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UNIT II : THE RANDOM VARIABLE

PROBABILITY :

Probability is the chance that something happens.

(or)

The amount of likelyhood(chance) of happening of an event is called Probability.

$$P = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}$$

Examples

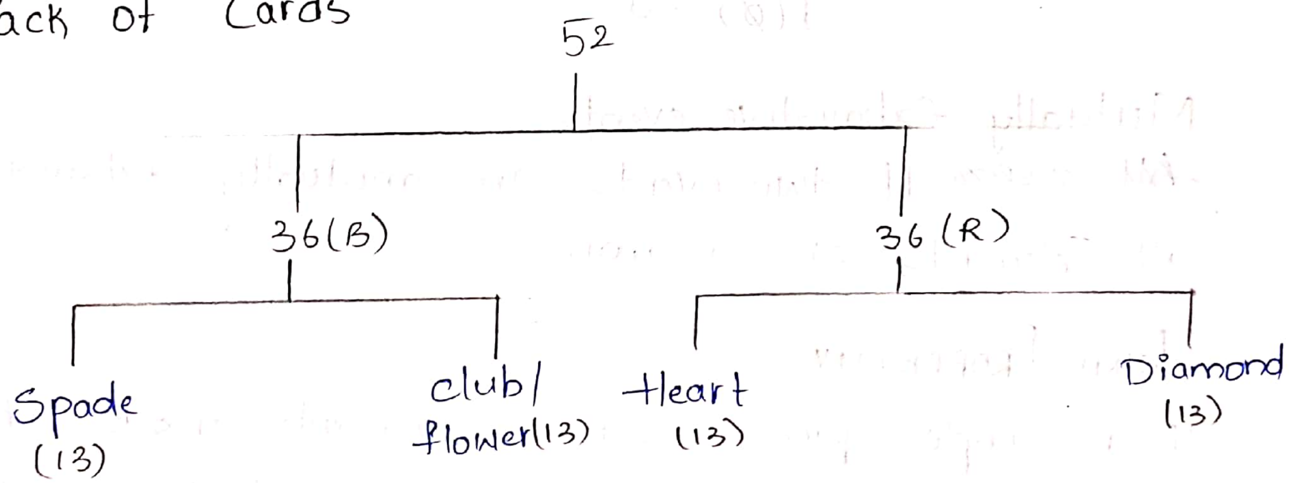
1. Tossing a coin

   Getting head $P = 1/2$

   Tail $P = 1/2$

2. Rolling a dice.

* Pack of Cards



$\underbrace{2, 3, 4, 5, 6, 7, 8, 9, 10}_{\text{No. cards}}$ $\underbrace{J\ Q\ K\ A}_{\text{face cards}}$

* **Axioms**

1. $P(A) \geq 0$
2. $P(S) = 1$ S – sample space / universal set
3. Probability of N. no. of mutually exclusive events is equal to probability of individual events

$$P\left(\bigcup_{n=1}^{N} A_n\right) = \sum_{n=1}^{N} P(A_n)$$

Mutually Exclusive events:

If two events / 2 sets are mutually exclusive if, The occurrence of one of the events includes the possibility of occurrence of any other element.

(or)

If there are no common elements they are called mutually exclusive

$$A \cap B = \emptyset$$

$$P(\emptyset) = 0$$

Mutually Exhaustive events:

~~All events~~ If two events are mutually exhaustive if all elements are common.

JOINT PROBABILITY

If a sample space consists of 2 events A & B which are not mutually exclusive, Then the probability of these events occurring jointly is called joint probability.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(or)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

* If A & B are mutually exclusive,

$$P(A \cup B) = P(A) + P(B) \qquad (\because P(A \cap B) = 0)$$

* If A & B are not mutually exclusive.

$$P(A \cup B) < P(A) + P(B)$$

* But always for any 2 sets A & B

$$P(A \cup B) \leq P(A) + P(B)$$

Q. One card is drawn from a regular deck of cards. What is the probability of either red or King

$$P(R) = \frac{26}{52} \qquad P(K) = \frac{4}{52}$$

$$P(R \cap K) = \frac{2}{52}.$$

$$\therefore P(R \cup K) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$$

* CONDITIONAL PROBABILITY

The Conditional probability of the event A given by event B is defined / denoted by $P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

→ The conditional probability must satisfy 3 Axioms.

1. $P(A/B) = 0$

If A & B are mutually exclusive events

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

2. $P(A/B) = 1$

Let $A = S$

$$P(A/B) = P(S/B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$\therefore P(A/B) = 1$$

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3. $P\left(\frac{A\cup C}{B}\right) = P(A/B) + P(C/B)$ if A, C are mutually exclusive and $A\cap B$, $B\cap C$ are mutually exclusive.

$$P\left(\frac{A\cup C}{B}\right) = \frac{P((A\cup C)\cap B)}{P(B)}$$

$$= \frac{P\{(A\cap B)\cup(C\cap B)\}}{P(B)}$$

$$= \frac{P(A\cap B) + P(C\cap B)}{P(B)}$$

$$= \frac{P(A\cap B)}{P(B)} + \frac{P(C\cap B)}{P(B)}$$

$$= P(A/B) + P(C/B)$$

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TOTAL PROBABILITY

Consider a sample space S that has n - mutually exclusive events B_n, $n = 1, 2, 3, \ldots, N$ such that,

$$B_m \cap B_n = \{\phi\}, \quad m \neq n = 1, 2, 3 \ldots N$$

The probability of any event A defined on this sample space can be expressed in terms of the conditional probability of events B_n. This probability is known as the total probability of event A

$$S = \bigcup_{n=1}^{N} B_n$$

$$A\cap S = A$$

$$A = A \cap \bigcup_{n=1}^{N} B_n$$

$$= \bigcup_{n=1}^{N} (A\cap B_n)$$

Taking Probability on both sides

$$P(A) = P\left[\bigcup_{n=1}^{N} (A\cap B_n)\right]$$

$$= \sum_{n=1}^{N} P(A\cap B_n) \quad \text{——— (1)}$$

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from Conditional probability,

$$P(A/B_n) = \frac{P(A\cap B_n)}{P(B_n)}$$

$$P(A\cap B_n) = P(A/B_n)\,P(B_n) \quad \text{—— (2)}$$

Substituting eq (2) in eq (1)

$$\boxed{P(A) = \sum_{n=1}^{N} P(A/B_n)\cdot P(B_n)} \quad \text{—— (3)}$$

* BAYE'S THEOREM:

Consider a sample space 'S' has N mutually exclusive events B_n, $n=1,2,3----N$ such that $B_m \cap B_n = \{\phi\}$, $m \neq n = 1,2,3---N$ and Any event A is defined on this sample space then the conditional probability of B_n and A can be written as

$$P(B_n/A) = \frac{P(A/B_n)\cdot P(B_n)}{\sum_{n=1}^{N} P(A/B_n)\cdot P(B_n)}$$

Proof:

