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adapted from KACTL and MIT $(^w^)$

2024-07-03

Contest (1)

TemplateShort.cpp 77aa31, 44 lines

```
#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using db = long double; // or double if tight TL
using str = string;

using pi = pair<int,int>;
#define mp make_pair
#define f first
#define s second

#define tcT template<class T
tcT> using V = vector<T>;
tcT, size_t SZ> using AR = array<T,SZ>;
using vi = V<int>;
using vb = V<bool>;
using vpi = V<pi>;

#define sz(x) int((x).size())
#define all(x) begin(x), end(x)
#define sor(x) sort(all(x))
#define rsz resize
#define pb push_back
#define ft front()
#define bk back()

#define FOR(i,a,b) for (int i = (a); i < (b); ++i)
#define F0R(i,a) FOR(i,0,a)
#define ROF(i,a,b) for (int i = (b)-1; i >= (a); --i)
#define R0F(i,a) ROF(i,0,a)
#define rep(a) F0R(_,a)
#define each(a,x) for (auto& a: x)

const int MOD = 1e9+7;
const db PI = acos((db)-1);
mt19937 rng(0); // or mt19937_64
tcT> bool ckmin(T& a, const T& b) {
  return b < a ? a = b, 1 : 0; } // set a = min(a,b)
tcT> bool ckmax(T& a, const T& b) {
  return a < b ? a = b, 1 : 0; } // set a = max(a,b)

int main() { cin.tie(0)->sync_with_stdio(0); }
```

.bashrc 14 lines

```
alias res='reset'
alias r='reset'
comp () {
  F="${1%.*}"
  g++ -g "${F}.cpp" -o "${F}" -Wall -Wextra -Wshadow -
    ↪Wconversion "${@:2}"
}
run () {
  F="${1%.*}"
  comp "${F}" "${@:2}" && ./"${F}"
}
all () {
  F="${1%.*}"
  comp "${F}" "${@:2}" && ./"${F}" < "${F}.in"
}
```

hash.sh 3 lines

```
# Hash file ignoring whitespace and comments. Verifies that
# code was correctly typed. Usage: 'sh hash.sh < A.cpp'
cpp -dD -P -fpreprocessed|tr -d '[:space:]'|md5sum|cut -c-6
```

Mathematics (2)

2.1 Equations

$$\begin{aligned} ax+by &= e \\ cx+dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed-bf}{ad-bc} \\ y &= \frac{af-ec}{ad-bc} \end{aligned}$$

Cramer's Rule: given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\begin{aligned} \sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w \\ \tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2} \end{aligned}$$

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $s = \frac{a+b+c}{2}$

Area: $A = \sqrt{s(s-a)(s-b)(s-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

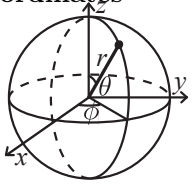
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$.

2.4.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \operatorname{atan2}(y,x)$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1+\tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums/Series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \le 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \le x \le 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x-\mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation.

$$\mathbb{E}(aX+bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

If X, Y are independent,

$$V(aX+bY) = a^2V(X) + b^2V(Y).$$

Expectation is linear If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

2.7.1 Discrete distributions

Binomial distribution

of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n,p)$, $n = 1,2,\ldots,0 \le p \le 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n,p) \approx \operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\operatorname{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1,2,\ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0,1,2,\ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

$\operatorname{Bin}(n,p) \approx \operatorname{Po}(np)$ for small p (binomial distribution with n coin flips, each of which is heads with probability p).

2.7.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\operatorname{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\operatorname{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu,\sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1,\sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2,\sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1+\mu_2+c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data Structures (3)

3.1 STL

HashMap.h

Description: Hash map with similar API as unordered_map. Initial capacity must be a power of 2 if provided.
Usage: `ht<int,int> h({},{},{},{},{1<<16});`
Memory: ~1.5x unordered map
Time: ~3x faster than unordered map

<ext/pb_ds/assoc_container.hpp> 5872b2, 9 lines

```
using namespace __gnu_pbds;
struct chash {
  const uint64_t C = ll(4e18*acos(0))+71; // large odd number
  const int RANDOM = rng();
  ll operator()(ll x) const { return __builtin_bswap64((x^
    ↪RANDOM)*C); }
};
template<class K,class V> using ht = gp_hash_table<K,V,chash>;
template<class K,class V> V get(ht<K,V>& u, K x) {
  auto it = u.find(x); return it == end(u) ? 0 : it->s; }
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. Change `null_type` to get a map.
Time: $\mathcal{O}(\log N)$

<ext/pb_ds/assoc_container.hpp> 018476, 5 lines

```
using namespace __gnu_pbds;
tcT> using Tree = tree<T, null_type, less<T>,
  rb_tree_tag, tree_order_statistics_node_update>;
#define ook order_of_key
#define fbo find_by_order
```

LineContainer.h

Description: Add lines of the form $ax + b$, query maximum y-coordinate for any x.

Time: $\mathcal{O}(\log N)$

df7386, 29 lines

```
using T = ll; const T INF = LLONG_MAX; // a/b rounded down
// ll fdiv(ll a, ll b) { return a/b-((a^b)<0&&a%b); }

bool _Q = 0;
struct Line {
  T a, b; mutable T lst;
  T eval(T x) const { return a*x+b; }
  bool operator<(const Line&o)const{return _Q?lst<o.lst:a<o.a;}
  T last_gre(const Line& o) const { assert(a <= o.a);
    // greatest x s.t. a*x+b >= o.a*x+o.b
    return lst=(a==o.a?(b>=o.b?INF:-INF):fdiv(b-o.b,o.a-a));}
};

struct LineContainer: multiset<Line> {
  bool isect(iterator it) { auto n_it = next(it);
    if (n_it == end()) return it->lst = INF, 0;
    return it->last_gre(*n_it) >= n_it->lst; }
  void add(T a, T b) {
    auto it = ins({a,b,0}); while (isect(it)) erase(next(it));
    if (it == begin()) return;
    if (isect(--it)) erase(next(it)), isect(it);
    while (it != begin()) {
      --it; if (it->lst < next(it)->lst) break;
      erase(next(it)); isect(it); }
  }
  T qmax(T x) { assert(!empty());
    _Q = 1; T res = lb({0,0,x})->eval(x); _Q = 0;
    return res; }
};
```

Bset.h

Description: bitset of variable size. Careful since it's slower than std::bitset

Usage: `tr2::dynamic_bitset<> bs;`
`bs.is_subset_of(), bs.is_proper_subset_of(), bs.find_first(), bs.find_next(pos)`

<tr2/dynamic_bitset>

IntervalContainer.h

Description: Stores disjoint intervals [a, b)

Time: $\mathcal{O}(\log N)$

"template.hpp" 5a1e5b, 17 lines

```
set<pi>::iterator addInterval(set<pi>& is, int L, int R) {
    if (L >= R) return is.end();
    auto it = is.lower_bound(mp(L,R)), bf = it;
    while (it != end(is) and it->f <= R)
        R = max(R, it->s), bf = it = is.erase(it);
    if (it != begin(is) and (--it)->s >= L)
        L = min(L, it->f), R = max(R, it->s), is.erase(it);
    return is.insert(bf, mp(L,R));
}
void removeInterval(set<pi>& is, int L, int R) {
    if (L >= R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->s;
    if (it->f == L) is.erase(it);
    else (int&)it->s = L;
    if (R != r2) is.emplace(R, r2);
}
```

MinDeque.h

Description: maintains minimum of deque while adding elements to back or deleting elements from front

98d063, 15 lines

```
template<class T> struct MinDeque {
  int lo = 0, hi = -1;
  deque<pair<T,int>> d;
  int size() { return hi-lo+1; }
  void push(T x) { // add to back
    while (sz(d) && d.back().f >= x) d.pop_back();
    d.pb({x,++hi});
  }
  void pop() { // delete from front
    assert(size());
    if (d.front().s == lo++) d.pop_front();
  }
  T mn() { return size() ? d.front().f : MOD; }
  // change MOD based on T
};
```

3.2 1D Range Queries

RMQ.h

Description: 1D range minimum query. If TL is an issue, use arrays instead of vectors and store values instead of indices.

Memory: $\mathcal{O}(N \log N)$

Time: $\mathcal{O}(1)$

"template.hpp" bcb9bf, 19 lines

```
tcT> struct RMQ { // floor(log_2(x))
  static constexpr int level(int x) { return 31-__builtin_clz(x
    ↪); }
  V<T> v; V<vi> jmp;
  int cmb(int a, int b) {
    return v[a]==v[b]?min(a,b):(v[a]<v[b]?a:b); }
  void init(const V<T>& _v) {
    v = _v; jmp = {vi(sz(v))};
    iota(all(jmp[0]),0);
    for (int j = 1; 1<<j <= sz(v); ++j) {
      jmp.pb(vi(sz(v)-(1<<j)+1));
      F0R(i,sz(jmp[j])) jmp[j][i] = cmb(jmp[j-1][i],
        jmp[j-1][i+(1<<(j-1))]);
    }
  }
  int index(int l, int r) {
    assert(l <= r); int d = level(r-l+1);
    return cmb(jmp[d][l],jmp[d][r-(1<<d)+1]); }
  T query(int l, int r) { return v[index(l,r)]; }
};
```

FenwickTree.h

Description: 1-based with closed ranges $[a, b]$. T = long long is prefered as you have to multiply elements by N.

Time: $\mathcal{O}(\log N)$

"template.hpp" b38e72, 14 lines

```
tcT> struct FT {
    V<T> B1, B2;
    FT(int N) : B1(N+1), B2(N+1) {};
    void add(V<T>& b, int p, T x) { for(;p<sz(b);p+=p&-p) b[p
      ↪]+=x; }
    void range_add(int l, int r, T x) {
        add(B1, l, x);
        add(B1, r + 1, -x);
        add(B2, l, x * (l - 1));
        add(B2, r + 1, -x * r);
    }
    T sum(V<T>& b, int p) { T s = 0; for(;p;p-=p&-p) s+=b[p];
      ↪return s; }
    T prefix_sum(int idx) { return sum(B1,idx)*idx - sum(B2,idx
      ↪); }
    T range_sum(int l, int r) {return prefix_sum(r) -
      ↪prefix_sum(l - 1);}
};
```

LazySegTree.h

Description: Segment Tree with lazy updates. Segments are [l,r)

Time: $\mathcal{O}(\log N)$ per query

"template.hpp" ad3fad, 75 lines

```
template<typename T, typename E, T (*f)(T, T), T (*g)(T, E), E
  ↪(*h)(E, E), T(*ti)(), E (*ei)()>
struct LazySegTree {
    int n, log, s;
    V<T> val; V<E> laz;
    LazySegTree() {}
    LazySegTree(V<T> const& v) { init(v); }
    void init(V<T> const& v) {
        n = 1, log = 0, s = sz(v);
        while (n < s) n <<= 1, ++log;
        val.rsz(2 * n, ti());
        laz.rsz(n, ei());
        F0R(i, s) val[i+n] = v[i];
        ROF(i,1,n) _update(i);
    }
    void update(int l, int r, E const& x) {
        if (l >= r) return;
        l += n, r += n;
        ROF(i,1,log+1) {
            if (((l>>i)<<i) != l) _push(l>>i);
            if (((r>>i)<<i) != r) _push((r-1)>>i);
        }
        int l2 = l, r2 = r;
        while (l < r) {
            if (l & 1) _apply(l++,x);
            if (r & 1) _apply(--r,x);
            l >>= 1, r >>= 1;
        }
        l = l2, r = r2;
        FOR(i,1,log+1) {
            if (((l>>i)<<i) != l) _update(l>>i);
            if (((r>>i)<<i) != r) _update((r-1)>>i);
        }
    }
    T query(int l, int r) {
        if (l >= r) return ti();
        l += n, r +=n;
        T L = ti(), R = ti();
        ROF(i,1,log+1) {
            if (((l>>i)<<i) != l) _push(l>>i);
            if (((r>>i)<<i) != r) _push((r-1)>>i);
        }
        while (l < r) {
            if (l & 1) L = f(L,val[l++]);
            if (r & 1) R = f(val[--r], R);
            l >>= 1, r >>= 1;
        }
        return f(L, R);
    }
    void set_val(int k, T const& x) {
        k += n;
        ROF(i,1,log+1)
            if (((k>>i)<<i) != k or (((k+1)>>i)<<i) != (k+1))
                _push(k>>i);
        val[k] = x;
        FOR(i,1,log+1)
            if (((k>>i)<<i) != k or (((k+1)>>i)<<i) != (k+1))
                _update(k>>i);
    }
private:
    void _push(int i) {
        if (laz[i] != ei()) {
            F0R(j,2) val[2*i+j] = g(val[2*i+j],laz[i]);
            if (2*i<n) F0R(j,2) compose(laz[2*i+j], laz[i]);
        }
```

```
      laz[i] = ei();
    }
    inline void _update(int i) { val[i] = f(val[2*i],val[2*i
      ↪+1]); }
    inline void _apply(int i, E const& x) {
      if (x != ei()) {
        val[i] = g(val[i], x);
        if (i < n) compose(laz[i], x);
      }
    }
    inline void compose(E& a, E const& b) { a = a == ei() ? b :
      ↪ h(a, b); }
};
```

SegTreeBeats.h

Description: Lazy SegTree supports modifications of the form `ckmin(a,i,t)` for all $l \le i \le r$, range max and sum queries. `SZ` is power of 2.

Time: $\mathcal{O}(\log N)$

a473ba, 61 lines

```
template<int SZ> struct SegTreeBeats { // declare globally
  int N, mx[2*SZ][2], maxCnt[2*SZ];
  ll sum[2*SZ];
  void pull(int ind) {
    F0R(i,2) mx[ind][i] = max(mx[2*ind][i],mx[2*ind+1][i]);
    maxCnt[ind] = 0;
    F0R(i,2) {
      if (mx[2*ind+i][0] == mx[ind][0])
        maxCnt[ind] += maxCnt[2*ind+i];
      else ckmax(mx[ind][1],mx[2*ind+i][0]);
    }
    sum[ind] = sum[2*ind]+sum[2*ind+1];
  }
  void build(vi& a, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) { R = (N = sz(a))-1; }
    if (L == R) {
      mx[ind][0] = sum[ind] = a[L];
      maxCnt[ind] = 1; mx[ind][1] = -1;
      return;
    }
    int M = (L+R)/2;
    build(a,2*ind,L,M); build(a,2*ind+1,M+1,R); pull(ind);
  }
  void push(int ind, int L, int R) {
    if (L == R) return;
    F0R(i,2) if (mx[2*ind^i][0] > mx[ind][0]) {
      sum[2*ind^i] -= (ll)maxCnt[2*ind^i]*
               (mx[2*ind^i][0]-mx[ind][0]);
      mx[2*ind^i][0] = mx[ind][0];
    }
  }
  void upd(int x, int y, int t, int ind=1, int L=0, int R=-1) {
    if (R == -1) R += N;
    if (R < x || y < L || mx[ind][0] <= t) return;
    push(ind,L,R);
    if (x <= L && R <= y && mx[ind][1] < t) {
      sum[ind] -= (ll)maxCnt[ind]*(mx[ind][0]-t);
      mx[ind][0] = t;
      return;
    }
    if (L == R) return;
    int M = (L+R)/2;
    upd(x,y,t,2*ind,L,M); upd(x,y,t,2*ind+1,M+1,R); pull(ind);
  }
  ll qsum(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return 0;
    push(ind,L,R);
    if (x <= L && R <= y) return sum[ind];
    int M = (L+R)/2;
    return qsum(x,y,2*ind,L,M)+qsum(x,y,2*ind+1,M+1,R);
  }
  int qmax(int x, int y, int ind = 1, int L = 0, int R = -1) {
    if (R == -1) R += N;
    if (R < x || y < L) return -1;
    push(ind,L,R);
    if (x <= L && R <= y) return mx[ind][0];
    int M = (L+R)/2;
    return max(qmax(x,y,2*ind,L,M),qmax(x,y,2*ind+1,M+1,R));
  }
};
```

RectUnion.h

Description: Area of rectangle union using segment tree that keeps track of min and number of mins. Rectangles are in form $(x_1, x_2), (y_1, y_2)$.

"template.hpp" f61bc5, 47 lines

```
typedef pi T;
T operator+(const T& l, const T& r) {
  return l.f != r.f ? min(l,r) : T{l.f,l.s+r.s}; }

const int SZ = 1<<18;
struct LazySeg {
  const T ID = {MOD,0}; T comb(T a, T b) { return a+b; }
  T seg[2*SZ]; int lazy[2*SZ];
  LazySeg() { F0R(i,2*SZ) seg[i] = {0,0}, lazy[i] = 0; }
  void push(int ind, int L, int R) {
    if (L != R) F0R(i,2) lazy[2*ind+i] += lazy[ind];
    seg[ind].f += lazy[ind]; // dependent on operation
    lazy[ind] = 0;
  } // recalc values for current node
  void pull(int ind) { seg[ind] = comb(seg[2*ind],seg[2*ind+1])
    ↪; }
  void build() { ROF(i,1,SZ) pull(i); }
  void upd(int lo,int hi,int inc,int ind=1,int L=0, int R=SZ-1)
    ↪ {
    push(ind,L,R); if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
      lazy[ind] = inc; push(ind,L,R); return; }
    int M = (L+R)/2; upd(lo,hi,inc,2*ind,L,M);
    upd(lo,hi,inc,2*ind+1,M+1,R); pull(ind);
  }
};

ll area(vector<pair<pi,pi>> v) {
  LazySeg L;
  vi y; each(t,v) y.pb(t.s.f), y.pb(t.s.s);
  sort(all(y)); y.erase(unique(all(y)),y.end());
  F0R(i,sz(y)-1) L.seg[SZ+i].s = y[i+1]-y[i];
  L.build();
  vector<array<int,4>> ev; // sweep line
  each(t,v) {
    t.s.f = lb(all(y),t.s.f)-begin(y);
    t.s.s = lb(all(y),t.s.s)-begin(y)-1;
    ev.pb({t.f.f,1,t.s.f,t.s.s});
    ev.pb({t.f.s,-1,t.s.f,t.s.s});
  }
  sort(all(ev));
  ll ans = 0;
  F0R(i,sz(ev)-1) {
    const auto& t = ev[i]; L.upd(t[2],t[3],t[1]);
    int len = y.bk-y.ft-L.seg[1].s; // L.mn[0].f should equal 0
    ans += (ll)(ev[i+1][0]-t[0])*len;
  }
  return ans;
}
```

Treap.h

Description: Easy BBST. Use split and merge to implement insert and delete.

Time: $\mathcal{O}(\log N)$

bdb758, 65 lines

```
using pt = struct tnode*;
struct tnode {
  int pri, val; pt c[2]; // essential
  int sz; ll sum; // for range queries
  bool flip = 0; // lazy update
  tnode(int _val) {
    pri = rng(); sum = val = _val;
    sz = 1; c[0] = c[1] = nullptr;
  }
  ~tnode() { F0R(i,2) delete c[i]; }
};
int getsz(pt x) { return x?x->sz:0; }
ll getsum(pt x) { return x?x->sum:0; }
pt prop(pt x) { // lazy propagation
  if (!x || !x->flip) return x;
  swap(x->c[0],x->c[1]);
  x->flip = 0; F0R(i,2) if (x->c[i]) x->c[i]->flip ^= 1;
  return x;
}
pt calc(pt x) {
  pt a = x->c[0], b = x->c[1];
  assert(!x->flip); prop(a), prop(b);
  x->sz = 1+getsz(a)+getsz(b);
  x->sum = x->val+getsum(a)+getsum(b);
  return x;
}
void tour(pt x, vi& v) { // print values of nodes,
  if (!x) return; // inorder traversal
  prop(x); tour(x->c[0],v); v.pb(x->val); tour(x->c[1],v);
}
pair<pt,pt> split(pt t, int v) { // >= v goes to the right
  if (!t) return {t,t};
  prop(t);
  if (t->val >= v) {
    auto p = split(t->c[0], v); t->c[0] = p.s;
    return {p.f,calc(t)};
  } else {
    auto p = split(t->c[1], v); t->c[1] = p.f;
    return {calc(t),p.s};
  }
}
pair<pt,pt> splitsz(pt t, int sz) { // sz nodes go to left
  if (!t) return {t,t};
  prop(t);
  if (getsz(t->c[0]) >= sz) {
    auto p = splitsz(t->c[0],sz); t->c[0] = p.s;
    return {p.f,calc(t)};
  } else {
    auto p=splitsz(t->c[1],sz-getsz(t->c[0])-1); t->c[1]=p.f;
    return {calc(t),p.s};
  }
}
pt merge(pt l, pt r) { //  keys in l < keys in r
  if (!l || !r) return l?:r;
  prop(l), prop(r); pt t;
  if (l->pri > r->pri) l->c[1] = merge(l->c[1],r), t = l;
  else r->c[0] = merge(l,r->c[0]), t = r;
  return calc(t);
}
pt ins(pt x, int v) { // insert v
  auto a = split(x,v), b = split(a.s,v+1);
  return merge(a.f,merge(new tnode(v),b.s)); }
pt del(pt x, int v) { // delete v
  auto a = split(x,v), b = split(a.s,v+1);
```

```
  return merge(a.f,b.s); }
```

LeftistHeap.h
Description: Persistent meldable heap.
Memory: $\mathcal{O}(\log N)$ per meld
Time: $\mathcal{O}(\log N)$ per meld

"template.hpp" 768fc5, 19 lines

```
tcTU> struct LeftistHeap {
    using self_t = LeftistHeap<T, U>;
    int rank; T key; U value;
    self_t *left, *right;
    LeftistHeap(int rank_, T key_, U value_, self_t* left_,
                self_t* right_)
        : rank{rank_}, key{key_}, value{value_}, left{left_},
          ↪right{right_} {}
    inline static deque<LeftistHeap> alloc;
    static self_t* insert(LeftistHeap* a, const T k, const U v)
       ↪ {
        if (not a or k < a->key) {
            alloc.emplace_back(1, k, v, a, nullptr);
            return &alloc.back();
        }
        auto l = a->left, r = insert(a->right, k, v);
        if (not l or r->rank > l->rank) swap(l, r);
        alloc.emplace_back(r ? r->rank + 1 : 0, a->key, a->
           ↪value, l, r);
        return &alloc.back();
    }
};
```

3.3 2D Range Queries

BIT2DOff.h
Description: point update and rectangle sum with offline 2D BIT. For each of the points to be updated, $x \in (0, SZ)$ and $y \neq 0$.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}\left(N \log^2 N\right)$

2b8415, 34 lines

```
template<class T, int SZ> struct OffBIT2D {
  bool mode = 0; // mode = 1 -> initialized
  vpi todo; // locations of updates to process
  int cnt[SZ], st[SZ];
  vi val; vector<T> bit; // store all BITs in single vector
  void init() { assert(!mode); mode = 1;
    int lst[SZ]; F0R(i,SZ) lst[i] = cnt[i] = 0;
    sort(all(todo),[](const pi& a, const pi& b) {
      return a.s < b.s; });
    each(t,todo) for (int x = t.f; x < SZ; x += x&-x)
      if (lst[x] != t.s) lst[x] = t.s, cnt[x] ++;
    int sum = 0; F0R(i,SZ) lst[i] = 0, st[i] = (sum += cnt[i]);
    val.rsz(sum); bit.rsz(sum); reverse(all(todo));
    each(t,todo) for (int x = t.f; x < SZ; x += x&-x)
      if (lst[x] != t.s) lst[x] = t.s, val[--st[x]] = t.s;
  }
  int rank(int y, int l, int r) {
    return ub(begin(val)+l,begin(val)+r,y)-begin(val)-l; }
  void UPD(int x, int y, T t) {
    for (y = rank(y,st[x],st[x]+cnt[x]); y <= cnt[x]; y += y&-y
       ↪)
      bit[st[x]+y-1] += t; }
  void upd(int x, int y, T t) {
    if (!mode) todo.pb({x,y});
    else for (;x<SZ;x+=x&-x) UPD(x,y,t); }
  T QUERY(int x, int y) { T res = 0;
    for (y = rank(y,st[x],st[x]+cnt[x]); y; y -= y&-y) res +=
       ↪bit[st[x]+y-1];
    return res; }
  T query(int x, int y) { assert(mode);
    T res = 0; for (;x;x-=x&-x) res += QUERY(x,y);
    return res; }
  T query(int xl, int xr, int yl, int yr) {
    return query(xr,yr)-query(xl-1,yr)
      -query(xr,yl-1)+query(xl-1,yl-1); }
};
```

Number Theory (4)

4.1 Modular Arithmetic

ModIntShort.h
Description: Modular arithmetic. Assumes MOD is prime.
Usage: `mi a = MOD+5; inv(a); // 400000003`

2672f9, 24 lines

