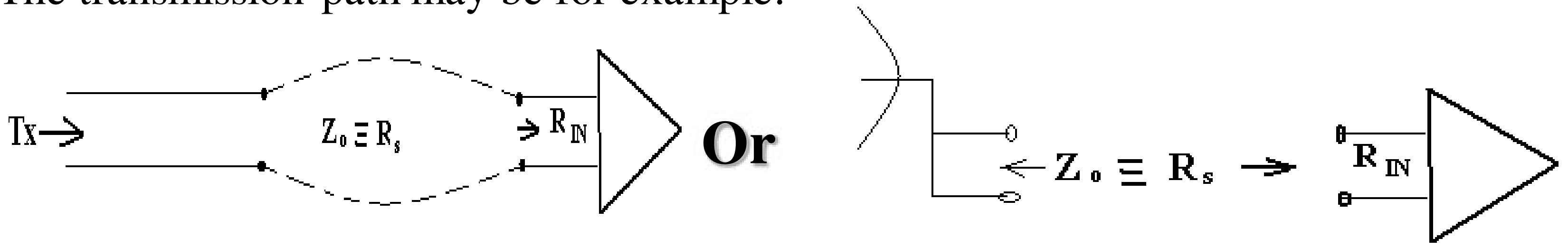
$$\overline{V_n^{\,2}} = 4kTB\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Matched Communication Systems

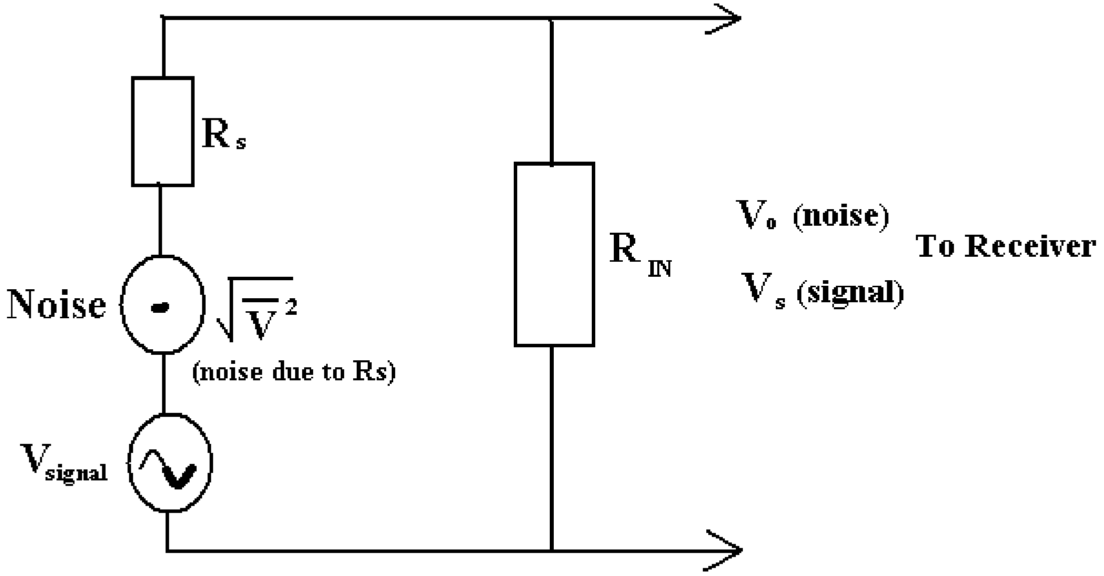
In communication systems we are usually concerned with the noise (i.e. S/N) at the receiver end of the system.



The transmission path may be for example:-



An equivalent circuit, when the line is connected to the receiver is shown below.



