Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 09

Computational Finance

Exercises for participants of the programme **'Quantitative Finance'**

C-Exercise 27 (Simulating paths of geometric Brownian motion)
Consider a geometric Brownian motion defined via

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t), \;\; X(0) = X_0,$$

with $\mu, X_0 \in \mathbb{R}$, $\sigma > 0$ and a Brownian motion $W(t)$. Write a function

```
[X_exact,X_Euler,X_Milshtein]=Sim_Paths_GeoBM(X0, mu, sigma, T, N)
```

that simulates the discrete path approximation $X(0), X(T/N), X(2T/N), \ldots, X(T)$ using the exact solution for the geometric Brownian motion SDE (`X_exact`), using the Euler method (`X_Euler`) and using the Milshtein method (`X_Milshtein`). Use the same simulations of $W(kT/N) - W((k-1)T/N$, $k = 1, \ldots, N$, for all three methods.
Test the function for

```
X0=100, mu=0.05, sigma=0.2, T=2
```

and $N = 10, 100, 1000, 10000$. For each value of N plot all three simulated paths into a single plot.
Useful scilab commands: subplot

Please turn over.

C-Exercise 28 (Valuation of European options in the Heston model using the Euler method)
Write a scilab function

```
[V0, c1, c2] = Heston_EuCall_MC_Euler (S0, r, gamma0, kappa,
               lambda, sigma_tilde, T, g, M, m)
```

that computes the price $V(0)$ of a European option with payoff $g(S_T)$ and maturity T in the Heston model via the Monte-Carlo method using M samples together with the 95%-confidence interval.
To this end, approximate the paths by the Euler method with a grid of m equidistant points in time by implementing your pseudo code from T-Exercise 29.
Test your function for the European call option using the same parameters as in C-Exercise 17, $M = 10000$ and $m = 250$. Compare your result with the result from C-Exercise 17.

T-Exercise 29 (Valuation of options in the Heston model using the Euler method)
Extend the Euler method for the Black-Scholes model presented from chapter 5.5 suitably such that it can generate paths of the price process in the Heston model.

(a) Write your solution for the path simulation in pseudo code e.g. as presented in the lecture notes.

(b) Write an algorithm in pseudo code that approximates using the Monte-Carlo method the price of an European up-and-out call option on the stock S with strike price K and barrier $B > K$. I.e., the option pays off at maturity T the amount

$$V(T) = 1_{\{S(t)<B \text{ for all } t\in[0,T]\}}(K - S(T))^+.$$

You may use your result from part a).

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 30.06.2017, 10:00
Discussion: Tue, 03/05.07.2017