Let us consider condition probability B_n given by A is

$$P(B_n/A) = \frac{P(A\cap B_n)}{P(A)}, \quad P(A) > 0 \quad \text{—— (1)}$$

Similarly,

$$P(A/B_n) = \frac{P(A\cap B_n)}{P(A)}, \quad P(A) > 0 \quad \text{—— (2)}$$

from eq (2)

$$P(A\cap B_n) = P(A/B_n)\cdot P(B_n) \quad \text{—— (3)}$$

substitute eq (3) in eq (1)

$$P(B_n/A) = \frac{P(A/B_n)\cdot P(B_n)}{P(A)} \quad \text{—— (4)}$$

from Total probability

$$P(A) = \sum_{n=1}^{N} P(A/B_n)\cdot P(B_n) \quad \text{—— (5)}$$

Sub (5) in eq (4)

$$P(B_n/A) = \frac{P(A/B_n)\cdot P(B_n)}{\sum_{n=1}^{N} P(A/B_n)\cdot P(B_n)}$$

$$P(B_n/A) = \frac{P(A/B_n)\cdot P(B_n)}{P(A/B_1)\cdot P(B_1)+P(A/B_2)\,P(B_2)+\cdots P(A/B_N)P(B_N)}$$

→ A Bag Contain 12 balls numbered from 1 to 12. If a ball is taken at random, what is the probability of having a ball with a number which is a multiple of either 2 (or) 5

Sample space $S = \{1,2,3,4,5,6,7,8,9,10,11,12\} \Rightarrow n(S) = 12$

Multiple of 2 $A = \{2,4,6,8,10,12\}$, $n(A) = 6$

Multiple of 3 $B = \{3,6,9,12\}$; $n(B) = 4$

$$A\cap B = \{6,12\}$$

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2} \qquad P(B) = \frac{n(B)}{n(S)} = \frac{4}{12} = \frac{1}{3}$$

$$P(A\cap B) = \frac{2}{12} = \frac{1}{6}$$

$$P(A\cup B) = \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{2}{3}$$

→ An experiment consists of the sum of the numbers showing up when two dices are draw. Define by 3 events. $A = \{sum = 7\}$, $B = \{8 < sum \le 11\}$, $C = \{10 < sum\}$. find the probability of A $(P(A))$, $P(B)$, $P(C)$, $P(A\cap B)$, $P(B\cap C)$, $P(C\cap A)$

$$S = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

$A = \{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)\}$

$B = \{(3,6)(4,5)(5,4)(6,3)(6,4)(5,5)(4,6)(6,5)(5,6)\}$

$C = \{(5,6)(6,5)(6,6)\}$

$n(S) = 6^2 = 36$ $\quad n(B) = 9$

$n(A) = 6$ $\quad n(C) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$ $\quad P(B) = \frac{n(B)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

$P(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

$A \cap B = \{\emptyset\}$, $\quad P(A \cap B) = 0$

$B \cap C = \{(6,5)(5,6)\}$ $\quad P(B \cap C) = 2/36$

$A \cap C = \{\emptyset\}$

$P(A \cap C) = 0$

→ In a box there are 100 resistors having resistance and tolerance as shown in table. Let a resistor be selected from the box and Assume each resistor has the same likelyhood of being choosen. Define 3 events A as "draw a 47 Ω resistor"

B as "draw a resistor with 5% tolerance"

C as "draw a 100Ω resistor"

find the probability of A (P(A)), P(B), P(C), $P(A \cap B)$, $P(B \cap C)$, $P(C \cap A)$, $P(A|B)$, $P(B|A)$, $P(C/A)$

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Resistance (Ω)	Tolerance		Total
	5%	10%	
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

$n(S) = 100$; $n(A) = 44$; $n(B) = 62$; $n(C) = 32$

$P(A) = \frac{n(A)}{n(S)} = \frac{44}{100}$ $\quad$ $P(B) = \frac{n(B)}{n(S)} = \frac{62}{100}$

$P(C) = \frac{n(C)}{n(S)} = \frac{32}{100}$

Joint Probability

$P(A \cap B) = \frac{28}{100}$ $\quad$ $P(B \cap C) = \frac{24}{100}$ $\quad$ $P(A \cap C) = 0$

Conditional probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{28/100}{62/100} = \frac{28}{62}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{28/100}{44/100} = \frac{28}{44}$$

$$P(C/A) = \frac{P(C \cap A)}{P(A)} = 0$$

$$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{24/100}{32/100} = \frac{24}{32}$$

* **PERMUTATION :**

If 'r' objects are choosen from a set of n distinct objects any particular arrangement or order of these objects is called permutation. The no. of permutations of r objects selected from a set of n objects is

$$^nP_r = \frac{n!}{(n-r)!} \quad (or) \quad \frac{\lfloor n}{\lfloor n-r}$$

* COMBINATIONS

To find the no. of ways in which r objects can be selected for a set of 'n' distinct objects is called the no. of combinations.

$$^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Q. How many permutations are there for 4 cards taken from a 52 cards deck.

$$^nP_r \Rightarrow {}^{52}P_4 = \frac{52!}{(52-4)!} = 52\times51\times50\times49$$

Q. ~~card is~~ What is the probability of drawing a 3 W & 4 G balls from a bag that contains 5W and 6G Balls. If 7 balls are drawn simultaneously at random.

$$\frac{^5C_3 + {}^6C_4}{^{11}C_7} = \frac{\left(\frac{5!}{2!\times3!}\right)\left(\frac{6!}{2!\,4!}\right)}{\frac{11!}{3!\,7!}}$$

Q. A box of 100 SC chips has 20 defective chips. 2 chips are selected at random ~~and~~ without replacement from the box.

a) Probability that 1st selected is defective

b) Probability that 2nd selected is defective, given that the 1st one is defective.

c) What probability that both are defective.

a) $\frac{^{20}C_1}{^{100}C_1}$

$\frac{20}{100}$

b) $\frac{(^{19}C_1)\cancel{(^{20}C_1)}}{^{99}C_1}$

$\frac{19}{99}$

c) $\cancel{\frac{^{20}C_2}{^{100}C_2}}$

$\frac{(^{20}C_1)(^{19}C_1)}{(^{100}C_1)(^{99}C_1)}$

→ A missile can be accidentally launched if 2 relays A & B both have failed. The P's of A & B failing are known to be 0.01 & 0.03 respectively. It is also known that B is more likely to fail ($P(B) = 0.06$). If A has failed. $P(B/A)$

(1) What is the probability of an accidental missile launch?

2) What is the probability that A will fail if B has failed

3) Or the events "A fails" & "B fails" statistically independent

1. $P(B \cap A) = P(B/A)\,P(A)$

$(0.06)(0.01) = 6\times10^{-4}$

2. $P(A/B) = \dfrac{6\times10^{-4}}{3\times10^{-2}} = 0.02$

⇒ If 2 events are statistically independent then the probability of occurence of 1 event is not affected by the occurence of other event. The P of the joint occurence of 2 events A & B must equal to the product of the 2 events probability

$\Rightarrow \boxed{P(A\cap B) = P(A).P(B)}$

$6\times10^{-4} \neq 3\times10^{-4}$ (Not statistically independent)

Q) 1 card is selected from an ordinary 52 card deck with an event A as "select a King", B as "select a jack or queen", C as "select a heart". Determine whether A, B, C are independent by pairs

$P(A) = \frac{4}{52} = \frac{1}{13}$ $\quad P(B) = \frac{8}{52} = \frac{2}{13}$ $\quad P(C) = \frac{13}{52} = \frac{1}{4}$

$P(A).P(B) = \frac{2}{169}$ $\quad P(A\cap B) = \cancel{\frac{4}{52}+\frac{8}{52}-\frac{12}{52}}$ 0

$P(B\cap C) = \frac{2}{26} = \frac{2}{13}.\frac{1}{4} = \frac{1}{26}$ $\quad P(C\cap A) = 1/52 = \frac{1}{4}\times\frac{1}{13}$

A, B, C are not statistically independent (∵ A & B are not SI)

* COMBINATIONS

To find the no. of ways in which r objects can be selected for a set of 'n' distinct objects is called the no. of combinations

$$^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

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Q. How many Permutations are there for 4 cards taken from a 52 cards deck.

