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Advanced Statistics II – Summer Term 2017

Problem Set 1

1. Let $X \sim \mathcal{N}\left(\frac{1}{n},\ \frac{1}{1+\frac{2}{n}}\right)$ and $Y \sim \mathcal{N}\left(2+\frac{1}{n},\ \frac{1}{n}\right)$. Discuss convergence in distribution for the random variables:

   a) $P = X \cdot Y$

   b) $Q = \frac{X}{Y}$

   Verify the convergence in distribution by using Monte-Carlo Simulation methods.

2. Let $X \sim \mathcal{N}(0,1)$ and consider $Y = |X|$. Verify that the density of Y is given by $f(y) = \sqrt{\frac{2}{\pi}} \exp\left(-\frac{y^2}{2}\right)$ by using Monte-Carlo Simulation methods.

3. Let $\{U_1, U_2, ..., U_n\}$ be a sequence of stochastically independent random variables from an exponential distribution with

   $$f(u;\theta) = \theta e^{-\theta u} I_{(0,\infty)}(u), \quad \theta > 0\ .$$

   Define

   $$Z_n = \frac{1}{n}\sum_{i=1}^{n} U_i\ .$$

   By using Monte-Carlo Simulation

   a) show that $Z_n \xrightarrow{p} c$, where c is a constant, and give c.

   b) find the asymptotic distribution of Z_n.

   c) find the asymptotic distribution of $Y_n = \exp\{-Z_n\}$.

What changes if θ changes?

Home Assignment (2 Points)

Let $(X_i, i : 1 \to n)$ be a random sample from a Uniform distribution on the interval $(0, 1]$ with the respective order statistics $X_{[1]} \leq X_{[2]} \leq \cdots \leq X_{[n]}$.

a) Find the density of the sample maximum as well as of the sample minimum

b) Find the probability limits of the maximum and the minimum

(as done in the PP-tutorial) and verify these results with Monte-Carlo Simulation methods. Choose $n = 10,\ 100,\ 1000$, $MC = 100,\ 500,\ 1000$. Comment (shortly) on the dependency of your results on n and MC. Plot your results in one figure using a 3x3 grid. Save this plot in a pdf file. You are only allowed to use functions (and packages) that are part of basic R. Please use the provided script file for the first home assignment and fill out the header!
Hand in your script file (with the necessary comments) until 2.5.17 8pm via mail.