```
template<int MOD, int RT> struct mint {
  static const int mod = MOD;
  static constexpr mint rt() { return RT; } // primitive root
  int v;
  explicit operator int() const { return v; }
  mint():v(0) {}
  mint(ll _v):v(int(_v%MOD)) { v += (v<0)*MOD; }
  mint& operator+=(mint o) {
    if ((v += o.v) >= MOD) v -= MOD;
    return *this; }
  mint& operator-=(mint o) {
    if ((v -= o.v) < 0) v += MOD;
    return *this; }
  mint& operator*=(mint o) {
    v = int((ll)v*o.v%MOD); return *this; }
  friend mint pow(mint a, ll p) { assert(p >= 0);
    return p==0?1:pow(a*a,p/2)*(p&1?a:1); }
  friend mint inv(mint a) { assert(a.v != 0); return pow(a,MOD
     ↪-2); }
  friend mint operator+(mint a, mint b) { return a += b; }
  friend mint operator-(mint a, mint b) { return a -= b; }
  friend mint operator*(mint a, mint b) { return a *= b; }
};
using mi = mint<(int)1e9+7, 5>;
using vmi = V<mi>;
```

ModMulLL.h
Description: Multiply two 64-bit integers mod another if 128-bit is not available. modMul is equivalent to `(ul)(__int128(a)*b%mod)`. Works for $0 \le a, b < mod < 2^{63}$.

530181, 9 lines

```
using ul = uint64_t;
ul modMul(ul a, ul b, const ul mod) {
  ll ret = a*b-mod*(ul)((db)a*b/mod);
  return ret+((ret<0)-(ret>=(ll)mod))*mod; }
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a,b/2,mod); res = modMul(res,res,mod);
  return b&1 ? modMul(res,a,mod) : res;
}
```

ModSqrt.h
Description: Tonelli-Shanks algorithm for square roots mod a prime. -1 if doesn't exist.
Usage: `sqrt(mi((ll)1e10)); // 100000`
Time: $\mathcal{O}\left(\log^2(MOD)\right)$

"ModInt.h" bcfa63, 14 lines

```
using T = int;
T sqrt(mi a) {
  mi p = pow(a,(MOD-1)/2);
  if (p.v != 1) return p.v == 0 ? 0 : -1;
  T s = MOD-1; int r = 0; while (s%2 == 0) s /= 2, ++r;
  mi n = 2; while (pow(n,(MOD-1)/2).v == 1) n = T(n)+1;
  // n non-square, ord(g)=2^r, ord(b)=2^m, ord(g)=2^r, m<r
  for (mi x = pow(a,(s+1)/2), b = pow(a,s), g = pow(n,s);;) {
    if (b.v == 1) return min(x.v,MOD-x.v); // x^2=ab
    int m = 0; for (mi t = b; t.v != 1; t *= t) ++m;
    rep(r-m-1) g *= g; // ord(g)=2^{m+1}
    x *= g, g *= g, b *= g, r = m; // ord(g)=2^m, ord(b)<2^m
  }
}
```

ModSum.h
Description: Counts # of lattice points (x, y) in the triangle $1 \le x, 1 \le y, ax + by \le s \pmod{2^{64}}$ and related quantities.
Time: $\mathcal{O}(\log ab)$

23cbf6, 20 lines

```
using ul = uint64_t;
ul sum2(ul n) { return n/2*((n-1)|1); } // sum(0..n-1)
// \return |{(x,y) | 1 <= x, 1 <= y, a*x+b*y <= S}|
//       = sum_{i=1}^{qs} (S-a*i)/b
ul triSum(ul a, ul b, ul s) { assert(a > 0 && b > 0);
  ul qs = s/a, rs = s%a; // ans = sum_{i=0}^{qs-1}(i*a+rs)/b
  ul ad = a/b*sum2(qs)+rs/b*qs; a %= b, rs %= b;
  return ad+(a?triSum(b,a,a*qs+rs):0); // reduce if a >= b
} // then swap x and y axes and recurse

// \return sum_{x=0}^{n-1} (a*x+b)/m
//       = |{(x,y) | 0 < m*y <= a*x+b < a*n+b}|
// assuming a*n+b does not overflow
ul divSum(ul n, ul a, ul b, ul m) { assert(m > 0);
  ul extra = b/m*n; b %= m;
  return extra+(a?triSum(m,a,a*n+b):0); }
// \return sum_{x=0}^{n-1} (a*x+b)%m
ul modSum(ul n, ll a, ll b, ul m) { assert(m > 0);
  a = (a%m+m)%m, b = (b%m+m)%m;
  return a*sum2(n)+b*n-m*divSum(n,a,b,m); }
```

DiscreteLog.h
Description: find least integer p such that $r^p \equiv x \mod MOD$
Time: $\mathcal{O}\left(\sqrt{mod}\right)$ per query

"template.hpp", "ModInt.h" 87c660, 21 lines

```
struct DiscreteLog {
  int root, block;
  unordered_map<int,int> u;
  mi cur;
  int query(mi x) {
    F0R(i,block) {
      if (u.count((int)x)) return i*block+u[(int)x];
      x *= cur;
    }
    return -1;
  }
  void init(int r) { // gcd(m,r) = 1
    root = r; block = sqrt(MOD)+1;
    u.clear(); cur = mi(1);
    F0R(i,block) {
      if (!u.count((int)cur)) u[(int)cur] = i;
      cur *= root;
    }
    cur = 1/cur;
  }
};
```

Order.h
Description: Calculates smallest P such that $x^P \equiv 1 \mod p$

"ModMulLL.h", "FactorBasic.h" e2c3b7, 7 lines

```
ll order(ll x, ll p) {
  if (gcd(x,p) != 1) return 0;
  ll P = phi(p); auto a = factor(P);
  each(t,a) while (P % t.f == 0
    && modPow(x,P/t.f,p) == 1) P /= t.f;
```

```
  return P;
}
```

4.2 Primality

4.2.1 Primes

$p = 962592769$ is such that $2^{21} \mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.2.2 Divisors

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Dirichlet Convolution: Given a function $f(x)$, let

$(f * g)(x) = \sum_{d|x} g(d) f(x/d)$.

If the partial sums $s_{f*g}(n), s_g(n)$ can be computed in $O(1)$ and $s_f(1 \dots n^{2/3})$ can be computed in $O\left(n^{2/3}\right)$ then all $s_f\left(\frac{n}{d}\right)$ can as well. Use

$$s_{f*g}(n) = \sum_{d=1}^{n} g(d) s_f(n/d).$$

If $f(x) = \mu(x)$ then $g(x) = 1, (f*g)(x) = (x == 1)$, and $s_f(n) = 1 - \sum_{i=2}^{n} s_f(n/i)$.

If $f(x) = \phi(x)$ then $g(x) = 1, (f*g)(x) = x$, and $s_f(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s_f(n/i)$.

Sieve.h

Description: Tests primality up to SZ. Runs faster if only odd indices are stored.

Time: $\mathcal{O}(SZ \log \log SZ)$ or $\mathcal{O}(SZ)$

41c6ed, 20 lines

```
template<int SZ> struct Sieve {
  bitset<SZ> is_prime; vi primes;
  Sieve() {
    is_prime.set(); is_prime[0] = is_prime[1] = 0;
    for (int i = 4; i < SZ; i += 2) is_prime[i] = 0;
    for (int i = 3; i*i < SZ; i += 2) if (is_prime[i])
      for (int j = i*i; j < SZ; j += i*2) is_prime[j] = 0;
    F0R(i,SZ) if (is_prime[i]) primes.pb(i);
  }
  // int sp[SZ]{}; // smallest prime that divides
  // Sieve() { // above is faster
  //  FOR(i,2,SZ) {
  //    if (sp[i] == 0) sp[i] = i, primes.pb(i);
  //    for (int p: primes) {
  //      if (p > sp[i] || i*p >= SZ) break;
  //      sp[i*p] = p;
  //    }
  //  }
  // }
};
```

MultiplicativePrefixSums.h

Description: $\sum_{i=1}^{N} f(i)$ where $f(i) = \prod \text{val}[e]$ for each p^e in the factorization of i. Must satisfy $\text{val}[1] = 1$. Generalizes to any multiplicative function with $f(p) = p^{\text{fixed power}}$.

Time: $\mathcal{O}\left(\sqrt{N}\right)$

"Sieve.h" 3151ea, 12 lines

```
vmi val;
mi get_prefix(ll N, int p = 0) {
  mi ans = N;
  for (; S.primes.at(p) <= N / S.primes.at(p); ++p) {
    ll new_N = N / S.primes.at(p) / S.primes.at(p);
    for (int idx = 2; new_N; ++idx, new_N /= S.primes.at(p)) {
      ans += (val.at(idx) - val.at(idx - 1))
           * get_prefix(new_N, p + 1);
    }
  }
  return ans;
}
```

PrimeCnt.h

Description: Counts number of primes up to N. Can also count sum of primes.

Time: $\mathcal{O}\left(N^{3/4}/\log N\right)$, 60ms for $N = 10^{11}$, 2.5s for $N = 10^{13}$

c04e96, 20 lines

```
ll count_primes(ll N) { // count_primes(1e13) == 346065536839
  if (N <= 1) return 0;
  int sq = (int)sqrt(N);
  vl big_ans((sq+1)/2), small_ans(sq+1);
  FOR(i,1,sq+1) small_ans[i] = (i-1)/2;
  F0R(i,sz(big_ans)) big_ans[i] = (N/(2*i+1)-1)/2;
  vb skip(sq+1); int prime_cnt = 0;
  for (int p = 3; p <= sq; p += 2) if (!skip[p]) { // primes
    for (int j = p; j <= sq; j += 2*p) skip[j] = 1;
    F0R(j,min((ll)sz(big_ans),(N/p/p+1)/2)) {
      ll prod = (ll)(2*j+1)*p;
      big_ans[j] -= (prod > sq ? small_ans[(double)N/prod]
            : big_ans[prod/2])-prime_cnt;
    }
    for (int j = sq, q = sq/p; q >= p; --q) for (;j >= q*p;--j)
      small_ans[j] -= small_ans[q]-prime_cnt;
    ++prime_cnt;
  }
  return big_ans[0]+1;
}
```

MillerRabin.h

Description: Deterministic primality test, works up to 2^{64}. For larger numbers, extend A randomly.

"ModMulLL.h" 89df33, 11 lines

```
bool prime(ul n) { // not ll!
  if (n < 2 || n % 6 % 4 != 1) return n-2 < 2;
  ul A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
     s = __builtin_ctzll(n-1), d = n>>s;
  each(a,A) {   // ^ count trailing zeroes
    ul p = modPow(a,d,n), i = s;
    while (p != 1 && p != n-1 && a%n && i--) p = modMul(p,p,n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

FactorFast.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(N^{1/4}\right)$, less for numbers with small factors

"MillerRabin.h", "ModMulLL.h" 99cf33, 16 lines

```
ul pollard(ul n) { // return some nontrivial factor of n
  auto f = [n](ul x) { return modMul(x, x, n) + 1; };
  ul x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modMul(prd, max(x,y)-min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return gcd(prd, n);
}
void factor_rec(ul n, map<ul,int>& cnt) {
  if (n == 1) return;
  if (prime(n)) { ++cnt[n]; return; }
  ul u = pollard(n);
  factor_rec(u,cnt), factor_rec(n/u,cnt);
}
```

4.3 Euclidean Algorithm

4.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = \gcd(a,b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

FracInterval.h

Description: Given fractions $a < b$ with non-negative numerators and denominators, finds fraction f with lowest denominator such that $a < f < b$. Should work with all numbers less than 2^{62}.

1860f3, 6 lines

```
pl bet(pl a, pl b) {
  ll num = a.f/a.s; a.f -= num*a.s, b.f -= num*b.s;
  if (b.f > b.s) return {1+num,1};
  auto x = bet({b.s,b.f},{a.s,a.f});
  return {x.s+num*x.f,x.f};
}
```

Euclid.h

Description: Generalized Euclidean algorithm. `euclid` and `invGeneral` work for $A, B < 2^{62}$.

Time: $\mathcal{O}(\log AB)$

"template.hpp" 90ce0a, 24 lines

```
// ceil(a/b)
// ll cdiv(ll a, ll b) { return a/b+((a^b)>0&&a%b); }
pl euclid(ll A, ll B) { // For A,B>=0, finds (x,y) s.t.
  // Ax+By=gcd(A,B), |Ax|,|By|<=AB/gcd(A,B)
  if (!B) return {1,0};
  pl p = euclid(B,A%B); return {p.s,p.f-A/B*p.s}; }
ll invGeneral(ll A, ll B) { // find x in [0,B) such that Ax=1
  ↪mod B
  pl p = euclid(A,B); assert(p.f*A+p.s*B == 1);
  return p.f+(p.f<0)*B; } // must have gcd(A,B)=1
ll numPositiveSols(ll A, ll B, ll C, bool strict) {
  // find number of (x,y) st Ax + By = C with x,y>=strict
  ll d = gcd(A, B);
```

```
  if (C % d) return 0;
  pl p = euclid(A, B); ll x = p.f, y = p.s;
  x *= C/d, y *= C/d;
  ll k1 = -x * d, k2 = y * d;
  if (k1 > 0) k1 = k1 / B + (k1 % B != 0);
  else k1 = k1 / B;
  if (k2 > 0) k2 = k2 / A;
  else k2 = -((-k2)/ A + (k2 % A != 0));
  ll ans = max(0LL, k2-k1+1);
  if (strict) ans -= ll(C%B==0) + ll(C%A==0);
  return ans;
}
```

CRT.h

Description: Chinese Remainder Theorem. $a.f \pmod{a.s}, b.f \pmod{b.s} \implies ? \pmod{\text{lcm}(a.s, b.s)}$. Should work for $ab < 2^{62}$.

"template.hpp", "Euclid.h" 23df64, 10 lines

```
pl CRT(pl a, pl b) { assert(0 <= a.f && a.f < a.s && 0 <= b.f
    ↪&& b.f < b.s);
  if (a.s < b.s) swap(a,b); // will overflow if b.s^2 > 2^{62}
  ll x,y; tie(x,y) = euclid(a.s,b.s);
  ll g = a.s*x+b.s*y, l = a.s/g*b.s;
  if ((b.f-a.f)%g) return {-1,-1}; // no solution
  // ?*a.s+a.f \equiv b.f \pmod{b.s}
  // ?=(b.f-a.f)/g*(a.s/g)^{-1} \pmod{b.s/g}
  x = (b.f-a.f)%b.s*x%b.s/g*a.s+a.f;
  return {x+(x<0)*l,l};
}
```

ModArith.h

Description: Statistics on mod'ed arithmetic series. `minBetween` and `minRemainder` both assume that $0 \le L \le R < B$, $AB < 2^{62}$

f68a6d, 40 lines

```
ll minBetween(ll A, ll B, ll L, ll R) {
  // min x s.t. exists y s.t. L <= A*x-B*y <= R
  A %= B;
  if (L == 0) return 0;
  if (A == 0) return -1;
  ll k = cdiv(L,A); if (A*k <= R) return k;
  ll x = minBetween(B,A,A-R%A,A-L%A); // min x s.t. exists y
  // s.t. -R <= Bx-Ay <= -L
  return x == -1 ? x : cdiv(B*x+L,A); // solve for y
}

// find min((Ax+C)%B) for 0 <= x <= M
// aka find minimum non-negative value of A*x-B*y+C
// where 0 <= x <= M, 0 <= y
ll minRemainder(ll A, ll B, ll C, ll M) {
  assert(A >= 0 && B > 0 && C >= 0 && M >= 0);
  A %= B, C %= B; ckmin(M,B-1);
  if (A == 0) return C;
  if (C >= A) { // make sure C<A
    ll ad = cdiv(B-C,A);
    M -= ad; if (M < 0) return C;
    C += ad*A-B;
  }
  ll q = B/A, new_B = B%A; // new_B < A
  if (new_B == 0) return C; // B-q*A

  // now minimize A*x-new_B*y+C
  // where 0 <= x,y and x+q*y <= M, 0 <= C < new_B < A
  // q*y -> C-new_B*y
  if (C/new_B > M/q) return C-M/q*new_B;
  M -= C/new_B*q; C %= new_B; // now C < new_B

  // given y, we can compute x = ceil[((B-q*A)*y-C)/A]
  // so x+q*y = ceil((B*y-C)/A) <= M
  ll max_Y = (M*A+C)/B; // must have y <= max_Y
  ll max_X = cdiv(new_B*max_Y-C,A); // must have x <= max_X
  if (max_X*A-new_B*max_Y+C >= new_B) --max_X;
  // now we can remove upper bound on y
  return minRemainder(A,new_B,C,max_X);
}
```

4.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

4.5 Lifting the Exponent

For $n > 0$, p prime, and ints x, y s.t. $p \nmid x, y$ and $p | x - y$:

- $p \neq 2$ or $p = 2, 4 | x - y \implies v_p(x^n - y^n) = v_p(x - y) + v_p(n)$.
- $p = 2, 2 | n \implies v_2(x^n - y^n) = v_2((x^2)^{n/2} - (y^2)^{n/2})$.

Combinatorial (5)

5.1 Permutations

5.1.1 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

5.1.2 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

5.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

5.3 General purpose numbers

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k) x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

5.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.4 Other

DeBruijnSeq.h

Description: Given alphabet $[0, k)$ constructs a cyclic string of length k^n that contains every length n string as substr. a6961b, 13 lines

```
vi deBruijnSeq(int k, int n) {
  if (k == 1) return {0};
  vi seq, aux(n+1);
  function<void(int,int)> gen = [&](int t, int p) {
    if (t > n) { // +lyndon word of len p
      if (n%p == 0) FOR(i,1,p+1) seq.pb(aux[i]);
    } else {
      aux[t] = aux[t-p]; gen(t+1,p);
      while (++aux[t] < k) gen(t+1,t);
    }
  };
  gen(1,1); return seq;
}
```

NimProduct.h

Description: Product of nimbers is associative, commutative, and distributive over addition (xor). Forms finite field of size 2^{2^k}. Defined by $ab = \text{mex}(\{a'b + ab' + a'b' : a' < a, b' < b\})$. Application: Given 1D coin turning games G_1, G_2 $G_1 \times G_2$ is the 2D coin turning game defined as follows. If turning coins at $x_1, x_2, \ldots, x_m$ is legal in G_1 and $y_1, y_2, \ldots, y_n$ is legal in G_2, then turning coins at all positions (x_i, y_j) is legal assuming that the coin at (x_m, y_n) goes from heads to tails. Then the grundy function $g(x, y)$ of $G_1 \times G_2$ is $g_1(x) \times g_2(y)$.
Time: 64^2 xors per multiplication, memorize to speed up. 5afe17, 46 lines

```
using ul = uint64_t;
struct Precalc {
  ul tmp[64][64], y[8][8][256];
  unsigned char x[256][256];
  Precalc() { // small nim products, all < 256
    F0R(i,256) F0R(j,256) x[i][j] = mult<8>(i,j);
    F0R(i,8) F0R(j,i+1) F0R(k,256)
      y[i][j][k] = mult<64>(prod2(8*i,8*j),k);
  }
  ul prod2(int i, int j) { // nim prod of 2^i, 2^j
    ul& u = tmp[i][j]; if (u) return u;
    if (!(i&j)) return u = 1ULL<<(i|j);
    int a = (i&j)&-(i&j); // a=2^k, consider 2^{2^k}
    return u=prod2(i^a,j)^prod2((i^a)|(a-1),(j^a)|(i&(a-1)));
    // 2^{2^k}*2^{2^k} = 2^{2^k}+2^{2^k-1}
  } // 2^{2^i}*2^{2^j} = 2^{2^i+2^j} if i<j
  template<int L> ul mult(ul a, ul b) {
    ul c = 0; F0R(i,L) if (a>>i&1)
      F0R(j,L) if (b>>j&1) c ^= prod2(i,j);
    return c;
  }
  // 2^{8*i}*(a>>(8*i)&255) * 2^{8*j}*(b>>(8*j)&255)
  // -> (2^{8*i}*2^{8*j})*((a>>(8*i)&255)*(b>>(8*j)&255))
  ul multFast(ul a, ul b) const { // faster nim product
    ul res = 0; auto f=[](ul c,int d) {return c>>(8*d)&255;};
    F0R(i,8) {
      F0R(j,i) res ^= y[i][j][x[f(a,i)][f(b,j)]
                  ^x[f(a,j)][f(b,i)]];
      res ^= y[i][i][x[f(a,i)][f(b,i)]];
    }
    return res;
  }
};
const Precalc P;

struct nb { // nimber
  ul x; nb() { x = 0; }
  nb(ul _x): x(_x) {}
  explicit operator ul() { return x; }
  nb operator+(nb y) { return nb(x^y.x); }
  nb operator*(nb y) { return nb(P.multFast(x,y.x)); }
  friend nb pow(nb b, ul p) {
    nb res = 1; for (;p;p/=2,b=b*b) if (p&1) res = res*b;
    return res; } // b^{2^{2^A}-1}=1 where 2^{2^A} > b
  friend nb inv(nb b) { return pow(b,-2); }
};
```

MatroidIsect.h

Description: Computes a set of maximum size which is independent in both graphic and colorful matroids, aka a spanning forest where no two edges are of the same color. In general, construct the exchange graph and find a shortest path. Can apply similar concept to partition matroid. M1 should have the slower matroid and M2 the faster matroid.
Usage: `MatroidIsect<Gmat,Cmat> M(Gmat(V,ed),Cmat(col),sz(ed)); M.solve();`
Time: $\mathcal{O}\left(GI^{1.5}\right)$ calls to oracles, where G is size of ground set and I is size of independent set.
"template.hpp", "DSU.h" f9dc7a, 73 lines

```
struct Gmat { // graphic matroid
  int n; vpi ed; DSU D;
  Gmat(int _n, vpi _ed): n(_n), ed(_ed) { D.init(n); }
  void clear() { D.init(n); }
  void ins(int i) { assert(D.unite(ed[i].f, ed[i].s)); }
  bool indep(int i) { return !D.sameSet(ed[i].f, ed[i].s); }
};
struct Cmat { // colorful matroid
  int C = 0; vi col; vb used;
  Cmat(vi col):col(col) {each(t,col) ckmax(C,t+1); clear(); }
  void clear() { used.assign(C,0); }
  void ins(int i) { used[col[i]] = 1; }
  bool indep(int i) { return !used[col[i]]; }
};
struct Xmat { // XOR lineal matroid
  vl v, b;
  Xmat(vl _v) : v(_v) {}
  ll fun(ll a) { each(x, b) ckmin(a, a ^ x); return a; }
  void clear() { b.clear(); }
  void ins(int i) {
    ll a = fun(v[i]);
    for (i = 0; i < sz(b) && a < b[i]; ++i);
    b.insert(b.begin() + i, a);
  }
  bool indep(int i) { return fun(v[i]) > 0; }
};
template<class M1, class M2> struct MatroidIsect {
    int n;
    vb iset;
    M1 m1; M2 m2;
    MatroidIsect(M1 _m1, M2 _m2, int _n) : n(_n), iset(_n + 1),
      ↪ m1(_m1), m2(_m2) {}
    vi solve() {
        F0R(i, n) if (m1.indep(i) && m2.indep(i))
            iset[i] = true, m1.ins(i), m2.ins(i);
        while (augment());
        vi ans;
        F0R(i, n) if (iset[i]) ans.pb(i);
        return ans;
    }
    vi frm;
    queue<int> q;
    vi fwdE(int a) {
        vi ans;
        m1.clear();
        F0R(v, n) if (iset[v] && v != a) m1.ins(v);
        F0R(b, n) if (!iset[b] && frm[b] == -1 && m1.indep(b))
            ans.pb(b), frm[b] = a;
        return ans;
    }
    int backE(int b) {
        m2.clear();
        F0R(c,2) F0R(v, n)
            if ((v == b || iset[v]) && (frm[v] == -1) == c) {
                if (!m2.indep(v))
                    return c ? q.push(v), frm[v] = b, v : -1;
                m2.ins(v);
            }
        return n;
    }
    bool augment() {
        frm.assign(n, -1);
        q = {}; q.push(n); // dummy node
        while (!q.empty()) {
      int a = q.front(), c; q.pop();
      for (int b : fwdE(a))
        while((c = backE(b)) >= 0) if (c == n) {
          while (b != n) iset[b] = !iset[b], b = frm[b];
          return true;
        }
    }
    return false;
    }
};
```

Numerical (6)

6.1 Matrix

Matrix.h

Description: 2D matrix operations.
"ModInt.h" b18e29, 21 lines

```
using T = mi;
using Mat = V<V<T>>; // use array instead if tight TL

Mat makeMat(int r, int c) { return Mat(r,V<T>(c)); }
Mat makeId(int n) {
  Mat m = makeMat(n,n); F0R(i,n) m[i][i] = 1;
  return m;
}
Mat operator*(const Mat& a, const Mat& b) {
  int x = sz(a), y = sz(a[0]), z = sz(b[0]);
  assert(y == sz(b)); Mat c = makeMat(x,z);
  F0R(i,x) F0R(j,y) F0R(k,z) c[i][k] += a[i][j]*b[j][k];
  return c;
}
Mat& operator*=(Mat& a, const Mat& b) { return a = a*b; }
Mat pow(Mat m, ll p) {
  int n = sz(m); assert(n == sz(m[0]) && p >= 0);
  Mat res = makeId(n);
  for (; p; p /= 2, m *= m) if (p&1) res *= m;
  return res;
}
```

MatrixInv.h

Description: Uses gaussian elimination to convert into reduced row echelon form and calculates determinant. For determinant via arbitrary modulos, use a modified form of the Euclidean algorithm because modular inverse may not exist. If you have computed $A^{-1} \pmod{p^k}$, then the inverse $\pmod{p^{2k}}$ is $A^{-1}(2I - AA^{-1})$.