$n = 52 \qquad r = 4$

$$^nP_r = {}^{52}P_4 = \frac{52!}{(52-4)!} = \frac{52!}{48!} = 52 \times 51 \times 49 \times 50$$

$$= 6497400$$

Q. What is the probability of drawing a 3 white and 4 Green balls from a bag that contains 5 white and 6 Green balls. If 7 balls are drawn simultaneously at random.

$$\frac{^5C_3 \times {}^6C_4}{^{11}C_7} = \frac{\frac{5!}{2!\,3!} \times \frac{6!}{2!\,4!}}{\frac{11!}{7!\,4!}}$$

$$= \frac{\cancel{120}^{\cancel{30}^{10}} \times \cancel{6}^{3} \times 120 \times 7!}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times 11!}$$

$$= \frac{30 \times 120 \times 7!}{11 \times 10 \times 9 \times 8 \times 7!}$$

$$= \frac{\cancel{30} \times \cancel{120}^{\cancel{40}\,5}}{11 \times \cancel{10} \times \cancel{9}_{3} \times 8} = \frac{5}{11}$$

Q. A box of 100 semiconductor chips has 20 defective chips. Two chips are selected at random, without replacement from the box.

a. What is the probability that the 1 st one is selected is defective

b. What is the probability that the 2nd one is selected is defective, given that the 1 st one was defective

c. What is probability that both are defective.

Given , $n(S) = 100$, defective $n(A) = 20$

a. $P(A) = \frac{n(A)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

b. $P(B/A) = 19/99$ $\quad P(B) = 19$ (defective) $n(S) = 99$

c. $P(A \cap B) = ?$ $\Rightarrow P(B/A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = P(B/A) \cdot P(A)$

$= \frac{19}{99} \times \frac{20}{100} = \frac{19}{495}$

Q. A missile can be accidentally launched if 2 relay's A & B Both have failed. The probability of A & B failing are known to be 0.01 & 0.03 respectively. It is also known that B is more likely to fail (probability = 0.06). If A has failed.

a. What is the probability of an accidental missile launch?

b. What is the probability that A will fail if B has failed?

c. On the events "A fails" & "B fails" statistically independent.

Given, $P(A) = 0.01$, $P(B) = 0.03$, $P(B/A) = 0.06$

a. $P(A \cap B) = P(B/A) \cdot P(A) = 0.06 \times 0.01 = 6 \times 10^{-4}$

b. $P(A/B) = P(A \cap B)/P(B) = 6 \times 10^{-4} / 3 \times 10^{-2} = 2 \times 10^{-2} = 0.02$

c. Statistically independent:

→ If 2 events are statistically independent then the probability of occurance of one even is not effected by the occurance of the other event

→ The probability of the joint occurance of 2 events A & B must equal the product of 2 event probabilities i.e,

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

$$P(A)\, P(B) = 0.01 \times 0.03 = 3 \times 10^{-4}$$

$\therefore$ $P(A \cap B) \neq P(A) \cdot P(B)$ $\therefore$ Not statistically independent.

Q. One card is selected from an ordinary 52 card deck with an event A as "select a King", B as "select a Jack or Queen" & C as "select a heart". Determine whether A, B & C are independent by pairs

$P(A) = 4/52$, $P(B) = 8/52$, $P(C) = 13/52$, $P(A \cap B) = 0$,

$P(B \cap C) = 2/52$ $\quad P(A \cap B) \neq P(A)P(B)$ $\therefore$ B, C & C, A pairs are statistically independent.

$P(A \cap C) = 1/52$ $\quad P(B \cap C) = P(B)P(C)$ $\quad$ $\therefore$ A, B, C are not statistically independent.

$P(A \cap C) = P(A)P(C)$

Sample Space

1. The set of all possible outcomes in any given experiment is known as sample space.
2. Sample space is of two types
   a. Continuous sample space
   b. Discrete sample space

Discrete Sample Space

A sample space is said to be discrete & finite, the set in the sample space have finite no. of elements.

Ex:- $A = \{1, 3, 4, 7\}$

If the sets in the sample space have infinite no. of elements then the sample space is called discrete & infinite sample space.

Ex:- $B = \{1, 2, 3, 4, \ldots\}$

Continuous Sample Space

If the sample space contains an infinite no. of elements with continuous values with in a given range then it is called as continuous sample space

Ex: $A = \{1 \le a \le 5\}$

Event: Subset of sample space is called event

Equally likely event: In a given experiment, if one of the event in a sample space does not depends on another event then the two events are called equally likely events

Ex:- Tossing a coin

* **Probability as a relative frequency**

If a coin is flipped many times (n) and heads shows up n_H times out of n flips, then the probability of H

$$P(H) = \lim_{n \to \infty} \frac{n_H}{n}$$

Where, $\frac{n_H}{n}$ is the probability as relative frequency (or) relative frequency of this event.

Bernoulli Trials:

Consider any experiment in which there are 2 possible outcomes. Assume that these events are statistically independent for every trial.

Let an event occur on any given trial with probability $P(A) = p$. If the exp. is repeated for n - trials called as Bernoulli Trials then the $P(A)$ occurring exactly 'r' times in 'n' independent Trials ($r <= n$) is given by

$$\boxed{P(A \text{ exactly } r \text{ time}) = {}^{n}C_{r}\, p^{r} (1-p)^{n-r}}$$

$p + q = 1$

success failure

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Q. A binary symmetrical channel is used for communication b/w a transmitter & receiver. A transmitter transmitts 2 possible i/p's 0 and 1 bits. The probability of transmitting a 0 bit is 0.45 and The probability of transmitting a 1 bit is 0.55. At the receiver end there are 4 possibilities

