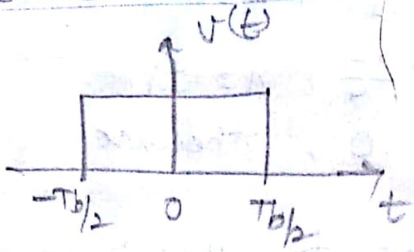
PSD of Polar NRZ Format



→ F.T. of v(t) is

Step 1:- $\boxed{V(f) = T_b \operatorname{sinc}(f T_b)}$

→ ACF of polar scheme

coefficient, $\boxed{A_k = \begin{cases} a, & \text{symbol 1} \\ -a, & \text{symbol 0} \end{cases}}$

→ $R_A[n] = E[A_k A_{k-n}]$

① If n=0

$$R_A[0] = E[A_k^2]$$

	A_k	A_k	A_k^2	Prob
0	$-a$	$-a$	a^2	$\frac{1}{2}$
1	a	a	a^2	$\frac{1}{2}$

$$\therefore R_A[0] = E[A_k A_{k-n}]$$

$$= E[A_k^2]$$

$$= \sum A_k^2 \, P[X=x]$$

$$= a^2 \cdot \frac{1}{2} + a^2 \cdot \frac{1}{2}$$

$$\boxed{R_A[0] = a^2}$$

② If $n \neq 0$; $R_A[n] = E[A_k A_{k-n}]$

	A_k	A_{k-n}	$A_k A_{k-n}$	Prob
0 0	$-a$	$-a$	a^2	$\frac{1}{4}$
0 1	$-a$	a	$-a^2$	$\frac{1}{4}$
1 0	a	$-a$	$-a^2$	$\frac{1}{4}$
1 1	a	a	a^2	$\frac{1}{4}$

$$R_A[n] = E[A_k A_{k-n}]$$

$$= a^2\frac{1}{4} - a^2 \tfrac{1}{4} - a^2 \tfrac{1}{4} + a^2(\tfrac{1}{4})$$

$$\boxed{R_A[n] = 0}$$

③ $$\therefore \boxed{R_A[n] = \begin{cases} a^2, & n=0 \\ 0, & n \neq 0 \end{cases}}$$

④ PSD

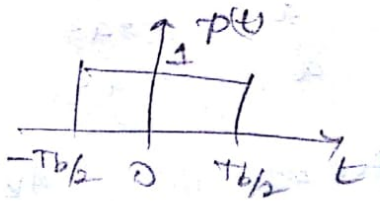
$$S_X(f) = \frac{1}{T_b}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi n f T_b}$$

$$= \frac{1}{\cancel{T_b}}[T_b^{\cancel{2}} \operatorname{sinc}^2(fT_b)] \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi n f T_b}$$

$$= T_b \operatorname{sinc}^2(fT_b)\left[a^2 + \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi n f T_b}\right]$$

$$\boxed{S_X(f) = a^2 T_b \operatorname{sinc}^2(fT_b)}$$

PSD of Bipolar NRZ format



① F.T. of v(t) is $\boxed{V(f) = T_b \operatorname{sinc}(fT_b)}$

② ACF

$$A_k = \begin{cases} a, -a & : \text{alternating } 1/0 \\ 0 & : \text{symbol } 0 \end{cases}$$

1 1 1

+a −a +a

$$R_A[n] = E[A_k A_{k-n}]$$

① If $n=0$

$$R_A[0] = E[A_k^2]$$

	A_k	A_k	A_k^2	Prob
0	0	0	0	1/2
1	a	a	a^2	1/4
	-a	-a	a^2	1/4

$$\therefore R_A[0] = 0 \cdot \tfrac{1}{2} + a^2 \cdot \tfrac{1}{4} + a^2 \tfrac{1}{4}$$