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The RMS voltage output, V_o (noise) is

$$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R_{IN}}{R_{IN}+R_S}\right)$$

Similarly, the signal voltage output due to V_{signal} at input is

$$Vs(_{signal}) = (V_{signal})\left(\frac{R_{IN}}{R_{IN}+R_s}\right)$$

For maximum power transfer, the input R_{IN} is matched to the source R_S, i.e. R_{IN} $= R_S$ = R (say)

Then $$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R}{2R}\right) = \frac{\sqrt{\overline{v^2}}}{2} \quad (RMS\ \ Value)$$

And signal, $V_{S(signal)} = \dfrac{V_{signal}}{2}$

Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = kT_S\, B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

We may also write $N_e = kT_e\, B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

$$\text{i.e. } kT_e\, B_n = (F\text{-}1)\, kT_S\, B_n$$

$$\text{or } \; T_e = (F\text{-}1)T_S$$

Noise Figure – Noise Factor for Passive Elements

Since $F = \dfrac{S_{IN}}{N_{IN}} \dfrac{N_{OUT}}{S_{OUT}}$ and $N_{OUT} = N_{IN}$.

$$F = \frac{S_{IN}}{G\,S_{IN}} = \frac{1}{G}$$

If we let L denote the insertion loss (ratio) of the network i.e. insertion loss

$$L_{dB} = 10 \log L$$

Then

$$L = \frac{1}{G}$$ and hence for passive network

$$F = L$$

Also, since $T_e = (F\text{-}1)T_S$

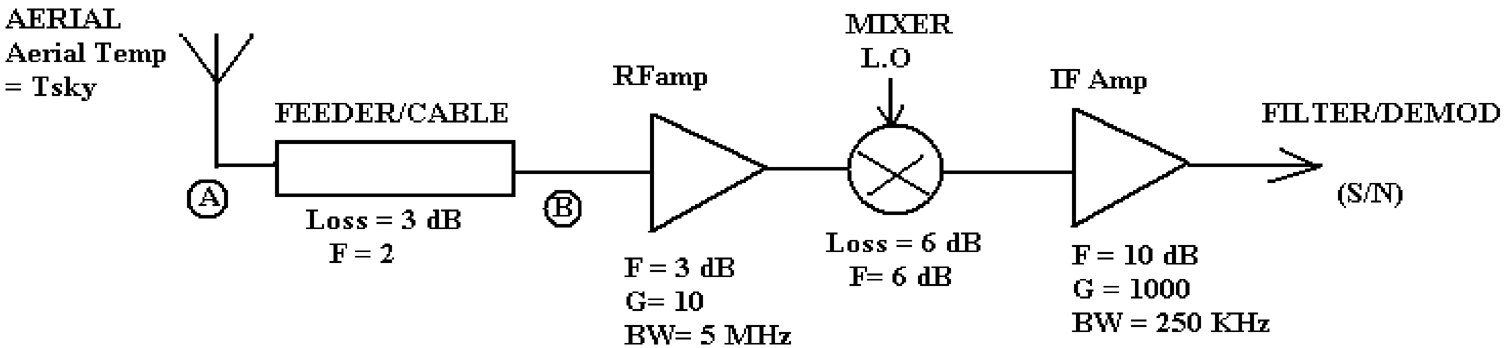
Then for passive network

$$T_e = (L\text{-}1)T_S$$

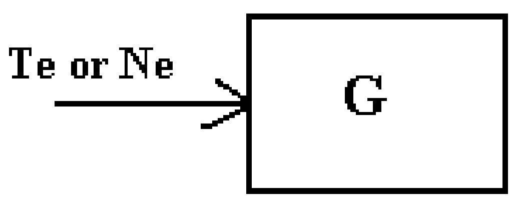
Where T_e is the equivalent noise temperature of a passive device referred to its input.

Cascaded Network

A receiver systems usually consists of a number of passive or active elements connected in series. A typical receiver block diagram is shown below, with example



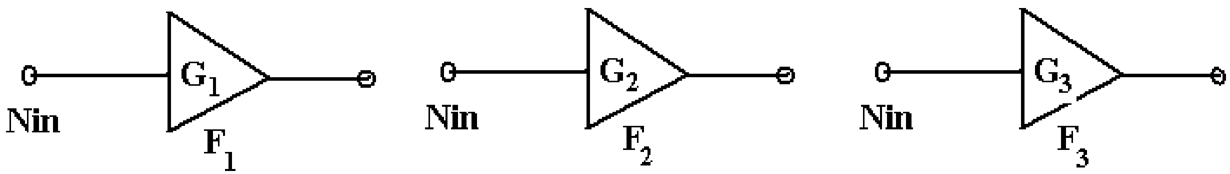
In order to determine the (S/N) at the input, the overall receiver noise figure or noise temperature must be determined. In order to do this all the noise must be referred to the same point in the receiver, for example to A, the feeder input or B, the input to the first amplifier.



