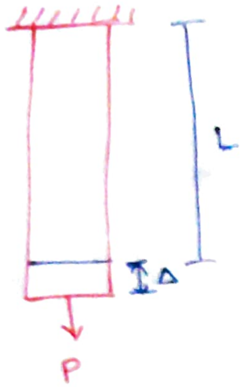
Flexibility Matrix

Flexibility is defined as the displacement due to unit force. Unit: (m/kN)



Consider a prismatic bar 'AB', subjected to axial force 'P'. The bar will deform, and the deformation 'Δ' is defined by the elementary equation of mechanics

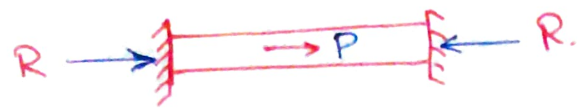
Geometry and Elastic Constants (m/kN)

$$\Delta = P \times \frac{L}{AE}$$

$$\Delta = f \times P$$

where f is flexibility of member.

Similarly, consider a axial system of bar, where external force is 'P' and the reaction will be 'R' at the ends.

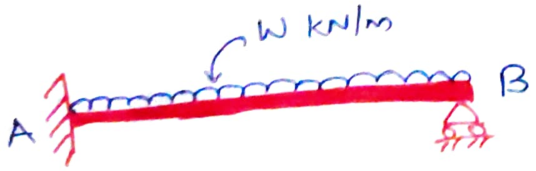


Then the relationship with the means of flexibility will be $\boxed{\Delta = f \times R}$

Hence, the equation is force displacement relation where displacement is function of force.

For Example

Consider a propped cantilever 'AB' with fixed support at 'A' and propped at 'B', subjected to external UDL 'w' kN/m.



As said In flexibility method, the forces are primary unknowns.

Determine the static indeterminacy.

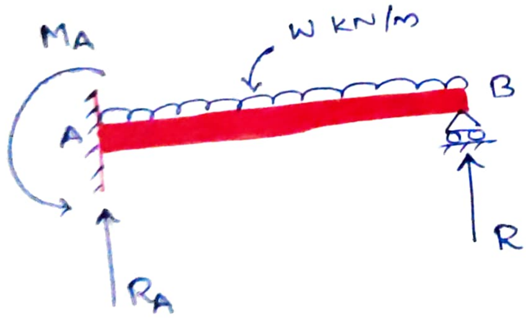
$$D_s = r - e$$

where r is total no. of reaction

e is equilibrium condition $\Rightarrow$ 2 ($\Sigma F = 0$
$\Sigma H = 0$)

As for our propped cantilever, the total no. of reactions are 3. (2 for fixed support and 1 at roller support).

$\therefore \; D_s = 3 - 2 = 1$



The static indeterminacy ($D_s = 1$) indicates the redundants i.e extra ~~reaction~~. unknown.

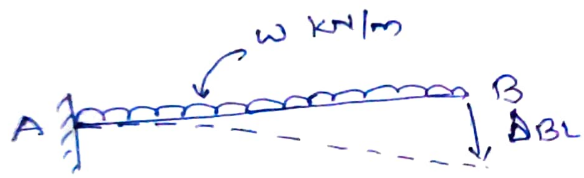
Next, identify the extra unknown (as $D_s = 1$).

Let us choose the propped reaction is the redundant

Now, by removing the redundant the structure remaining is called basic determinat structure

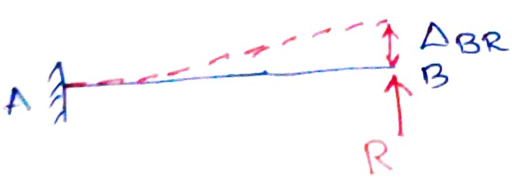
Redundant Released
(Basic Determinate structure)

Considering the basic determinate structure (BDS) apply all external loads.



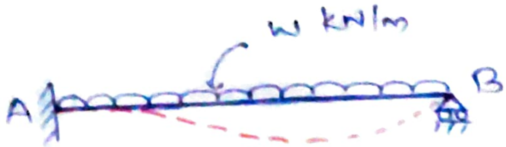
BDS - All external Loads

Again consider the BDS and apply only Redundant force



BDS - Redundant only

So, if the principal of superposition is valied then BDS with external load + BDS with Redundant then it will be original structure



$\Delta_B = \Delta_{BL} + \Delta_{BR} = 0$

If i consider the vertical reaction at the propped support is my redundant, then in that direction the displacement when external load are applied and the displacement when redundant is applied

$\Delta_B \Rightarrow$ Resultant displacement

$\Delta_{BL} \Rightarrow$ Displacement due to load

$\Delta_{BL} \Rightarrow$ Displacement due to redundant

$$\Delta_B = \Delta_{BL} + \Delta_{BR} = 0$$

This condition is known as compatibility condition of displacement.

If the support at B is unyielding then Δ_B becomes zero.

In this compatibility condition equation Δ_{BL} is known because of the external loads, but Δ_{BR} is the unknown, it is need to be determined.

Note:

① As Redundant forces are primary unknowns, it is also called as force method.

② Degree of static indeterminacy = No. of Redundants

③ BDS is obtained by releasing the selected redundants.

④ BDS must be stable. Cantilever / Simply supported / overhang / determinate hybrid structures are the forms of BDS.

⑤ Solution of problem to get redundants is obtained by compatibility condition of displacements

$$\Delta_i = \Delta_{iL} + \Delta_{iR}$$

Degree of static Indeterminacy

① Indeterminate Beams and Frames

$D_s = r - e$ (For general beams and frames)

$D_s = r - e - \Sigma(m_h - 1)$ (For hybrid beams and frames)

When any internal pin is involved

where,

r = No. of reactions (Horizontal reactions in beams are neglected)

e = No. of equilibrium conditions

= 2 for beams and 3 for plane frames

m_h = No. of members meeting at internal pin.

② Indeterminate Truss:

$D_{se} = r - e$ (external)

$D_{si} = m - (2j - 3)$ (internal)

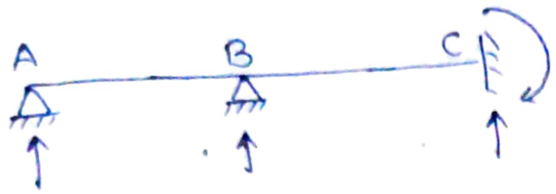
m = no of members

j = no. of joints

Example 1 :

Target 1. To find D_s

$D_s = r - e$



$D_s = 4 - 2 = 2$

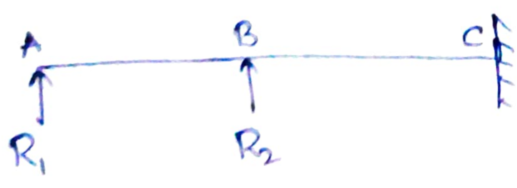
Thus No. of redundants = 2

So out of these 4 reactions we have release 2 redundants

Target 2: Identify Redundants and obtain BDS

Option 1: BDS is cantilever:

~~If~~ In order to cantilever be the BDS, the redundants at A and B are released.