$$\boxed{R_A[0] = \frac{a^2}{2}}$$

② If $n=1$

$$R_A[n] = E[A_k \cdot A_{k-1}]$$

	A_k	A_{k-1}	A_{k-1}	Prob
0 0	0	0	0	1/4
0 1	0	$\pm a$	0	1/4
1 0	$\pm a$	0	0	1/4
1 1	+a	-a	$-a^2$	1/8
	-a	a	$-a^2$	1/8

for bits 11 $\quad A_k \; A_{k-n} = -a^2 \quad$ Prob 1/4

$$\therefore R_A[1] = \sum A_k A_{k-1} [P(X=x)]$$

$$= 0 \cdot \tfrac{1}{4} + 0 \cdot \tfrac{1}{4} + 0 \cdot \tfrac{1}{4} - a^2(\tfrac{1}{8}) - a^2(\tfrac{1}{8})$$

$$\boxed{R_A[1] = -\frac{a^2}{4}}$$

③ From the properties of ACF we know that

$$R_A(-n) = R_A(n)$$

$$\boxed{\therefore R_A(-1) = R_A(1)}$$

④ If $n > 1$

	A_k	A_{k-n}	$A_k A_{k-n}$	Prob
00	0	0	0	1/4
01	0	$\pm a$	0	1/4
10	$\pm a$	0	0	1/4
11	$+a$	$+a$	a^2	1/16
	$-a$	$+a$	$-a^2$	1/16
	$+a$	$-a$	$-a^2$	1/16
	$-a$	$-a$	a^2	1/16

$$\therefore R_A[n] = 0\cdot\tfrac{1}{4} + 0\cdot\tfrac{1}{4} + 0\cdot\tfrac{1}{4} + a^2\cdot\frac{1}{16} - a^2\frac{1}{16} - a^2\frac{1}{16} + a^2\frac{1}{16}$$

$$\boxed{\therefore R_A(n) = 0 \quad ; n > 1}$$

⑤

$$\therefore R_A[n] = \begin{cases} +\frac{a^2}{2} & , n = 0 \\ -\frac{a^2}{4} & , n = \pm 1 \\ 0 & , n > 1 \end{cases}$$

⑥

$$S_X(f) = \frac{1}{T_b}\,|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi n f T_b}$$

$$= \frac{1}{T_b}\left[T_b^2 \operatorname{sinc}^2(fT_b)\right] \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi n f T_b}$$

$$= T_b \operatorname{sinc}^2(fT_b)\left[R_A(0) + R_A(1)e^{-j2\pi f T_b} + R_A(-1)e^{j2\pi n f T_b}\right]$$

$$= T_b \operatorname{sinc}^2(fT_b)\left[\frac{a^2}{2} - \frac{a^2}{4}e^{-j2\pi f T_b} - \frac{a^2}{4}e^{j2\pi f T_b}\right]$$

$$= T_b \operatorname{sinc}^2(fT_b)\left\{\frac{a^2}{2} - \frac{a^2}{4}\left[e^{-j2\pi f T_b} + e^{j2\pi f T_b}\right]\right\}$$

$$= T_b \operatorname{sinc}^2(fT_b)\left\{\frac{a^2}{2} - \frac{a^2}{2}\left[\frac{e^{-j2\pi f T_b} + e^{j2\pi f T_b}}{2}\right]\right\}$$

$$= T_b \operatorname{sinc}^2(fT_b)\left\{\frac{a^2}{2} - \frac{a^2}{2}\left[\cos(2\pi f T_b)\right]\right\}$$

$$= \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b) - \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b)\cos(2\pi f T_b)$$

