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Advanced Statistics II – Summer Term 2017

Problem Set 4

1. Home Assignment 3 Consider the random sample $\{X_i, i : 1 \to n\}$ from density

$$f(x;\theta) = \frac{1}{\theta}\exp\{-\frac{x}{\theta}\} \cdot I_{(0,\infty)}(x)\ , \quad \theta > 0\,.$$

   a) Write a function for the Cramér-Rao Lower Bound for the variance of unbiased estimators for θ, $1/\theta$ and $p(X > c)$.

   b) Write a function for each of the estimators derived in class.
   Hint: Use $\frac{1}{n}\sum_{i=1}^{n} \mathcal{I}_{(c,\infty)}(x_i)$ as an estimator for $p(X > c)$.

   c) Conduct a simulation study for the following parameters: $MC = 1000$; $n = 1, \cdots, 100$; $\theta = 0.1,\ 1,\ 2,\ 5$; $c = 1,\ 10$. Plot the simulated variance for each estimator as a function of n as well as for every θ (and c, for the last estimator). Add the CRLB to your plot. Arrange all graphs on one single figure. Save this figure as a pdf. Name all axes appropriately, add a meaningful legend and indicate in each graph to which parameter combination it refers. Use the estimators derived in class. For every estimator you have to create a pdf file.

   d) Is the Cramer-Rao variance inequality valid? Which of the estimators reaches the CRLB? Does this depend on θ, c?

2. Let $\{X_i, i : 1 \to n\}$ be a random sample from a distribution with pdf

$$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!} I_{\{0,1,\ldots\}}(x)\ , \quad \theta > 0\ .$$

   (a) Write a function that computes the log-likelihood.

   (b) Construct a sample from this distribution with n and θ of your choice.

   (c) Compute the ML estimator numerically using the likelihood function of part a) and compare the result with the closed form solution derived in the PP tutorial.

   (d) Estimate and compare for both methods the computation time.

3. Assume you have a random sample from $N(\mu, 1)$.

   a) Write a function that computes the log-likelihood.

   b) The following parameters are given: $n = 10,\ 100,\ 1000,\ 10000$; $\mu \in [-5, 5]$. Compute the ML estimator for μ for all possible parameter combinations. Plot $\hat{\mu}_{ML}$ against μ for every n. Save the results in one pdf file containing one figure (with several subgraphs). Do the results depend on the starting value and/or n?

   c) Estimate the computation time for the procedure in c).

Home Assignment 4 (2 points)

Assume you have a random sample from $N(\mu, \sigma^2)$. The following parameters are given: $\mu_i \in [-5, 5]$, $\sigma_i \in [0.1, 25]$, $i \in \{1,\ 2, \cdots,\ 100\}$ and $n \in \{100,\ 200, \cdots,\ 1000\}$.

a) Write a function that computes the log-likelihood of the given distribution.

b) Write a function that computes the euclidean distance between two points (vectors).

c) Write a procedure that computes numerically the ML estimator for μ **and** σ for every possible parameter combination. You might find the function *optim*() useful. As starting values choose $\hat{\mu} = 0$ and $\hat{\sigma} = 1$.

d) Compute the euclidean distance between your ML estimator $\hat{\theta}_{ML} = (\hat{\mu}_{ML}, \hat{\sigma}_{ML})$ and the true value $\theta = (\mu, \sigma)$ by using your function from part b). Save all distances in one suitable R-object.

e) Do the results from part d) depend on the starting value? Justify your answer!

You are only allowed to use functions (and packages) that are part of basic R. Please use the provided script file for the first home assignment and fill out the header! Hand in your script file (with the necessary comments) until 13.6.17 8pm via mail.