PTSP QUESTION BANK

UNIT-I

1. Distinguish between Joint Probability and Conditional Probability.
2. State and Prove that Total Probability and Bayes Theorem.
3. In a box there are 100 Resistors having resistance and tolerance as shown in Table1. Let a resistor be selected from the box and assume each resistor has the same likelihood of being chosen. Define three events: A as "draw a 47-Ω resistor ", B as "draw a resistor with 5% tolerance ", C as "draw a 100- Ω resistor". Find the Joint and Conditional Probabilities.

Resistance(Ω)	Tolerance		
	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

4. i) A single card is drawn from a 52- card deck.
   a) What is the probability that the card is a jack?
   b) What is the probability the card will be a 5 or smaller?
   c) What is the probability that the card is a red 10?
   d) What is the probability that the card is being either a red or a king?

   ii) A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03, respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed.
   i) What is the probability of an accidental missile launch?
   ii) What is the probability that A will fail if B has failed?
   iii) Are the Events 'A' fails and 'B' fails statistically independent?
5. i) Write short notes on "poission distribution function" and "Binomial distribution function".

   ii) Write short notes on "Gaussian distribution function" and "Uniform distribution function".
6. Distinguish between Probability Distribution and Probability Density functions and their properties.
7. Distinguish between Conditional Probability Distribution and Conditional Probability Density functions and their properties.
8. **(a)** Define Probability and give its three Axioms?

   (b) S.T the chances of throwing "six" with 4, 3 or 2 dice respectively are as 1:6:18.

9. If 'X' is normally distributed with mean 70 and standard deviation (σ), 16

   Find (i) P ($38 \leq X \leq 46$). (ii) P ($82 \leq X \leq 94$).

10. Consider the probability density $f_{(}x) = ae^{-b|x|}$, where x is a random variable whose allowable values range from x = -∞ to ∞. Find

   i) CDF

   ii) Relationship between a and b

   Probability that the outcome x lies between 1 and 2

UNIT-II

1. Distinguish between Moment generating function and Characteristic function and their properties.
2. Explain about Moments. State and prove that Variance and their properties.
3. Show that the mean value E[X] and variance σ_X^2 of the Rayleigh random variable, with density given by $f_X(x) = \frac{2}{b}(x-a)e^{-(x-a)^2/b}, x \geq a$, are E[X] = $a + \sqrt{\pi b/4}$, $\sigma_X^2 = b(4-\pi)/4$.
4. A random variable X has a probability density $f_X(x) = \frac{\pi}{16}\cos(\pi x/8)$ $-4 \leq x \leq 4$

   0 Elsewhere.

   Find: a) Mean value E[X], b) Second order moment E[X^2] and c) Variance σ_X^2
5. The random variable X has the characteristic function given by $\phi_x(w) = 1-|w|, |w| \leq 1$

   0 , $|w|>1$

   Find density function of random variable X.
6. Find the Mean and Variance& characteristic function $\emptyset_x(\omega)$ and moment generating function M_x(t) for uniform distribution?
7. Show that the distribution function for which the characteristic function $e^{-|t|}$ has the density function $f_X(x) = \frac{1}{\pi(1+x^2)}$;$-\infty < x < \infty$.
8. (a)If **'X'** is a R.V, Show that Var (ax+b) = a^2 Var(X).

   (b) If 'X' and 'Y' are two independent random variables, such that

   E(X)= λ_1, variance (X) = σ_1^2, E(Y) = λ_2, Variance (Y)= σ_2^2.

   Prove that variance (XY) = $\sigma_1^2\sigma_2^2$+$\lambda_1^2\sigma_2^2$+$\lambda_2^2\sigma_1^2$.

9. The density function of a random variable X is

$$f(x) = 5e^{-5x} \quad 0 \le x \le \infty$$
$$= 0 \quad \text{else where}$$

Find $E[(X-1)^2]$

UNIT-III

1. Distinguish between Joint Probability Distribution and Probability Density functions and their properties.
2. The Joint probabilities of two random variables X & Y are given in table2.