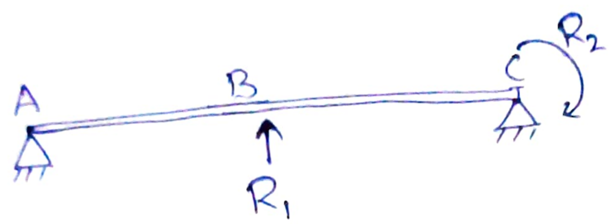


$R_1 \Rightarrow$ Coordinate axis which represents reaction at A

$R_2 \Rightarrow$ Coordinate axis which represents reaction at B

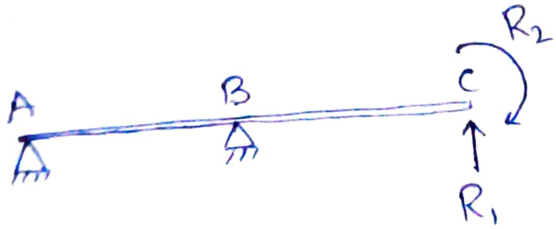
Option 2: BDS is Simply Supported Beam.

In this option, the redundants to be selected are at B and momentary reaction at C. Then the remaining structure will be



Option 3: BDS is overhang.

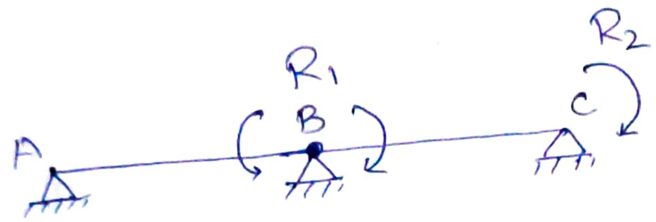
If this option is considered, then the two reactions at supported C is choosen as redundants.



Option 4: BDS is hybrid element.

In order to make the redundant force as internal moments, then a pin must be inserted at the internal hinge.

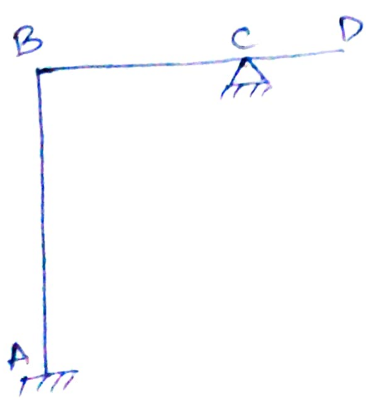
As, if we insert a pin, an internal moment at at that point will be released.



For internal redundant, we have to show the redundant in pair.

Example 2

- Consider a frame ABCD fixed at A and hinged at C.



Target 1 : To find D_s

$$D_s = r - e$$

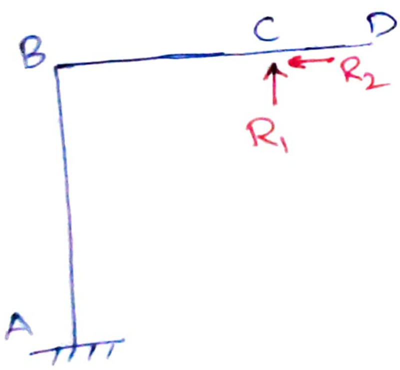
$$D_s = 5 - 3 = 2$$

Thus No. of redundants = 2

That means we have to find 2 unknown.

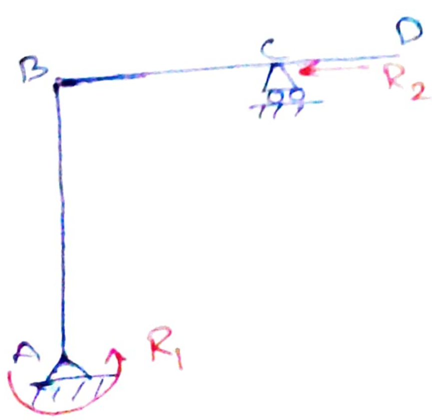
Target 2 : Identify Redundants and obtain BDS

option 1: BDS is cantilever



To maintain the cantilever let keep support A as it is and the two reactions at C that identified as redundant

Option 2: BDS is overhanging:



In case of over hang, the moment reaction at A and horizontal reaction at C are to be released

Unit load Method for Displacements

After learning how to find degree of indeterminacy and how to obtain BDs by releasing the redundant. The next thing is to understand how to find the displacement.

Unit load method for displacements

→ The method is based on principal of virtual work.

→ Best suitable to find slope and deflections of determinate structures (BDS) subjected to external loads and unit loads.

→ Displacement due to external Loads at any point 'i' is given by

$$\Delta_{iL} = \sum \int \frac{M \cdot m_i}{EI} \, dx$$

→ Displacement due to any point 'i' due to unit load at j is given by (Called as flexibility coefficient).

$$\delta_{ij} = \sum \int \frac{m_i \cdot m_j}{EI} \, dx$$

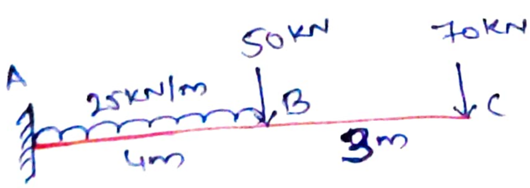
Where M are the equations of BM in BDS subjected to external loads.

m are the equations of BM in BDS subjected to unit loads.

Example:

Consider a ~~cat~~ Cantilever beam.

objective → Find Δ_{CL}



Step 1: Draw following figures of BDS and find reactions

(i) M diagram: BDS - all external loads

(ii) m diagram: BDS — unit load/moment at point where displacement/rotation is required

Step2: BM equation table

Take section in each segment and write BM equation of each segment in tabular form.

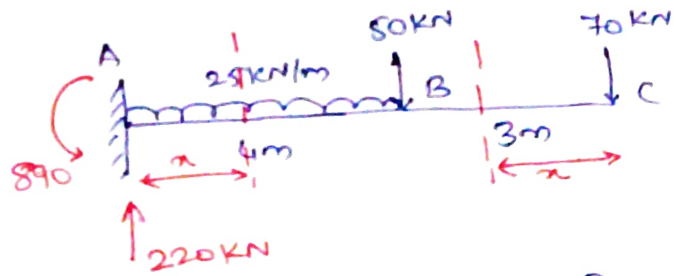
Step 3: Find displacements at desired points

$$\Delta_{iL} = \sum \int \frac{M \cdot m_i}{EI} dx \qquad \delta_{ij} = \sum \int \frac{m_i \cdot m_j}{EI} dx$$

1. Draw diagrams

M diagram:

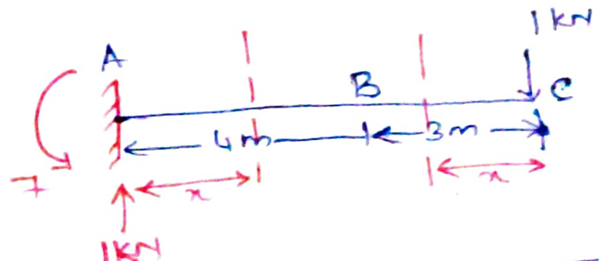
BDS - all external loads



Find the reactions $\Sigma M = 0$ and $\Sigma F_y = 0$ at the fixed end.

m. diagram:

BDS - Unit load at point C



Find the reaction at A, using $\Sigma F_y = 0$ and $\Sigma M = 0$.

- After this, So as to facilitate BM equations we have to take sections. We have two ~~seg~~ segments namely AB and BC.

- In segment AB, take a section at distance x from A. with a limits of the section 0 to 4m.