(i) The probability of transmitting a 0 bit & ~~or~~ receiving a 0 bit is 0.8

(ii) The probability of transmitting a 0 bit & receiving a 1 bit is 0.2

(iii) The probability of transmitting a 1 bit & receiving a 0 bit is 0.2

(iv) The probability of transmitting a 1 bit & receiving a 1 bit is 0.8

find the probabilities of possible bits at receiver end

$P > 0.5$ – correct transmission

$P < 0.5$ – error transmission

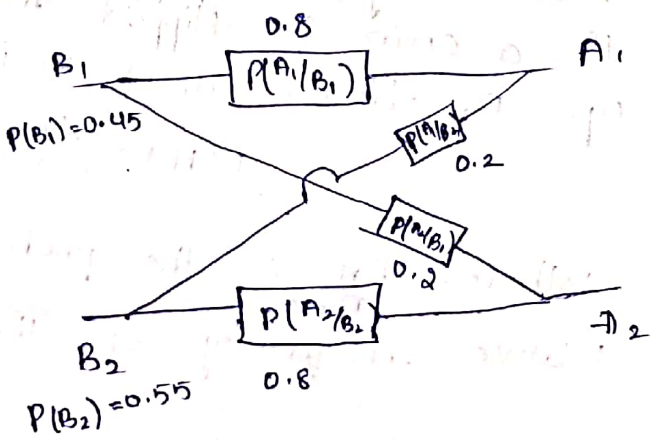
Let us consider 2 events B_1 & B_2

B_1 – Transmitting a '0' bit

B_2 – Transmitting a '1' bit

A_1 – Receiving a '0' bit

A_2 – Receiving a '1' bit



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Given,

Probability of transmitting a '0' bit is $P(B_1) = 0.45$

Probability of trasmitting a '1' bit is $P(B_2) = 0.55$

$P(A_1/B_1) = 0.8$ $\qquad P(A_2/B_1) = 0.2$

$P(A_1/B_2) = 0.2$ $\qquad P(A_2/B_2) = 0.8$

$$P(A_1) = \sum_{n=1}^{2} P(A_1/B_n).P(B_n)$$

$$P(A_1) = P(A_1/B_1).P(B_1) + P(A_1/B_2).P(B_2)$$

$$= 0.8 \times 0.45 + 0.2 \times 0.55$$

$$= 0.47$$

$$P(A_2) = P(A_2/B_1)P(B_1) + P(A_2/B_2).P(B_2)$$

$$= 0.2 \times 0.45 + 0.8 \times 0.55$$

$$= 0.53$$

Using Baye's Theorem

To find $P(B_1/A_1)$

Probability of transmitting a '0' bit from receiving a '0' bit

$$P(B_1/A_1) = \frac{P(A_1/B_1)P(B_1)}{P(A_1)} = \frac{0.8 \times 0.45}{0.47} = 0.76$$

$$P(B_2/A_1) = \frac{P(A_1/B_2).P(B_2)}{P(A_1)} = \frac{0.2 \times 0.55}{0.47} = 0.234$$

$$P(B_2/A_2) = \frac{P(A_2/B_2).P(B_2)}{P(A_2)} = \frac{0.8 \times 0.55}{0.53} = 0.83$$

$$P(B_1/A_2) = \frac{P(A_2/B_1).P(B_1)}{P(A_2)} = \frac{0.2 \times 0.45}{0.53} = 0.16$$

$P(B_1/A_1)$ & $P(B_2/A_2)$ have the value > 0.5

$\therefore$ $P(B_1/A_1)$ & $P(B_2/A_2)$ are correct probability and

$P(B_1/A_2)$ & $P(B_2/A_1)$ are error paobability.

* RANDOM VARIABLE

→ A Random variable is a real funcn. for the events of a given sample space 'S' Thus for a given exp. defined by a sample space 'S' with events 's', the random variable is a funcn of 's'. It is denoted with a Capital letter like

$X(s)$ or $X \leq x$

→ A Random variable X can be considered to be a funcn that maps all events of the sample space into points on the real axis.

→ Typical Random variables are the no. of hits in a shooting game, no. of heads when tossing a coin, temp. or pressure variations of a physical system, no. of missile hits on the target, no. of favourable outcomes in Bernoulli trials, Percentage of marks obtained by students

* Conditions for a function to be a Random Variable

1. The set $\{X \leq x\}$ shall be an event for any real no's x. The probability of this event is equal to the sum of the probabilities of all the elementary events corresponding to $\{X \leq x\}$.
2. The probabilities of events $\{X = \infty\}$ & $\{X = -\infty\}$ are zero

Classification of Random Variable

There are 3 types of Random Variables

1. Continuous
2. discrete
3. Mixed

1. Continuous Random Variable

A Random Variable is said to be continuous if it can take all possible values in an interval

Ex:- $X = \{1 \leq x \leq 5\}$

2. Discrete Random Variable.

→ A Random variable which can take only a discrete no's or values is called Discrete random variable.

Ex:- A dice thrown.

$$X = \{1, 2, 3, 4, 5, 6\}$$

3. Mixed Random Variable.

→ A mixed random variable is one for which sum of it's values are discrete & sum are continuous

$$X = \{1, 2, 4, 5 \leq x \leq 10\}$$

* Probability distribution function

(Cumulative distribution funcn) (CDF)

The PDF associated with a random variable X is defined will be as the probability that, The outcome of an exp. is one of the outcomes for which $X \leq x$, where 'x' is any real no. i.e

$$F_X(x) = P(X \leq x)$$

Properties

1. $F_X(\infty) = 1$ & $F_X(-\infty) = 0$

Proof : $F_X(\infty) = P(X \leq \infty) = P(S) = 1$

$F_X(-\infty) = P(X \leq -\infty) = 0$

2. $0 \leq F_X(x) \leq 1$ ($\because$ from Above property)

3. If $x_1 < x_2$ then, $F_X(x_1) \leq F_X(x_2)$

Proof :- The event $\{X \leq x_1\}$ is a subset of the event $\{X \leq x_2\}$

$\therefore P(X \leq x_1) \leq P(X \leq x_2)$

$\therefore F_X(x_1) \leq F_X(x_2)$

4. $F_X(x_2) - F_X(x_1) = P(x_1 < X \leq x_2)$

Proof :- Two events $\{X \leq x_1\}$ and $\{x_1 < X \leq x_2\}$ are mutually exclusive events i.e a random variable X cannot have a value less than x_1 & in b/w x_1 & x_2 simultaneously. i.e

$$\{X \le x_2\} = \{X \le x_1\} + \{x_1 < X \le x_2\}$$

Take probability on both sides

$$P\{X \le x_2\} = P\{X \le x_1\} + P\{x_1 < X \le x_2\} \quad (\because P(A \cap B) = 0)$$

$$P\{X \le x_2\} - P\{X \le x_1\} = P\{x_1 < X \le x_2\}$$

$$F_X(x_2) - F_X(x_1) = P(x_1 < X \le x_2)$$

5. $F_X(x^+) = F_X(x)$

x^+ is infinitely small value $(0.0000 \cdots 1)$

$x^+ \to x + \varepsilon$

6. $P(X > x) = 1 - F_X(x)$

Proof :-

Q events $\{X \le x_1\}$ & $\{X > x\}$ are mutually exclusive events Then,

$$P\{X \le x\} + P\{X > x\} = P(S)$$

$$P\{X \le x\} + P\{X > x\} = 1$$

$$\therefore P\{X > x\} = 1 - P\{X \le x\}$$

$$P\{X > x\} = 1 - F_X(x)$$

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* Note :-

1. If X be a discretel random variable then consideration of it's CDF is defined as,

$$F_X(x) = \sum_{i=1}^{N} P(x_i) \cdot u(x - x_i)$$

Probability density function (PDF)

The probability density function is denoted by $f_X(x)$ & is defined as the derivative of Probability distribution function.

$$f_X(x) = \frac{d}{dx}(F_X(x))$$

Properties:

1. $f_X(x) \geq 0$, for all x.