Time: $\mathcal{O}\left(N^3\right)$, determinant of 1000×1000 matrix of modints in 1 second if you reduce # of operations by half

73ec43, 38 lines

```
const db EPS = 1e-9; // adjust?
int getRow(V<V<db>>& m, int R, int i, int nex) {
  pair<db,int> bes{0,-1}; // find row with max abs value
  FOR(j,nex,R) ckmax(bes,{abs(m[j][i]),j});
  return bes.f < EPS ? -1 : bes.s; }
int getRow(V<vmi>& m, int R, int i, int nex) {
  FOR(j,nex,R) if (m[j][i] != 0) return j;
  return -1; }
pair<T,int> gauss(Mat& m) { // convert to reduced row echelon
    ↪form
  if (!sz(m)) return {1,0};
  int R = sz(m), C = sz(m[0]), rank = 0, nex = 0;
  T prod = 1; // determinant
  F0R(i,C) {
    int row = getRow(m,R,i,nex);
    if (row == -1) { prod = 0; continue; }
    if (row != nex) prod *= -1, swap(m[row],m[nex]);
    prod *= m[nex][i]; rank++;
    T x = 1/m[nex][i]; FOR(k,i,C) m[nex][k] *= x;
    F0R(j,R) if (j != nex) {
      T v = m[j][i]; if (v == 0) continue;
      FOR(k,i,C) m[j][k] -= v*m[nex][k];
    }
    nex++;
  }
  return {prod,rank};
}
Mat inv(Mat m) {
  int R = sz(m); assert(R == sz(m[0]));
  Mat x = makeMat(R,2*R);
  F0R(i,R) {
    x[i][i+R] = 1;
    F0R(j,R) x[i][j] = m[i][j];
  }
  if (gauss(x).s != R) return Mat();
  Mat res = makeMat(R,R);
  F0R(i,R) F0R(j,R) res[i][j] = x[i][j+R];
  return res;
}
```

MatrixTree.h

Description: Kirchhoff's Matrix Tree Theorem. Given adjacency matrix, calculates # of spanning trees.

"MatrixInv.h" 48363d, 11 lines

```
T numSpan(const Mat& m) {
  int n = sz(m); Mat res = makeMat(n-1,n-1);
  F0R(i,n) FOR(j,i+1,n) {
    mi ed = m[i][j]; res[i][i] += ed;
    if (j != n-1) {
      res[j][j] += ed;
      res[i][j] -= ed, res[j][i] -= ed;
    }
  }
  return gauss(res).f;
}
```

ShermanMorrison.h

Description: Calculates $(A + uv^T)^{-1}$ given $B = A^{-1}$. Not invertible if sum=0.

"MatrixInv.h" 3a3f34, 7 lines

```
void ad(Mat& B, const V<T>& u, const V<T>& v) {
  int n = sz(A); V<T> x(n), y(n);
  F0R(i,n) F0R(j,n)
    x[i] += B[i][j]*u[j], y[j] += v[i]*B[i][j];
  T sum = 1; F0R(i,n) F0R(j,n) sum += v[i]*B[i][j]*u[j];
  F0R(i,n) F0R(j,n) B[i][j] -= x[i]*y[j]/sum;
}
```

6.2 Polynomials

Poly.h

Description: Basic poly ops including division. Can replace T with double, complex.

"ModInt.h" cd218a, 73 lines

```
using T = mi; using poly = V<T>;
void remz(poly& p) { while (sz(p)&&p.bk==T(0)) p.pop_back(); }
poly REMZ(poly p) { remz(p); return p; }
poly rev(poly p) { reverse(all(p)); return p; }
poly shift(poly p, int x) {
  if (x >= 0) p.insert(begin(p),x,0);
  else assert(sz(p)+x >= 0), p.erase(begin(p),begin(p)-x);
  return p;
}
poly RSZ(const poly& p, int x) {
  if (x <= sz(p)) return poly(begin(p),begin(p)+x);
  poly q = p; q.rsz(x); return q;  }
T eval(const poly& p, T x) { // evaluate at point x
  T res = 0; R0F(i,sz(p)) res = x*res+p[i];
  return res; }
poly dif(const poly& p) { // differentiate
  poly res; FOR(i,1,sz(p)) res.pb(T(i)*p[i]);
  return res; }
poly integ(const poly& p) { // integrate
  static poly invs{0,1};
  for (int i = sz(invs); i <= sz(p); ++i)
    invs.pb(-MOD/i*invs[MOD%i]);
  poly res(sz(p)+1); F0R(i,sz(p)) res[i+1] = p[i]*invs[i+1];
  return res;
}

poly& operator+=(poly& l, const poly& r) {
  l.rsz(max(sz(l),sz(r))); F0R(i,sz(r)) l[i] += r[i];
  return l; }
poly& operator-=(poly& l, const poly& r) {
  l.rsz(max(sz(l),sz(r))); F0R(i,sz(r)) l[i] -= r[i];
  return l; }
poly& operator*=(poly& l, const T& r) { each(t,l) t *= r;
  return l;  }
poly& operator/=(poly& l, const T& r) { each(t,l) t /= r;
  return l; }
poly operator+(poly l, const poly& r) { return l += r; }
poly operator-(poly l, const poly& r) { return l -= r; }
poly operator-(poly l) { each(t,l) t *= -1; return l; }
poly operator*(poly l, const T& r) { return l *= r; }
poly operator*(const T& r, const poly& l) { return l*r; }
poly operator/(poly l, const T& r) { return l /= r;  }
poly operator*(const poly& l, const poly& r) {
  if (!min(sz(l),sz(r))) return {};
  poly x(sz(l)+sz(r)-1);
  F0R(i,sz(l)) F0R(j,sz(r)) x[i+j] += l[i]*r[j];
  return x;
}
poly& operator*=(poly& l, const poly& r) { return l = l*r; }

pair<poly,poly> quoRemSlow(poly a, poly b) {
  remz(a); remz(b); assert(sz(b));
  T lst = b.bk, B = T(1)/lst; each(t,a) t *= B;
  each(t,b) t *= B;
  poly q(max(sz(a)-sz(b)+1,0));
  for (int dif; (dif=sz(a)-sz(b)) >= 0; remz(a)) {
    q[dif] = a.bk; F0R(i,sz(b)) a[i+dif] -= q[dif]*b[i]; }
  each(t,a) t *= lst;
  return {q,a}; // quotient, remainder
}
poly operator%(const poly& a, const poly& b) {
  return quoRemSlow(a,b).s; }
T resultant(poly a, poly b) { // R(A,B)
  // =b_m^n*prod_{j=1}^mA(mu_j)
  // =b_m^na_n^m*prod_{i=1}^nprod_{j=1}^m(mu_j-lambda_i)
  // =(-1)^{mn}a_n^m*prod_{i=1}^nB(lambda_i)
  // =(-1)^{nm}R(B,A)
  // Also, R(A,B)=b_m^{deg(A)-deg(A-CB)}R(A-CB,B)
  int ad = sz(a)-1, bd = sz(b)-1;
  if (bd <= 0) return bd < 0 ? 0 : pow(b.bk,ad);
  int pw = ad; a = a%b; pw -= (ad = sz(a)-1);
  return resultant(b,a)*pow(b.bk,pw)*T((bd&ad&1)?-1:1);
}
```

PolyInterpolate.h

Description: n points determine unique polynomial of degree $\leq n-1$. For numerical precision pick $v[k].f = c * \cos(k/(n-1) * \pi), k = 0 \ldots n-1$.

Time: $\mathcal{O}\left(n^2\right)$

"Poly.h" aada3a, 8 lines

```
poly interpolate(V<pair<T,T>> v) {
  poly res, tmp{1};
  F0R(i,sz(v)) { T prod = 1; // add one point at a time
    F0R(j,i) v[i].s -= prod*v[j].s, prod *= v[i].f-v[j].f;
    v[i].s /= prod; res += v[i].s*tmp; tmp *= poly{-v[i].f,1};
  } // add multiple of (x-v[0].f)*(x-v[1].f)*...*(x-v[i-1].f)
  return res;
}
```

PolyRoots.h

Description: Finds the real roots of a polynomial.

Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`

Time: $\mathcal{O}\left(N^2 \log(1/\epsilon)\right)$

"Poly.h" c9127a, 20 lines

```
typedef db T;
poly polyRoots(poly p, T xmin, T xmax) {
  if (sz(p) == 2) { return {-p[0]/p[1]}; }
  auto dr = polyRoots(dif(p),xmin,xmax);
  dr.pb(xmin-1); dr.pb(xmax+1); sort(all(dr));
  poly ret;
```

```
  F0R(i,sz(dr)-1) {
    T l = dr[i], h = dr[i+1];
    bool sign = eval(p,l) > 0;
    if (sign^(eval(p,h) > 0)) {
      F0R(it,60) { // while (h-l > 1e-8)
        auto m = (l+h)/2, f = eval(p,m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.pb((l+h)/2);
    }
  }
  return ret;
}
```

FFT.h

Description: Multiply polynomials of ints for any modulus $< 2^{31}$. For XOR convolution ignore `m` within `fft`.
Time: $\mathcal{O}(N \log N)$. For $N = 10^6$, conv ~0.13ms, conv_general ~320ms.

"template.hpp", "ModInt.h" 19ab20, 39 lines

```
// const int MOD = 998244353;
tcT> void fft(V<T>& A, bool invert = 0) { // NTT
  int n = sz(A); assert((T::mod-1)%n == 0); V<T> B(n);
  for(int b = n/2; b; b /= 2, swap(A,B)) { // w = n/b'th root
    T w = pow(T::rt(),(T::mod-1)/n*b), m = 1;
    for(int i = 0; i < n; i += b*2, m *= w) F0R(j,b) {
      T u = A[i+j], v = A[i+j+b]*m;
      B[i/2+j] = u+v; B[i/2+j+n/2] = u-v;
    }
  }
  if (invert) { reverse(1+all(A));
    T z = inv(T(n)); each(t,A) t *= z; }
} // for NTT-able moduli
tcT> V<T> conv(V<T> A, V<T> B) {
  if (!min(sz(A),sz(B))) return {};
  int s = sz(A)+sz(B)-1, n = 1; for (; n < s; n *= 2);
  A.rsz(n), fft(A); B.rsz(n), fft(B);
  F0R(i,n) A[i] *= B[i];
  fft(A,1); A.rsz(s); return A;
}
template<class M, class T> V<M> mulMod(const V<T>& x, const V<T
  ↪>& y) {
  auto con = [](const V<T>& v) {
    V<M> w(sz(v)); F0R(i,sz(v)) w[i] = (int)v[i];
    return w; };
  return conv(con(x), con(y));
} // arbitrary moduli
tcT> V<T> conv_general(const V<T>& A, const V<T>& B) {
  using m0 = mint<(119<<23)+1,62>; auto c0 = mulMod<m0>(A,B);
  using m1 = mint<(5<<25)+1,  62>; auto c1 = mulMod<m1>(A,B);
  using m2 = mint<(7<<26)+1,  62>; auto c2 = mulMod<m2>(A,B);
  int n = sz(c0); V<T> res(n); m1 r01 = inv(m1(m0::mod));
  m2 r02 = inv(m2(m0::mod)), r12 = inv(m2(m1::mod));
  F0R(i,n) { // a=remainder mod m0::mod, b fixes it mod m1::mod
    int a = c0[i].v, b = ((c1[i]-a)*r01).v,
      c = (((c2[i]-a)*r02-b)*r12).v;
    res[i] = (T(c)*m1::mod+b)*m0::mod+a; // c fixes m2::mod
  }
  return res;
}
```

PolyInvSimpler.h

Description: computes A^{-1} such that $AA^{-1} \equiv 1 \pmod{x^n}$. Newton's method: If you want $F(x) = 0$ and $F(Q_k) \equiv 0 \pmod{x^a}$ then $Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2a}}$ satisfies $F(Q_{k+1}) \equiv 0 \pmod{x^{2a}}$. Application: if $f(n), g(n)$ are the #s of forests and trees on n nodes then $\sum_{n=0}^{\infty} f(n)x^n = \exp\left(\sum_{n=1}^{\infty} \frac{g(n)}{n!}\right)$.
Usage: `vmi v{1,5,2,3,4}; ps(exp(2*log(v,9),9)); // squares v`
Time: $\mathcal{O}(N \log N)$. For $N = 5 \cdot 10^5$, inv~270ms, log ~350ms, exp~550ms

"FFT.h", "Poly.h" 6e5362, 30 lines

```
poly inv(poly A, int n) { // Q-(1/Q-A)/(-Q^{-2})
  poly B{inv(A[0])};
  for (int x = 2; x/2 < n; x *= 2)
    B = 2*B-RSZ(conv(RSZ(A,x),conv(B,B)),x);
  return RSZ(B,n);
}
poly sqrt(const poly& A, int n) {  // Q-(Q^2-A)/(2Q)
  assert(A[0].v == 1); poly B{1};
  for (int x = 2; x/2 < n; x *= 2)
    B = inv(T(2))*RSZ(B+conv(RSZ(A,x),inv(B,x)),x);
  return RSZ(B,n);
}
// return {quotient, remainder}
pair<poly,poly> quoRem(const poly& f, const poly& g) {
  if (sz(f) < sz(g)) return {{},f};
  poly q = conv(inv(rev(g),sz(f)-sz(g)+1),rev(f));
  q = rev(RSZ(q,sz(f)-sz(g)+1));
  poly r = RSZ(f-conv(q,g),sz(g)-1); return {q,r};
}
poly log(poly A, int n) { assert(A[0].v == 1); // (ln A)' = A'/
  ↪A
  A.rsz(n); return integ(RSZ(conv(dif(A),inv(A,n-1)),n-1)); }
poly exp(poly A, int n) { assert(A[0].v == 0);
  poly B{1}, IB{1}; // inverse of B
  for (int x = 1; x < n; x *= 2) {
    IB = 2*IB-RSZ(conv(B,conv(IB,IB)),x);
    poly Q = dif(RSZ(A,x)); Q += RSZ(conv(IB,dif(B)-conv(B,Q))
      ↪,2*x-1);
    B = B+RSZ(conv(B,RSZ(A,2*x)-integ(Q)),2*x);
  }
  return RSZ(B,n);
}
```

6.3 Misc

LinearRecurrence.h

Description: Berlekamp-Massey. Computes linear recurrence `C` of order N for sequence `s` of $2N$ terms. $C[0] = 1$ and for all $i \geq sz(C) - 1$, $\sum_{j=0}^{sz(C)-1} C[j]s[i-j] = 0$.
Usage: `LinRec L; L.init({0,1,1,2,3}); L.eval(5); L.eval(6); // 5, 8`
Time: `init` $\Rightarrow \mathcal{O}(N|C|)$, `eval` $\Rightarrow \mathcal{O}\left(|C|^2 \log p\right)$ or faster with FFT

"Poly.h" 39ea71, 29 lines

```
struct LinRec {
  poly s, C, rC;
  void BM() {
    int x = 0; T b = 1;
    poly B; B = C = {1}; // B is fail vector
    F0R(i,sz(s)) { // update C after adding a term of s
      ++x; int L = sz(C), M = i+3-L;
      T d = 0; F0R(j,L) d += C[j]*s[i-j]; // [D^i]C*s
      if (d.v == 0) continue; // [D^i]C*s=0
      poly _C = C; T coef = d*inv(b);
      C.rsz(max(L,M)); F0R(j,sz(B)) C[j+x] -= coef*B[j];
      if (L < M) B = _C, b = d, x = 0;
    }
  }
  void init(const poly& _s) {
    s = _s; BM();
    rC = C; reverse(all(rC));
    C.erase(begin(C)); each(t,C) t *= -1;
  } // now s[i]=sum_{j=0}^{sz(C)-1}C[j]*s[i-j-1]
  poly getPow(ll p) { // get x^p mod rC
    if (p == 0) return {1};
    poly r = getPow(p/2); r = (r*r)%rC;
    return p&1?(r*poly{0,1})%rC:r;
  }
  T dot(poly v) { // dot product with s
    T ans = 0; F0R(i,sz(v)) ans += v[i]*s[i];
    return ans; } // get p-th term of rec
  T eval(ll p) { assert(p >= 0); return dot(getPow(p)); }
};
```

Integrate.h

Description: Integration of a function over an interval using Simpson's rule, exact for polynomials of degree up to 3. The error should be proportional to dif^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.
Usage: `quad([](db x) { return x*x+3*x+1; }, 2, 3) // 14.833...`

3eba8b, 6 lines

```
template<class F> db quad(F f, db a, db b) {
  const int n = 1000;
  db dif = (b-a)/2/n, tot = f(a)+f(b);
  FOR(i,1,2*n) tot += f(a+i*dif)*(i&1?4:2);
  return tot*dif/3;
}
```

IntegrateAdaptive.h

Description: Unused. Fast integration using adaptive Simpson's rule, exact for polynomials of degree up to 5.
Usage: `db z, y;`

```
db h(db x) { return x*x + y*y + z*z <= 1; }
db g(db y) { ::y = y; return quad(h, -1, 1); }
db f(db z) { ::z = z; return quad(g, -1, 1); }
db sphereVol = quad(f,-1,1), pi = sphereVol*3/4;
```

3b316e, 10 lines

```
template<class F> db simpson(F f, db a, db b) {
  db c = (a+b)/2; return (f(a)+4*f(c)+f(b))*(b-a)/6; }
template<class F> db rec(F& f, db a, db b, db eps, db S) {
  db c = (a+b)/2;
  db S1 = simpson(f,a,c), S2 = simpson(f,c,b), T = S1+S2;
  if (abs(T-S)<=15*eps || b-a<1e-10) return T+(T-S)/15;
  return rec(f,a,c,eps/2,S1)+rec(f,c,b,eps/2,S2);
}
template<class F> db quad(F f, db a, db b, db eps = 1e-8) {
  return rec(f,a,b,eps,simpson(f,a,b)); }
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns `-inf` if there is no solution, `inf` if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage:

```
vvd A{{1,-1}, {-1,1}, {-1,-2}};
vd b{1,1,-4}, c{-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
```

Time: $\mathcal{O}(NM \cdot \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^N\right)$ in the general case.

"template.hpp" c99f9c, 67 lines

```
using T = db; // double probably suffices
using vd = V<T>; using vvd = V<vd>;
const T eps = 1e-8, inf = 1/.0;

#define ltj(X) if (s==-1 || mp(X[j],N[j])<mp(X[s],N[s])) s=j
struct LPSolver {
  int m, n; // # m = contraints, # n = variables
  vi N, B; // N[j] = non-basic variable (j-th column), = 0
  vvd D; // B[j] = basic variable (j-th row)
  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
    F0R(i,m) F0R(j,n) D[i][j] = A[i][j];
    F0R(i,m) B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
```

```
    // B[i]: basic variable for each constraint
    // D[i][n]: artificial variable for testing feasibility
    F0R(j,n) N[j] = j, D[m][j] = -c[j];
    // D[m] stores negation of objective,
    // which we want to minimize
    N[n] = -1; D[m+1][n] = 1; // to find initial feasible
  } // solution, minimize artificial variable
  void pivot(int r, int s) { // swap B[r] (row)
    T inv = 1/D[r][s]; // with N[r] (column)
    F0R(i,m+2) if (i != r && abs(D[i][s]) > eps) {
      T binv = D[i][s]*inv;
      F0R(j,n+2) if (j != s) D[i][j] -= D[r][j]*binv;
      D[i][s] = -binv;
    }
    D[r][s] = 1; F0R(j,n+2) D[r][j] *= inv; // scale r-th row
    swap(B[r],N[s]);
  }
  bool simplex(int phase) {
    int x = m+phase-1;
    while (1) { // if phase=1, ignore artificial variable
      int s = -1; F0R(j,n+1) if (N[j] != -phase) ltj(D[x]);
      // find most negative col for nonbasic (NB) variable
      if (D[x][s] >= -eps) return 1;
      // can't get better sol by increasing NB variable
      int r = -1;
      F0R(i,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
              < mp(D[r][n+1] / D[r][s], B[r])) r = i;
        // find smallest positive ratio
      } // -> max increase in NB variable
      if (r == -1) return 0; // objective is unbounded
      pivot(r,s);
    }
  }
  T solve(vd& x) { // 1. check if x=0 feasible
    int r = 0; FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { // if not, find feasible start
      pivot(r,n); // make artificial variable basic
      assert(simplex(2)); // I think this will always be true??
      if (D[m+1][n+1] < -eps) return -inf;
      // D[m+1][n+1] is max possible value of the negation of
      // artificial variable, optimal value should be zero
      // if exists feasible solution
      F0R(i,m) if (B[i] == -1) { // artificial var basic
        int s = 0; FOR(j,1,n+1) ltj(D[i]); // -> nonbasic
        pivot(i,s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    F0R(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

Graphs (7)

Erdos-Gallai: $d_1 \geq \cdots \geq d_n$ can be degree sequence of simple graph on n vertices iff their sum is even and $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k), \forall 1 \leq k \leq n$.

7.1 Basics

DSU.h

Description: Disjoint Set Union with path compression and union by size. Add edges and test connectivity. Use for Kruskal's or Boruvka's minimum spanning tree.