- Similarly, in segment BC, take a section at a distance of xm from C, with a limits of the section 0 to 3m.

2. BM Table $\boxed{+ve}$

Segment	Origin	Lim	I	M	m
AB	A	0 to 4	I	$220x - 890 - 25x^2/2$	$x - 7$
BC	C	0 to 3	I	$-70x$	$-x$

3. Find displacement:

$$A_{CL} = \sum \int \frac{M.m_c}{EI} dx$$

$$\Delta_{CL} = \overbrace{\int_0^4 \frac{(220x - 890 - 12.5x^2).(x-7)}{EI}\,dx}^{\text{Segment AB}} + \overbrace{\int_0^3 \frac{(-70x).(-x)}{EI}\,dx}^{\text{Segment BC}}$$

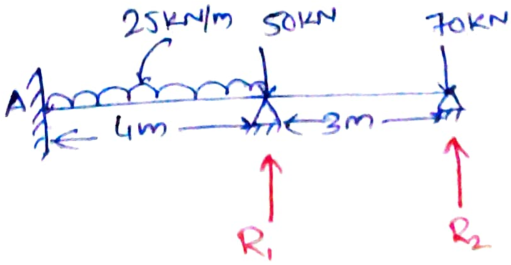
$$\Delta_{CL} = \frac{11870}{EI}\ \text{m} \downarrow$$

This displacement is downward as the unit load assumed is downward.

For suppose if we get the displacement value as (-ve) negative then the direction of displacement is upward.

Flexibility matrix

→ Given Indeterminate Structure



→ $D_s = r - e = 2$

R_1 and R_2 are the two identified redundants

→ Compatibility condition of displacement is

$$\Delta_i = \Delta_{iL} + \Delta_{iR} \rightarrow (1a)$$

$$\Delta_i = \Delta_{iL} + f_{ij} R_i \rightarrow (1b)$$

Here, Δ_{iL} can be found directly because the loads are given unknown.

While, Δ_{iR} i.e., displacement due to redundant R_1 and R_2 are unknowns.

So Δ_{iR} is converted into flexibility (f_{ij}) multiplied by R_i

Expanding equation (1b) for two redundants will be

$$\Delta_1 = \Delta_{1L} + f_{11} R_1 + f_{12} R_2 + \dots \quad (2a)$$

$$\Delta_2' = \Delta_{2L} + f_{21} R_1 + f_{22} R_2 + \dots \quad (2b)$$

$\Delta_1 \rightarrow$ displacement at coordinate axis 1 at R_1

$\Delta_2 \rightarrow$ displacement at coordinate axis 2 at R_2

Note:

$f_{11} \rightarrow$ displacement at 1 due to unit load at 1

$f_{12} \rightarrow$ displacement at 1 due to unit load at 2.

The equation 2a and 2b in matrix form will be

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}^{n\times 1} = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{nL} \end{bmatrix}^{n\times 1} + \underbrace{\begin{bmatrix} f_{11} & f_{12} & f_{1n} \\ f_{21} & f_{22} & f_{2n} \\ f_{31} & f_{32} & f_{3n} \end{bmatrix}^{n\times n}}_{\text{flexibility matrix}} \times \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}^{n\times 1}$$

Flexibility matrix is square matrix of $n \times n$ size where 'n' indicates the number of redundants.

Note :

1. All the elements in principal diagonal are positive.
2. Maxwell's reciprocal theorem can be applicable here, so $f_{ij} = f_{ji}$ means f_{12} is always equal to f_{21}.
3. The flexibility matrix is always a square, symmetric and all elements in principal diagonal are allways positive.

Properties of Flexibility matrix:

1. As per Maxwell's Reciprocal Theorem $f_{ij} = f_{ji}$
2. Symmetric matrix
3. Always square matrix of order n. where, $n = D_s$.
4. Elements on principal diagonal are always positive

$$[R] = \{[\Delta_i] - [\Delta_{iL}]\} \times [f_{ij}]^{-1} \longrightarrow (3)$$

Bullet points:

→ Flexibility or Force method is classical elastic method of structural analysis.

→ Redundant forces are primary unknowns hence

$$D_s = \text{No. of Redundants}$$

(When yielding of any support is given then corresponding reaction must be chosen as one of the redundant.)

→ Displacements due to external loads at coordinates of redundants and flexibility coefficients are derived by unit load method.

→ Flexibility matrix is always square and symmetric having positive elements on its principal diagonal.

→ Compatibility condition of displacement gives the solution of redundants.

Analysis of indeterminate structure : Flexibility method

1. Determine D_s

2. Identify Redundants and obtain BDS (No. of Redundants $= D_s$)

3. Draw following diagrams

   M dia : BDS - All external loads, find reactions

   m_1 dia : BDS - unit load at redundant 1, find reactions

   m_2 dia : BDS - unit load at redundant 2, find reactions

4. BM equations table.

   Take section in each segment of loading and write BM equations with proper sign conventions of BM.

5. Find BDS displacements due to external loads

$$\Delta_{iL} = \sum \int \frac{M . m_i}{EI} dx$$

6. Find flexibility coefficient and derive flexibility matrix

$$f_{ij} = \sum \int \frac{m_i . m_j}{EI} dx$$

Note: $f_{ij} = f_{ji}$ hence find upper triangular elements only.

7. Determine Redundants by compatibility condition

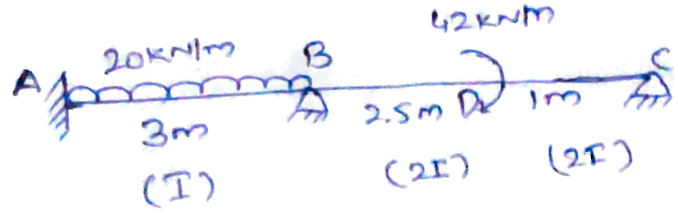
$$\Delta_i = \Delta_{iL} + f_{ij} R_i$$

Thus redundants are obtained by solving matrix equation below

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_n \end{bmatrix}^{n\times 1} = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{nL} \end{bmatrix}^{n\times 1} + \begin{bmatrix} f_{11} & f_{12} & f_{1n} \\ f_{21} & f_{22} & f_{2n} \\ f_{n1} & f_{n2} & f_{nn} \end{bmatrix}^{n\times n} \times \begin{bmatrix} R_1 \\ R_2 \\ R_n \end{bmatrix}^{n\times 1}$$

8. Draw final FBD, SFD and BMD.

Example



Given: $\delta_B = 2mm = 0.002m \downarrow$

$EI = 38000 KN.m^2$.

thus $\delta_B = 76/EI \; m \downarrow$

Consider a continuous beam ABC. fixed at A, supported at B and C.

Step 1: Find Ds

$D_s = r - e$

No. of reactions are 4 i.e 2 at fixed support A and one each at Support B and C.

$D_s = r - e = 4 - 2 = 2.$

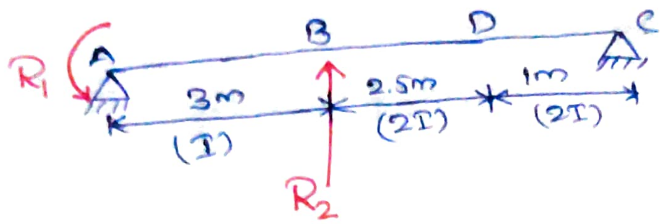
Step 2: Identify Redundants, Release them and obtain BDS

As yielding of support B is given, we must select R_B as one of our Redundant.