2. $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

Proof :- $\int_{-\infty}^{\infty} f_X(x)dx = \int_{-\infty}^{\infty} \frac{d}{dx}(F_X(x))\,dx = F_X(x)\Big|_{-\infty}^{\infty} = F_X(\infty) - F_X(-\infty)$

$= 1 - 0 = 1$

3. $F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(p)\,dp$

4. $\int_{x_1}^{x_2} f_X(x)\,dx = P(x_1 < X \leq x_2)$

Proof :-

$$\int_{x_1}^{x_2} f_X(x)\,dx = \int_{x_1}^{x_2} \frac{d}{dx}(F_X(x))\,dx = F_X(x)\Big|_{x_1}^{x_2}$$

$$= F_X(x_2) - F_X(x_1)$$

$$= P(X \leq x_2) - P(X \leq x_1)$$

$$= P(x_1 < X \leq x_2)$$

5. If X is a continuous random variable, $P(X = x) = 0$ for all values of x.

Note :-

If X has discrete random variable, consideration of it's probability density function is defined as,

$$f_X(x) = \sum_{i=1}^{N} P(x_i) \cdot \delta(x - x_i)$$

Q. Let us take the exp. of rolling 2 fair dice. There are 36 possible outcomes since each die may show any no. from 1 to 6. find the probability distribution func (CDF) & PDF of sum of 2 dice.

$$S = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ | & | & | & | & | & | \\ | & | & | & | & | & | \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

$x = i + j \Rightarrow$ 2 to 12 (sum range)

CDF

$F_X(x) = P(X \le x)$

$F_X(0) = P(X \le 0) = 0$

$F_X(1) = P(X \le 1) = 0$

$F_X(2) = P(X \le 2) = 1/36$

$F_X(3) = P(X \le 3) = P(2) + P(3)$

$= \frac{1}{36} + \frac{2}{36} = \frac{1}{12}$

$F_X(4) = P(X \le 4) = P(2) + P(3) + P(4)$

$= P(X \le 3) + P(4)$

$= \frac{3}{36} + \frac{3}{36} = \frac{1}{6}$

$F_X(5) = P(X \le 5) = P(X \le 4) + P(5)$

$= \frac{6}{36} + \frac{4}{36} = \frac{5}{13}$

$F_X(6) = P(X \le 6) = P(X \le 5) + P(6)$

$= \frac{5}{13} + \frac{5}{36} = \frac{15}{36}$

$F_X(7) = P(X \le 7) = P(X \le 6) + P(7)$

$= \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$

$F_X(8) = P(X \le 8) = P(X \le 7) + P(8)$

$= \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$

$F_X(9) = P(X \le 9) = P(X \le 8) + P(9)$

$= \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$

$F_X(10) = P(X \le 10)$

$= P(X \le 9) + P(10)$

$= \frac{30}{36} + \frac{3}{36}$

$= \frac{33}{36}$

$F_X(11) = P(X \le 11)$

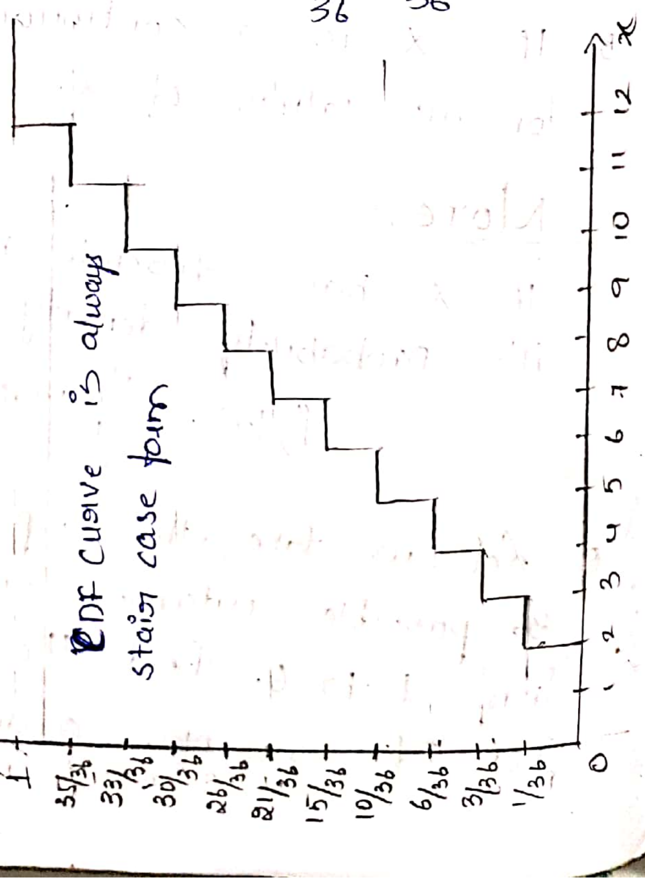
$= P(X \le 10) + P(11)$

$= \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$

$F_X(12) = P(X \le 12)$

$= P(X \le 11) + P(12)$

$= \frac{35}{36} + \frac{1}{36} = 1$



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PDF

$f_x(x) = \frac{d}{dx}(F_x(x)) \Rightarrow f_x(x) = P(X = x)$

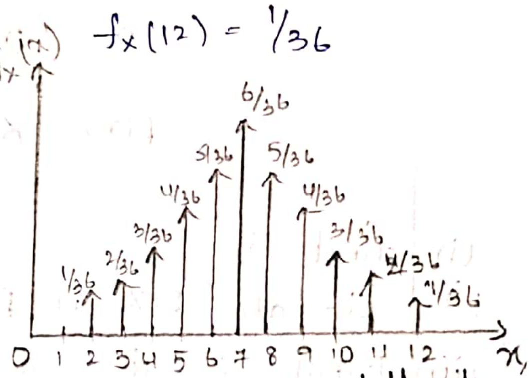
$f_x(2) = 1/36$ $f_x(7) = 6/36$ $f_x(12) = 1/36$

$f_x(3) = 2/36$ $f_x(8) = 5/36$

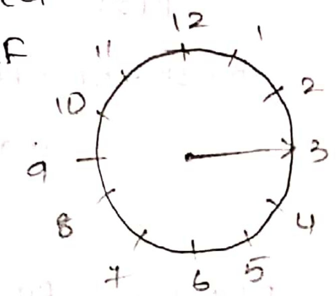
$f_x(4) = 3/36$ $f_x(9) = 4/36$

$f_x(5) = 4/36$ $f_x(10) = 3/36$

$f_x(6) = 5/36$ $f_x(11) = 2/36$



Q. Let us consider an expt. where a pointer on a wheel of chance is spun. Assume that wheel is numbered from 1 to 12 find the CDF & PDF



CDF

$F_x(1) = P(X \leq 1) = 1/12$ $F_x(7) = P(X \leq 7) = 7/12$

$F_x(2) = P(X \leq 2) = 2/12$ $F_x(8) = P(X \leq 8) = 8/12$

$F_x(3) = P(X \leq 3) = 3/12$ $F_x(9) = P(X \leq 9) = 9/12$

$F_x(4) = P(X \leq 4) = 4/12$ $F_x(10) = P(X \leq 10) = 10/12$

$F_x(5) = P(X \leq 5) = 5/12$ $F_x(11) = P(X \leq 11) = 11/12$

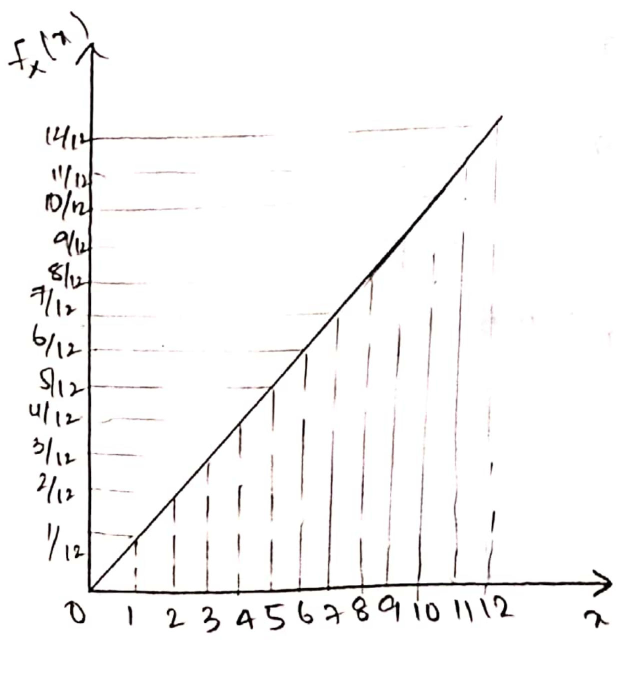
$F_x(6) = P(X \leq 6) = 6/12$ $F_x(12) = P(X \leq 12) = 12/12 = 1$