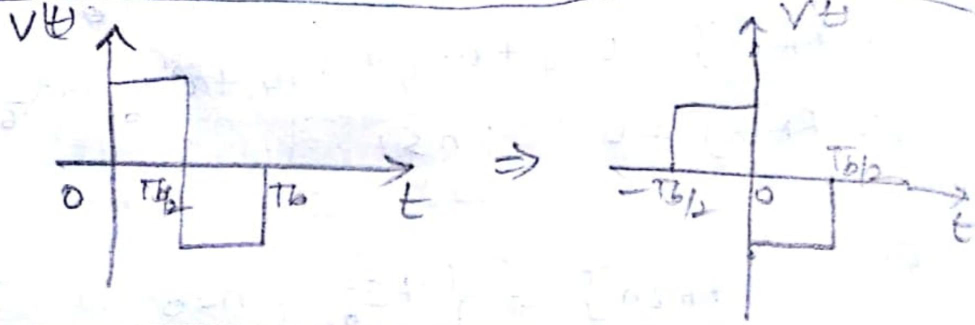
$$= \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b)\left[1 - \cos(2\pi f T_b)\right]$$

$$\because \; 1-\cos 2\theta = 2\sin^2\theta$$

$$S_x(f) = \frac{a^2 T_b}{\cancel{2}}\,\text{sinc}^2(fT_b)\,\cancel{2}\sin^2(\pi f T_b)$$

$$\boxed{\therefore \; S_x(f) = a^2 T_b\,\text{sinc}^2(fT_b)\sin^2(\pi f T_b)}$$

PSD of Manchester Format



(i) F.T. of $v(t)$

$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-T_b/2}^{0} (1)\, e^{-j2\pi f t}\, dt + \int_{0}^{T_b/2} (-1)\, e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-T_b/2}^{0} + \left[-\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{0}^{T_b/2}$$

$$= \frac{1}{-j2\pi f}\left[1 - e^{j\cancel{2}\pi f \cdot \frac{T_b}{\cancel{2}}} - \left(e^{-j\cancel{2}\pi f \cdot \frac{T_b}{\cancel{2}}} - 1\right)\right]$$

$$= \frac{1}{-j2\pi f}\left[1 - e^{j\pi f T_b} - e^{-j\pi f T_b} + 1\right]$$

$$= \frac{-1}{j2\pi f}\left[2 - \left(e^{j\pi f T_b} + e^{-j\pi f T_b}\right)\right]$$

$$= \frac{-1}{j\cancel{2}\pi f}\left[\cancel{2} - \cancel{2}\cos(\pi f T_b)\right]$$

$$= \frac{-1}{j\pi f}\left[1 - \cos(\pi f T_b)\right]$$

$$= \frac{-1}{j\pi f}\left[2\sin^2\left(\frac{\pi f T_b}{2}\right)\right]$$

$\because 1-\cos 2\theta = 2\sin^2\theta$

$$= \frac{j}{\pi f/2}\left[\sin\left(\frac{\pi f T_b}{2}\right)\sin\left(\frac{\pi f T_b}{2}\right)\right]$$

$$= \frac{j}{\pi f/2}\times\frac{T_b}{T_b}\left[\sin\left(\frac{\pi f T_b}{2}\right)\sin\left(\frac{\pi f T_b}{2}\right)\right]$$

$$\boxed{V(f) = j\,T_b\,\mathrm{sinc}\left(\frac{f T_b}{2}\right)\sin\left(\frac{\pi f T_b}{2}\right)}$$