Y\X	1	2	3
1	0.2	0.1	0.2
2	0.15	0.2	0.15

Find out 1). Joint & Marginal Distribution function and Plot.
2). Joint & Marginal Density function and Plot.

3. Write short notes on "Point conditioning", and "Interval conditioning".

4. The Joint Probability Density function $f_{X,Y}(x,y) = \frac{1}{18} e^{-\left(\frac{x}{6}+\frac{y}{3}\right)}; x \ge 0, y \ge 0$ Find Marginal density functions and show that X and Y are independent random variables.

5. Find the Conditional Density Function $f_{X/Y}(x/y)$ and $f_{Y/X}(y/x)$ for the Joint Density function $f_{X,Y}(x,y) = xe^{-x(y+1)}u(x)u(y)$.
6. Write short notes on "Jointly Gaussian Random variables".
7. Consider random variables Y_1&Y_2 related to arbitrary random variables X & Y by the coordinate rotation $Y_1 = X\cos\theta + Y\sin\theta$, $Y_2 = -X\sin\theta + Y\cos\theta$.
   i) Find the covariance of Y_1&Y_2
   ii) For what value of θ, the random variables Y_1&Y_2 uncorrelated
8. A Joint density is given by

$$f_{XY}(x,y) = \begin{cases} \frac{2}{43}(x+0.5y)^2, & 0 < x < 2 \text{ and } 0 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

   i) Find all the first and second order moment
   ii) Find the Covariance
   iii) Are X and Y uncorrelated?
9. Random variables X and Y have Joint density function

$$f_{XY}(x,y) = (x+y)^2/40, \text{ for } -1 < x < 1, \ -3 < y < 3$$
$$= 0, \quad \text{else where}$$

   i) Find all the second order moments of X and Y
   ii) What are the variances of X and Y
   iii) What is the correlation coefficient?

10. Write about the "Linear Transformation of Gaussian Random Variables"?

11. a) What is the "Joint moments about Origin" and give its First order and second order moments?
    b) Briefly explain about the "Joint Central Moments"?

12. Two Random Variable 'X' and 'Y' have the joint characteristic function $\emptyset_{X,Y}$ (ω_1, ω_2) = exp $(-2\omega_1^2 - 8\omega_2^2)$ Show that X and Y are both Zero-Mean Random Variables and they are uncorrelated?

13. Let X and Y be jointly continuous random variable with joint

    PDF $f_{XY}(X, Y) = X^2 + \frac{XY}{3}$; $0 \leq X \leq 1$; $0 \leq Y \leq 2$.

    Find (i) $f_X(X)$ (Marginal Density function of X). (ii) $f_X(Y)$.
    (iii) Are X and Y are Independent. (iv) $f_X(\frac{X}{Y})$. (v) $f_Y(\frac{Y}{X})$.

14. For the following Joint Distribution of R.V 'X' & 'Y' Find

    i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y = 2)$ (iv) $P(X<3, Y \leq 4)$

Y \ X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

UNIT-IV

1. State and prove that auto correlation & cross correlation functions and their properties.
2. Explain the following with respect to Random process
   i) Wide sense stationary ii) Time average & Ergodic Random Processer
3. Given the random process $X(t)=A \cos(w_o t) + B \sin(w_o t)$ where w_o is a constant and A and B are uncorrelated zero mean random variables having different density functions but the same variances σ^2 show that X(t) wide sense stationary but not strictly stationary.
4. Given the ACF for a stationary ergodic process with no periodic components is $R_{XX}(\tau)=25+4/(1+6\tau)$. Find mean & variance of process X(t).
5. A random process X(t) is defined as $X(t) = A; 0 \leq t \leq 1$
   Where A is random variable that is uniformly distributed from $-\theta$ to θ. Prove that auto correlation function of X(t) is $\theta^2/3$.