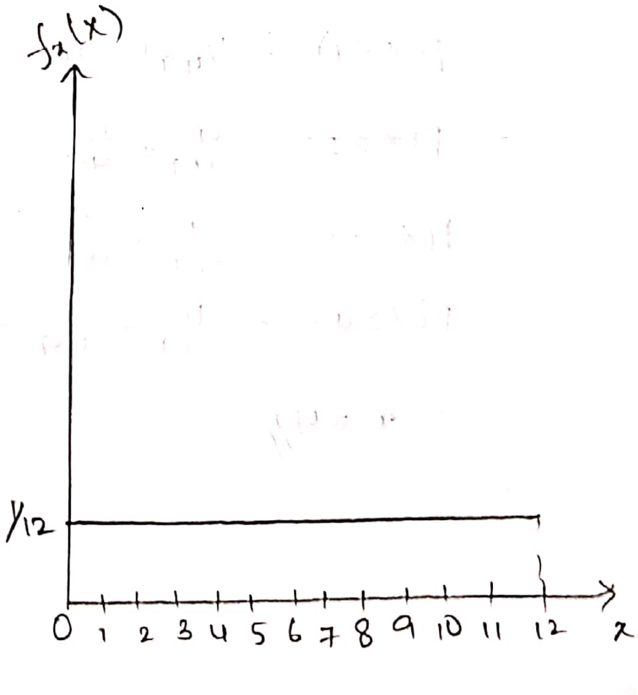
PDF

$f_x(x) = \frac{d}{dx}(F_x(x)) = P(X = x)$

$\forall\ x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

$\therefore\ f_x(x) = 1/12$





Q. A random variable has the following probability function values of x

x	0	1	2	3	4	5	6
$p(x)$	K	$3k$	$5k$	$7k$	$9K$	$11K$	$13K$

(i) find K
(ii) Evaluate $P(X<4)$, $P(x\geq 5)$, $P(3<x\leq 6)$
(iii) What is the smallest value of x for which $P(X\leq x)$ is $>1/2$

(i) WKT, $P(S) = 1$

$$K+3K+5K+7K+9K+11K+13K = 1$$

$$49K = 1$$

$$K = 1/49$$

(ii) $P(x<4) = P(0)+P(1)+P(2)+P(3)$

$$= \frac{1}{49}+\frac{3}{49}+\frac{5}{49}+\frac{7}{49} = \frac{16}{49}$$

$P(x\geq 5) = P(5)+P(6)$

$$= \frac{11}{49}+\frac{13}{49} = \frac{24}{49}$$

$P(3<x\leq 6) = P(4)+P(5)+P(6)$

$$= 9K+11K+13K = 33K = 33/49$$

(iii) $P(x\leq 0) = 1/49$

$P(x\leq 1) = 1/49+3/49 = 4/49$

$P(x\leq 2) = \frac{4}{49}+\frac{5}{49} = 9/49$

$P(x\leq 3) = \frac{9}{49}+\frac{7}{49} = \frac{16}{49}$

$P(x\leq 4) = \frac{16}{49}+\frac{9}{49} = \frac{25}{49} > 0.5$

$x = 4$//

16/12/19 Q. Is the funcn defined as follows a density funcn

$$f_x(x) = 0, \quad x < 2$$

$$f_x(x) = \frac{1}{18}(3+2x), \quad 2 \le x \le 4$$

$$f_x(x) = 0, \quad x > 4$$

For PDF, then $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$

$\therefore \int_2^4 \frac{1}{18}(3+2x)\,dx \Rightarrow \frac{1}{18}(3x+x^2)\Big|_2^4$ $(\because x<2 \;\&\; x>4 = 0)$

$$\Rightarrow \frac{1}{18}(6+12) = 1$$

$\therefore$ The above funcn is probability density funcn.

Q. A random variable X has the density funcn,

$$f_x(x) = \begin{cases} \frac{1}{4} & -2<x<2 \\ 0 & \text{otherwise} \end{cases}$$

find

(i) $P(X<1)$

(ii) $P(|X|>1)$

(iii) $P(3X+2>5)$

(i) WKT, $F_x(x) = P(X \le x) = \int_{-\infty}^{\infty} f_x(p)\,dp$

$$P(X<1) = \int_{-\infty}^{1} f_x(x)\,dx$$

$$= \int_{-2}^{1} \frac{1}{4}dx + \int_{-\infty}^{-2} 0 \cdot dx = \frac{1}{4}(1+2) = 3/4$$

(ii) $P(|X|>1) = P(\pm X > 1)$

$+X>1$
$X>1 \rightarrow 1 \text{ to } \infty$
$-X>1$
$X<-1 \rightarrow -\infty \text{ to } -1$

$$= P(X<-1) + P(X>1)$$

$$= \int_{-\infty}^{-1} f_x(x)\,dx + \int_{1}^{\infty} f_x(x)\,dx$$

$$= \int_{-2}^{-1} \frac{1}{4}dx + \int_{1}^{2} \frac{1}{4}dx$$

$$\frac{1}{4}(x)\Big|_{-2}^{-1} + \frac{1}{4}(x)\Big|_{1}^{2}$$

$$\frac{1}{4}(-1+2) + \frac{1}{4}(-1+2)$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(iii) $P((3x+2) > 5)$

$P(3x > 3) \Rightarrow P(x > 1) = \int_{1}^{\infty} f_x(x)\,dx$

$= \int_{1}^{2} \frac{1}{4}\,dx + 0 = \frac{1}{4}(2-1) = \frac{1}{4}$

Q. A Continuous random variable x has a probability density funcn, $f_x(x) = 3x^2, 0 \le x \le 1$. find 'a' & 'b' such that,

(i) $P(x \le a) = P(x > a)$

(ii) $P(x > b) = 0.05$

(i) $P(x \le a) = \int_{-\infty}^{a} f_x(x)\,dx$ & $P(x > a) = \int_{a}^{\infty} f_x(x)\,dx$

$$\int_{0}^{a} f_x(x)\,dx = \int_{a}^{1} f_x(x)\,dx$$

$$\int_{0}^{a} 3x^2\,dx = \int_{a}^{1} 3x^2\,dx$$

$$\Rightarrow x^3\Big|_{0}^{a} = x^3\Big|_{a}^{1}$$

$$a^3 = 1 - a^3$$

$$a^3 = \frac{1}{2} \Rightarrow a = (\frac{1}{2})^{1/3}$$

(or)

$P(x \le a) + P(x > a) = 1$

$P(x \le a) = \frac{1}{2} - P(x > a)$ (∵ Total probability = 1)