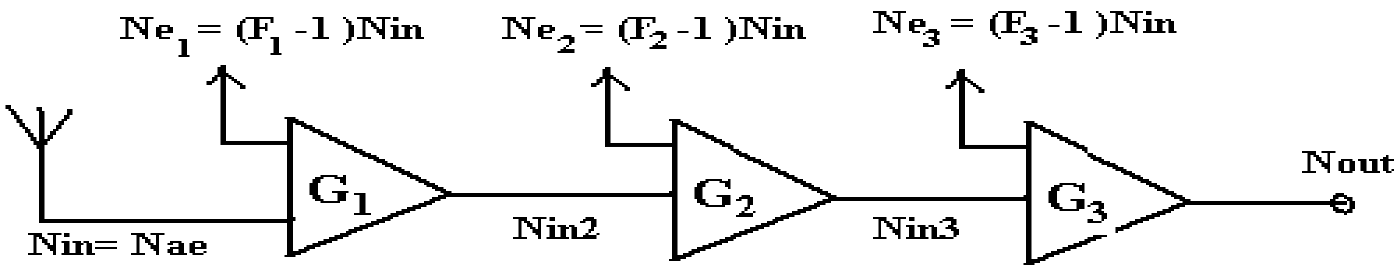
T_e or N_e is the noise referred to the input.

System Noise Figure

Assume that a system comprises the elements shown below,



Assume that these are now cascaded and connected to an aerial at the input, with $N_{IN} = N_{ae}$ from the aerial.



Now ,
$$N_{OUT} = G_3\left(N_{IN3} + N_{e3}\right)$$
$$= G_3\left(N_{IN3} + \left(F_3 - 1\right)N_{IN}\right)$$

Since
$$N_{IN3} = G_2\left(N_{IN2} + N_{e2}\right) = G_2\left(N_{IN2} + \left(F_2 - 1\right)N_{IN}\right)$$

similarly
$$N_{IN2} = G_1\left(N_{ae} + \left(F_1 - 1\right)N_{IN}\right)$$

Cont'd….

$$N_{OUT} = G_3\left[G_2\left[G_1 N_{ae} + G_1\left(F_1 - 1\right)N_{IN}\right] + G_2\left(F_2 - 1\right)N_{IN}\right] + G_3\left(F_3 - 1\right)N_{IN}$$

The overall system Noise Factor is

$$F_{sys} = \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1 G_2 G_3 N_{ae}}$$

$$= 1 + \left(F_1 - 1\right)\frac{N_{IN}}{N_{ae}} + \frac{\left(F_2 - 1\right)}{G_1}\frac{N_{IN}}{N_{ae}} + \frac{\left(F_3 - 1\right)}{G_1 G_2}\frac{N_{IN}}{N_{ae}}$$

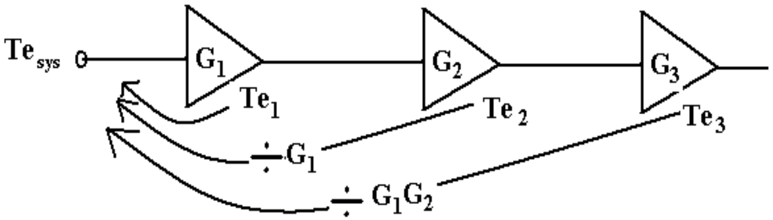
If we assume N_{ae} is $\approx N_{IN}$, i.e. we would measure and specify F_{sys} under similar conditions as F_1, F_2 etc (i.e. at 290 K), then for *n* elements in cascade.

$$F_{sys} = F_1 + \frac{\left(F_2 - 1\right)}{G_1} + \frac{\left(F_3 - 1\right)}{G_1 G_2} + \frac{\left(F_4 - 1\right)}{G_1 G_2 G_3} + \ldots\ldots\ldots\ldots + \frac{\left(F_n - 1\right)}{G_1 G_2 \ldots\ldots\ldots G_{n-1}}$$

The equation is called FRIIS Formula.