Let BDS is S S Beam as shown

Vertical reaction at B as one Redundant and momentary reaction at A as 2nd redundant.

Hence, the structure is

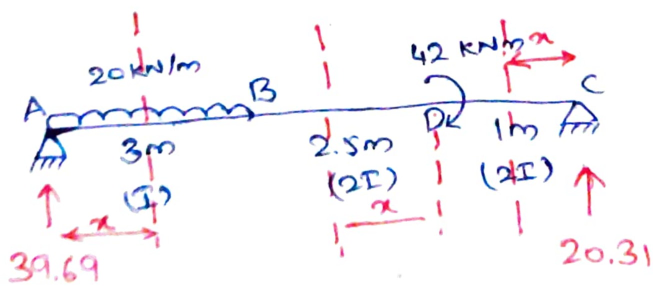


(a)

R_1 and R_2 are the redundants.

<u>Step 3:</u> Draw BDS diagrams, find reactions and take sections

(i) M diagram: BDS - All external Loads



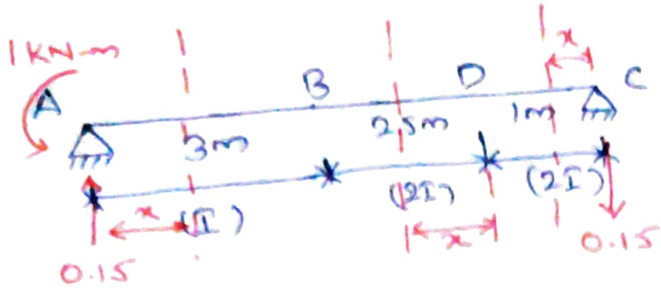
(b)

Find out reactions by using simple equilibrium conditions

Vertical reaction at A = 39.69 kN

Vertical reaction at B = 20.31 kN

Now, 2nd moment diagram (m_1)



m_1 diagram: BDS - Unit load at R_1

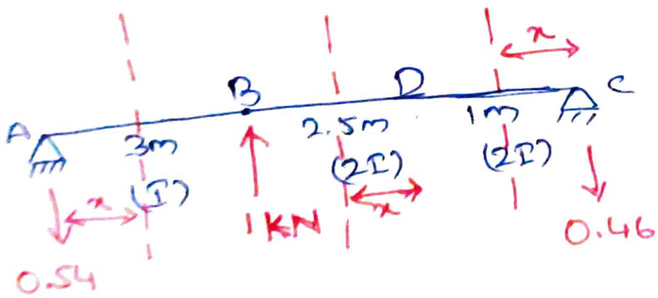
Redundant R_1 is a momentary axis, so here unit moment is applied at point A.

Find out reactions by using equilibrium equations

vertical reaction at A = +0.15 (↑)

vertical reaction at C = 0.15 (↓)

Now, m_2 diagram: BDS - unit load at R_2



Redundant R_2 is a vertical axis, so here unit vertical force is applied at joint B

Find out the reactions

vertical reaction at A = 0.54 (↓)

Vertical reaction at C = 0.46 (↓)

Step 4: BM equation table.

+ve

(i) M dia: BDS – All external loads [Fig. (b)]

Segment	Origin	Limit	I	(i) M	(ii) m_1	(iii) m_2
AB	A	0 – 3	I	$36.69x - 20x^2/2$	$0.15x - 1$	$-0.54x$
BD	D	0 – 2.5	2I	$20.31(x+1) - 42$ $= 20.31x - 21.69$	$-0.15(x+1)$ $= -0.15x - 0.15$	$-0.46(x+1)$ $= -0.46x - 0.46$
CD	C	0 – 1	2I	$20.31x$	$-0.15x$	$-0.46x$

(ii) m_1 dia: BDS – unit load at R_1 [fig.(c)]

(iii) m_2 dia: BDS – unit load at R_2 [fig.(d)]

Step 5: BDS displacements due to external load.

To find

(i) Displacement in BDS due to all external loads

(ii) Displacements in BDS due to unit loads.

$$\Delta_{1L} = \sum \int \frac{M.m_1}{EI}\,dx$$ displacement at R_1 due to external loads

[From table above]

$$\Delta_{1L} = \int_0^3 \frac{\overset{(i)}{(39.69x - 20x^2/2).(0.15x-1)}}{E(I)}\,dx$$

$$+ \int_0^{2.5} \frac{\overset{(ii)}{(20.31x - 21.69).(-0.15x - 0.15)}}{E(2I)}\,dx$$

$$+ \int_0^{1} \frac{\overset{(iii)}{(20.31x).(-0.15x)}}{E(2I)}\,dx$$

$$\Rightarrow \quad \Delta_{1L} = \frac{-69.17}{EI}$$

observe that

(i) for segment AB, the limits are 0 to 3.

(ii) for segment BD, the limits are 0 to 2.5

(iii) for segment DC, the limits are 0 to 1.

Respective values are substituted in the equation (M, m_1)

For AB → value of I is I

BD and DC → value of I is (2I)

Similarly,

$$\Delta_{2L} = \sum \int \frac{M m_2}{EI} dx$$ displacement at R_2 due to external loads.

[From, tabe above, substitute the values of $M_2 m_2$ for respective segments].

$$\Delta_{2L} = \int_0^3 \frac{(39.69x - 10x^2)(-0.54x)}{E(I)} dx + \int_0^{2.5} \frac{(20.31x - 21.69).(-0.46x - 0.46)}{E(2I)} dx$$

$$+ \int_0^1 \frac{(20.31x).(-0.46x)}{E(2I)} dx.$$

$$\Rightarrow \Delta_{2L} = \frac{-95.5}{EI}$$

Step 6: BDS displacements due to unit load (Flexibility Coefficient)

$$f_{11} = \sum \int \frac{m_1 m_1}{EI} dx$$ displacement at R_1 due to unit load at R_1

Substitute the values of m_1 from the table

$$f_{11} = \int_0^3 \frac{(0.15x-1).(0.15x-1)}{E(I)} dx + \int_0^{2.5} \frac{(-0.15x-0.15).(-0.15x-0.15)}{E(2I)} dx$$

$$+ \int_0^1 \frac{(-0.15x).(-0.15x)}{E(2I)} dx$$

$$f_{11} = \frac{2.01}{EI}$$

Similarly for

$$f_{22} = \sum \int \frac{m_2 . m_2}{EI} . dx$$

Displacement at R_2 due to unit load at R_2.

$$f_{22} = \int_0^3 \frac{(-0.54)x . (-0.54x)}{E(I)} dx + \int_0^{2.5} \frac{(-0.46x-0.46).(-0.46x-0.46)}{E(2I)} dx$$

$$+ \int_0^1 \frac{(-0.46x).(-0.46x)}{E(2I)} dx$$

$$f_{22} = \frac{4.136}{EI}$$

$$\delta_{12} = \delta_{21} = \sum \int \frac{m_1 . m_2}{EI} dx$$

Displacement at R_1 due to unit load at R_2.

$$\delta_{12} = \delta_{21} = \int_0^3 \frac{(0.15x - 1).(-0.54x)}{E(I)} dx + \int_0^{2.5} \frac{(0.15x - 0.15).(-0.46x - 0.46)}{E(2I)} . dx$$

$$+ \int_0^1 \frac{(-0.15x).(-0.46x)}{E(2I)} dx$$

$$\delta_{12} = \delta_{21} = \frac{2.19}{EI}$$

Step 7: Compatibility condition to find Redundants

$$\Delta_i = \Delta_{iL} + \delta_{ij} R_i$$

Then $[R_i] = [\delta_{ij}]^{-1} \times \{[\Delta_i] - [\Delta_{iL}]\}$

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \end{bmatrix} \longrightarrow (1)$$

where Δ_1 = Final displacement at R_1
= 0 (as R_1 has no rotational yield)

Δ_2 = Final displacement at R_2
= $-76/EI$ m (as R_2 has settlement yield in opposite sense of R_2)

Putting values in equation 1.