Time: $\mathcal{O}(\alpha(N))$

"template.hpp" e42a83, 11 lines

```
struct DSU {
  vi e; void init(int N) { e = vi(N,-1); }
  int get(int x) { return e[x] < 0 ? x : e[x] = get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union by size
    x = get(x), y = get(y); if (x == y) return 0;
    if (e[x] > e[y]) swap(x,y);
    e[x] += e[y]; e[y] = x; return 1;
  }
};
```

DSUrb.h

Description: Disjoint Set Union with Rollback

"template.hpp" 7d0297, 18 lines

```
struct DSUrb {
  vi e; void init(int n) { e = vi(n,-1); }
  int get(int x) { return e[x] < 0 ? x : get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  V<AR<int,4>> mod;
  bool unite(int x, int y) { // union-by-rank
    x = get(x), y = get(y);
    if (x == y) { mod.pb({-1,-1,-1,-1}); return 0; }
    if (e[x] > e[y]) swap(x,y);
    mod.pb({x,y,e[x],e[y]});
    e[x] += e[y]; e[y] = x; return 1;
  }
  void rollback() {
    auto a = mod.bk; mod.pop_back();
    if (a[0] != -1) e[a[0]] = a[2], e[a[1]] = a[3];
  }
};
```

NegativeCycle.h

Description: use Bellman-Ford (make sure no underflow)

688ec8, 11 lines

```
vi negCyc(int N, V<pair<pi,int>> ed) {
  vl d(N); vi p(N); int x = -1;
  rep(N) {
    x = -1; each(t,ed) if (ckmin(d[t.f.s],d[t.f.f]+t.s))
      p[t.f.s] = t.f.f, x = t.f.s;
    if (x == -1) return {};
  }
  rep(N) x = p[x]; // enter cycle
  vi cyc{x}; while (p[cyc.bk] != x) cyc.pb(p[cyc.bk]);
  reverse(all(cyc)); return cyc;
}
```

TopoSort.h

Description: sorts vertices such that if there exists an edge x->y, then x goes before y

"template.hpp" 55695d, 15 lines

```
struct TopoSort {
  int N; vi in, res;
  V<vi> adj;
  void init(int _N) { N = _N; in.rsz(N); adj.rsz(N); }
  void ae(int x, int y) { adj[x].pb(y), ++in[y]; }
  bool sort() {
    queue<int> todo;
    F0R(i,N) if (!in[i]) todo.push(i);
    while (sz(todo)) {
      int x = todo.ft; todo.pop(); res.pb(x);
      each(i,adj[x]) if (!(--in[i])) todo.push(i);
    }
    return sz(res) == N;
  }
};
```

ArtPointsBridges.h

Description: Finds bridges and articulation points of an undirected graph. Be careful with double counting.

Time: $\mathcal{O}(N+M)$

"template.hpp" ca2bd3, 21 lines

```
void artPointsBridges(V<vi> const& g) {
    int n = sz(g), cnt = 0, root = 0;
    vi num(n), low(n);
    auto dfs = [&](auto&& self, int u, int p)->void{
        num[u] = low[u] = cnt++;
        each(v, g[u]) if (!num[v]) {
            if (p == -1) ++root;
            self(self, v, u);
            if (low[v] >= num[u] and p != -1)
                // u is articulation point
            if (low[v] > num[v])
                // u<->v is a bridge
            ckmin(low[u],low[v]);
        } else ckmin(low[u],num[v]);
    };
    F0R(i,n) if (!num[i]) {
        root = 0; dfs(dfs, i, -1);
        if (root > 1) {}
            // i is an articulation point
    }
}
```

7.2 Trees

LCAjump.h

Description: Calculates lowest common ancestor in tree with verts $0 \ldots N-1$ and root R using binary jumping.

Memory: $\mathcal{O}(N \log N)$

Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ query

"template.hpp" 6b0ee9, 28 lines

```
struct LCA {
  int N; V<vi> par, adj; vi depth;
  void init(int _N) {  N = _N;
    int d = 1; while ((1<<d) < N) ++d;
    par.assign(d,vi(N)); adj.rsz(N); depth.rsz(N);
  }
  void ae(int x, int y) { adj[x].pb(y), adj[y].pb(x); }
  void gen(int R = 0) { par[0][R] = R; dfs(R); }
  void dfs(int x = 0) {
    FOR(i,1,sz(par)) par[i][x] = par[i-1][par[i-1][x]];
    each(y,adj[x]) if (y != par[0][x])
      depth[y] = depth[par[0][y]=x]+1, dfs(y);
  }
  int jmp(int x, int d) {
    F0R(i,sz(par)) if ((d>>i)&1) x = par[i][x];
    return x; }
  int lca(int x, int y) {
    if (depth[x] < depth[y]) swap(x,y);
    x = jmp(x,depth[x]-depth[y]); if (x == y) return x;
    R0F(i,sz(par)) {
      int X = par[i][x], Y = par[i][y];
      if (X != Y) x = X, y = Y;
    }
    return par[0][x];
  }
  int dist(int x, int y) { // # edges on path
    return depth[x]+depth[y]-2*depth[lca(x,y)]; }
};
```

LCArmq.h

Description: Euler Tour LCA. Compress takes a subset S of nodes and computes the minimal subtree that contains all the nodes pairwise LCAs and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.

Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(1)$ LCA, $\mathcal{O}(|S| \log |S|)$ compress

"template.hpp", "RMQ.h" e5a035, 28 lines

```
struct LCA {
  int N; V<vi> adj;
  vi depth, pos, par, rev; // rev is for compress
  vpi tmp; RMQ<pi> r;
  void init(int _N) { N = _N; adj.rsz(N);
    depth = pos = par = rev = vi(N); }
  void ae(int x, int y) { adj[x].pb(y), adj[y].pb(x); }
  void dfs(int x) {
    pos[x] = sz(tmp); tmp.eb(depth[x],x);
    each(y,adj[x]) if (y != par[x]) {
      depth[y] = depth[par[y]=x]+1, dfs(y);
      tmp.eb(depth[x],x); }
  }
  void gen(int R = 0) { par[R] = R; dfs(R); r.init(tmp); }
  int lca(int u, int v){
    u = pos[u], v = pos[v]; if (u > v) swap(u,v);
    return r.query(u,v).s; }
  int dist(int u, int v) {
    return depth[u]+depth[v]-2*depth[lca(u,v)]; }
  vpi compress(vi S) {
    auto cmp = [&](int a, int b) { return pos[a] < pos[b]; };
    sort(all(S),cmp); R0F(i,sz(S)-1) S.pb(lca(S[i],S[i+1]));
    sort(all(S),cmp); S.erase(unique(all(S)),end(S));
    vpi ret{{0,S[0]}}; F0R(i,sz(S)) rev[S[i]] = i;
    FOR(i,1,sz(S)) ret.eb(rev[lca(S[i-1],S[i])],S[i]);
    return ret;
  }
};
```

HLD.h

Description: Heavy-Light Decomposition, add val to verts and query sum in path/subtree. @todo change so that it works for new LazySegTree!!!!

Time: any tree path is split into $\mathcal{O}(\log N)$ parts

"LazySegTree.h" 15bbde, 57 lines

```
template<typename T, typename E, T (*f)(T, T), T (*g)(T, E), E
  ↪(*h)(E, E), T(*ti)(), E (*ei)(), bool VALS_IN_EDGES>
struct HLD {
    int n, t, log, s;
    V<vi> adj;
    vi par, root, depth, sz, pos, rpos;
    LazySegTree<T, E, f, g, h, ti, ei> tree;
    HLD(int _N) { n = _N, s = 1, log = 0;
        while (s < n) s <<= 1, ++log;
        adj.rsz(s), par.rsz(s), root.rsz(s), depth.rsz(s), sz.
          ↪rsz(s), pos.rsz(s);
        V<T> v(n,ti());
        tree = LazySegTree<T,E,f,g,h,ti,ei>(v);
    }
    void ae(int x, int y) { adj[x].pb(y), adj[y].pb(x); }
    void dfsSz(int x) {
        sz[x] = 1;
        each(y,adj[x]) {
            par[y] = x, depth[y] = depth[x] + 1;
            adj[y].erase(find(all(adj[y]),x)); // remove parent
              ↪ from adj list
            dfsSz(y); sz[x] += sz[y];
            if (sz[y] > sz[adj[x][0]]) swap(y,adj[x][0]);
        }
    }
    void dfsHld(int x) {
        pos[x] = t++; rpos.pb(x);
        each(y,adj[x]) {
            root[y] = (y == adj[x][0] ? root[x] : y);
            dfsHld(y); }
    }
    void init(int R = 0) {
        par[R] = depth[R] = t = 0; dfsSz(R);
        root[R] = R; dfsHld(R);
    }
    int lca(int x, int y) {
        for(; root[x] != root[y]; y = par[root[y]])
            if (depth[root[x]] > depth[root[y]]) swap(x,y);
        return depth[x] < depth[y] ? x : y;
    }
    void set(int x, T const& v) {
        tree.set_val(pos[x],v);
    }
    template<class BinaryOp>
    void processPath(int x, int y, BinaryOp op) {
        for (; root[x] != root[y]; y = par[root[y]]) {
            if (depth[root[x]] > depth[root[y]]) swap(x,y);
            op(pos[root[y]], pos[y]); }
        if (depth[x] > depth[y]) swap(x,y);
        op(pos[x]+VALS_IN_EDGES,pos[y]);
    }
    void modifyPath(int x, int y, E const& v) {
        processPath(x,y,[this,&v](int l, int r){tree.update(l,r
          ↪+1,v);}); }
    T queryPath(int x, int y) {
        T res = ti();
        processPath(x,y,[this,&res](int l,int r){res=f(res,tree
          ↪.query(l,r+1));});
        return res; }
    void modifySubtree(int x, E const& v) {
        tree.update(pos[x]+VALS_IN_EDGES,pos[x]+sz[x],v); }
};
```

Centroid.h

Description: The centroid of a tree of size N is a vertex such that after removing it, all resulting subtrees have size at most $\frac{N}{2}$. Supports updates in the form "add 1 to all verts v such that $dist(x, v) \leq y$."

Memory: $\mathcal{O}(N \log N)$

Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ update and query

907e21, 54 lines

```
void ad(vi& a, int b) { ckmin(b,sz(a)-1); if (b>=0) a[b]++; }
void prop(vi& a) { R0F(i,sz(a)-1) a[i] += a[i+1]; }
template<int SZ> struct Centroid {
  vi adj[SZ]; void ae(int a,int b){adj[a].pb(b),adj[b].pb(a);}
  bool done[SZ]; // processed as centroid yet
  int N,sub[SZ],cen[SZ],lev[SZ]; // subtree size, centroid anc
  int dist[32-__builtin_clz(SZ)][SZ]; // dists to all ancs
  vi stor[SZ], STOR[SZ];
  void dfs(int x, int p) { sub[x] = 1;
    each(y,adj[x]) if (!done[y] && y != p)
      dfs(y,x), sub[x] += sub[y];
  }
  int centroid(int x) {
    dfs(x,-1);
    for (int sz = sub[x];;) {
      pi mx = {0,0};
      each(y,adj[x]) if (!done[y] && sub[y] < sub[x])
        ckmax(mx,{sub[y],y});
      if (mx.f*2 <= sz) return x;
      x = mx.s;
    }
  }
  void genDist(int x, int p, int lev) {
    dist[lev][x] = dist[lev][p]+1;
    each(y,adj[x]) if (!done[y] && y != p) genDist(y,x,lev); }
  void gen(int CEN, int _x) { // CEN = centroid above x
    int x = centroid(_x); done[x] = 1; cen[x] = CEN;
    sub[x] = sub[_x]; lev[x] = (CEN == -1 ? 0 : lev[CEN]+1);
    dist[lev[x]][x] = 0;
    stor[x].rsz(sub[x]),STOR[x].rsz(sub[x]+1);
    each(y,adj[x]) if (!done[y]) genDist(y,x,lev[x]);
    each(y,adj[x]) if (!done[y]) gen(x,y);
  }
  void init(int _N) { N = _N; FOR(i,1,N+1) done[i] = 0;
    gen(-1,1); } // start at vert 1
  void upd(int x, int y) {
    int cur = x, pre = -1;
    R0F(i,lev[x]+1) {
      ad(stor[cur],y-dist[i][x]);
      if (pre != -1) ad(STOR[pre],y-dist[i][x]);
      if (i > 0) pre = cur, cur = cen[cur];
    }
  } // call propAll() after all updates
  void propAll() { FOR(i,1,N+1) prop(stor[i]), prop(STOR[i]); }
  int query(int x) { // get value at vertex x
    int cur = x, pre = -1, ans = 0;
    R0F(i,lev[x]+1) { // if pre != -1, subtract those from
      ans += stor[cur][dist[i][x]]; // same subtree
      if (pre != -1) ans -= STOR[pre][dist[i][x]];
      if (i > 0) pre = cur, cur = cen[cur];
    }
    return ans;
  }
};
```

7.2.1 SqrtDecompton

HLD generally suffices. If not, here are some common strategies:

- Rebuild the tree after every $\sqrt{N}$ queries.
- Consider vertices with $>$ or $<\sqrt{N}$ degree separately.
- For subtree updates, note that there are $O(\sqrt{N})$ distinct sizes among child subtrees of any node.

Block Tree: Use a DFS to split edges into contiguous groups of size $\sqrt{N}$ to $2\sqrt{N}$.

Mo's Algorithm for Tree Paths: Maintain an array of vertices where each one appears twice, once when a DFS enters the vertex (st) and one when the DFS exists (en). For a tree path $u \leftrightarrow v$ such that st[u]<st[v],

- If u is an ancestor of v, query [st[u],st[v]].
- Otherwise, query [en[u],st[v]] and consider $LCA(u, v)$ separately.

Solutions with worse complexities can be faster if you optimize the operations that are performed most frequently. Use arrays instead of vectors whenever possible. Iterating over an array in order is faster than iterating through the same array in some other order (ex. one given by a random permutation) or DFSing on a tree of the same size. Also, the difference between $\sqrt{N}$ and the optimal block (or buffer) size can be quite large. Try up to 5x smaller or larger (at least).

7.3 DFS Algorithms

EulerPath.h

Description: Eulerian path starting at `src` if it exists, visits all edges exactly once. Works for both directed and undirected. Returns vector of {vertex,label of edge to vertex}. Second element of first pair is always -1.
Time: $\mathcal{O}(N+M)$

"template.hpp" 9c222d, 23 lines

```
template<bool directed> struct Euler {
  int N; V<vpi> adj; V<vpi::iterator> its; vb used;
  void init(int _N) { N = _N; adj.rsz(N); }
  void ae(int a, int b) {
    int M = sz(used); used.pb(0);
    adj[a].eb(b,M); if (!directed) adj[b].eb(a,M); }
  vpi solve(int src = 0) {
    its.rsz(N); F0R(i,N) its[i] = begin(adj[i]);
    vpi ans, s{{src,-1}}; // {{vert,prev vert},edge label}
    int lst = -1; // ans generated in reverse order
    while (sz(s)) {
      int x = s.bk.f; auto& it=its[x], en=end(adj[x]);
      while (it != en && used[it->s]) ++it;
      if (it == en) { // no more edges out of vertex
        if (lst != -1 && lst != x) return {};
        // not a path, no tour exists
        ans.pb(s.bk); s.pop_back(); if (sz(s)) lst=s.bk.f;
      } else s.pb(*it), used[it->s] = 1;
    } // must use all edges
    if (sz(ans) != sz(used)+1) return {};
    reverse(all(ans)); return ans;
  }
};
```

SCCT.h

Description: Tarjan's, DFS once to generate strongly connected components in topological order. a, b in same component if both $a \to b$ and $b \to a$ exist. Uses less memory than Kosaraju b/c doesn't store reverse edges.
Time: $\mathcal{O}(N+M)$

a36e0c, 22 lines

```
struct SCC {
  int N, ti = 0; V<vi> adj;
  vi disc, comp, stk, comps;
  void init(int _N){ N = _N, adj.rsz(N);
    disc.rsz(N), comp.rsz(N,-1);}
  void ae(int x, int y) { adj[x].pb(y); }
  int dfs(int x) {
    int low = disc[x] = ++ti; stk.pb(x);
    each(y,adj[x]) if (comp[y] == -1) // comp[y] == -1,
      ckmin(low,disc[y]?:dfs(y));  // disc[y] != 0 -> in stack
    if (low == disc[x]) { // make new SCC
      // pop off stack until you find x
      comps.pb(x); for (int y = -1; y != x;)
        comp[y = stk.bk] = x, stk.pop_back();
    }
    return low;
  }
  void gen() {
    F0R(i,N) if (!disc[i]) dfs(i);
    reverse(all(comps));
  }
};
```

TwoSAT.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||!b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (`~x`).

Usage: `TwoSat ts;`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setVal(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(N); // Returns true iff it is solvable`
`ts.ans[0..N-1] holds the assigned values to the vars`

"SCC.h" ff0f3d, 31 lines

```
struct TwoSAT {
  int N = 0; vpi edges;
  void init(int _N) { N = _N; }
  int addVar() { return N++; }
  void either(int x, int y) {
    x = max(2*x,-1-2*x), y = max(2*y,-1-2*y);
    edges.eb(x,y); }
  void implies(int x, int y) { either(~x,y); }
  void must(int x) { either(x,x); }
  void atMostOne(const vi& li) {
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = addVar();
      either(cur,~li[i]); either(cur,next);
      either(~li[i],next); cur = ~next;
    }
    either(cur,~li[1]);
  }
  vb solve() {
    SCC S; S.init(2*N);
    each(e,edges) S.ae(e.f^1,e.s), S.ae(e.s^1,e.f);
    S.gen(); reverse(all(S.comps)); // reverse topo order
    for (int i = 0; i < 2*N; i += 2)
      if (S.comp[i] == S.comp[i^1]) return {};
    vi tmp(2*N); each(i,S.comps) if (!tmp[i])
      tmp[i] = 1, tmp[S.comp[i^1]] = -1;
    vb ans(N); F0R(i,N) ans[i] = tmp[S.comp[2*i]] == 1;
    return ans;
  }
};
```

BCC.h

Description: Biconnected components of edges. Removing any vertex in BCC doesn't disconnect it. To get block-cut tree, create a bipartite graph with the original vertices on the left and a vertex for each BCC on the right. Draw edge $u \leftrightarrow v$ if u is contained within the BCC for v. Self-loops are not included in any BCC while BCCS of size 1 represent bridges.
Time: $\mathcal{O}(N+M)$

"template.hpp" 0625a6, 35 lines

```
struct BCC {
  V<vpi> adj; vpi ed;
  V<vi> edgeSets, vertSets; // edges for each bcc
  int N, ti = 0; vi disc, stk;
  void init(int _N) { N = _N; disc.rsz(N), adj.rsz(N); }
  void ae(int x, int y) {
    adj[x].eb(y,sz(ed)), adj[y].eb(x,sz(ed)), ed.eb(x,y); }
  int dfs(int x, int p = -1) { // return lowest disc
    int low = disc[x] = ++ti;
    each(e,adj[x]) if (e.s != p) {
      if (!disc[e.f]) {
        stk.pb(e.s); // disc[x] < LOW -> bridge
        int LOW = dfs(e.f,e.s); ckmin(low,LOW);
        if (disc[x] <= LOW) { // get edges in bcc
          edgeSets.eb(); vi& tmp = edgeSets.bk; // new bcc
          for (int y = -1; y != e.s; )
            tmp.pb(y = stk.bk), stk.pop_back();
        }
      } else if (disc[e.f] < disc[x]) // back-edge
        ckmin(low,disc[e.f]), stk.pb(e.s);
    }
    return low;
  }
  void gen() {
    F0R(i,N) if (!disc[i]) dfs(i);
    vb in(N);
    each(c,edgeSets) { // edges contained within each BCC
      vertSets.eb(); // so you can easily create block cut tree
      auto ad = [&](int x) {
        if (!in[x]) in[x] = 1, vertSets.bk.pb(x); };
      each(e,c) ad(ed[e].f), ad(ed[e].s);
      each(e,c) in[ed[e].f] = in[ed[e].s] = 0;
    }
  }
};
```

MaximalCliques.h

Description: Used only once. Finds all maximal cliques.
Time: $\mathcal{O}\left(3^{N/3}\right)$

f5cd93, 16 lines

```
using B = bitset<128>; B adj[128];
int N;
// possibly in clique, not in clique, in clique
void cliques(B P = ~B(), B X={}, B R={}) {
  if (!P.any()) {
    if (!X.any()) // do smth with R
    return;
  }
  int q = (P|X)._Find_first();
  // clique must contain q or non-neighbor of q
  B cands = P&~adj[q];
  F0R(i,N) if (cands[i]) {
    R[i] = 1; cliques(P&adj[i],X&adj[i],R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

7.4 Flows

Konig's Theorem: In a bipartite graph, max matching = min vertex cover.

Dilworth's Theorem: For any partially ordered set, the sizes of the max antichain and of the min chain decomposition are equal. Equivalent to Konig's theorem on the bipartite graph (U, V, E) where $U = V = S$ and (u, v) is an edge when $u < v$. Those vertices outside the min vertex cover in both U and V form a max antichain.

Dinic.h

Description: Fast flow. After computing flow, edges $\{u, v\}$ such that $lev[u] \neq -1$, $lev[v] = -1$ are part of min cut. Use `reset` and `rcap` for Gomory-Hu.
Time: $\mathcal{O}\left(N^2M\right)$ flow, $\mathcal{O}\left(M\sqrt{N}\right)$ bipartite matching

"template.hpp" b7b370, 38 lines

```
struct Dinic {
  using F = ll; // flow type
  struct Edge { int to; F flo, cap; };
  int N; V<Edge> eds; V<vi> adj;
  void init(int _N) { N = _N; adj.rsz(N), cur.rsz(N); }
  void ae(int u, int v, F cap, F rcap = 0) { assert(min(cap,
    ↪rcap) >= 0);
    adj[u].pb(sz(eds)); eds.pb({v,0,cap});
    adj[v].pb(sz(eds)); eds.pb({u,0,rcap});
  }
  vi lev; V<vi::iterator> cur;
```

```
  bool bfs(int s, int t) { // level = shortest distance from
    ↪source
    lev = vi(N,-1); F0R(i,N) cur[i] = begin(adj[i]);
    queue<int> q({s}); lev[s] = 0;
    while (sz(q)) { int u = q.ft; q.pop();
      each(e,adj[u]) { const Edge& E = eds[e];
        int v = E.to; if (lev[v] < 0 && E.flo < E.cap)
          q.push(v), lev[v] = lev[u]+1;
      }
    }
    return lev[t] >= 0;
  }
  F dfs(int v, int t, F flo) {
    if (v == t) return flo;
    for (; cur[v] != end(adj[v]); cur[v]++) {
      Edge& E = eds[*cur[v]];
      if (lev[E.to]!=lev[v]+1||E.flo==E.cap) continue;
      F df = dfs(E.to,t,min(flo,E.cap-E.flo));
      if (df) { E.flo += df; eds[*cur[v]^1].flo -= df;
        return df; } // saturated >=1 one edge
    }
    return 0;
  }
  F maxFlow(int s, int t) {
    F tot = 0; while (bfs(s,t)) while (F df =
      dfs(s,t,numeric_limits<F>::max())) tot += df;
    return tot;
  }
};
```

GomoryHu.h

Description: Returns edges of Gomory-Hu tree (second element is weight). Max flow between pair of vertices of undirected graph is given by min edge weight along tree path. Uses the fact that for any i, j, k, $\lambda_{ik} \geq \min(\lambda_{ij}, \lambda_{jk})$, where λ_{ij} denotes the flow between i and j.

Time: $N - 1$ calls to Dinic

"Dinic.h" 0d712e, 16 lines

