CPSC 340 Practice Exam

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Problems From Homework

1. Give DFA's accepting the following languages over the alphabet {0,1}
   (a) The set of all strings ending in 00.
   (b) The set of all strings with three consecutive 0's.
   (c) the set of strings with 011 as a sub string.

2. Let A be a DFA and a a particular input symbol of A, such that for all states a of A we have $\delta(q, a) = q$.
   (a) Show by induction on n that for all $n \geq 0, \delta(q, a^n) = 1$, where a^n is the string consisting of n a's.
   (b) Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^a \cap L(A) = \emptyset$

3. Consider the DFA with the following transition table:

	0	1
$\rightarrow A$	A	B
$*B$	B	A

   Informally describe the language accepted by this DFA and prove by induction on the length of an input string that your description is correct Hint: When setting up the inductive hypothesis it is wise to make a statement about what inputs get you to each state, not just what inputs get you to the accepting state.

4. write a program to solve the following problem:
   given an NFA $M = (Q, \Sigma, \delta, q0, F)$ and an input string of M w, you are able to determine if w is in L(M)

5. Convert to a DFA the following NFA:

$\rightarrow p$	p,q	p
q	{r}	{r}
r	{s}	$\emptyset$
$*s$	{s}	{s}

6. Give a nondeterministic finite automata to accept the following language. Try to take advantage of nondeterminism as much as possible.

   The set of strings over the alphabet {0,1,..,9} such that the final digit has appeared before.

7. Design NFA's to recognize the following sets of stings.
   (a) abc, abd, and aacd. Assume the alphabet is {a,b,c,d}.
   (b) 0101, 101, 011.
   (c) ab, bc, and ca.

8. Convert each of your NFA's from question 7 to DFA's

9. Here is a transition table for a DFA:

	0	1
$\rightarrow q_2$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

Construct the transition diagram for the DFA and give a regular expression for its language by eliminating state

10. Convert the following DFA to a regular expression, using the state-elimination technique.

	0	1
$\rightarrow * p$	s	p
q	p	s
r	r	q
s	q	r

11. Prove that the following are not a regular languages
    (a) $\{0^n 1 0^n | n \geq 1\}$
    (b) $\{0^n | n$ is a perfect square$\}$

12. (a) Construct a right-linear grammar for the language denoted by regex $((aab^*ba)^*)$
    (b) Construct an NFA or ϵ-NFA that accepts the language generated by the grammar:

$$\text{G} = (\{\text{S,A,B}\},\{\text{a,b}\},\text{P,S}) whereP:$$
$$\rightarrow abA$$
$$A \rightarrow baB$$
$$B \rightarrow aA|bb$$

(c) Design context-free grammars for the following languages:
    (a) The set $\{0^n 1^n | n \geq 1\}$
    (b) The set $\{a^i b^j c^k | i \neq j \text{ or } j \neq k\}$

(c) The following grammar generates the language of regular expression $0^*1(0+1)^*$:

$$S \rightarrow A1B$$
$$A \rightarrow 0A|\epsilon$$
$$B \rightarrow 0B|1B|\epsilon$$

Give the leftmost derivation of the following string:

00101

(d) Design a PDA to accept the following language. You may accept either by final state or by empty stack, whichever is more convenient:

The set of all strings of 0's and 1's with an equal number of 0's and 1's

(e) Design a PDA to accept each of the following language

$$\{a^i b^j c^k | i = j \text{ or } j = k\}$$

(f) PDA P $= (\{q_0, q_1, q_2, q_3, f\}, \{Z_0, A, B\}, \delta, q_0, Z_0, \{f\})$ has the following rules defining δ:

$$\begin{array}{lll}
\delta(q_0, a, Z_0) = (q_1, AAZ_0) & \delta(q_0, b, Z_0) = (q_2, BZ_0) & \delta(q_0, \epsilon, Z_0) = (f, \epsilon) \\
\delta(q_1, a, A) = (q_1, AAA) & \delta(q_1, b, A) = (q_1, \epsilon) & \delta(q_1, \epsilon, Z_0) = (q_0, Z_0) \\
\delta(q_2, a, B) = (q_3, \epsilon) & \delta(q_2, b, B) = (q_2, BB) & \delta(q_2, \epsilon, Z_0) = (q_0, Z_0) \\
\delta(q_3, \epsilon, B) = (q_2, \epsilon) & \delta(q_3, \epsilon, Z_0) = (q_1, AZ_0) &
\end{array}$$

Note that, since each of the sets above has only one choice of move, we have omitted the set brackets from each set of rules.