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = EI \begin{bmatrix} 2.01 & 2.19 \\ 2.19 & 4.136 \end{bmatrix}^{-1} \times \frac{1}{EI} \begin{bmatrix} 0-(-69.17) \\ -76-(-95.5) \end{bmatrix}$$

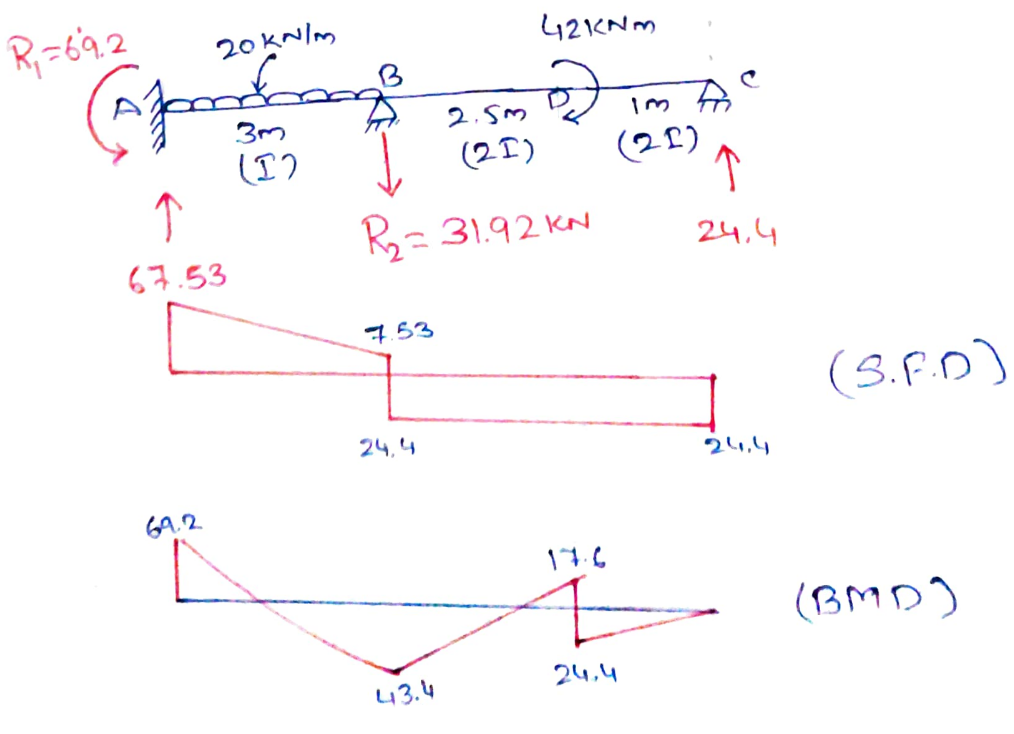
$$\Rightarrow \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = EI \begin{bmatrix} 1.176 & -0.623 \\ -0.623 & 0.571 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 69.71 \\ 19.5 \end{bmatrix}$$

After solving, we get

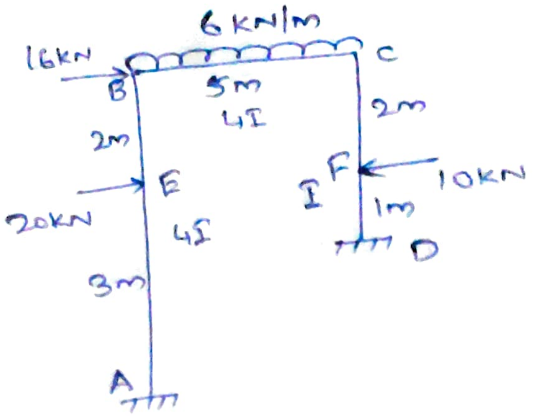
$R_1 = 69.196$ KN.m ↺

$R_2 = -31.92$ KN ↓

Step 8: Final FBD



(2)



Step 1:

Find the degree of static indeterminacy & marking redundant Reaction.

$D_s = D_{si} + D_{se}$

$D_s = 0 + (R - e)$

$D_s = 0 + (6 - 3) = 3$

Therefore there are 3 numbers of addition reaction which is making the streecture indeterminate.

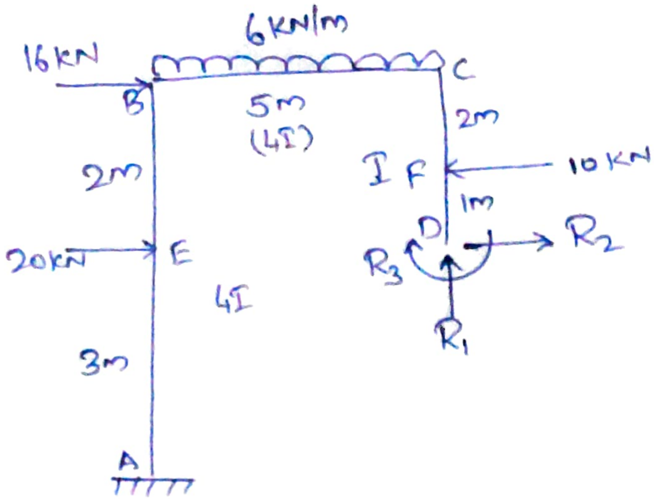
Step 2: Identifying Redundants, release them and obtain BDS.