6. Two random processes X(t) and Y(t) be defined by $X(t) = A\cos\omega t + B\sin\omega t$ and $Y(t) = B\cos\omega t - A\sin\omega t$, where A & B are uncorrelated random variables with zero mean and same variance and ω is a constant. Find the cross-correlation function and show that X(t) and Y(t) are jointly WSS.
7. Define "Stationary Random Process" and explain the First order Stationary processes?
8. (a)Explain the Wide-Sense Stationary?
   (b) S.T the Random Processes X(t)=A$\cos(\omega_0 t + \theta)$ is wide-sense stationary, if it is assumed that A and ω_0 are constants and 'θ' is uniformly distributed R.V as the interval $(0,2\pi)$.
9. (a) $R_{XY}(\tau) = 36+16/(1+8\tau)$. Find Mean and Mean Square, Variance of Random process X(t)?
   (b) Give short notes on Covariance and give its properties?

UNIT-V

1. Explain Wiener Khinchine theorem.
2. Prove that cross power spectrum and cross correlation function form Fourier transform pair.
3. If the Autocorrelation function of a loss process is $R_{XX}(\tau) = Ke^{-K|\tau|}$. Show that its spectral density is given by
   $$S_{XX}(\omega)=\frac{2}{1+\left(\frac{\omega}{K}\right)^2}$$
4. Find the power spectral density of a WSS random process X(t) where ACF is $R_{XX}(\tau)=ae^{-b|\tau|}$.
5. State and prove that power density spectrum and their properties.
6. Find the cross correlation function for the cross power spectral density is
   $$S_{XY}(\omega)=\frac{1}{25+\omega^2}.$$
7. State and prove that Cross power density spectrum and their properties.
8. An Ergodic Random process is known to have an Auto Correlation function of the form
   $R_{XX}(\tau) = 1- |\tau|$; for $|\tau| \le 1$
   $=0$; for $|\tau| \ge 1$
   Show that the Spectral density is given by $S_{XX}(\omega)=\left[\frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}\right]^2$.

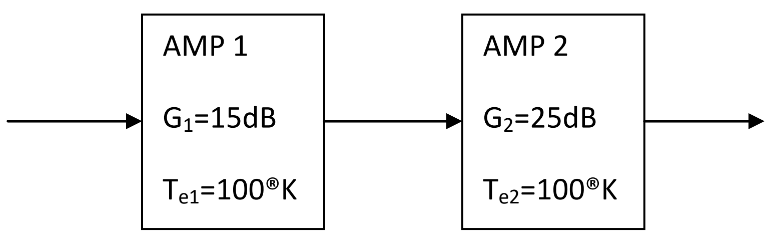
9. Find the Power Spectral density of the random process if that has the Auto Correlation function $R_{XX}(\tau) = \frac{A_0^2}{2}\cos(\omega_0 \tau)$ where A_0 and ω_0 are constants.

10. Find the cross power spectral density, if

    i) $R_{XY}(\tau) = \frac{A^2}{2} sin(\omega_o \tau)$ ii) $R_{XY}(\tau) = \frac{A^2}{2} cos(\omega_o \tau)$

UNIT-VI

1. Explain the following any two
   i) Effective Noise temperature
   ii) Resistive Noise
   iii) Average Noise figure
   iv) Thermal noise
2. Prove that the output power spectral density equals the input power spectral density multiplied by the squared magnitude of the transform of the filter.
3. Define Band limited processes and state its properties.
4. A random process X(t) is applied as input to a system whose impulse response is $h(t) = 3u(t)t^2 \exp(-8t)$. If $E[X(t)] = 2$, what is the mean value of the system response y (t).
5. Explain the Spectral characteristics of linear system response.
6. a) Find overall noise figure and equivalent input noise temperature of following figure at room temperature.



   b) Derive noise bandwidth (ω_N) equation.

7. a) Briefly explain the "White Noise"?

   b) Write about the "Average Noise Figure of Cascaded Network"?

   c) Briefly explain about the "Effective Noise Temperature"?

8. Define the Average Noise Band Width and explain the Noise Band Width of R C Low Pass Filter?