```
template<class F> V<pair<pi,F>> gomoryHu(int N,
    const V<pair<pi,F>>& ed) {
  vi par(N); Dinic<F> D; D.init(N);
  vpi ed_locs; each(t,ed)ed_locs.pb(D.ae(t.f.f,t.f.s,t.s,t.s));
  V<pair<pi,F>> ans;
  FOR(i,1,N) {
    each(p,ed_locs) { // reset capacities
      auto& e = D.adj.at(p.f).at(p.s);
      auto& e_rev = D.adj.at(e.to).at(e.rev);
      e.cap = e_rev.cap = (e.cap+e_rev.cap)/2;
    }
    ans.pb({{i,par[i]},D.maxFlow(i,par[i])});
    FOR(j,i+1,N) if (par[j] == par[i] && D.lev[j]) par[j] = i;
  }
  return ans;
}
```

MCMF.h

Description: Minimum-cost maximum flow, assumes no negative cycles. It is possible to choose negative edge costs such that the first run of Dijkstra is slow, but this hasn't been an issue in the past. Edge weights ≥ 0 for every subsequent run. To get flow through original edges, assign ID's during ae.

Time: Ignoring first run of Dijkstra, $\mathcal{O}(FM \log M)$ if caps are integers and F is max flow.

"template.hpp" 072c1b, 40 lines

```
struct MCMF {
  using F = ll; using C = ll; // flow type, cost type
  struct Edge { int to; F flo, cap; C cost; };
  int N; V<C> p, dist; vi pre; V<Edge> eds; V<vi> adj;
  void init(int _N) { N = _N;
    p.rsz(N), dist.rsz(N), pre.rsz(N), adj.rsz(N); }
  void ae(int u, int v, F cap, C cost) { assert(cap >= 0);
    adj[u].pb(sz(eds)); eds.pb({v,0,cap,cost});
    adj[v].pb(sz(eds)); eds.pb({u,0,0,-cost});
  } // use asserts, don't try smth dumb
  bool path(int s, int t) { // find lowest cost path to send
    ↪flow through
    const C inf = numeric_limits<C>::max(); F0R(i,N) dist[i] =
      ↪inf;
    using T = pair<C,int>; priority_queue<T,vector<T>,greater<T
      ↪>> todo;
    todo.push({dist[s] = 0,s});
    while (sz(todo)) { // Dijkstra
      T x = todo.top(); todo.pop(); if (x.f > dist[x.s])
        ↪continue;
      each(e,adj[x.s]) { const Edge& E = eds[e]; // all weights
        ↪ should be non-negative
        if (E.flo < E.cap && ckmin(dist[E.to],x.f+E.cost+p[x.s
          ↪]-p[E.to]))
          pre[E.to] = e, todo.push({dist[E.to],E.to});
      }
    } // if costs are doubles, add some EPS so you
    // don't traverse ~0-weight cycle repeatedly
    return dist[t] != inf; // return flow
  }
  pair<F,C> calc(int s, int t) { assert(s != t);
    F0R(_,N) F0R(e,sz(eds)) { const Edge& E = eds[e]; //
      ↪Bellman-Ford
      if (E.cap) ckmin(p[E.to],p[eds[e^1].to]+E.cost); }
    F totFlow = 0; C totCost = 0;
    while (path(s,t)) { // p -> potentials for Dijkstra
      F0R(i,N) p[i] += dist[i]; // don't matter for unreachable
        ↪ nodes
      F df = numeric_limits<F>::max();
      for (int x = t; x != s; x = eds[pre[x]^1].to) {
        const Edge& E = eds[pre[x]]; ckmin(df,E.cap-E.flo); }
      totFlow += df; totCost += (p[t]-p[s])*df;
      for (int x = t; x != s; x = eds[pre[x]^1].to)
        eds[pre[x]].flo += df, eds[pre[x]^1].flo -= df;
    } // get max flow you can send along path
    return {totFlow,totCost};
  }
};
```

7.5 Matching

Hungarian.h

Description: Given J jobs and W workers (J <= W), computes the minimum cost to assign each prefix of jobs to distinct workers.
@tparam T a type large enough to represent integers on the order of J * max(—C—)
@param C a matrix of dimensions JxW such that C[j][w] = cost to assign j-th job to w-th worker (possibly negative)
@return a vector of length J, with the j-th entry equaling the minimum cost to assign the first (j+1) jobs to distinct workers

Time: $\mathcal{O}(J^2W)$

"template.hpp" 3aece9, 36 lines

```
template <class T> vector<T> hungarian(const vector<vector<T>>
  ↪&C) {
  const int J = sz(C), W = sz(C[0]);
  assert(J <= W);
  vector<int> job(W + 1, -1);
  vector<T> ys(J), yt(W + 1);
  vector<T> answers;
  const T inf = numeric_limits<T>::max();
  for (int j_cur = 0; j_cur < J; ++j_cur) {
    int w_cur = W;
    job[w_cur] = j_cur;
    vector<T> min_to(W + 1, inf);
    vector<int> prv(W + 1, -1);
    vector<bool> in_Z(W + 1);
    while (job[w_cur] != -1) {
      in_Z[w_cur] = true;
      const int j = job[w_cur];
      T delta = inf;
      int w_next;
      for (int w = 0; w < W; ++w) {
        if (!in_Z[w]) {
          if (ckmin(min_to[w], C[j][w] - ys[j] - yt[w]))
            prv[w] = w_cur;
          if (ckmin(delta, min_to[w])) w_next = w;
        }
      }
      for (int w = 0; w <= W; ++w) {
        if (in_Z[w]) ys[job[w]] += delta, yt[w] -= delta;
        else min_to[w] -= delta;
      }
      w_cur = w_next;
    }
    for (int w; w_cur != -1; w_cur = w) job[w_cur] = job[w =
      ↪prv[w_cur]];
    answers.push_back(-yt[W]);
  }
  return answers;
}
```

GeneralMatchBlossom.h

Description: Variant on Gabow's Impl of Edmond's Blossom Algorithm. General unweighted max matching with 1-based indexing. If `white[v] = 0` after `solve()` returns, v is part of every max matching.

Time: $\mathcal{O}(NM)$, faster in practice

fd5cc7, 50 lines

```
struct MaxMatching {
  int N; V<vi> adj;
  V<int> mate, first; vb white; vpi label;
  void init(int _N) { N = _N; adj = V<vi>(N+1);
    mate = first = vi(N+1); label = vpi(N+1); white = vb(N+1);
      ↪}
  void ae(int u, int v) { adj.at(u).pb(v), adj.at(v).pb(u); }
  int group(int x) { if (white[first[x]]) first[x] = group(
    ↪first[x]);
    return first[x]; }
  void match(int p, int b) {
    swap(b,mate[p]); if (mate[b] != p) return;
    if (!label[p].s) mate[b] = label[p].f, match(label[p].f,b);
      ↪ // vertex label
    else match(label[p].f,label[p].s), match(label[p].s,label[p
      ↪].f); // edge label
  }
  bool augment(int st) { assert(st);
    white[st] = 1; first[st] = 0; label[st] = {0,0};
    queue<int> q; q.push(st);
    while (!q.empty()) {
      int a = q.ft; q.pop(); // outer vertex
      each(b,adj[a]) { assert(b);
        if (white[b]) { // two outer vertices, form blossom
          int x = group(a), y = group(b), lca = 0;
          while (x||y) {
            if (y) swap(x,y);
            if (label[x] == pi{a,b}) { lca = x; break; }
            label[x] = {a,b}; x = group(label[mate[x]].first);
          }
          for (int v: {group(a),group(b)}) while (v != lca) {
            assert(!white[v]); // make everything along path
              ↪white
            q.push(v); white[v] = true; first[v] = lca;
            v = group(label[mate[v]].first);
          }
        } else if (!mate[b]) { // found augmenting path
          mate[b] = a; match(a,b); white = vb(N+1); // reset
```

```
        return true;
      } else if (!white[mate[b]]) {
        white[mate[b]] = true; first[mate[b]] = b;
        label[b] = {0,0}; label[mate[b]] = pi{a,0};
        q.push(mate[b]);
      }
    }
  }
  return false;
 }
 int solve() {
  int ans = 0;
  FOR(st,1,N+1) if (!mate[st]) ans += augment(st);
  FOR(st,1,N+1) if (!mate[st] && !white[st]) assert(!augment(
    ↪st));
  return ans;
 }
};
```

GeneralWeightedMatch.h

Description: General max weight max matching with 1-based indexing. Edge weights must be positive, combo of UnweightedMatch and Hungarian.
Time: $\mathcal{O}\left(N^3\right)$?

120873, 145 lines

```
template<int SZ> struct WeightedMatch {
  struct edge { int u,v,w; }; edge g[SZ*2][SZ*2];
  void ae(int u, int v, int w) { g[u][v].w = g[v][u].w = w; }
  int N,NX,lab[SZ*2],match[SZ*2],slack[SZ*2],st[SZ*2];
  int par[SZ*2],floFrom[SZ*2][SZ],S[SZ*2],aux[SZ*2];
  vi flo[SZ*2]; queue<int> q;
  void init(int _N) { N = _N; // init all edges
    FOR(u,1,N+1) FOR(v,1,N+1) g[u][v] = {u,v,0}; }
  int eDelta(edge e) { // >= 0 at all times
    return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2; }
  void updSlack(int u, int x) { // smallest edge -> blossom x
    if (!slack[x] || eDelta(g[u][x]) < eDelta(g[slack[x]][x]))
      slack[x] = u; }
  void setSlack(int x) {
    slack[x] = 0; FOR(u,1,N+1) if (g[u][x].w > 0
     && st[u] != x && S[st[u]] == 0) updSlack(u,x); }
  void qPush(int x) {
    if (x <= N) q.push(x);
    else each(t,flo[x]) qPush(t); }
  void setSt(int x, int b) {
    st[x] = b; if (x > N) each(t,flo[x]) setSt(t,b); }
  int getPr(int b, int xr) { // get even position of xr
    int pr = find(all(flo[b]),xr)-begin(flo[b]);
    if (pr&1) { reverse(1+all(flo[b])); return sz(flo[b])-pr; }
    return pr; }
  void setMatch(int u, int v) { // rearrange flo[u], matches
    edge e = g[u][v]; match[u] = e.v; if (u <= N) return;
    int xr = floFrom[u][e.u], pr = getPr(u,xr);
    F0R(i,pr) setMatch(flo[u][i],flo[u][i^1]);
    setMatch(xr,v); rotate(begin(flo[u]),pr+all(flo[u])); }
  void augment(int u, int v) { // set matches including u->v
    while (1) { // and previous ones
      int xnv = st[match[u]]; setMatch(u,v);
      if (!xnv) return;
      setMatch(xnv,st[par[xnv]]);
      u = st[par[xnv]], v = xnv;
    }
  }
  int lca(int u, int v) { // same as in unweighted
    static int t = 0; // except maybe return 0
    for (++t;u||v;swap(u,v)) {
      if (!u) continue;
      if (aux[u] == t) return u;
      aux[u] = t; u = st[match[u]];
      if (u) u = st[par[u]];
    }
    return 0;
  }
  void addBlossom(int u, int anc, int v) {
    int b = N+1; while (b <= NX && st[b]) ++b;
    if (b > NX) ++NX; // new blossom
    lab[b] = S[b] = 0; match[b] = match[anc]; flo[b] = {anc};
    auto blossom = [&](int x) {
      for (int y; x != anc; x = st[par[y]])
        flo[b].pb(x), flo[b].pb(y = st[match[x]]), qPush(y);
    };
    blossom(u); reverse(1+all(flo[b])); blossom(v); setSt(b,b);
    // identify all nodes in current blossom
    FOR(x,1,NX+1) g[b][x].w = g[x][b].w = 0;
    FOR(x,1,N+1) floFrom[b][x] = 0;
    each(xs,flo[b]) { // find tightest constraints
      FOR(x,1,NX+1) if (g[b][x].w == 0 || eDelta(g[xs][x]) <
        eDelta(g[b][x])) g[b][x]=g[xs][x], g[x][b]=g[x][xs];
      FOR(x,1,N+1) if (floFrom[xs][x]) floFrom[b][x] = xs;
    } // floFrom to deconstruct blossom
    setSlack(b); // since didn't qPush everything
  }
  void expandBlossom(int b) {
    each(t,flo[b]) setSt(t,t); // undo setSt(b,b)
    int xr = floFrom[b][g[b][par[b]].u], pr = getPr(b,xr);
    for(int i = 0; i < pr; i += 2) {
      int xs = flo[b][i], xns = flo[b][i+1];
      par[xs] = g[xns][xs].u; S[xs] = 1; // no setSlack(xns)?
      S[xns] = slack[xs] = slack[xns] = 0; qPush(xns);
    }
    S[xr] = 1, par[xr] = par[b];
    FOR(i,pr+1,sz(flo[b])) { // matches don't change
      int xs = flo[b][i]; S[xs] = -1, setSlack(xs); }
    st[b] = 0; // blossom killed
  }
  bool onFoundEdge(edge e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) { // v unvisited, matched with smth else
      par[v] = e.u, S[v] = 1; slack[v] = 0;
      int nu = st[match[v]]; S[nu] = slack[nu] = 0; qPush(nu);
    } else if (S[v] == 0) {
      int anc = lca(u,v); // if 0 then match found!
      if (!anc) return augment(u,v),augment(v,u),1;
      addBlossom(u,anc,v);
    }
    return 0;
  }
  bool matching() {
    q = queue<int>();
    FOR(x,1,NX+1) {
      S[x] = -1, slack[x] = 0; // all initially unvisited
      if (st[x] == x && !match[x]) par[x] = S[x] = 0, qPush(x);
    }
    if (!sz(q)) return 0;
    while (1) {
      while (sz(q)) { // unweighted matching with tight edges
        int u = q.ft; q.pop(); if (S[st[u]] == 1) continue;
        FOR(v,1,N+1) if (g[u][v].w > 0 && st[u] != st[v]) {
          if (eDelta(g[u][v]) == 0) { // condition is strict
            if (onFoundEdge(g[u][v])) return 1;
          } else updSlack(u,st[v]);
        }
      }
      int d = INT_MAX;
      FOR(b,N+1,NX+1) if (st[b] == b && S[b] == 1)
        ckmin(d,lab[b]/2); // decrease lab[b]
      FOR(x,1,NX+1) if (st[x] == x && slack[x]) {
        if (S[x] == -1) ckmin(d,eDelta(g[slack[x]][x]));
        else if (S[x] == 0) ckmin(d,eDelta(g[slack[x]][x])/2);
      } // edge weights shouldn't go below 0
      FOR(u,1,N+1) {
        if (S[st[u]] == 0) {
          if (lab[u] <= d) return 0; // why?
          lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
      } // lab has opposite meaning for verts and blossoms
      FOR(b,N+1,NX+1) if (st[b] == b && S[b] != -1)
        lab[b] += (S[b] == 0 ? 1 : -1)*d*2;
      q = queue<int>();
      FOR(x,1,NX+1) if (st[x]==x && slack[x] // new tight edge
        && st[slack[x]] != x && eDelta(g[slack[x]][x]) == 0)
          if (onFoundEdge(g[slack[x]][x])) return 1;
      FOR(b,N+1,NX+1) if (st[b]==b && S[b]==1 && lab[b]==0)
        expandBlossom(b); // odd dist blossom taken apart
    }
    return 0;
  }
  pair<ll,int> calc() {
    NX = N; st[0] = 0; FOR(i,1,2*N+1) aux[i] = 0;
    FOR(i,1,N+1) match[i] = 0, st[i] = i, flo[i].clear();
    int wMax = 0;
    FOR(u,1,N+1) FOR(v,1,N+1)
      floFrom[u][v] = (u == v ? u : 0), ckmax(wMax,g[u][v].w);
    FOR(u,1,N+1) lab[u] = wMax; // start high and decrease
    int num = 0; ll wei = 0; while (matching()) ++num;
    FOR(u,1,N+1) if (match[u] && match[u] < u)
      wei += g[u][match[u]].w; // edges in matching
    return {wei,num};
  }
};
```

MaxMatchFast.h

Description: Fast bipartite matching.
Time: $\mathcal{O}\left(M\sqrt{N}\right)$

ec6c96, 31 lines

```
vpi maxMatch(int L, int R, const vpi& edges) {
  V<vi> adj = V<vi>(L);
  vi nxt(L,-1), prv(R,-1), lev, ptr;
  F0R(i,sz(edges)) adj.at(edges[i].f).pb(edges[i].s);
  while (true) {
    lev = ptr = vi(L); int max_lev = 0;
    queue<int> q; F0R(i,L) if (nxt[i]==-1) lev[i]=1, q.push(i);
    while (sz(q)) {
      int x = q.ft; q.pop();
      for (int y: adj[x]) {
        int z = prv[y];
        if (z == -1) max_lev = lev[x];
        else if (!lev[z]) lev[z] = lev[x]+1, q.push(z);
      }
      if (max_lev) break;
    }
    if (!max_lev) break;
    F0R(i,L) if (lev[i] > max_lev) lev[i] = 0;
    auto dfs = [&](auto self, int x) -> bool {
      for (;ptr[x] < sz(adj[x]);++ptr[x]) {
        int y = adj[x][ptr[x]], z = prv[y];
        if (z == -1 || (lev[z] == lev[x]+1 && self(self,z)))
          return nxt[x]=y, prv[y]=x, ptr[x]=sz(adj[x]), 1;
      }
      return 0;
    };
    F0R(i,L) if (nxt[i] == -1) dfs(dfs,i);
  }
  vpi ans; F0R(i,L) if (nxt[i] != -1) ans.pb({i,nxt[i]});
  return ans;
}
```

7.6 Advanced

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). To find maximum independent set consider complement.
Time: Runs in about 1s for $n = 155$ and worst case random graphs ($p = .90$). Faster for sparse graphs.

"template.hpp" e80bc7, 41 lines

```
struct MaxClique {
  db limit = 0.025, pk = 0; // # of steps
  struct Vertex { int i, d=0; Vertex(int _i):i(_i){} };
  typedef vector<Vertex> vv; vv V;
  vector<bitset<200>> e; vector<vi> C; // colors
  vi qmax,q,S,old; // max/current clique, sum # steps up to lev
  void init(vv& r) { // v.d -> degree
    each(v,r) { v.d = 0; each(j,r) v.d += e[v.i][j.i]; }
    sort(all(r),[](Vertex a,Vertex b) { return a.d > b.d; });
    int mxD = r[0].d; F0R(i,sz(r)) r[i].d = min(i,mxD)+1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev-1]-old[lev]; old[lev] = S[lev-1];
    while (sz(R)) {
      if (sz(q)+R.bk.d <= sz(qmax)) return; // no larger clique
      q.pb(R.bk.i); // insert node with max col into clique
      vv T; each(v,R) if (e[R.bk.i][v.i]) T.pb({v.i});
      if (sz(T)) {
        if (S[lev]++/++pk < limit) init(T); // recalc degs
        int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1,1);
        C[1].clear(), C[2].clear();
        each(v,T) {
          int k = 1; auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]),f)) k ++;
          if (k > mxk) mxk = k, C[mxk+1].clear(); // new set
          if (k < mnk) T[j++].i = v.i;
          C[k].pb(v.i);
        }
        if (j > 0) T[j-1].d = 0; // >=1 vert >=j part of clique
        FOR(k,mnk,mxk+1) each(i,C[k]) T[j].i = i, T[j++].d = k;
        expand(T,lev+1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back(); // R.bk not in set
    }
  }
  vi solve(vector<bitset<200>> conn) {
    e = conn; C.rsz(sz(e)+1), S.rsz(sz(C)), old = S;
    F0R(i,sz(e)) V.pb({i});
    init(V), expand(V); return qmax;
  }
};
```

ChordalGraphRecognition.h

Description: Recognizes graph where every induced cycle has length exactly 3 using maximum adjacency search.

6cc97d, 58 lines

```
int N,M;
set<int> adj[MX];
int cnt[MX];
vi ord, rord;

vi find_path(int x, int y, int z) {
  vi pre(N,-1);
  queue<int> q; q.push(x);
  while (sz(q)) {
    int t = q.ft; q.pop();
    if (adj[t].count(y)) {
      pre[y] = t; vi path = {y};
      while (path.bk != x) path.pb(pre[path.bk]);
      path.pb(z);
      return path;
    }
    each(u,adj[t]) if (u != z && !adj[u].count(z) && pre[u] ==
      ↪-1) {
      pre[u] = t;
      q.push(u);
    }
  }
  assert(0);
}

int main() {
  setIO(); re(N,M);
  F0R(i,M) {
    int a,b; re(a,b);
    adj[a].insert(b), adj[b].insert(a);
  }
  rord = vi(N,-1);
  priority_queue<pi> pq;
  F0R(i,N) pq.push({0,i});
  while (sz(pq)) {
    pi p = pq.top(); pq.pop();
    if (rord[p.s] != -1) continue;
    rord[p.s] = sz(ord); ord.pb(p.s);
    each(t,adj[p.s]) pq.push({++cnt[t],t});
  }
  assert(sz(ord) == N);
  each(z,ord) {
    pi big = {-1,-1};
    each(y,adj[z]) if (rord[y] < rord[z])
      ckmax(big,mp(rord[y],y));
    if (big.f == -1) continue;
    int y = big.s;
    each(x,adj[z]) if (rord[x] < rord[y]) if (!adj[y].count(x))
      ↪ {
      ps("NO");
      vi v = find_path(x,y,z);
      ps(sz(v));
      each(t,v) pr(t,' ');
      exit(0);
    }
  }
  ps("YES");
  reverse(all(ord));
  each(z,ord) pr(z,' ');
}
```

DominatorTree.h

Description: Used only a few times. Assuming that all nodes are reachable from *root*, a dominates b iff every path from *root* to b passes through a.
Time: $\mathcal{O}(M \log N)$

4b8836, 41 lines

```
template<int SZ> struct Dominator {
  vi adj[SZ], ans[SZ]; // input edges, edges of dominator tree
  vi radj[SZ], child[SZ], sdomChild[SZ];
  int label[SZ], rlabel[SZ], sdom[SZ], dom[SZ], co = 0;
  int par[SZ], bes[SZ];
  void ae(int a, int b) { adj[a].pb(b); }
  int get(int x) { // DSU with path compression
    // get vertex with smallest sdom on path to root
    if (par[x] != x) {
      int t = get(par[x]); par[x] = par[par[x]];
      if (sdom[t] < sdom[bes[x]]) bes[x] = t;
    }
    return bes[x];
  }
  void dfs(int x) { // create DFS tree
    label[x] = ++co; rlabel[co] = x;
    sdom[co] = par[co] = bes[co] = co;
    each(y,adj[x]) {
      if (!label[y]) {
        dfs(y); child[label[x]].pb(label[y]); }
      radj[label[y]].pb(label[x]);
    }
  }
  void init(int root) {
    dfs(root);
    ROF(i,1,co+1) {
      each(j,radj[i]) ckmin(sdom[i],sdom[get(j)]);
      if (i > 1) sdomChild[sdom[i]].pb(i);
      each(j,sdomChild[i]) {
        int k = get(j);
        if (sdom[j] == sdom[k]) dom[j] = sdom[j];
        else dom[j] = k;
      }
      each(j,child[i]) par[j] = i;
    }
    FOR(i,2,co+1) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
      ans[rlabel[dom[i]]].pb(rlabel[i]);
    }
  }
};
```

EdgeColor.h

Description: Used only once. Naive implementation of Misra & Gries edge coloring. By Vizing's Theorem, a simple graph with max degree d can be edge colored with at most $d + 1$ colors
Time: $\mathcal{O}\left(N^2M\right)$, faster in practice

cc2b29, 40 lines