The Redundants are

(i) Vertical reaction at support D

(ii) Horizontal reaction at support D

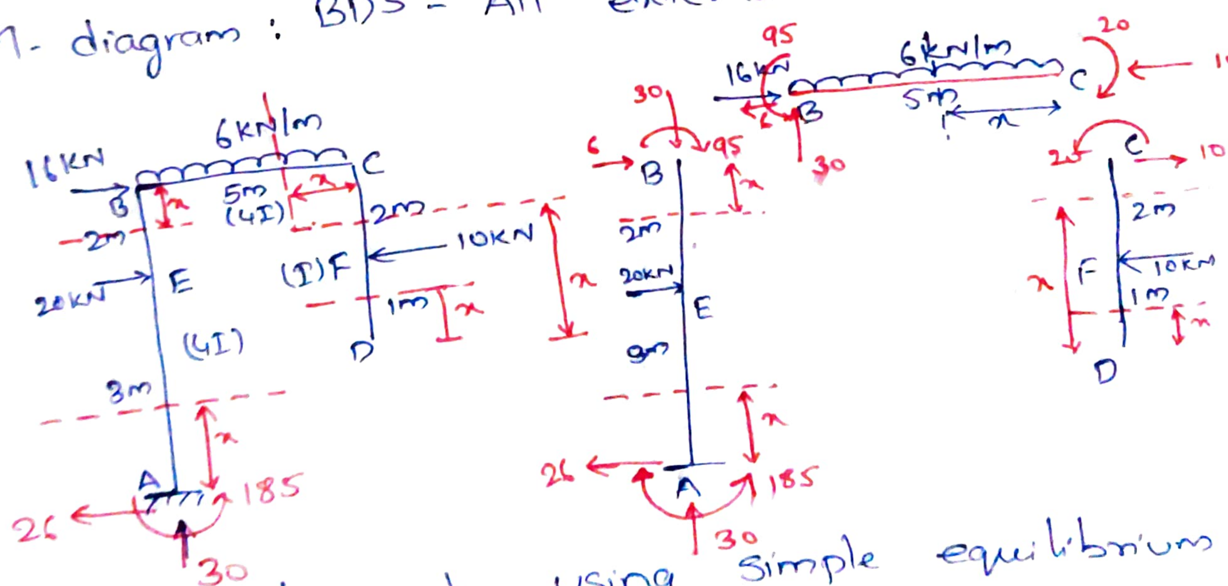
(iii) Moment reaction at Support D



R_1, R_2, R_3 are the Redundants.

Step 3: Draw BDS diagrams. Find reactions and take sections.

(i) M- diagram : BDS - All external loads



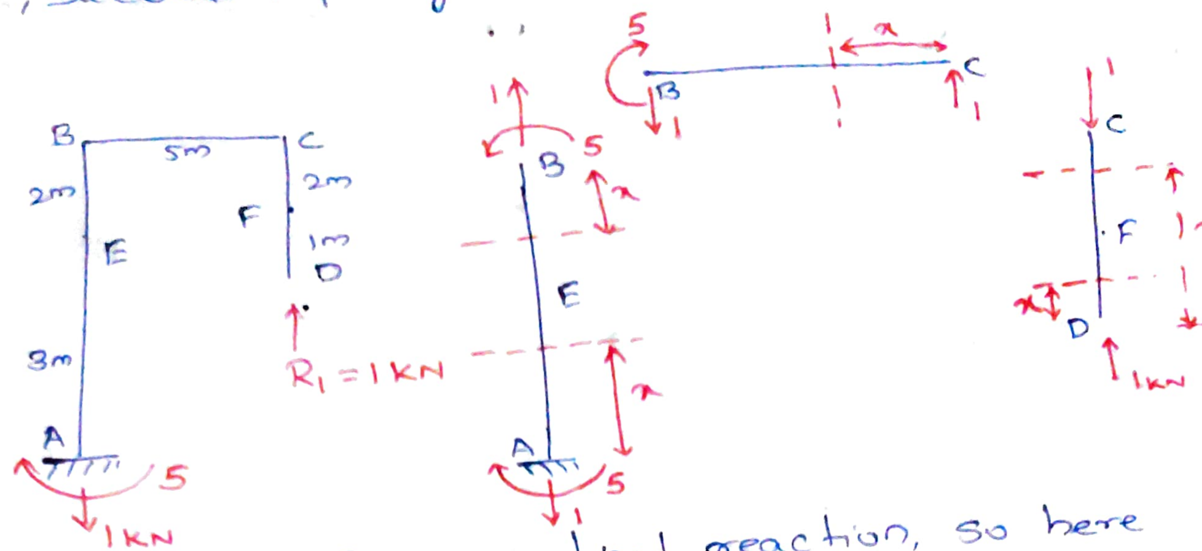
Find out reactions by using simple equilibrium conditions.

Vertical reaction at A = 30 KN

Horizontal reaction at A = 26 KN

Moment reaction at A = 185 KNm

Now, ~~second~~ m_1 diagram : BDS - unit load at R_1



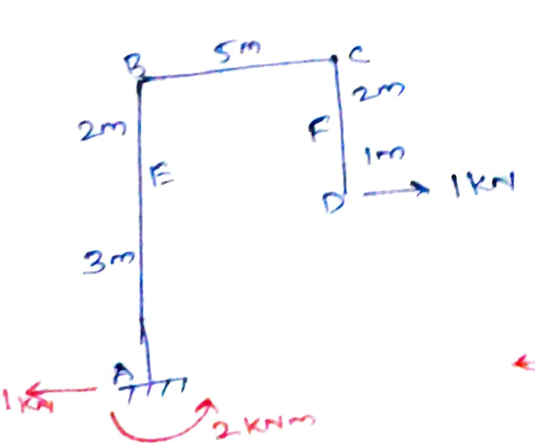
Redundant R_1 is a vertical reaction, so here a unit force is applied at point D.

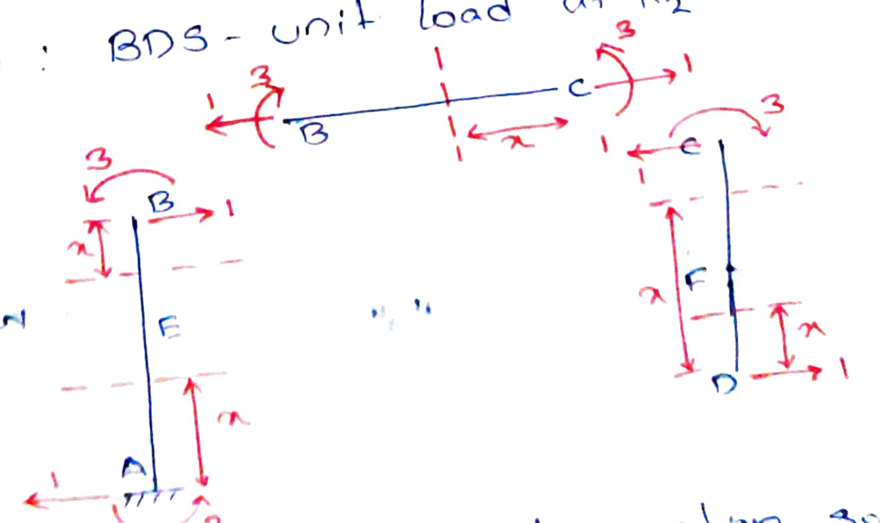
Find out the reactions by using equilibrium equation.

Vertical reaction at A = 1 kN

Moment reaction at A = 5 kNm.