Give an execution trace (sequence of ID's) showing that string *bab* is in L(P)

(g) Use the CFL pumping lemma to show each of these languages not to be context-free

(a) $\{a^i b^j c^k | i < j < k\}$

(b) $\{0^i 1^j | j = i^2\}$

(c) Given the turing machine:

State	0	1	X	Y	B
q_0	(q_1, X, R)	—	—	(q_3, Y, R)	—
q_1	$(q_1, 0, R)$	(q_2, Y, L)	—	(q_1, Y, R)	—
q_2	$(q_2, 0, L)$	—	(q_0, X, R)	(q_2, Y, L)	—
q_3	—	—	—	(q_3, Y, R)	(q_4, B, R)
q_4	—	—	—	—	—

show the Id of the Turing machine if the input tape contains *00*

(d) Consider the Turning machine.

$$M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

informally but clearly describe the language L(M) if δ consists of the following set of rules:

$$\delta(q_0, 0) = (q_1, q, R); \delta(q_1, 1) = (q_0, 0, R); \delta(q_1, B) = (q_f, B, R)$$

(e) Design the Turing machine for the following language

The set of strings with an equal number of 0's and 1's

Given In Exam Prep

1. the turing machine $(\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, B\}, \delta, B, \{q_2\})$ has the following transitions and no others

$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_2, 0, L)$$
$$\delta(q_2, 1) = (q_0, 1, R)$$

starting with ID $q_0 0110$ show the entire sequence of ID's entered by this TM until it halts

2. Is the language $\{a^n b^n c^{n-1} | n \geq 1\}$ context free? prove it.
3. Is the language $\{a^n b^n c^{n-1} | n \geq 1\}$ regular? prove it.
4. Design a PDA to accept the language $\{ww^R | w \in \{a, b\}^*\}$
5. Is the language $L = \{abw | w \in \{a, b\}^*\}$ regular? if it is find a DFA to accept it, otherwise prove it's not.
6. design a context-free grammar $G = (S, A, B, a, b, P, S)$ where P is the set of productions:

$$S \to AB|C$$
$$\text{A} \to aAb|ab$$
$$\text{B} \to cBd|cd$$
$$\text{C} \to aCd|aDd$$
$$\text{D} \to bDc|bc$$

the grammar is ambiguous. show in particular that the string aabbbccdd has two

(a) parse trees

(b) leftmost derivations

(c) rightmost derivations

7. Consider the grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$, where P is the set of productions:

$$S \to abA$$
$$\text{A} \to baB$$
$$\text{B} \to aA|bb$$

let L=L(G). is L a regular language? if L is a regular language, construct a DFA accepting L. if it is not why?

8. give a DFA accepting the language $L = \{w|w \in \{a, b\}^*|$ w ends in ab$\}$

9. suppose $L_1 = \{a^{2n}|n \geq 1\}$ $L_2 = \{b^n|n \geq 1\}$, prove or disprove $L_1L_2 = \{a^{2n}b^n\}$

10. L_1 and L_2 are recursivly enumarable. is $L_1 \cap L_2$ RW? Why?

Last Years Final

1. Consider the grammar $G = (\{S\}, \{a, b, +, *\}, P, S)$ where P:

$$S \to S + S$$
$$\text{S} \to a$$
$$\text{S} \to S * S$$
$$\text{S} \to b$$

show that "a+b*a" has two

(a) parse trees

(b) leftmost derivations

(c) rightmost derivations

2. Construct a regular grammar for $b^*a(bba^*ab)^*$

3. Design PDA to accept the language $\{w|w \in \{a, b, c, d\}^*$ and $n_a(w) = n_b(w) + n_c(w) + 2\}$ where $n_a(w)$ is the number of a's in w

4. Is there a program H with input (x,y) with the definition:

$$\{x(y) \downarrow \text{return no, } x(y) \uparrow \text{return yes}\}$$

5. Is the language $L = \{a^{i_1}b^jc^k|i \leq k - j, i, j, k \geq 0\}$ regular? Prove or disprove.

6. Design CFG for $\{a^{n+2}b^n|n \geq 1\} \cup \{a^{2n}b^{n+1}|n \geq 0\}$

7. Grammar G has P:

$$S \to Aab|baB$$
$$\text{A} \to bB|aA|\epsilon$$
$$\text{B} \to Bb|aB|\epsilon$$

(a) Is G context free?

(b) Is G regular?

(c) Is L(G) context free

(d) is L(G) regular?