

Advanced Statistics II – Summer Term 2017

Problem Set 5

1. Home Assignment 4: Assume you have a random sample from $N(\mu, \sigma^2)$. The following parameters are given: $\mu_i \in [-5, 5]$, $\sigma_i \in [0.1, 25]$, $i \in \{1, 2, \dots, 100\}$ and $n \in \{100, 200, \dots, 1000\}$.
 - a) Write a function that computes the log-likelihood of the given distribution.
 - b) Write a function that computes the euclidean distance between two points (vectors).
 - c) Write a procedure that computes numerically the ML estimator for μ **and** σ for every possible parameter combination. You might find the function `optim()` useful. As starting values choose $\hat{\mu} = 0$ and $\hat{\sigma} = 1$.
 - d) Compute the euclidean distance between your ML estimator $\hat{\theta}_{ML} = (\hat{\mu}_{ML}, \hat{\sigma}_{ML})$ and the true value $\theta = (\mu, \sigma)$ by using your function from part b). Save all distances in one suitable R-object.
 - e) Do the results from part d) depend on the starting value? Justify your answer!
2. Let $\{X_i, i : 1 \rightarrow n\} \sim \mathcal{N}(\mu, 1)$
 - (a) Write a function that computes the log-likelihood.
 - (b) Write a function that computes $q(\theta, x)$. Use the first two non-central moments for this.
 - (c) Construct a sample from this distribution with $n = 100$ and μ of your choice.
 - (d) Compute the LS, ML and GMM estimators numerically.
 - (e) Compare your results.
3. Let $\{X_i, i : 1 \rightarrow n\} \sim \Gamma(1, \beta)$
 - (a) Write a function that computes the [log-likelihood](#).
 - (b) Write a function that computes $q(\theta, x)$. Use the first two non-central moments for this.
 - (c) Construct a sample from this distribution with $n = 100$ and β of your choice.
 - (d) Compute the LS, ML and GMM estimators numerically.
 - (e) Compare your results.

Home Assignment 5 (2 points)

Assume you have a random sample from $N(\mu, \sigma^2)$. The following parameters are given: $\mu = 5$, $\sigma = 2$ and $n = 100$.

- a) Write a function that computes the [log-likelihood](#) of the given distribution.
- b) Write a function that computes $q(\theta, x)$. Use the first two non-central moments for this.
- c) Construct a sample from this distribution.
- d) Compute the LS, ML and GMM estimators numerically, choose as starting values $(\hat{\mu}, \hat{\sigma}) = (0, 1)$.
- e) Compare your results in terms of MSE.

Please use the provided script file for the first home assignment and fill out the header!

Hand in your script file (with the necessary comments) until 27.6.17 8pm via [mail](#).