$\because \mathrm{sinc}\,\theta = \frac{\sin(\pi\theta)}{\pi\theta}$

② ACF

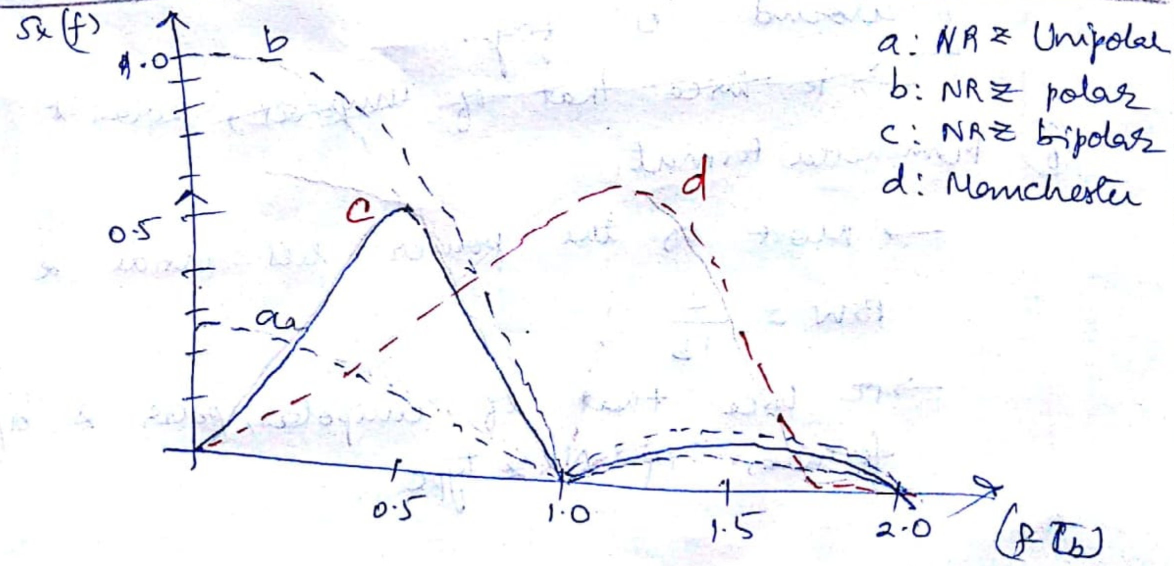
$$\boxed{R_A(n) = \begin{cases} a^2 & n=0 \\ 0, & n\neq 0\end{cases}}$$

③ PSD

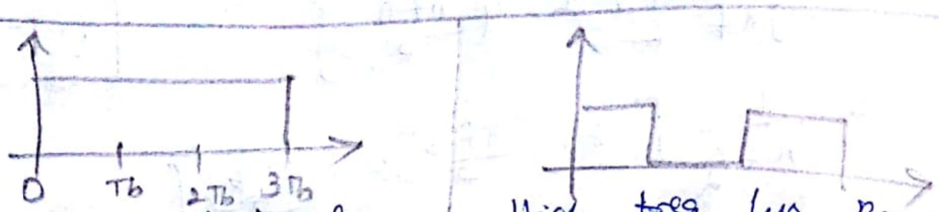
$$S_x(f) = \frac{1}{T_b}|V(f)|^2\sum_{n=-\infty}^{\infty} R_A(n)e^{-j2\pi f n T_b}$$

$$= \frac{1}{\cancel{T_b}}\left[T_b^{\cancel{2}}\,\mathrm{sinc}^2\left(\frac{f T_b}{2}\right)\sin^2\left(\frac{\pi f T_b}{2}\right)\right]a^2$$

$$\boxed{S_x(f) = a^2 T_b\,\mathrm{sinc}^2\left(\frac{f T_b}{2}\right)\sin^2\left(\frac{\pi f T_b}{2}\right)}$$



Power Spectra of different binary data formats



Avg. Power More for Low freq | High freq less Power

1. PSD is normalized with respect to $a^2 T_b$
2. freq 'f' is normalized w.r.t. bit rate $1/T_b$.
3. NRZ Unipolar Format
   → Most of power lies b/w dc and bit rate of the input data.
4. NRZ Polar Format
   → Most of power lies inside Main lobe of sinc shaped curve which extends to the bit rate $1/T_b$.
5. NRZ Bipolar Format
   → Most of the Power lies inside a
   $BW = \frac{1}{T_b}$
   → Spectral content is relatively small around '0' freq.
   ~~→ i.e. twice that of unipolar, polar~~
6. Manchester Format
   → Most of the power lies inside a
   $BW = \frac{2}{T_b}$
   → i.e. twice that of unipolar, polar & bipolar formats of NRZ type