Lecture #10

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Modeling differential equations

$$\frac{dN}{dt} = rN$$
$$N_t = N_0 e^{rt}$$

$$n_t = n_0 e^{rt} \overset{\text{differentiate}}{\longrightarrow} \frac{dn}{dt} = rN$$

$$n_t = n_0 e^{rt} \overset{\text{integrate}}{\longleftarrow} \frac{dn}{dt} = rN$$

General form for coding of differential equations

$$\frac{dN}{dt} = \text{"change"}$$

Definition of derivative and graph of it

- change in N measured over a small time interval t
- in a graph of N vs t, dN/dt is the slope of the function at t.

Coding a derivative in R

```
N[t] <- N[t - 1] + "change"
```

Other details and model complexity

- dN/dt will often be a function of N at time t
- dN/dt may also be a function of the environment, or of the population sizes of other species also measured at time t
- if there is a time lag, then growth of N at time t will often depend on growth of N at a time further in the past
- model parameters can be constants or they can be random variables

```
# two species (N and P) with two parameters (r and a)
N[t] <- N[t - 1] + r*N[t-1] - a*P[t-1]*N[t-1]

# one species with multiple time lags and
# associated parameters (r1,r2,r3)
N[t} <- N[t -1] + r1*N[t-1] + r2*N[t-2] + r3*N[t-3]

# one species stochastic exponential model with normal distribution
# instead of a constant parameter r

N[t] <- N[t-1] + rnorm(n=1,mean=r,sd=1)*N[t-1]
```

Matrix columns for standard plots

```
N          # population size or response variable
t          # time (usually an integer value 1..Tmax)
dN/dt      # growth rate (N[t] - N[t-1])
(1/N)dNdt  # per capita growth rate
```

Create plots

- N versus t
- dN/dt versus N
- $(1/N)dN/dt$ versus N
- possibly show N as a function of parameters or other variables