$P(x \le a) = \int_{-\infty}^{a} f_x(x)\,dx = \int_{0}^{a} 3x^2\,dx$

$$\Rightarrow a^3 = \frac{1}{2} \Rightarrow (\frac{1}{2})^{1/3} = a$$

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(ii) $P(X>b) = 0.05$

$$P(X>b) = \int_b^\infty f_X(x)\,dx \Rightarrow \int_b^1 3x^2\,dx = 0.05$$

$$x^3 \Big|_b^1 = 0.05$$

$$1 - b^3 = 0.05$$

$$b^3 = 0.95 \Rightarrow b = (0.95)^{1/3} \text{ or } (19/20)^{1/3}$$

Q. A random variable X has the following probability funcn,

x	-2	-1	0	1
$P(x)$	$1/8$	$1/8$	$1/4$	$1/2$

find & draw the PDF & CDF

CDF,

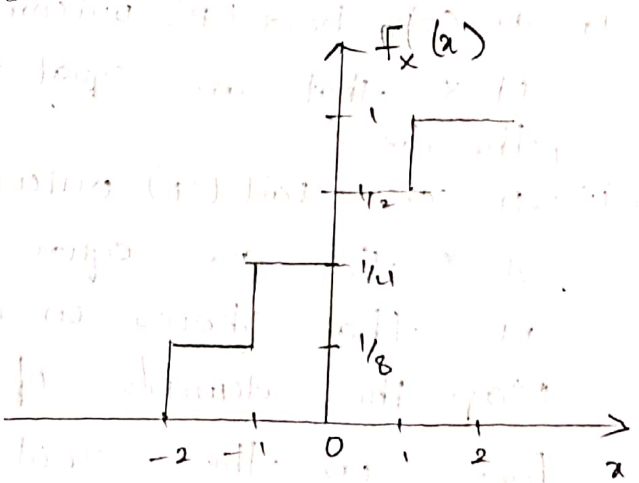
$F_X(x) = P(X \le x)$

$F_X(-2) = P(X \le -2) = 1/8$

$F_X(-1) = P(X \le -1) = 1/8 + 1/8 = 1/4$

$F_X(0) = P(X \le 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$F_X(1) = P(X \le 1) = 1/2 + 1/2 = 1$



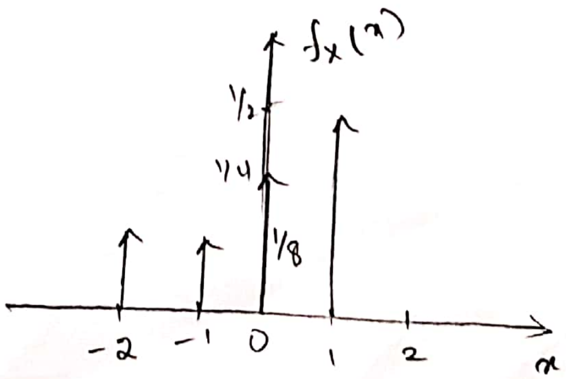
PDF

$f_X(-2) = P(-2) = 1/8$

$f_X(-1) = P(-1) = 1/8$

$f_X(0) = P(0) = 1/4$

$f_X(1) = P(1) = 1/2$



Q. Consider the distribution funcn for X defined by,

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ 1 - \frac{1}{4}e^{-x} & , x \geq 0 \end{cases}$$. Determine $P(X=0)$ & $P(X \geq 0)$

$$P(X=0) = \frac{d}{dx} F_X(x) = f_X(0) = \left[\frac{d}{dx}\left(1 - \frac{1}{4}e^{-x}\right)\right]_{x=0}$$

$$= -\frac{1}{4}(-e^{-x})\Big|_{x=0}$$

$$= \frac{1}{4}$$

$$P(X > x) = 1 - P(X \leq x)$$

$$P(X \geq 0) = 1 - P(X < 0) \Rightarrow 1 - 0 = 1$$

Q. In a exp. of rolling a die & flipping a coin. The random variable X is choosen such that

(i) A coin head (H) outcome corresponds to positive values of X that are equal to the numbers that show upon the die.

(ii) A coin tail (T) outcome corresponds to negative values of X that are equal in magnitude to ~~th~~ twice the no. that shows on die.

Map the elements of random variable X into the points on the real line & explain

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20/12/19 Verify the following is a distribution funcn or not

$$F_X(x) = \begin{cases} 0, & x < -a \\ \frac{1}{2}\left(\frac{x}{a}+1\right), & -a \leq x \leq a \\ 1, & x > a \end{cases}$$

$$f_X(x) = \frac{d}{dx} F_X(x) = \frac{1}{2}\left(\frac{1}{a}\right) = \frac{1}{2a}, \quad -a \leq x \leq a$$

from and property;

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow \int_{-a}^{a} \frac{1}{2a}\,dx \Rightarrow \frac{1}{2a}(2a) = 1$$

$\therefore$ distributive funcn

* CONDITIONAL DISTRIBUTION & DENSITY FUNCTIONS.

Conditional probability of a 'A' given by 'B' is written as,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

→ This concept can be applied to conditional distribution also. Let 'A' be an event $\{X \leq x\}$ for the random variable 'X'.

If 'B' is given the condition distribution funcn of the Random variable X is

$$F_X(x/B) = \frac{P(X \leq x \cap B)}{P(B)}$$

Methods of defining Conditioning Event

The Event 'B' can be defined from some characteristic of the physical experiment. It may be defined in terms of R.V 'X' or other than 'X'.

Case 1: If 'B' is defined interms of 'X'

$$B = \{X \leq b\} \quad -\infty < b < \infty$$

$$F_X(x/B) = P\left(\frac{X \leq x}{X \leq b}\right) = \frac{P(X \leq x \cap X \leq b)}{P(X \leq b)}, \quad P(X \leq b) \neq 0$$

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Here there are 2 cases

a) $b \leq x$ (or) $x \geq b$

$$F_X\left(\frac{x}{X \leq b}\right) = \frac{P(X \leq b)}{P(X \leq b)} = 1$$

b) If $b > x$ or $x < b$ Then,

$$F_X\left(\frac{x}{X \leq b}\right) = \frac{P(X \leq x)}{P(X \leq b)} = \frac{F_X(x)}{F_X(b)}$$

$$\boxed{F_X\left(x/X \leq b\right) = \begin{cases} \dfrac{F_X(x)}{F_X(b)}, & x < b \\ 1, & x \geq b \end{cases}}$$

The corresponding coditional density funcn is,

$$f_X\left(x/X \leq b\right) = \begin{cases} \dfrac{f_X(x)}{\int_{-\infty}^{b} f_X(x)\,dx}, & x < b \\ 0, & x \geq b \end{cases}$$

Case 2: If the event B is defined interms of another random variable 'Y' i.e if the random variable X is conditioned by a second random variable Y Where, $B = \{Y \leq y\}$ It is called point conditioning

i.e,
$$F_{\frac{X}{Y}}\left(\frac{x}{y}\right) = P\left(\frac{X \leq x}{Y \leq y}\right) = \frac{P[X \leq x, Y \leq y)}{P(Y \leq y)}$$

$$= \frac{F_{X,Y}(x,y)}{F_Y(y)}, \quad F_Y(y) \neq 0$$

The corresponding conditional density is,

$$f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}, \quad f_Y(y) \neq 0$$

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* Properties

1. $F_X\left(\frac{-\infty}{B}\right) = P\left(\frac{X \leq -\infty}{B}\right) = 0$

2. $F_X\left(\frac{\infty}{B}\right) = 1$

3. $0 \leq F_X\left(\frac{x}{B}\right) < 1$

4. If $x_1 < x_2$ then $F_X\left(\frac{x_1}{B}\right) \leq F_X\left(\frac{x_2}{B}\right)$

5. $F_X\left(\frac{x_2}{B}\right) - F_X\left(\frac{x_1}{B}\right) = P\left(\frac{x_1}{B} < X \leq \frac{x_2}{B}\right)$

6. $F_X\left(\frac{x^+}{B}\right) = F_X\left(\frac{x}{B}\right)$

7. $P(X > x/B) = 1 - F_X\left(\frac{x}{B}\right)$

* CONDITIONAL DENSITY FUNCTION.