System Noise Temperature

Since $T_e = (L-1)T_S$, i.e. $F = 1 + \dfrac{T_e}{T_s}$



Then

$$F_{sys} = 1 + \frac{T_{e\,sys}}{T_s} \qquad \begin{cases} where\,T_{e\ sys}\ is\ the\ equivalent\ Noise\ temperature\ of\ the\ system \\ and\,T_s\ is\ the\ noise\ temperature\ of\ the\ source \end{cases}$$

and

$$\left(1 + \frac{T_{e\,sys}}{T_s}\right) = \left(1 + \frac{T_{e1}}{T_s}\right) + \frac{\left(1 + \dfrac{T_{e\,2}}{T_s} - 1\right)}{G_1} + \ldots etc$$

$$i.e.\ \ from\,F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \ldots\ldots etc$$

$which\ gives$

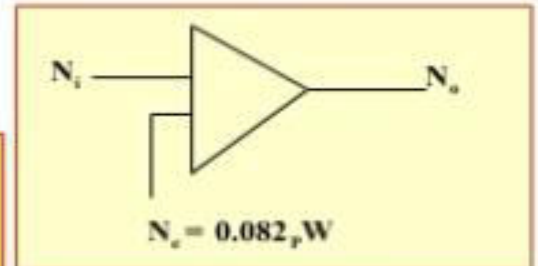
$$T_{e\,sys} = T_{e1} + \frac{T_{e\,2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Example 2

Noise generated in amplifier of 5 MHz bandwidth is represented by amplifier input noise power of 0.082 pW. Calculate noise factor and noise figure if the amplifier was fed with the

(a) source input signal match the temperature of 300 K

(b) source input signal match the temperature of 100 K



(a) Noise power from the source input = KT_iB

$$= 1.38 \times 10^{-23} \times 300 \times 5 \times 10^{6}$$

$$= 0.021 \text{ pW}$$

Ni

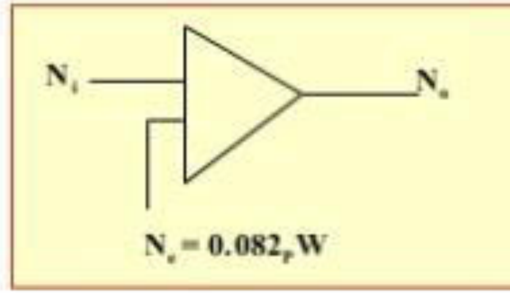
$$\textbf{Noise Factor} = \frac{Ni + Ne}{Ni} = \frac{0.021 + 0.082}{0.021} = \frac{0.103}{0.021} = 4.9$$

Noise Figure **= $10\log_{10} 4.9$ = 6.9 dB**

(b) Noise power from the source input = KT_iB

$= 1.38 \times 10^{-23} \times 100 \times 5 \times 10^{6}$

$= 0.007$ pW



$$\textbf{Noise Factor} = \frac{Ni + Ne}{Ni} = \frac{0.007 + 0.082}{0.007} = \frac{0.103}{0.007} = 12.7$$

Noise Figure = $10\log_{10} 12.7$ = 11.04 dB

Noise factor and noise figure were less when operated at room temperature.

Example 3

An antenna is connected to an amplifier with noise temperature, T_e = 125 °K, gain, G = 10^8. Given the bandwidth, B = 10 MHz and output receiver noise, N_o = 10 µW. Determine the antenna temperature, T_i and noise figure, F of the receiver.

$$
\begin{aligned}
N_o &= (N_i + N_e)G \\
&= (KT_iB + KT_eB)G \\
&= KB(T_i + T_e)G
\end{aligned}
$$

$$10\mu = 1.38\times10^{-23}\times10\times10^{6}(T_i + 125)10^{8}$$

$$\therefore\ T_i = 600\ ^{\circ}K$$

$$F = 1 + \frac{T_e}{T_i} = 1 + \frac{125}{600} = 1.2$$

or

$$F = \frac{N_i + N_e}{N_i} = \frac{100}{82.8} = 1.2$$

(600+125)/600