```
template<int SZ> struct EdgeColor {
  int N = 0, maxDeg = 0, adj[SZ][SZ], deg[SZ];
  void init(int _N) { N = _N;
    F0R(i,N) { deg[i] = 0; F0R(j,N) adj[i][j] = 0; } }
  void ae(int a, int b, int c) {
    adj[a][b] = adj[b][a] = c; }
  int delEdge(int a, int b) {
    int c = adj[a][b]; adj[a][b] = adj[b][a] = 0;
    return c; }
  V<bool> genCol(int x) {
    V<bool> col(N+1); F0R(i,N) col[adj[x][i]] = 1;
    return col; }
  int freeCol(int u) {
    auto col = genCol(u); int x = 1;
    while (col[x]) ++x; return x; }
  void invert(int x, int d, int c) {
    F0R(i,N) if (adj[x][i] == d)
      delEdge(x,i), invert(i,c,d), ae(x,i,c); }
  void ae(int u, int v) {
    // check if you can add edge w/o doing any work
    assert(N); ckmax(maxDeg,max(++deg[u],++deg[v]));
    auto a = genCol(u), b = genCol(v);
    FOR(i,1,maxDeg+2) if (!a[i] && !b[i])
      return ae(u,v,i);
    V<bool> use(N); vi fan = {v}; use[v] = 1;
    while (1) {
      auto col = genCol(fan.bk);
      if (sz(fan) > 1) col[adj[fan.bk][u]] = 0;
      int i=0; while (i<N && (use[i] || col[adj[u][i]])) i++;
      if (i < N) fan.pb(i), use[i] = 1;
      else break;
    }
    int c = freeCol(u), d = freeCol(fan.bk); invert(u,d,c);
    int i = 0; while (i < sz(fan) && genCol(fan[i])[d]
      && adj[u][fan[i]] != d) i ++;
    assert (i != sz(fan));
    F0R(j,i) ae(u,fan[j],delEdge(u,fan[j+1]));
    ae(u,fan[i],d);
```

```
  }
};
```

DirectedMST.h

Description: Chu-Liu-Edmonds algorithm. Computes minimum weight directed spanning tree rooted at r, edge from $par[i] \to i$ for all $i \neq r$. Use DSU with rollback if need to return edges.

Time: $\mathcal{O}(M \log M)$

"DSUrb.h" 5d5c10, 61 lines

```
struct Edge { int a, b; ll w; };
struct Node { // lazy skew heap node
  Edge key; Node *l, *r; ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, a->r = merge(b, a->r));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll,vi> dmst(int n, int r, const vector<Edge>& g) {
  DSUrb dsu; dsu.init(n);
  vector<Node*> heap(n); // store edges entering each vertex
  // in increasing order of weight
  each(e,g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0; vi seen(n,-1); seen[r] = r;
  vpi in(n,{-1,-1}); // edge entering each vertex in MST
  vector<pair<int,vector<Edge>>> cycs;
  F0R(s,n) {
    int u = s, w;
    vector<pair<int,Edge>> path;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      seen[u] = s;
      Edge e = heap[u]->top(); path.pb({u,e});
      heap[u]->delta -= e.w, pop(heap[u]);
      res += e.w, u = dsu.get(e.a);
      if (seen[u] == s) { // found cycle, contract
        Node* cyc = 0; cycs.eb();
        do {
          cyc = merge(cyc, heap[w = path.bk.f]);
          cycs.bk.s.pb(path.bk.s);
          path.pop_back();
        } while (dsu.unite(u,w));
        u = dsu.get(u); heap[u] = cyc, seen[u] = -1;
        cycs.bk.f = u;
      }
    }
    each(t,path) in[dsu.get(t.s.b)] = {t.s.a,t.s.b};
  } // found path from root to s, done
  while (sz(cycs)) { // expand cycs to restore sol
    auto c = cycs.bk; cycs.pop_back();
    pi inEdge = in[c.f];
    each(t,c.s) dsu.rollback();
    each(t,c.s) in[dsu.get(t.b)] = {t.a,t.b};
    in[dsu.get(inEdge.s)] = inEdge;
  }
  vi par(n); F0R(i,n) par[i] = in[i].f;
  // i == r ? in[i].s == -1 : in[i].s == i
  return {res,par};
}
```

LCT.h

Description: Link-Cut Tree. Given a function $f(1 \dots N) \to 1 \dots N$, evaluates $f^b(a)$ for any a, b. `sz` is for path queries; `sub`, `vsub` are for subtree queries. `x->access()` brings `x` to the top and propagates it; its left subtree will be the path from `x` to the root and its right subtree will be empty. Then `sub` will be the number of nodes in the connected component of `x` and `vsub` will be the number of nodes under `x`. Use `makeRoot` for arbitrary path queries.

Usage: `FOR(i,1,N+1)LCT[i]=new snode(i); link(LCT[1],LCT[2],1);`

Time: $\mathcal{O}(\log N)$

e24bf7, 115 lines

```
typedef struct snode* sn;
struct snode { //////// VARIABLES
  sn p, c[2]; // parent, children
  sn extra; // extra cycle node for "The Applicant"
  bool flip = 0; // subtree flipped or not
  int val, sz; // value in node, # nodes in current splay tree
  int sub, vsub = 0; // vsub stores sum of virtual children
  snode(int _val) : val(_val) {
    p = c[0] = c[1] = extra = NULL; calc(); }
  friend int getSz(sn x) { return x?x->sz:0; }
  friend int getSub(sn x) { return x?x->sub:0; }
  void prop() { // lazy prop
    if (!flip) return;
    swap(c[0],c[1]); flip = 0;
    F0R(i,2) if (c[i]) c[i]->flip ^= 1;
  }
  void calc() { // recalc vals
    F0R(i,2) if (c[i]) c[i]->prop();
    sz = 1+getSz(c[0])+getSz(c[1]);
    sub = 1+getSub(c[0])+getSub(c[1])+vsub;
  }
  //////// SPLAY TREE OPERATIONS
  int dir() {
    if (!p) return -2;
    F0R(i,2) if (p->c[i] == this) return i;
    return -1; // p is path-parent pointer
  } // -> not in current splay tree
  // test if root of current splay tree
  bool isRoot() { return dir() < 0; }
  friend void setLink(sn x, sn y, int d) {
    if (y) y->p = x;
    if (d >= 0) x->c[d] = y; }
  void rot() { // assume p and p->p propagated
    assert(!isRoot()); int x = dir(); sn pa = p;
    setLink(pa->p, this, pa->dir());
    setLink(pa, c[x^1], x); setLink(this, pa, x^1);
    pa->calc();
  }
  void splay() {
    while (!isRoot() && !p->isRoot()) {
      p->p->prop(), p->prop(), prop();
      dir() == p->dir() ? p->rot() : rot();
      rot();
    }
    if (!isRoot()) p->prop(), prop(), rot();
    prop(); calc();
  }
  sn fbo(int b) { // find by order
    prop(); int z = getSz(c[0]); // of splay tree
    if (b == z) { splay(); return this; }
    return b < z ? c[0]->fbo(b) : c[1] -> fbo(b-z-1);
  }
  //////// BASE OPERATIONS
  void access() { // bring this to top of tree, propagate
    for (sn v = this, pre = NULL; v; v = v->p) {
      v->splay(); // now switch virtual children
      if (pre) v->vsub -= pre->sub;
      if (v->c[1]) v->vsub += v->c[1]->sub;
      v->c[1] = pre; v->calc(); pre = v;
    }
    splay(); assert(!c[1]); // right subtree is empty
  }
  void makeRoot() {
    access(); flip ^= 1; access(); assert(!c[0] && !c[1]); }
  //////// QUERIES
  friend sn lca(sn x, sn y) {
    if (x == y) return x;
    x->access(), y->access(); if (!x->p) return NULL;
    x->splay(); return x->p?:x; // y was below x in latter case
  } // access at y did not affect x -> not connected
  friend bool connected(sn x, sn y) { return lca(x,y); }
  // # nodes above
  int distRoot() { access(); return getSz(c[0]); }
  sn getRoot() { // get root of LCT component
    access(); sn a = this;
    while (a->c[0]) a = a->c[0], a->prop();
    a->access(); return a;
  }
  sn getPar(int b) { // get b-th parent on path to root
    access(); b = getSz(c[0])-b; assert(b >= 0);
    return fbo(b);
  } // can also get min, max on path to root, etc
  //////// MODIFICATIONS
  void set(int v) { access(); val = v; calc(); }
  friend void link(sn x, sn y, bool force = 0) {
    assert(!connected(x,y));
    if (force) y->makeRoot(); // make x par of y
    else { y->access(); assert(!y->c[0]); }
    x->access(); setLink(y,x,0); y->calc();
  }
  friend void cut(sn y) { // cut y from its parent
    y->access(); assert(y->c[0]);
    y->c[0]->p = NULL; y->c[0] = NULL; y->calc(); }
  friend void cut(sn x, sn y) { // if x, y adj in tree
    x->makeRoot(); y->access();
    assert(y->c[0] == x && !x->c[0] && !x->c[1]); cut(y); }
};
sn LCT[MX];

//////// THE APPLICANT SOLUTION
void setNex(sn a, sn b) { // set f[a] = b
  if (connected(a,b)) a->extra = b;
  else link(b,a); }
void delNex(sn a) { // set f[a] = NULL
  auto t = a->getRoot();
  if (t == a) { t->extra = NULL; return; }
  cut(a); assert(t->extra);
  if (!connected(t,t->extra))
    link(t->extra,t), t->extra = NULL;
}
sn getPar(sn a, int b) { // get f^b[a]
  int d = a->distRoot(); if (b <= d) return a->getPar(b);
  b -= d+1; auto r = a->getRoot()->extra; assert(r);
  d = r->distRoot()+1; return r->getPar(b%d);
}
```

Geometry (8)

8.1 Primitives

ComplexComp.h

Description: Allows you to sort complex numbers.

6f828b, 5 lines

```
#define x real()
```

```
#define y imag()
using P = complex<db>;
namespace std {
  bool operator<(P l,P r) { return mp(l.x,l.y)<mp(r.x,r.y); } }
```

PointShort.h
Description: Use in place of `complex<T>`.
268ecf, 36 lines

```
using T = db; // or ll
const T EPS = 1e-9; // adjust as needed
using P = pair<T,T>; using vP = V<P>; using Line = pair<P,P>;
int sgn(T a) { return (a>EPS)-(a<-EPS); }
T sq(T a) { return a*a; }

T abs2(P p) { return sq(p.f)+sq(p.s); }
T abs(P p) { return sqrt(abs2(p)); }
T arg(P p) { return atan2(p.s,p.f); }
P conj(P p) { return P(p.f,-p.s); }
P perp(P p) { return P(-p.s,p.f); }
P dir(T ang) { return P(cos(ang),sin(ang)); }

P operator+(P l, P r) { return P(l.f+r.f,l.s+r.s); }
P operator-(P l, P r) { return P(l.f-r.f,l.s-r.s); }
P operator*(P l, T r) { return P(l.f*r,l.s*r); }
P operator/(P l, T r) { return P(l.f/r,l.s/r); }
P operator*(P l, P r) { // complex # multiplication
  return P(l.f*r.f-l.s*r.s,l.s*r.f+l.f*r.s); }
P operator/(P l, P r) { return l*conj(r)/abs2(r); }

P unit(const P& p) { return p/abs(p); }
T dot(const P& a, const P& b) { return a.f*b.f+a.s*b.s; }
T dot(const P& p, const P& a, const P& b) { return dot(a-p,b-p)
  ↪; }
T cross(const P& a, const P& b) { return a.f*b.s-a.s*b.f; }
T cross(const P& p, const P& a, const P& b) {
  return cross(a-p,b-p); }
P reflect(const P& p, const Line& l) {
  P a = l.f, d = l.s-l.f;
  return a+conj((p-a)/d)*d; }
P foot(const P& p, const Line& l) {
  return (p+reflect(p,l))/(T)2; }
bool onSeg(const P& p, const Line& l) {
  return sgn(cross(l.f,l.s,p)) == 0 && sgn(dot(p,l.f,l.s)) <= 0
    ↪; }
ostream& operator<<(ostream& os, P p) {
  return os << "(" << p.f << "," << p.s << ")"; }
```

AngleCmp.h
Description: Sorts points in ccw order about origin in the same way as `atan2`, which returns real in $(-\pi, \pi]$ so points on negative x-axis come last.
Usage: `vP v; sort(all(v),angleCmp);`
"Point.h" 2df5fc, 8 lines

```
// WARNING: you will get unexpected results if you mistype this
  ↪ as bool instead of int
// -1 if lower half, 0 if origin, 1 if upper half
int half(P x) { return x.s != 0 ? sgn(x.s) : -sgn(x.f); }
bool angleCmp(P a, P b) { int A = half(a), B = half(b);
  return A == B ? cross(a,b) > 0 : A < B; }

// equivalent to: sort(all(v),[](P a, P b) {
//    return atan2(a.s,a.f) < atan2(b.s,b.f); });
```

SegDist.h
Description: computes distance between P and line (segment) AB
"Point.h" 0cb69a, 6 lines

```
T lineDist(const P& p, const Line& l) {
  return abs(cross(p,l.f,l.s))/abs(l.f-l.s); }
T segDist(const P& p, const Line& l) {
  if (dot(l.f,p,l.s) <= 0) return abs(p-l.f);
  if (dot(l.s,p,l.f) <= 0) return abs(p-l.s);
  return lineDist(p,l); }
```

SegIsect.h
Description: computes the intersection point(s) of line (segments) a and b
"Point.h" 7f6ba0, 26 lines

```
// {unique intersection point} if it exists
// {b.f,b.s} if input lines are the same
// empty if lines do not intersect
vP lineIsect(const Line& a, const Line& b) {
  T a0 = cross(a.f,a.s,b.f), a1 = cross(a.f,a.s,b.s);
  if (a0 == a1) return a0 == 0 ? vP{b.f,b.s} : vP{};
  return {(b.s*a0-b.f*a1)/(a0-a1)};
}

// point in interior of both segments a and b, if it exists
vP strictIsect(const Line& a, const Line& b) {
  T a0 = cross(a.f,a.s,b.f), a1 = cross(a.f,a.s,b.s);
  T b0 = cross(b.f,b.s,a.f), b1 = cross(b.f,b.s,a.s);
  if (sgn(a0)*sgn(a1) < 0 && sgn(b0)*sgn(b1) < 0)
    return {(b.s*a0-b.f*a1)/(a0-a1)};
  return {};
}

// intersection of segments, a and b may be degenerate
vP segIsect(const Line& a, const Line& b) {
  vP v = strictIsect(a,b); if (sz(v)) return v;
  set<P> s;
  #define i(x,y) if (onSeg(x,y)) s.ins(x)
  i(a.f,b); i(a.s,b); i(b.f,a); i(b.s,a);
  return {all(s)};
}
```

8.2 Polygons

PolygonCenArea.h
Description: centroid (center of mass) of a polygon with constant mass per unit area and SIGNED area
Time: $\mathcal{O}(N)$
"Point.h" 4ca221, 7 lines

```
pair<P,T> cenArea(const vP& v) { assert(sz(v) >= 3);
  P cen{}; T area{};
  F0R(i,sz(v)) {
    int j = (i+1)%sz(v); T a = cross(v[i],v[j]);
    cen += a*(v[i]+v[j]); area += a; }
  return {cen/area/(T)3,area/2}; // area is SIGNED
}
```

InPolygon.h
Description: Tests whether point is inside, on, or outside of a polygon (returns -1, 0, or 1). Both CW and CCW polygons are ok.
Time: $\mathcal{O}(N)$
"Point.h" 6ebff0, 9 lines

```
int inPoly(const P& p, const vP& poly) {
  int n = sz(poly), ans = 0;
  F0R(i,n) {
    P x = poly[i], y = poly[(i+1)%n]; if (x.s > y.s) swap(x,y);
    if (onSeg(p,{x,y})) return 0;
    ans ^= (x.s <= p.s && p.s < y.s && cross(p,x,y) > 0);
  }
  return ans ? -1 : 1;
}
```

ConvexHull.h
Description: top-bottom convex hull
Time: $\mathcal{O}(N \log N)$
"Point.h" 868655, 18 lines

```
pair<vi,vi> ulHull(const vP& v) {
  vi p(sz(v)), u, l; iota(all(p), 0);
  sort(all(p), [&v](int a, int b) { return v[a] < v[b]; });
  each(i,p) {
    #define ADDP(C, cmp) while (sz(C) > 1 && cross(\
      v[C[sz(C)-2]],v[C.bk],v[i]) cmp 0) C.pop_back(); C.pb(i);
    ADDP(u, >=); ADDP(l, <=);
  }
  return {u,l};
}
vi hullInd(const vP& v) { // returns indices in CCW order
  vi u,l; tie(u,l) = ulHull(v); if (sz(l) <= 1) return l;
  if (v[l[0]] == v[l[1]]) return {0};
  l.insert(end(l),1+rall(u)-1); return l;
}
vP hull(const vP& v) {
  vi w = hullInd(v); vP res; each(t,w) res.pb(v[t]);
  return res; }
```

MinkowskiSum.h
Description: Minkowski sum of two convex polygons given in CCW order.
Time: $\mathcal{O}(N)$
"ConvexHull.h" e5d935, 29 lines

```
vP minkowski_sum(vP a, vP b) {
  if (sz(a) > sz(b)) swap(a, b);
  if (!sz(a)) return {};
  if (sz(a) == 1) {
    each(t, b) t += a.ft;
    return b;
  }
  rotate(begin(a), min_element(all(a)), end(a));
  rotate(begin(b), min_element(all(b)), end(b));
  a.pb(a[0]), a.pb(a[1]);
  b.pb(b[0]), b.pb(b[1]);
  vP result;
  int i = 0, j = 0;
  while (i < sz(a)-2 || j < sz(b)-2) {
    result.pb(a[i]+b[j]);
    T crs = cross(a[i+1]-a[i],b[j+1]-b[j]);
    i += (crs >= 0);
    j += (crs <= 0);
  }
  return result;
}

T diameter2(vP p) { // example application: squared diameter
  vP a = hull(p);
  vP b = a; each(t, b) t *= -1;
  vP c = minkowski_sum(a, b);
  T ret = 0; each(t, c) ckmax(ret, abs2(t));
  return ret;
}
```

HullDiameter.h
Description: Rotating calipers. Returns square of greatest distance between two points in P.
Time: $\mathcal{O}(N)$ given convex hull
"ConvexHull.h" 213a2a, 9 lines

```
db diameter2(vP P) {
  P = hull(P);
  int n = sz(P), ind = 1; T ans = 0;
  if (n > 1) F0R(i,n) for (int j = (i+1)%n;;ind = (ind+1)%n) {
    ckmax(ans, abs2(P[i]-P[ind]));
    if (cross(P[j]-P[i],P[(ind+1)%n]-P[ind]) <= 0) break;
  }
  return ans;
}
```

HullTangents.h

Description: Given convex polygon with no three points collinear and a point strictly outside of it, computes the lower and upper tangents.
Time: $\mathcal{O}(\log N)$

"Point.h" 85b807, 36 lines

```
bool lower;
bool better(P a, P b, P c) {
  T z = cross(a,b,c);
  return lower ? z < 0 : z > 0; }
int tangent(const vP& a, P b) {
  if (sz(a) == 1) return 0;
  int lo, hi;
  if (better(b,a[0],a[1])) {
    lo = 0, hi = sz(a)-1;
    while (lo < hi) {
      int mid = (lo+hi+1)/2;
      if (better(b,a[0],a[mid])) lo = mid;
      else hi = mid-1;
    }
    lo = 0;
  } else {
    lo = 1, hi = sz(a);
    while (lo < hi) {
      int mid = (lo+hi)/2;
      if (!better(b,a[0],a[mid])) lo = mid+1;
      else hi = mid;
    }
    hi = sz(a);
  }
  while (lo < hi) {
    int mid = (lo+hi)/2;
    if (better(b,a[mid],a[(mid+1)%sz(a)])) lo = mid+1;
    else hi = mid;
  }
  return lo%sz(a);
}
pi tangents(const vP& a, P b) {
  lower = 1; int x = tangent(a,b);
  lower = 0; int y = tangent(a,b);
  return {x,y};
}
```

LineHull.h

Description: `lineHull` accepts line and ccw convex polygon. If all vertices in poly lie to one side of the line, returns a vector of closest vertices to line as well as orientation of poly with respect to line (± 1 for above/below). Otherwise, returns the range of vertices that lie on or below the line. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log N)$

"Point.h" 40e5a6, 41 lines

```
using Line = AR<P,2>;
#define cmp(i,j) sgn(-dot(dir,poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i+1,i) >= 0 && cmp(i,i-1+n) < 0
int extrVertex(const vP& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo+1 < hi) {
    int m = (lo+hi)/2;
    if (extr(m)) return m;
    int ls = cmp(lo+1,lo), ms = cmp(m+1,m);
    (ls < ms || (ls == ms && ls == cmp(lo,m)) ? hi : lo) = m;
  }
  return lo;
}
vi same(Line line, const vP& poly, int a) {
  // points on same parallel as a
  int n = sz(poly); P dir = perp(line[0]-line[1]);
  if (cmp(a+n-1,a) == 0) return {(a+n-1)%n,a};
  if (cmp(a,a+1) == 0) return {a,(a+1)%n};
  return {a};
}
#define cmpL(i) sgn(cross(line[0],line[1],poly[i]))
pair<int,vi> lineHull(Line line, const vP& poly) {
  int n = sz(poly); assert(n>1);
  int endA = extrVertex(poly,perp(line[0]-line[1])); // lowest
  if (cmpL(endA) >= 0) return {1,same(line,poly,endA)};
  int endB = extrVertex(poly,perp(line[1]-line[0])); // highest
  if (cmpL(endB) <= 0) return {-1,same(line,poly,endB)};
  AR<int,2> res;
  F0R(i,2) {
    int lo = endA, hi = endB; if (hi < lo) hi += n;
    while (lo < hi) {
      int m = (lo+hi+1)/2;
      if (cmpL(m%n) == cmpL(endA)) lo = m;
      else hi = m-1;
    }
    res[i] = lo%n; swap(endA,endB);
  }
  if (cmpL((res[0]+1)%n) == 0) res[0] = (res[0]+1)%n;
  return {0,{(res[1]+1)%n,res[0]}};
}
```

HalfPlaneIsect.h

Description: Returns vertices of half-plane intersection. A half-plane is the area to the left of a ray, which is defined by a point `p` and a direction `dp`. Area of intersection should be sufficiently precise when all inputs are integers with magnitude $\leq 10^5$. Intersection must be bounded. Probably works with floating point too (but EPS might need to be adjusted?).
Time: $\mathcal{O}(N \log N)$

"AngleCmp.h" 18f712, 52 lines

```
struct Ray {
  P p, dp; // origin, direction
  P isect(const Ray& L) const {
    return p+dp*(cross(L.dp,L.p-p)/cross(L.dp,dp)); }
  bool operator<(const Ray& L) const {
    return angleCmp(dp,L.dp); }
};

vP halfPlaneIsect(V<Ray> rays, bool add_bounds = false) {
  if (add_bounds) { // bound input by rectangle [0,DX] x [0,DY]
    int DX = 1e9, DY = 1e9;
    rays.pb({P{0,0},P{1,0}});
    rays.pb({P{DX,0},P{0,1}});
    rays.pb({P{DX,DY},P{-1,0}});
    rays.pb({P{0,DY},P{0,-1}});
  }
  sor(rays); // sort rays by angle
  { // remove parallel rays
    V<Ray> nrays;
    each(t,rays) {
      if (!sz(nrays) || cross(nrays.bk.dp,t.dp) > EPS) { nrays.
        ↪pb(t); continue; }
      // last two rays are parallel, keep only one
      if (cross(t.dp,t.p-nrays.bk.p) > 0) nrays.bk = t;
    }
    swap(rays, nrays);
  }
  auto bad = [&](const Ray& a, const Ray& b, const Ray& c) {
    P p1 = a.isect(b), p2 = b.isect(c);
    if (dot(p2-p1,b.dp) <= EPS) {
      if (cross(a.dp,c.dp) <= 0) return 2; // isect(a,b,c) =
        ↪empty
      return 1; // isect(a,c) == isect(a,b,c)
    }
    return 0; // all three rays matter
  };
  #define reduce(t) \
    while (sz(poly) > 1) { \
      int b = bad(poly.at(sz(poly)-2),poly.bk,t); \
      if (b == 2) return {}; \
      if (b == 1) poly.pop_back(); \
      else break; \
    }
  deque<Ray> poly;
  each(t,rays) { reduce(t); poly.pb(t); }
  for(;;poly.pop_front()) {
    reduce(poly[0]);
    if (!bad(poly.bk,poly[0],poly[1])) break;
  }
  assert(sz(poly) >= 3); // expect nonzero area
  vP poly_points; F0R(i,sz(poly))
    poly_points.pb(poly[i].isect(poly[(i+1)%sz(poly)]));
  return poly_points;
}
```

HalfPlaneSet.h

Description: Online Half-Plane Intersection

d2027d, 77 lines