Now, m_2 diagram : BDS - unit load at R_2





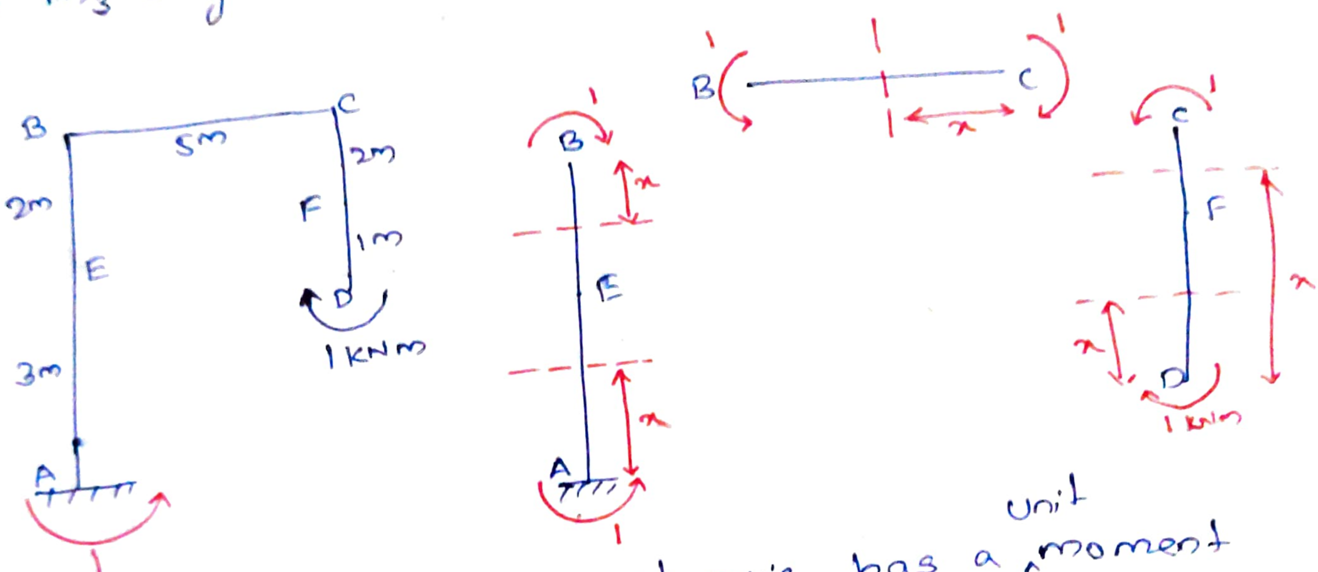
Redundant R_2 is a horizontal reaction, so a unit load is applied at point D.

Finding the reactions using equilibrium equations

Horizontal reaction at A = 1 kN.

moment reaction at A = 2 kNm

Now m_3 diagram: BDS - unit load R_3.



Redundant R_3 is a moment axis, has a unit moment is applied at support D.

finding the reactions using equilibrium equations

Moment Reaction at A = 1 kN-m (↺)

Step 4: BM equation table (↶ ↑ +ve)

Segment	Origin	Limit	I	M_0	m_1	m_2	m_3
AE	A	0-3	4I	$26x - 185$	5	$x-2$	-1
EB	B	0-2	4I	$-6x - 95$	5	$3-x$	-1
BC	C	0-5	4I	$-3x^2 - 20$	x	3	-1
CF	D	1-3	I	$-10(x-1)$	0	x	-1
FD	D	0-1	I	0	0	x	-1

Step 5: BDS displacements due to external load

$$\Delta_{1L} = \Sigma \int \frac{M.m_1}{EI} dx$$

Displacement at R_1 due to external load.

$$= \int_0^3 \frac{(26x-185)(5)}{E(4I)} dx + \int_0^2 \frac{(-6x-95)(5)}{E(4I)} dx + \int_0^5 \frac{(-3x^2-20).(x)}{E(4I)} dx$$

$$+ \int_{\phi}^{3} \frac{(-10(x-1)).(0)}{E(I)} dx + 0$$

$$\Rightarrow \frac{-979.687}{EI}$$

$$\Delta_{2L} = \sum \int \frac{M.m_2}{EI} dx$$

Displacement at R_2 due to external Load.

$$\Delta_{2L} = \int_0^3 \frac{(26x-185)(x-2)}{E(4I)} dx + \int_0^2 \frac{(-6x-95)(3-x)}{E(4I)} dx$$

$$+ \int_0^5 \frac{(-3x^2-20).(3)}{E(4I)} dx + \int_1^3 \frac{(-10(x-1)).x}{EI} dx + 0$$

$$= \frac{-246.042}{EI}$$

Displacement at R_3 due to external load.

$$\Delta_{3L} = \sum \int \frac{M.m_3}{EI} dx$$

$$= \int_0^3 \frac{(26x-185)(-1)}{E(4I)} dx + \int_0^2 \frac{(-6x-95)(-1)}{E(4I)} dx + \int_0^5 \frac{(-3x^2-20)(-1)}{E(4I)} dx$$

$$+ \int_1^3 \frac{(-10(x-1)).(-1)}{E(I)} dx + 0$$

$$= \frac{236.25}{EI}$$

Step 6: BDS displacements due to unit load (Flexibility coefficients)

$$\text{flexibility matrix } f_{ij} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}_{3\times 3}$$

$$\therefore f_{11} = \sum \int \frac{m_1 m_1}{EI} dx$$

displacement at R_1 due to unit load R_1

$$= \int_0^3 \frac{5\times 5}{E(4I)} dx + \int_0^2 \frac{5\times 5}{4EI} dx + \int_0^5 \frac{x.x}{4EI} dx + 0 + 0$$

$$= \frac{125}{3EI}$$

$$f_{12} = f_{21} = \sum \int \frac{m_1 m_2}{EI} dx$$

displacement at R_1/R_2 due to unit load R_1/R_2

$$= \int_0^3 \frac{5\times(x-2)}{E(4I)} dx + \int_0^2 \frac{5\times(3-x)}{4EI} dx + \int_0^5 \frac{x\times 3}{4EI} dx + 0 + 0$$

$$= \frac{12.5}{EI}$$

$$\delta_{13} = \delta_{31} = \Sigma \int \frac{m_1 m_3}{EI}\, dx$$

displacment at R_1 / R_3 due to unit load R_1 / R_3.

$$= \int_0^3 \frac{5 \times (-1)}{4EI}\, dx + \int_0^2 \frac{5 \times (-1)}{4EI}\, dx + \int_0^5 \frac{x \times (-1)}{4EI}\, dx + 0 + 0$$

$$= \frac{-15}{4EI} + \left(-\frac{5}{2EI}\right) + \left(\frac{-25}{8EI}\right)$$

$$\Rightarrow \frac{-9.375}{EI}$$

$$\delta_{23} = \delta_{32} = \Sigma \int \frac{m_2 m_3}{EI}\, dx$$

displacement at R_2 / R_3 due to unit load R_2 / R_3.

$$\Rightarrow \int_0^3 \frac{(x-2)(-1)}{4EI}\, dx + \int_0^2 \frac{(3-x)(-1)}{4EI}\, dx + \int_0^5 \frac{(3)(-1)}{4EI}\, dx$$

$$+ \int_1^3 \frac{(x)(-1)}{EI}\, dx + \int_0^1 \frac{(x)(-1)}{EI}\, dx$$

$$\Rightarrow \frac{3}{8EI} + \left(\frac{-1}{EI}\right) + \left(\frac{-15}{4EI}\right) + \left(\frac{-4}{EI}\right) + \left(\frac{-1}{2EI}\right)$$

$$\Rightarrow \frac{-8.875}{EI}$$

$$\delta_{22} = \Sigma \int \frac{m_2 m_2}{EI} dx$$

Displacement at R_2 due to unit load R_2

$$= \int_0^3 \frac{(x-2).(x-2)}{4EI} dx + \int_0^2 \frac{(3-x)(3-x)}{4EI} dx + \int_0^5 \frac{3\times 3}{4EI} dx$$

$$+ \int_1^3 \frac{x \times x}{EI} dx + \int_0^1 \frac{x\times x}{EI} dx$$

$$\Rightarrow \frac{3}{4EI} + \frac{13}{6EI} + \frac{45}{4EI} + \frac{26}{3EI} + \frac{1}{3EI}$$

$$\Rightarrow \frac{139}{6EI}$$

$$\delta_{33} = \Sigma \int \frac{m_3 m_3}{EI} dx$$

Displacement at R_3 due to unit load R_3

$$= \int_0^3 \frac{(-1).(-1)}{4EI} dx + \int_0^2 \frac{(-1).(-1)}{4EI} dx + \int_0^5 \frac{(-1)\times(-1)}{4EI} dx +$$

$$\int_1^3 \frac{(-1)\times(-1)}{EI} dx + \int_0^1 \frac{(-1)\times(-1)}{EI} dx$$

$$\Rightarrow \frac{3}{4EI} + \frac{1}{2EI} + \frac{5}{4EI} + \frac{2}{EI} + \frac{1}{EI}$$

$$\Rightarrow \frac{5.5}{EI}$$

Step 7: Compatibility condition to find Redundants

$$\Delta_i = \Delta_{iL} + f_{ij} R_i$$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{3L} \end{bmatrix} + \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