It is defined as the derivative of conditional distribution funcn.

$$f_X\left(\frac{x}{B}\right) = \frac{d}{dx} F_X\left(\frac{x}{B}\right)$$

Properties:

1. $f_X\left(\frac{x}{B}\right) \geq 0$, for all x.

2. $\int_{-\infty}^{\infty} f_X\left(\frac{x}{B}\right) dx = 1$

3. $F_X\left(\frac{x}{B}\right) = \int_{-\infty}^{x} f_X(p/B)\, dp$

4. $P\left(\frac{x_1}{B} < X \leq \frac{x_2}{B}\right) = \int_{x_1}^{x_2} f_X\left(\frac{x}{B}\right) dx$

Q. 2 boxes have Red, Green & blue balls in them, The no. of balls of each is given in table. Consider an expt. will be to select a box and then a ball from the selected box. One box (B_2) is slightly larger than the other (B_1), causing it to be selected more frequently. ~~Let~~ ~~B_2~~ Assume $P(B_1) = 2/10$, $P(B_2) = 8/10$. (B_1 & B_2 are mutually exclusive & $B_1 \cup B_2$ is the certain event, since some box must be selected. Therefore $P(B_1) + P(B_2)$ must equal unity). Find the conditional distributions & conditional density of B_1 & B_2 and also find the Total probability of all colours (Red, Green, blue)

x_i	Ball colour	Box		Total
		B_1	B_2	
1	Red	5	80	85
2	Green	65	60	125
3	Blue	60	10	70
	Total	130	150	280

Conditional probabilities

$$P(x=1/B_1) = \frac{P(x=1 \cap x=B_1)}{P(x=B_1)} = \frac{5/280}{130/280} = \frac{5}{130}$$

$$P(x=2/B_1) = \frac{65/280}{130/280} = 65/130$$

$$P(x=3/B_1) = \frac{60/280}{130/280} = 60/130$$

$$P(x=1/B_2) = 80/150 \qquad P(x=2/B_2) = 60/150$$

$$P(x=3/B_2) = 10/150$$

$$f_x(x/B) = \sum_{x_i=1}^{3} P\left(\frac{x_i}{B}\right) \delta(x-x_i)$$

$$F_x(x/B) = \sum_{x_i=1}^{3} P\left(\frac{x_i}{B}\right) \cdot u(x-x_i)$$

2/12/19 8) In an expt. there are 2 boxes. Each box contains balls as shown in table. The event is 2 " select a box randomly & then select a ball from the selected box". If the probability of selecting the 1st box is 0.3, Then find

a. The conditional probability & density funcn's

b. The probability distribution & density funcn's

c. Plot the function

Ball Colours	Boxes 1	Boxes 2	Total
Red	10	50	60
Blue	20	40	60
White	50	30	80
Total	80	120	200

Let 1st box be B_1.

and 2nd box be B_2

Given $P(B_1) = 0.3$ $\therefore P(B_1) + P(B_2) = 1 \Rightarrow P(B_2) = 0.7$

Consider that the discrete R.V 'x' is the event of selecting a coloured ball.

The values of x are $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ when a red, a blue & a white ball is selected respectively

a. Conditional probability

" Probability of getting a red ball when box B_1 is selected is $P(x_1/B_1) = \frac{P(x_1 \cap B_1)}{P(B_1)} = \frac{10/200}{\cancel{80/200}\ 0.3} = \frac{1}{8}$

" Probability of getting blue ball when B_1 is selected is"

$$P(x_2/B_1) = \frac{P(x_2 \cap B_1)}{P(B_1)} = \frac{20/200}{\cancel{80/200}\ 0.3} = \frac{1}{4}$$

Probability of getting a white ball when B_1 is selected is

$$P(x_3/B_1) = \frac{P(x_3 \cap B_1)}{P(B_1)} = \frac{50/200}{80/\cancel{2000}\cdot 3} = \frac{5}{8}$$

lly

$$P(x_1/B_2) = \frac{50/200}{120/2000\cdot 7} = \frac{5}{12} \quad P\left(\frac{x_2}{B_2}\right) = \frac{40}{\cancel{120}\,0.7\times 200} \quad P\left(\frac{x_3}{B_2}\right) = \frac{3}{\cancel{12}\,0.7\times 200}$$

b. $$F_X\left(\frac{x}{B_1}\right) = \sum_{i=1}^{3} P(x_i/B_i)\, u(x - x_i)$$

$$= \frac{1}{8} u(x-1) + \frac{2}{8} u(x-2) + \frac{5}{8} (x-3)$$

$$f_X(x/B_1) = \sum_{i=1}^{3} P(x_i/B_i)\, \delta(x - x_i)$$

$$= \frac{1}{8} \delta(x - \cancel{x_1}) + \delta(x-2)\frac{2}{8} + \frac{5}{8}\delta(x-3)$$

Similarly,

$$F_X(x/B_2) = \frac{5}{12} u(x-1) + \frac{4}{12} u(x-2) + \frac{3}{12} u(x-3)$$

$$f_X(x/B_2) = \frac{5}{12}\delta(x-1) + \frac{4}{12}\delta(x-2) + \frac{3}{12}\delta(x-3)$$

Total probabilities :

$$P(X = x_1) = \sum_{n=1}^{2} P(x_1/B_n)\, P(B_n)$$

$$= P(x_1/B_1)\, P(B_1) + P(x_1/B_2)\, P(B_2)$$

$$= \frac{1}{8}(0.3) + \frac{5}{12}(0.7)$$

$$= \frac{3}{80} + \frac{35}{120} = 0.329$$

23/12/19 PROBABILITY MASS FUNCTION

Consider the discrete random variable x with infinite no. of possible outcomes i.e $x = x_1, x_2, x_3, \ldots, x_n$

If the probability of x is $p(x_i)$ for $i = 1, 2, 3, \ldots \infty$ satisfies the following conditions

1. $p(x_i) \geq 0$, for all i

2. $\sum_{i=1}^{\infty} p(x_i) = 1$

Then the funcn $p(x)$ is called the probability mass funcn. It is same as the probability density funcn. It can be written as $f_X(x_i) = P(X = x_i)$

DISTRIBUTION AND DENSITY FUNCTIONS:

There are 6 distribution funcn's

1. Binomial distribution funcn
2. Poison distribution funcn

} discrete random variables

3. Uniform distribution funcn
4. Gaussian distribution funcn
5. Exponential distribution funcn
6. Rayliegh distribution funcn.

} continuous random variables