```
using T = int;
using T2 = long long;
using T4 = __int128_t;
const T2 INF = 2e9;

struct Line {
  T a, b;
  T2 c;
};

bool operator<(Line m, Line n) {
  auto half = [&](Line m) { return m.b < 0 || m.b == 0 && m.a <
    ↪ 0; };
  return make_tuple(half(m), (T2)m.b * n.a) <
       make_tuple(half(n), (T2)m.a * n.b);
}
tuple<T4, T4, T2> LineIntersection(Line m, Line n) {
  T2 d = (T2)m.a * n.b - (T2)m.b * n.a; // assert(d);
  T4 x = (T4)m.c * n.b - (T4)m.b * n.c;
  T4 y = (T4)m.a * n.c - (T4)m.c * n.a;
  return {x, y, d};
}
Line LineFromPoints(T x1, T y1, T x2, T y2) {
  // everything to the right of ray {x1, y1} -> {x2, y2}
  T a = y1 - y2, b = x2 - x1;
  T2 c = (T2)a * x1 + (T2)b * y1;
  return {a, b, c}; // ax + by <= c
}
ostream &operator<<(ostream &out, Line l) {
  out << "Line " << l.a << " " << l.b << " " << -l.c;
  // out << "(" << l.a << " * x + " << l.b << " * y <= " << l.c
    ↪ << ")";
  return out;
}

struct HalfplaneSet : multiset<Line> {
  HalfplaneSet() {
    insert({+1, 0, INF});
    insert({0, +1, INF});
    insert({-1, 0, INF});
    insert({0, -1, INF});
  };
  auto adv(auto it, int z) { // z = {-1, +1}
    return (z == -1 ? --(it == begin() ? end() : it)
            : (++it == end() ? begin() : it));
  }
  bool chk(auto it) {
    Line l = *it, pl = *adv(it, -1), nl = *adv(it, +1);
```

```
    auto [x, y, d] = LineIntersection(pl, nl);
    T4 sat = l.a * x + l.b * y - (T4)l.c * d;
    if (d < 0 && sat < 0) return clear(), 0; // unsat
    if ((d > 0 && sat <= 0) || (d == 0 && sat < 0)) return
      ↪erase(it), 1;
    return 0;
  }
  void Cut(Line l) { // add ax + by <= c
    if (empty()) return;
    auto it = insert(l);
    if (chk(it)) return;
    for (int z : {-1, +1})
      while (size() && chk(adv(it, z)))
        ;
  }
  double Maximize(T a, T b) { // max ax + by (UNTESTED)
    if (empty()) return -1 / 0.;
    auto it = lower_bound({a, b});
    if (it == end()) it = begin();
    auto [x, y, d] = LineIntersection(*adv(it, -1), *it);
    return (1.0 * a * x + 1.0 * b * y) / d;
  }
  double Area() {
    double total = 0.;
    for (auto it = begin(); it != end(); ++it) {
      auto [x1, y1, d1] = LineIntersection(*adv(it, -1), *it);
      auto [x2, y2, d2] = LineIntersection(*it, *adv(it, +1));
      total += (1.0 * x1 * y2 - 1.0 * x2 * y1) / d1 / d2;
    }
    return total * 0.5;
  }
};
```

8.3 Circles

Circle.h

Description: represent circle as {center,radius}

"Point.h" 91f3fc, 6 lines

```
using Circ = pair<P,T>;
int in(const Circ& x, const P& y) { // -1 if inside, 0, 1
  return sgn(abs(y-x.f)-x.s); }
T arcLength(const Circ& x, P a, P b) {
    // precondition: a and b on x
  P d = (a-x.f)/(b-x.f); return x.s*acos(d.f); }
```

CircleIsect.h

Description: Circle intersection points and intersection area. Tangents will be returned twice.

"Circle.h" 21a173, 22 lines

```
vP isect(const Circ& x, const Circ& y) { // precondition: x!=y
  T d = abs(x.f-y.f), a = x.s, b = y.s;
  if (sgn(d) == 0) { assert(a != b); return {}; }
  T C = (a*a+d*d-b*b)/(2*a*d);
  if (abs(C) > 1+EPS) return {};
  T S = sqrt(max(1-C*C,(T)0)); P tmp = (y.f-x.f)/d*x.s;
  return {x.f+tmp*P(C,S),x.f+tmp*P(C,-S)};
}
vP isect(const Circ& x, const Line& y) {
  P c = foot(x.f,y); T sq_dist = sq(x.s)-abs2(x.f-c);
  if (sgn(sq_dist) < 0) return {};
  P offset = unit(y.s-y.f)*sqrt(max(sq_dist,T(0)));
  return {c+offset,c-offset};
}
T isect_area(Circ x, Circ y) { // not thoroughly tested
  T d = abs(x.f-y.f), a = x.s, b = y.s; if (a < b) swap(a,b);
  if (d >= a+b) return 0;
  if (d <= a-b) return PI*b*b;
  T ca = (a*a+d*d-b*b)/(2*a*d), cb = (b*b+d*d-a*a)/(2*b*d);
  T s = (a+b+d)/2, h = 2*sqrt(s*(s-a)*(s-b)*(s-d))/d;
  return a*a*acos(ca)+b*b*acos(cb)-d*h;
}
```

CircleTangents.h

Description: internal and external tangents between two circles

"Circle.h" d9a76f, 22 lines

```
P tangent(P x, Circ y, int t = 0) {
  y.s = abs(y.s); // abs needed because internal calls y.s < 0
  if (y.s == 0) return y.f;
  T d = abs(x-y.f);
  P a = pow(y.s/d,2)*(x-y.f)+y.f;
  P b = sqrt(d*d-y.s*y.s)/d*y.s*unit(x-y.f)*dir(PI/2);
  return t == 0 ? a+b : a-b;
}
V<pair<P,P>> external(Circ x, Circ y) {
  V<pair<P,P>> v;
  if (x.s == y.s) {
    P tmp = unit(x.f-y.f)*x.s*dir(PI/2);
    v.eb(x.f+tmp,y.f+tmp);
    v.eb(x.f-tmp,y.f-tmp);
  } else {
    P p = (y.s*x.f-x.s*y.f)/(y.s-x.s);
    F0R(i,2) v.eb(tangent(p,x,i),tangent(p,y,i));
  }
  return v;
}
V<pair<P,P>> internal(Circ x, Circ y) {
  return external({x.f,-x.s},y); }
```

Circumcenter.h

Description: returns {circumcenter,circumradius}

"Circle.h" a2c6a6, 5 lines

```
Circ ccCenter(P a, P b, P c) {
  b -= a; c -= a;
  P res = b*c*(conj(c)-conj(b))/(b*conj(c)-conj(b)*c);
  return {a+res,abs(res)};
}
```

MinEnclosingCirc.h

Description: minimum enclosing circle

Time: expected $\mathcal{O}(N)$

"Circumcenter.h" 53963d, 13 lines

```
circ mec(vP ps) {
  shuffle(all(ps), rng);
  P o = ps[0]; T r = 0, EPS = 1+1e-8;
  F0R(i,sz(ps)) if (abs(o-ps[i]) > r*EPS) {
    o = ps[i], r = 0; // point is on MEC
    F0R(j,i) if (abs(o-ps[j]) > r*EPS) {
      o = (ps[i]+ps[j])/2, r = abs(o-ps[i]);
      F0R(k,j) if (abs(o-ps[k]) > r*EPS)
        tie(o,r) = ccCenter(ps[i],ps[j],ps[k]);
    }
  }
  return {o,r};
}
```

8.4 Misc

ClosestPair.h

Description: Line sweep to find two closest points .

Time: $\mathcal{O}(N \log N)$

"Point.h" 2b60fa, 17 lines

```
pair<P,P> solve(vP v) {
  pair<db,pair<P,P>> bes; bes.f = INF;
  set<P> S; int ind = 0;
  sort(all(v));
  F0R(i,sz(v)) {
    if (i && v[i] == v[i-1]) return {v[i],v[i]};
    for (; v[i].f-v[ind].f >= bes.f; ++ind)
      S.erase({v[ind].s,v[ind].f});
    for (auto it = S.ub({v[i].s-bes.f,INF});
      it != end(S) && it->f < v[i].s+bes.f; ++it) {
      P t = {it->s,it->f};
      ckmin(bes,{abs(t-v[i]),{t,v[i]}});
    }
    S.insert({v[i].s,v[i].f});
  }
  return bes.s;
}
```

DelaunayFast.h

Description: Fast Delaunay triangulation assuming no duplicates and not all points collinear (in latter case, result will be empty). Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in ccw order. Each circumcircle will contain none of the input points. If coordinates are ints at most B then `T` should be large enough to support ints on the order of B^4.

Time: $\mathcal{O}(N \log N)$

"Point.h" 7ff542, 82 lines

```
// using lll = ll; (if coords are < 2e4)
using lll = __int128;
// returns true if p strictly within circumcircle(a,b,c)
bool inCircle(P p, P a, P b, P c) {
  a -= p, b -= p, c -= p; // assert(cross(a,b,c)>0);
  lll x = (lll)abs2(a)*cross(b,c)+(lll)abs2(b)*cross(c,a)
      +(lll)abs2(c)*cross(a,b);
  return x*(cross(a,b,c)>0?1:-1) > 0;
}

P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point
using Q = struct Quad*;
struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};
Q makeEdge(P orig, P dest) {
  Q q[]{new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
      new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  F0R(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) { swap(a->o->rot->o, b->o->rot->o); swap(
  ↪a->o, b->o); }
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next()); splice(q->r(), b);
  return q;
}
pair<Q,Q> rec(const vP& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.bk);
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = cross(s[0], s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s)-half});
```

```
  tie(B, rb) = rec({sz(s)-half+all(s)});
  while ((cross(B->p,H(A)) < 0 && (A = A->next())) ||
       (cross(A->p,H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (inCircle(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  while (1) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && inCircle(H(RC), H(LC))))
      base = connect(RC, base->r());
    else base = connect(base->r(), LC->r());
  }
  return {ra, rb};
}
V<AR<P,3>> triangulate(vP pts) {
  sor(pts); assert(unique(all(pts)) == end(pts)); // no
    ↪duplicates
  if (sz(pts) < 2) return {};
  Q e = rec(pts).f; V<Q> q = {e};
  while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  int qi = 0; while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  V<AR<P,3>> ret(sz(pts)/3);
  F0R(i,sz(pts)) ret[i/3][i%3] = pts[i];
  return ret;
}
```

ManhattanMST.h

Description: Given N points, returns up to $4N$ edges which are guaranteed to contain a MST for graph with edge weights $w(p,q) = |p.x-q.x|+|p.y-q.y|$. Edges are in the form {dist, {src, dst}}.

Time: $\mathcal{O}(N \log N)$

"DSU.h" b7a3bd, 24 lines

```
// use standard MST algorithm on result to find final MST
V<pair<int,pi>> manhattanMst(vpi v) {
  vi id(sz(v)); iota(all(id),0);
  V<pair<int,pi>> ed;
  F0R(k,4) {
    sort(all(id),[&](int i, int j) {
      return v[i].f+v[i].s < v[j].f+v[j].s; });
    map<int,int> sweep; // find first octant neighbors
    each(i,id) { // those in sweep haven't found neighbor yet
      for (auto it = sweep.lb(-v[i].s);
        it != end(sweep); sweep.erase(it++)) {
        int j = it->s;
        pi d{v[i].f-v[j].f,v[i].s-v[j].s};if (d.s>d.f)break;
        ed.pb({d.f+d.s,{i,j}});
      }
      sweep[-v[i].s] = i;
    }
    each(p,v) {
      if (k&1) p.f *= -1;
      else swap(p.f,p.s);
    }
  }
  return ed;
}
```

8.5 3D

Point3D.h

Description: Basic 3D geometry.

"Point.h" d7d537, 82 lines

```
using P3 = AR<T,3>; using Tri = AR<P3,3>; using vP3 = V<P3>;
T abs2(const P3& x) {
  T sum = 0; F0R(i,3) sum += sq(x[i]);
  return sum; }
T abs(const P3& x) { return sqrt(abs2(x)); }

P3& operator+=(P3& l, const P3& r) { F0R(i,3) l[i] += r[i];
  return l; }
P3& operator-=(P3& l, const P3& r) { F0R(i,3) l[i] -= r[i];
  return l; }
P3& operator*=(P3& l, const T& r) { F0R(i,3) l[i] *= r;
  return l; }
P3& operator/=(P3& l, const T& r) { F0R(i,3) l[i] /= r;
  return l; }
P3 operator-(P3 l) { l *= -1; return l; }
P3 operator+(P3 l, const P3& r) { return l += r; }
P3 operator-(P3 l, const P3& r) { return l -= r; }
P3 operator*(P3 l, const T& r) { return l *= r; }
P3 operator*(const T& r, const P3& l) { return l*r; }
P3 operator/(P3 l, const T& r) { return l /= r; }

P3 unit(const P3& x) { return x/abs(x); }
T dot(const P3& a, const P3& b) {
  T sum = 0; F0R(i,3) sum += a[i]*b[i];
  return sum; }
P3 cross(const P3& a, const P3& b) {
  return {a[1]*b[2]-a[2]*b[1],a[2]*b[0]-a[0]*b[2],
      a[0]*b[1]-a[1]*b[0]}; }
P3 cross(const P3& a, const P3& b, const P3& c) {
  return cross(b-a,c-a); }
P3 perp(const P3& a, const P3& b, const P3& c) {
  return unit(cross(a,b,c)); }

bool isMult(const P3& a, const P3& b) { // for long longs
  P3 c = cross(a,b); F0R(i,sz(c)) if (c[i] != 0) return 0;
  return 1; }
bool collinear(const P3& a, const P3& b, const P3& c) {
  return isMult(b-a,c-a); }

T DC(const P3&a,const P3&b,const P3&c,const P3&p) {
  return dot(cross(a,b,c),p-a); }
bool coplanar(const P3&a,const P3&b,const P3&c,const P3&p) {
  return DC(a,b,c,p) == 0; }
bool op(const P3& a, const P3& b) {
  int ind = 0; // going in opposite directions?
  FOR(i,1,3) if (std::abs(a[i]*b[i])>std::abs(a[ind]*b[ind]))
    ind = i;
  return a[ind]*b[ind] < 0;
}
// coplanar points, b0 and b1 on opposite sides of a0-a1?
bool opSide(const P3&a,const P3&b,const P3&c,const P3&d) {
  return op(cross(a,b,c),cross(a,b,d)); }
// coplanar points, is a in Triangle b
bool inTri(const P3& a, const Tri& b) {
  F0R(i,3)if(opSide(b[i],b[(i+1)%3],b[(i+2)%3],a))return 0;
  return 1; }

// point-seg dist
T psDist(const P3&p,const P3&a,const P3&b) {
  if (dot(a-p,a-b) <= 0) return abs(a-p);
  if (dot(b-p,b-a) <= 0) return abs(b-p);
  return abs(cross(p,a,b))/abs(a-b);
}
// projection onto line
P3 foot(const P3& p, const P3& a, const P3& b) {
  P3 d = unit(b-a); return a+dot(p-a,d)*d; }
// rotate p about axis
P3 rotAxis(const P3& p, const P3& a, const P3& b, T theta) {
  P3 dz = unit(b-a), f = foot(p,a,b);
  P3 dx = p-f, dy = cross(dz,dx);
  return f+cos(theta)*dx+sin(theta)*dy;
}
// projection onto plane
P3 foot(const P3& a, const Tri& b) {
  P3 c = perp(b[0],b[1],b[2]);
  return a-c*(dot(a,c)-dot(b[0],c)); }
// line-plane intersection
P3 lpIntersect(const P3&a0,const P3&a1,const Tri&b) {
  P3 c = unit(cross(b[2]-b[0],b[1]-b[0]));
  T x = dot(a0,c)-dot(b[0],c), y = dot(a1,c)-dot(b[0],c);
  return (y*a0-x*a1)/(y-x);
}
```

Hull3D.h

Description: Incremental 3D convex hull where not all points are coplanar. Normals to returned faces point outwards. If coordinates are ints at most B then T should be large enough to support ints on the order of B^3. Changes order of points. The number of returned faces may depend on the random seed, because points that are on the boundary of the convex hull may or may not be included in the output.

Time: $\mathcal{O}\left(N^2\right)$, $\mathcal{O}(N \log N)$

5f4f0e, 91 lines

```
// using T = ll;
bool above(const P3&a,const P3&b,const P3&c,const P3&p) {
  return DC(a,b,c,p) > 0; } // is p strictly above plane
void prep(vP3& p) { // rearrange points such that
  shuffle(all(p),rng); // first four are not coplanar
  int dim = 1;
  FOR(i,1,sz(p))
    if (dim == 1) {
      if (p[0] != p[i]) swap(p[1],p[i]), ++dim;
    } else if (dim == 2) {
      if (!collinear(p[0],p[1],p[i]))
        swap(p[2],p[i]), ++dim;
    } else if (dim == 3) {
      if (!coplanar(p[0],p[1],p[2],p[i]))
        swap(p[3],p[i]), ++dim;
    }
  assert(dim == 4);
}

using F = AR<int,3>; // face
V<F> hull3d(vP3& p) {
  // s.t. first four points form tetra
  prep(p); int N = sz(p); V<F> hull; // triangle for each face
  auto ad = [&](int a, int b, int c) { hull.pb({a,b,c}); };
  // +new face to hull
  ad(0,1,2), ad(0,2,1); // initialize hull as first 3 points
  V<vb> in(N,vb(N)); // is zero before each iteration
  FOR(i,3,N) { // incremental construction
    V<F> def, HULL; swap(hull,HULL);
    // HULL now contains old hull
    auto ins = [&](int a, int b, int c) {
      if (in[b][a]) in[b][a] = 0; // kill reverse face
      else in[a][b] = 1, ad(a,b,c);
    };
    each(f,HULL) {
      if (above(p[f[0]],p[f[1]],p[f[2]],p[i]))
        F0R(j,3) ins(f[j],f[(j+1)%3],i);
        // recalc all faces s.t. point is above face
      else def.pb(f);
    }
    each(t,hull) if (in[t[0]][t[1]]) // edge exposed,
```

```
      in[t[0]][t[1]] = 0, def.pb(t); // add a new face
    swap(hull,def);
  }
  return hull;
}
V<F> hull3dFast(vP3& p) {
  prep(p); int N = sz(p); V<F> hull;
  vb active; // whether face is active
  V<vi> rvis; // points visible from each face
  V<AR<pi,3>> other; // other face adjacent to each edge of
    ↪face
  V<vi> vis(N); // faces visible from each point
  auto ad = [&](int a, int b, int c) {
    hull.pb({a,b,c}); active.pb(1); rvis.eb(); other.eb(); };
  auto ae = [&](int a, int b) { vis[b].pb(a), rvis[a].pb(b); };
  auto abv = [&](int a, int b) {
    F f=hull[a]; return above(p[f[0]],p[f[1]],p[f[2]],p[b]);};
  auto edge = [&](pi e) -> pi {
    return {hull[e.f][e.s],hull[e.f][(e.s+1)%3]}; };
  auto glue = [&](pi a, pi b) { // link two faces by an edge
    pi x = edge(a); assert(edge(b) == mp(x.s,x.f));
    other[a.f][a.s] = b, other[b.f][b.s] = a;
  }; // ensure face 0 is removed when i=3
  ad(0,1,2), ad(0,2,1); if (abv(1,3)) swap(p[1],p[2]);
  F0R(i,3) glue({0,i},{1,2-i});
  FOR(i,3,N) ae(abv(1,i),i); // coplanar points go in rvis[0]
  vi label(N,-1);
  FOR(i,3,N) { // incremental construction
    vi rem; each(t,vis[i]) if (active[t]) active[t]=0, rem.pb(t
      ↪);
    if (!sz(rem)) continue; // hull unchanged
    int st = -1;
    each(r,rem) F0R(j,3) {
      int o = other[r][j].f;
      if (active[o]) { // create new face!
        int a,b; tie(a,b) = edge({r,j}); ad(a,b,i); st = a;
        int cur = sz(rvis)-1; label[a] = cur;
        vi tmp; set_union(all(rvis[r]),all(rvis[o]),
                  back_inserter(tmp));
        // merge sorted vectors ignoring duplicates
        each(x,tmp) if (abv(cur,x)) ae(cur,x);
        glue({cur,0},other[r][j]); // glue old w/ new face
      }
    }
    for (int x = st, y; ; x = y) { // glue new faces together
      int X = label[x]; glue({X,1},{label[y=hull[X][1]],2});
      if (y == st) break;
    }
  }
  V<F> ans; F0R(i,sz(hull)) if (active[i]) ans.pb(hull[i]);
  return ans;
}
```

PolySaVol.h

Description: surface area and volume of polyhedron, normals to faces must point outwards

"Hull3D.h" 52fc2b, 8 lines

```
pair<T,T> SaVol(vP3 p, V<F> faces) {
  T s = 0, v = 0;
  each(i,faces) {
    P3 a = p[i[0]], b = p[i[1]], c = p[i[2]];
    s += abs(cross(a,b,c)); v += dot(cross(a,b),c);
  }
  return {s/2,v/6};
}
```

Strings (9)

9.1 Light

KMP.h

Description: f[i] is length of the longest proper suffix of the i-th prefix of s that is a prefix of s

Time: $\mathcal{O}(N)$

"template.hpp" 4538e4, 13 lines

```
vi kmp(str s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    for (f[i]=f[i-1];f[i]!=-1&&s[f[i]]!=s[i-1];)f[i]=f[f[i]];
    ++f[i]; }
  return f;
}
vi getOc(str a, str b) { // find occurrences of a in b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a))
    ret.pb(i-sz(a));
  return ret;
}
```

Z.h

Description: f[i] is the max len such that s.substr(0,len) == s.substr(i,len)

Time: $\mathcal{O}(N)$

"template.hpp" 566170, 15 lines

```
vi z(str s) {
  int N = sz(s), L = 1, R = 0; s += '#';
  vi ans(N); ans[0] = N;
  FOR(i,1,N) {
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
    while (s[i+ans[i]] == s[ans[i]]) ++ans[i];
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}
vi getPrefix(str a, str b) { // find prefixes of a in b
  vi t = z(a+b); t = vi(sz(a)+all(t));
  each(u,t) ckmin(u,sz(a));
  return t;
}
```

Manacher.h

Description: length of largest palindrome centered at each character of string and between every consecutive pair

Time: $\mathcal{O}(N)$

"template.hpp" fcc3f7, 13 lines

```
vi manacher(str _S) {
  str S = "@"; each(c,_S) S += c, S += "#";
  S.bk = '&';
  vi ans(sz(S)-1); int lo = 0, hi = 0;
  FOR(i,1,sz(S)-1) {
    if (i != 1) ans[i] = min(hi-i,ans[hi-i+lo]);
    while (S[i-ans[i]-1] == S[i+ans[i]+1]) ++ans[i];
    if (i+ans[i] > hi) lo = i-ans[i], hi = i+ans[i];
  }
  ans.erase(begin(ans));
  F0R(i,sz(ans)) if (i%2 == ans[i]%2) ++ans[i];
  return ans;
}
```

LyndonFactor.h

Description: A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string s is a factorization $s = w_1 w_2 \dots w_k$ where all strings w_i are simple and $w_1 \geq w_2 \geq \dots \geq w_k$. Min rotation gets min index i such that cyclic shift of s starting at i is minimum.

Time: $\mathcal{O}(N)$

"template.hpp" af38ba, 19 lines