Since there are no external deflection,

therefore $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \frac{1}{EI} \begin{bmatrix} -979.6875 \\ -246.0417 \\ 236.25 \end{bmatrix} + \frac{1}{EI} \begin{bmatrix} \frac{125}{3} & 12.5 & -9.375 \\ 12.5 & \frac{139}{6} & -8.875 \\ -9.375 & -8.875 & 5.5 \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

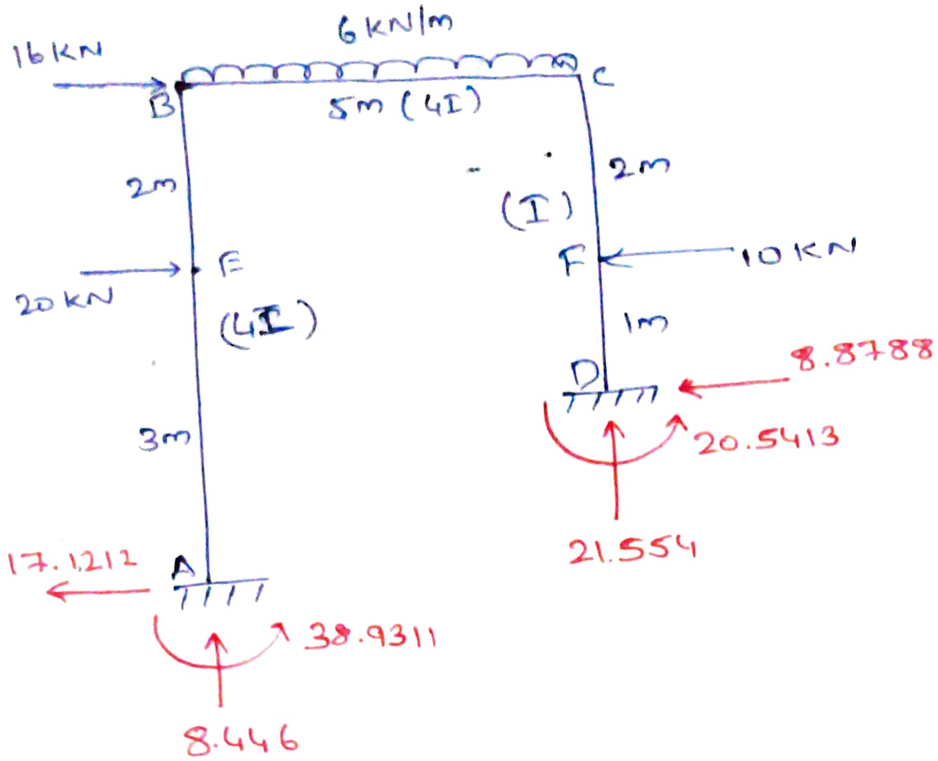
Solving this equation we get R_1, R_2, R_3 as

$R_1 = 21.554$ kN

$R_2 = -8.8788$ kN

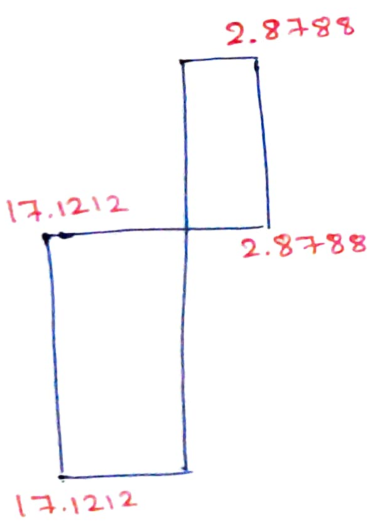
$R_3 = -20.5413$ kNm.

Step 8: Final BMD

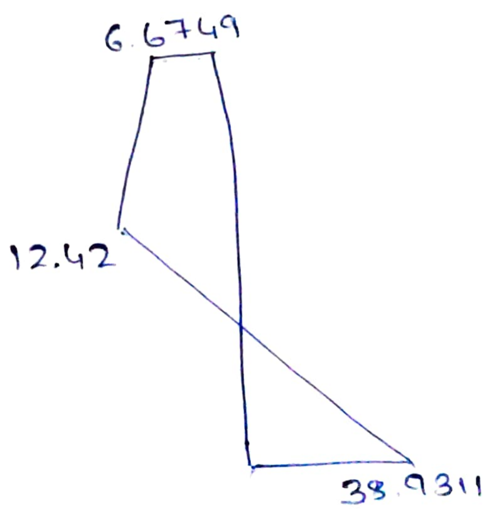


For AB

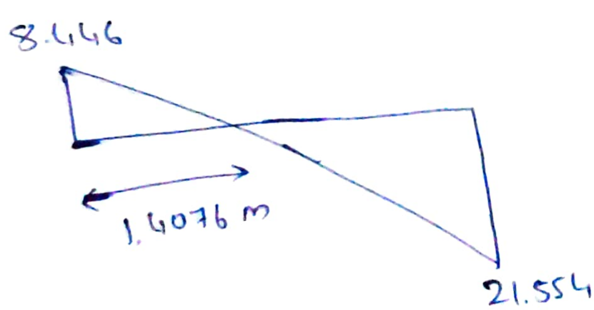
S.F.D.

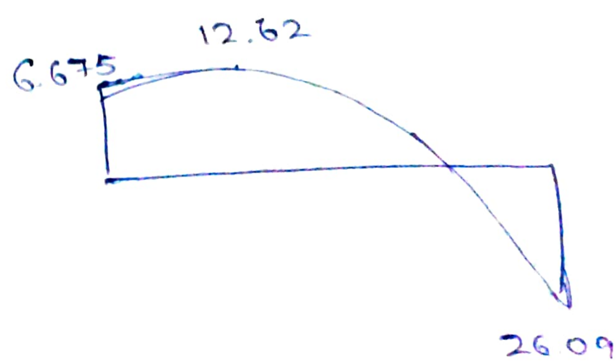


BMD



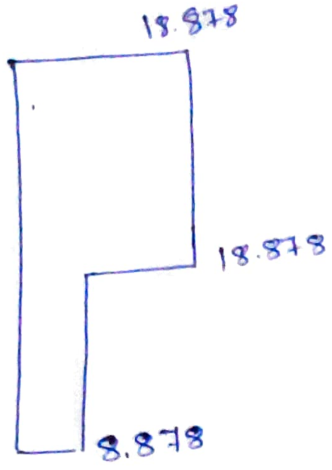
For BC





For CD

S.F.D



B.MD