```
vs duval(str s) {
  int N = sz(s); vs factors;
  for (int i = 0; i < N; ) {
    int j = i+1, k = i;
    for (; j < N && s[k] <= s[j]; ++j) {
      if (s[k] < s[j]) k = i;
      else ++k;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i,j-k));
  }
  return factors;
}
int minRotation(str s) {
  int N = sz(s); s += s;
  vs d = duval(s); int ind = 0, ans = 0;
  while (ans+sz(d[ind]) < N) ans += sz(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= sz(d[ind--]);
  return ans;
}
```

HashRange.h

Description: Polynomial hash for substrings with two bases.

"template.hpp" fc0b90, 27 lines

```
using H = AR<int,2>; // bases not too close to ends
H makeH(char c) { return {c,c}; }
uniform_int_distribution<int> BDIST(0.1*MOD,0.9*MOD);
const H base{BDIST(rng),BDIST(rng)};
H operator+(H l, H r) {
  F0R(i,2) if ((l[i] += r[i]) >= MOD) l[i] -= MOD;
  return l; }
H operator-(H l, H r) {
  F0R(i,2) if ((l[i] -= r[i]) < 0) l[i] += MOD;
  return l; }
H operator*(H l, H r) {
  F0R(i,2) l[i] = (ll)l[i]*r[i]%MOD;
  return l; }
// H& operator+=(H& l, H r) { return l = l+r; }
// H& operator-=(H& l, H r) { return l = l-r; }
// H& operator*=(H& l, H r) { return l = l*r; }

V<H> pows{{1,1}};
struct HashRange {
  str S; V<H> cum{{}};
  void add(char c) { S += c; cum.pb(base*cum.bk+makeH(c)); }
  void add(str s) { each(c,s) add(c); }
  void extend(int len) { while (sz(pows) <= len)
    pows.pb(base*pows.bk); }
  H hash(int l, int r) { int len = r+1-l; extend(len);
    return cum[r+1]-pows[len]*cum[l]; }
};
```

ReverseBW.h

Description: Used only once. Burrows-Wheeler Transform appends # to a string, sorts the rotations of the string in increasing order, and constructs a new string that contains the last character of each rotation. This function reverses the transform.

Time: $\mathcal{O}(N \log N)$

"template.hpp" e400d8, 7 lines

```
str reverseBW(str t) {
  vi nex(sz(t)); iota(all(nex),0);
  stable_sort(all(nex),[&t](int a,int b){return t[a]<t[b];});
```

```
  str ret; for (int i = nex[0]; i; )
    ret += t[i = nex[i]];
  return ret;
}
```

AhoCorasickFixed.h

Description: Aho-Corasick for fixed alphabet. For each prefix, stores link to max length suffix which is also a prefix. solve() returns a list with all appearances of the words of v in s
Time: $\mathcal{O}\left(N \sum\right)$

"template.hpp" 0e09a9, 44 lines

```
template<char A, size_t ASZ> struct ACfixed {
  struct Node { AR<int, ASZ> to; int link; vpi endsHere; bool
    ↪end; int terminal; };
  V<Node> d{{}};
  vi bfs;
  ACfixed(vector<str> v) { // Initialize with patterns
    F0R(i, sz(v)) ins(v[i], i);
    pushLinks();
  }
  void ins(str& s, int i) {
    int v = 0;
    each(C,s) {
      int c = C-A;
      if (!d[v].to[c]) d[v].to[c] = sz(d), d.eb();
      v = d[v].to[c];
    }
    d[v].end = true;
    d[v].endsHere.eb(i, sz(s));
  }
  void pushLinks() {
    d[0].link = -1;
    queue<int> q; q.push(0);
    while (sz(q)) {
      int v = q.ft; q.pop(); bfs.pb(v);
      d[v].terminal = d[v].link == -1 ? 0 : d[d[v].link].end ?
        ↪d[v].link : d[d[v].link].terminal;
      each(x, d[d[v].terminal].endsHere) d[v].endsHere.pb(x);
      F0R(c,ASZ) {
        int u = d[v].to[c]; if (!u) continue;
        d[u].link = d[v].link == -1 ? 0 : d[d[v].link].to[c];
        q.push(u);
      }
      if (v) F0R(c,ASZ) if (!d[v].to[c])
        d[v].to[c] = d[d[v].link].to[c];
    }
  }
  V<vi> solve(str s, int n) {
    V<vi> ans(n);
    int cur = 0;
    F0R(i, sz(s)) {
      cur = d[cur].to[s[i] - A];
      each(p, d[cur].endsHere) ans[p.f].pb(i - p.s + 1);
    }
    return ans;
  }
};
```

SuffixArray.h

Description: Sort suffixes. First element of `sa` is `sz(S)`, `isa` is the inverse of `sa`, and `lcp` stores the longest common prefix between every two consecutive elements of `sa`.
Time: $\mathcal{O}\left(N \log N\right)$

"RMQ.h" 27a566, 30 lines

```
struct SuffixArray {
  str S; int N; vi sa, isa, lcp;
  void init(str _S) { N = sz(S = _S)+1; genSa(); genLcp(); }
  void genSa() { // sa has size sz(S)+1, starts with sz(S)
    sa = isa = vi(N); sa[0] = N-1; iota(1+all(sa),0);
    sort(1+all(sa),[&](int a, int b) { return S[a] < S[b]; });
    FOR(i,1,N) { int a = sa[i-1], b = sa[i];
      isa[b] = i > 1 && S[a] == S[b] ? isa[a] : i; }
    for (int len = 1; len < N; len *= 2) { // currently sorted
      // by first len chars
      vi s(sa), is(isa), pos(N); iota(all(pos),0);
      each(t,s) {int T=t-len;if (T>=0) sa[pos[isa[T]]++] = T;}
      FOR(i,1,N) { int a = sa[i-1], b = sa[i];
        isa[b] = is[a]==is[b]&&is[a+len]==is[b+len]?isa[a]:i; }
    }
  }
  void genLcp() { // Kasai's Algo
    lcp = vi(N-1); int h = 0;
    F0R(b,N-1) { int a = sa[isa[b]-1];
      while (a+h < sz(S) && S[a+h] == S[b+h]) ++h;
      lcp[isa[b]-1] = h; if (h) h--; }
    R.init(lcp);
  }
  RMQ<int> R;
  int getLCP(int a, int b) { // lcp of suffixes starting at a,b
    if (a == b) return sz(S)-a;
    int l = isa[a], r = isa[b]; if (l > r) swap(l,r);
    return R.query(l,r-1);
  }
};
```

TandemRepeats.h

Description: Find all (i, p) such that `s.substr(i,p) == s.substr(i+p,p)`. No two intervals with the same period intersect or touch.
Usage: `tandem_repeats("aaabababa") // {{0, 1, 1}, {2, 5, 2}}`
Time: $\mathcal{O}\left(N \log N\right)$

"SuffixArray.h" 35e6eb, 13 lines

```
V<AR<int,3>> tandem_repeats(str s) {
  int N = sz(s); SuffixArray A,B;
  A.init(s); reverse(all(s)); B.init(s);
  V<AR<int,3>> runs;
  for (int p = 1; 2*p <= N; ++p) { // do in O(N/p) for period p
    for (int i = 0, lst = -1; i+p <= N; i += p) {
      int l = i-B.getLCP(N-i-p,N-i), r = i-p+A.getLCP(i,i+p);
      if (l > r || l == lst) continue;
      runs.pb({lst = l,r,p}); // for each i in [l,r],
    } // s.substr(i,p) == s.substr(i+p,p)
  }
  return runs;
}
```

9.2 Heavy

PalTree.h

Description: Used infrequently. Palindromic tree computes number of occurrences of each palindrome within string. `ans[i][0]` stores min even x such that the prefix $s[1..i]$ can be split into exactly x palindromes, `ans[i][1]` does the same for odd x.
Time: $\mathcal{O}\left(N \sum\right)$ for addChar, $\mathcal{O}\left(N \log N\right)$ for updAns

"template.hpp" 8a7d31, 41 lines

```
struct PalTree {
  static const int ASZ = 26;
  struct node {
    AR<int,ASZ> to = AR<int,ASZ>();
    int len, link, oc = 0; // # occurrences of pal
    int slink = 0, diff = 0;
    AR<int,2> seriesAns;
    node(int _len, int _link) : len(_len), link(_link) {}
  };
  str s = "@"; V<AR<int,2>> ans = {{0,MOD}};
  V<node> d = {{0,1},{-1,0}}; // dummy pals of len 0,-1
  int last = 1;
  int getLink(int v) {
    while (s[sz(s)-d[v].len-2] != s.bk) v = d[v].link;
    return v;
  }
  void updAns() { // serial path has O(log n) vertices
    ans.pb({MOD,MOD});
    for (int v = last; d[v].len > 0; v = d[v].slink) {
      d[v].seriesAns=ans[sz(s)-1-d[d[v].slink].len-d[v].diff];
      if (d[v].diff == d[d[v].link].diff)
        F0R(i,2) ckmin(d[v].seriesAns[i],
            d[d[v].link].seriesAns[i]);
      // start of previous oc of link[v]=start of last oc of v
      F0R(i,2) ckmin(ans.bk[i],d[v].seriesAns[i^1]+1);
    }
  }
  void addChar(char C) {
    s += C; int c = C-'a'; last = getLink(last);
    if (!d[last].to[c]) {
      d.eb(d[last].len+2,d[getLink(d[last].link)].to[c]);
      d[last].to[c] = sz(d)-1;
      auto& z = d.bk; z.diff = z.len-d[z.link].len;
      z.slink = z.diff == d[z.link].diff
        ? d[z.link].slink : z.link;
    } // max suf with different dif
    last = d[last].to[c]; ++d[last].oc;
    updAns();
  }
  void numOc() { ROF(i,2,sz(d)) d[d[i].link].oc += d[i].oc; }
};
```

SuffixAutomaton.h

Description: Used infrequently. Constructs minimal deterministic finite automaton (DFA) that recognizes all suffixes of a string. `len` corresponds to the maximum length of a string in the equivalence class, `pos` corresponds to the first ending position of such a string, `lnk` corresponds to the longest suffix that is in a different class. Suffix links correspond to suffix tree of the reversed string!
Time: $\mathcal{O}\left(N \log \sum\right)$

"template.hpp" 76a99a, 67 lines

```
struct SuffixAutomaton {
  int N = 1; vi lnk{-1}, len{0}, pos{-1}; // suffix link,
  // max length of state, last pos of first occurrence of state
  V<map<char,int>> nex{1}; V<bool> isClone{0};
  // transitions, cloned -> not terminal state
  V<vi> iLnk; // inverse links
  int add(int p, char c) { // ~p nonzero if p != -1
    auto getNex = [&]() {
      if (p == -1) return 0;
      int q = nex[p][c]; if (len[p]+1 == len[q]) return q;
      int clone = N++; lnk.pb(lnk[q]); lnk[q] = clone;
      len.pb(len[p]+1), nex.pb(nex[q]),
      pos.pb(pos[q]), isClone.pb(1);
      for (; ~p && nex[p][c] == q; p = lnk[p]) nex[p][c]=clone;
      return clone;
    };
    // if (nex[p].count(c)) return getNex();
    // ^ need if adding > 1 string
    int cur = N++; // make new state
    lnk.eb(), len.pb(len[p]+1), nex.eb(),
    pos.pb(pos[p]+1), isClone.pb(0);
    for (; ~p && !nex[p].count(c); p = lnk[p]) nex[p][c] = cur;
    int x = getNex(); lnk[cur] = x; return cur;
  }
  void init(str s) { int p = 0; each(x,s) p = add(p,x); }
  // inverse links
  void genIlnk() {iLnk.rsz(N);FOR(v,1,N)iLnk[lnk[v]].pb(v);}
  // APPLICATIONS
  void getAllOccur(vi& oc, int v) {
    if (!isClone[v]) oc.pb(pos[v]); // terminal position
```

```
    each(u,iLnk[v]) getAllOccur(oc,u); }
  vi allOccur(str s) { // get all occurrences of s in automaton
    int cur = 0;
    each(x,s) {
      if (!nex[cur].count(x)) return {};
      cur = nex[cur][x]; }
    // convert end pos -> start pos
    vi oc; getAllOccur(oc,cur); each(t,oc) t += 1-sz(s);
    sort(all(oc)); return oc;
  }
  vl distinct;
  ll getDistinct(int x) {
    // # distinct strings starting at state x
    if (distinct[x]) return distinct[x];
    distinct[x]=1;each(y,nex[x]) distinct[x]+=getDistinct(y.s);
    return distinct[x]; }
  ll numDistinct() { // # distinct substrings including empty
    distinct.rsz(N); return getDistinct(0); }
  ll numDistinct2() { // assert(numDistinct()==numDistinct2());
    ll ans = 1; FOR(i,1,N) ans += len[i]-len[lnk[i]];
    return ans; }
};

// // SuffixAutomaton S;
// // vi sa; str s;
// // void dfs(int x) {
// //   if (!S.isClone[x]) sa.pb(sz(s)-1-S.pos[x]);
// //   V<pair<char,int>> chr;
// //   each(t,S.iLnk[x]) chr.pb({s[S.pos[t]-S.len[x]],t});
// //   sort(all(chr)); each(t,chr) dfs(t.s);
// // }

// // int main() {
// //   re(s); reverse(all(s));
// //   S.init(s); S.genIlnk();
// //   dfs(0); ps(sa); // generating suffix array for s
// // }
```

SuffixTree.h

Description: Used infrequently. Ukkonen's algorithm for suffix tree. Longest non-unique suffix of `s` has length `len[p]+lef` after each call to `add` terminates. Each iteration of loop within `add` decreases this quantity by one.

Time: $\mathcal{O}\left(N \log \sum\right)$

"template.hpp" 39751c, 51 lines

```
struct SuffixTree {
  str s; int N = 0;
  vi pos, len, lnk; V<map<char,int>> to;
  int make(int POS, int LEN) { // lnk[x] is meaningful when
    // x!=0 and len[x] != MOD
    pos.pb(POS);len.pb(LEN);lnk.pb(-1);to.eb();return N++; }
  void add(int& p, int& lef, char c) { // longest
    // non-unique suffix is at node p with lef extra chars
    s += c; ++lef; int lst = 0;
    for (;lef;p?p=lnk[p]:lef--) { // if p != root then lnk[p]
      // must be defined
      while (lef>1 && lef>len[to[p][s[sz(s)-lef]]])
        p = to[p][s[sz(s)-lef]], lef -= len[p];
      // traverse edges of suffix tree while you can
      char e = s[sz(s)-lef]; int& q = to[p][e];
      // next edge of suffix tree
      if (!q) q = make(sz(s)-lef,MOD), lnk[lst] = p, lst = 0;
      // make new edge
      else {
        char t = s[pos[q]+lef-1];
        if (t == c) { lnk[lst] = p; return; } // suffix not
          ↪unique
        int u = make(pos[q],lef-1);
        // new node for current suffix-1, define its link
        to[u][c] = make(sz(s)-1,MOD); to[u][t] = q;
        // new, old nodes
        pos[q] += lef-1; if (len[q] != MOD) len[q] -= lef-1;
        q = u, lnk[lst] = u, lst = u;
      }
    }
  }
  void init(str _s) {
    make(-1,0); int p = 0, lef = 0;
    each(c,_s) add(p,lef,c);
    add(p,lef,'$'); s.pop_back(); // terminal char
  }
  int maxPre(str x) { // max prefix of x which is substring
    for (int p = 0, ind = 0;;) {
      if (ind == sz(x) || !to[p].count(x[ind])) return ind;
      p = to[p][x[ind]];
      F0R(i,len[p]) {
        if (ind == sz(x) || x[ind] != s[pos[p]+i]) return ind;
        ind ++;
      }
    }
  }
  vi sa; // generate suffix array
  void genSa(int x = 0, int Len = 0) {
    if (!sz(to[x])) sa.pb(pos[x]-Len); // found terminal node
    else each(t,to[x]) genSa(t.s,Len+len[x]);
  }
};
```

Various (10)

10.1 Dynamic programming

10.1.1 Knuth DP

When doing DP on intervals:
$a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j,

- one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$.
- This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$.
- Consider also: Monotone queues, ternary search.

10.1.2 Divide and Conquer DP

If the recurrence of the dynamic programming is of the form

$$dp(i,j) = \min_{0 \leq k \leq j} dp(i-1,k-1) + C(k,j)$$

where $C(k,j)$ is a cost function and $dp(i,j) = 0$ for $j < 0$ and $opt(i,j) \leq opt(i,j+1)$ for all i,j, then we can apply Divide and Conquer DP.

DnC.h

Description: Solves Divide and Conquer DP

Time: $\mathcal{O}\left(NM \log M\right)$

"template.hpp" dc7557, 23 lines

```
using T = ll;
V<T> dp_before, dp_cur;
T C(int k, int j); // Cost function
// Computes dp_cur[l] ... dp_cur[r]
void compute(int l, int r, int optl, int optr) {
    if (l > r) return;
    int m = (l + r) >> 1;
    pair<T,int> best = mp(numeric_limits<T>::max(), -1);
    FOR(k,optl,min(m,optr+1))
        ckmin(best,mp((k?dp_before[k-1]:T(0))+C(k, m), k));
    dp_cur[m] = best.f;
    int opt = best.s;
    compute(l,m-1,optl,opt);
    compute(m+1,r,opt,optr);
}

T solve(int N, int M) {
    dp_before.assign(M,T(0));
    dp_cur.assign(M,T(0));
    F0R(i,M) dp_before[i] = C(0,i);
    FOR(i,1,N) compute(0,M-1,0,M-1), dp_before = dp_cur;
    return dp_before[M-1];
}
```

10.1.3 Line Container DP

If the recurrence of the dynamic programming is of the form

$$dp(i) = \max_{j<i} g(i) \cdot h(j) + dp(j)$$

We can define $y = dp(i), x = g(i), m_j = h(j), c_j = d(j)$ and the problem reduces to finding the minimum y that we can obtain with all the lines $y = m_j \cdot x + c_j$. We can solve that problem using a Li Chao Tree (see Line Container in Data Structures).

10.2 Misc

LIS.h

Description: Returns indices of a longest increasing sequence

Time: $\mathcal{O}\left(N \log N\right)$

"template.hpp" 35d0e7, 14 lines

```
tcT> vi lis(V<T> const& S) {
    if (!sz(S)) return {};
    vi prev(sz(S)); V<pair<T,int>> res;
    F0R(i,sz(S)) {
        // Change 0 to i for non-decreasing
        auto it = lb(all(res), mp(S[i],0));
        if (it == end(res)) res.eb(), it = res.end() - 1;
        *it = mp(S[i],i);
        prev[i] = it == begin(res) ? 0 : (it-1)->s;
    }
    int L = sz(res), cur = res.bk.s; vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
}
```

CircularLCS.h

Description: Used only twice. For strs A, B calculates longest common subsequence of A with all rotations of B

Time: $\mathcal{O}\left(|A| \cdot |B|\right)$

db21cf, 26 lines

```
int circular_lcs(str A, str B) {
  B += B;
  int max_lcs = 0;
  V<vb> dif_left(sz(A)+1,vb(sz(B)+1)), dif_up(sz(A)+1,vb(sz(B)
    ↪+1));
  auto recalc = [&](int x, int y) { assert(x && y);
    int res = (A.at(x-1) == B.at(y-1)) |
      dif_up[x][y-1] | dif_left[x-1][y];
    dif_left[x][y] = res-dif_up[x][y-1];
```

```
    dif_up[x][y]   = res-dif_left[x-1][y];
  };
  FOR(i,1,sz(A)+1) FOR(j,1,sz(B)+1) recalc(i,j);
  F0R(j,sz(B)/2) {
    // 1. zero out dp[.][j], update dif_left and dif_right
    if (j) for (int x = 1, y = j; x <= sz(A) && y <= sz(B); ) {
      int pre_up = dif_up[x][y];
      if (y == j) dif_up[x][y] = 0;
      else recalc(x,y);
      (pre_up == dif_up[x][y]) ? ++x : ++y;
    }
    // 2. calculate LCS(A[0:sz(A)),B[j:j+sz(B)/2))
    int cur_lcs = 0;
    FOR(x,1,sz(A)+1) cur_lcs += dif_up[x][j+sz(B)/2];
    ckmax(max_lcs,cur_lcs);
  }
  return max_lcs;
}
```

SMAWK.h

Description: Given negation of totally monotone matrix with entries of type `D`, find indices of row maxima (their indices increase for every submatrix). If tie, take lesser index. `f` returns matrix entry at (r, c) in $O(1)$. Use in place of divide & conquer to remove a log factor.

Time: $\mathcal{O}(R + C)$, can be reduced to $\mathcal{O}(C(1 + \log R/C))$ evaluations of `f`

0a9abb, 25 lines

```
template<class F, class D=ll> vi smawk (F f, vi x, vi y) {
  vi ans(sz(x),-1); // x = rows, y = cols
  #define upd() if (ans[i] == -1 || w > mx) ans[i] = c, mx = w
  if (min(sz(x),sz(y)) <= 8) {
    F0R(i,sz(x)) { int r = x[i]; D mx;
      each(c,y) { D w = f(r,c); upd(); } }
    return ans;
  }
  if (sz(x) < sz(y)) { // reduce subset of cols to consider
    vi Y; each(c,y) {
      for (;sz(Y);Y.pop_back()) { int X = x[sz(Y)-1];
        if (f(X,Y.bk) >= f(X,c)) break; }
      if (sz(Y) < sz(x)) Y.pb(c);
    } y = Y;
  } // recurse on half the rows
  vi X; for (int i = 1; i < sz(x); i += 2) X.pb(x[i]);
  vi ANS = smawk(f,X,y); F0R(i,sz(ANS)) ans[2*i+1] = ANS[i];
  for (int i = 0, k = 0; i < sz(x); i += 2){
    int to = i+1 < sz(ans) ? ans[i+1] : y.bk; D mx;
    for(int r = x[i];;++k) {
      int c = y[k]; D w = f(r,c); upd();
      if (c == to) break; }
  }
  return ans;
};
```

TernarySearch.h

Description: solve for min on functions which are strictly decreasing then strictly increasing

Time: $\mathcal{O}(\log_3 N)$

"template.hpp" f8ad85, 7 lines

```
db eval(db x);

db ternary(db l, db r) {
    if (abs(r-l) <= 1e-9) return (l+r)/2;
    db l1 = (2*l+r)/3, r1 = (l+2*r)/3;
    return eval(l1) < eval(r1) ? ternary(l,r1) : ternary(l1,r);
}
```

Calendar.h

Description: Years between 1 and 9999. Assumes Gregorian Calendar

5536b8, 12 lines

```
int dateToInt(int d, int m, int y) {
    return 1461*(y+4800+(m-14)/12)/4+367*(m-2-(m-14)/12*12)
      ↪/12-3*((y+4900+(m-14)/12)/100)/4+d-32075;
}
void intToDate(int jd, int& d, int& m, int& y) {
    int x, n, i, j;
    x = jd + 68569; n = 4*x/146097;
    x -= (146097*n+3)/4;
    i = (4000*(x+1))/1461001;
    x -= 1461*i/4-31; j = 80*x/2447;
    d = x-2447*j/80; x = j/11;
    m = j+2-12*x; y = 100*(n-49)+i+x;
}
```

10.3 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

10.4 Optimization tricks

10.4.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `F0R(b,k) F0R(i,1<<K) if (i&1<<b) D[i] += D[i^(1<<b)];` computes all sums of subsets.

10.4.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines. Also consider older `#pragma GCC target ("sse4")`.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).