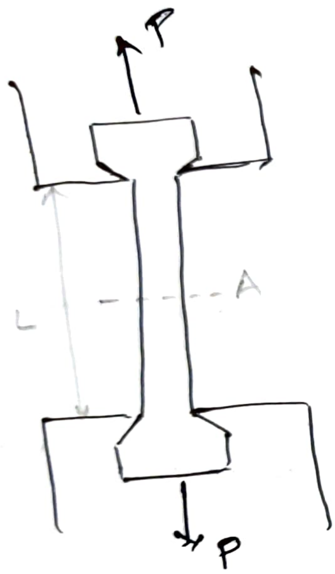
Plastic Analysis

Plasticity: It is the property of the material by virtue of which on loading the material undergoes permanent deformation ~~w~~ without any breakage.

Ex: Mild steel.



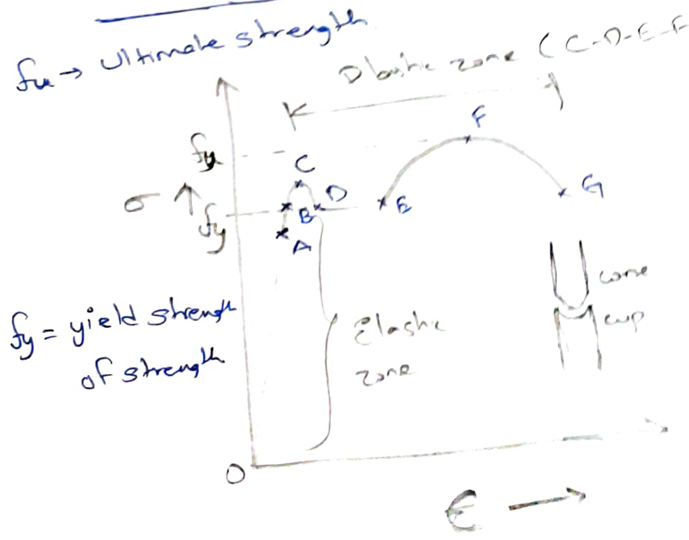
Load applied per unit cross-sectional area ~~i.e~~ is known as stress

i.e $\frac{P}{A} = \sigma$

The change in ~~along~~ length caused due to the load ~~per~~ to its original length is known as strain.

i.e $\frac{\Delta}{L} = \epsilon$.

Stress . strain curve of mild steel.



A → Proportional limit

B → Elastic limit.

C → Upper yield Point

D → Lower yield Point.

D-E → Yield zone / Plateau

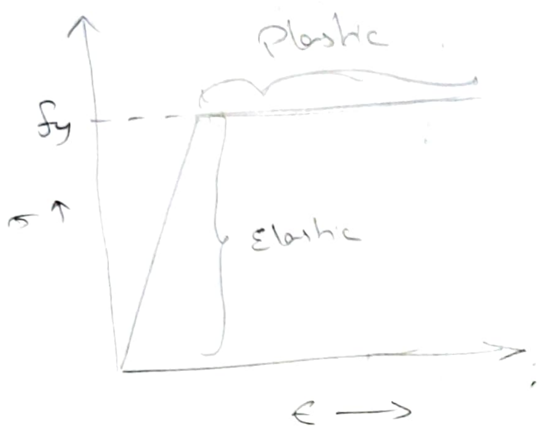
E-F → Strain hardening

F-G → Necking

G → Fracture.

Assumption in plastic Analysis

① Idealized stress - strain curve.



* This is a favourable assumption as we ignore the impact of strain hardening.

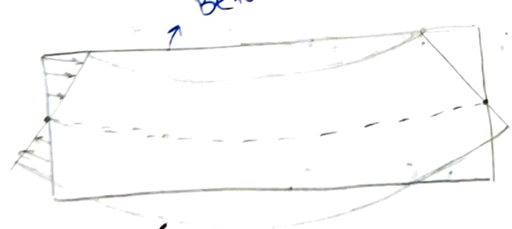
we under estimate the strength of material which leads to safer design.

The strain hardening zone is completely ignored.

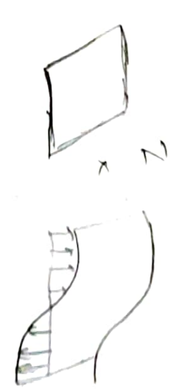
This curve is also called as Bilinear curve

② Plane section before bending remains plane after bending.

→ strain diagram will be linear.

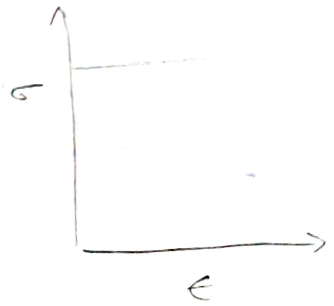


Considered only flexural stress.



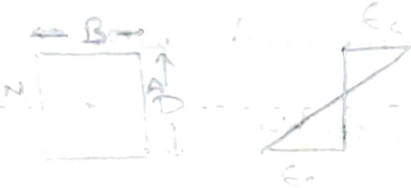
* No warping due to shear stress

*



Ideally Plastic

Theory



strain dig

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{My}{I}$$

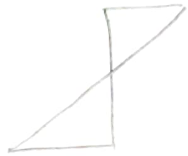
$$= \frac{M}{(I/y)} = \frac{M}{Z}$$

$Z \rightarrow$ Elastic section modulus

Case - I

(elastic state)

$f < f_y$



f

$M < M_y$

(M_y = yielding moment)

Case II

yield state

f_y

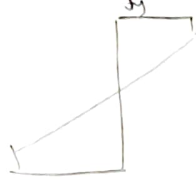
f_y

$M = M_y$

Case III

Elasto-Plastic ~~state~~ state

f_y



f_y

$M_p > M > M_y$

Case IV

Plastic state

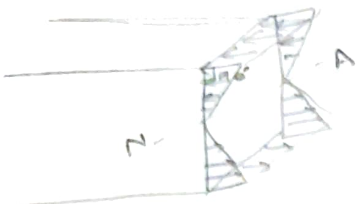
f_y

f_y

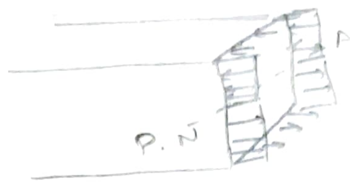
$M = M_p > M_y$

Plastic Moment (M_p):

It is the moment which generates Plastic state at a cross-section.

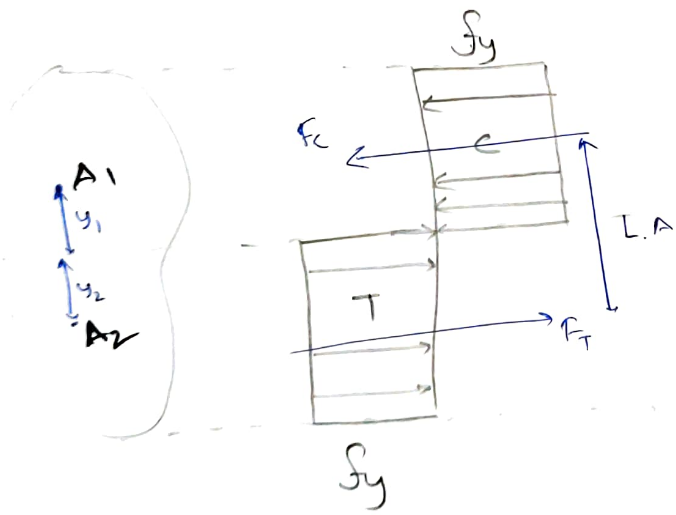


Elastic state



Plastic state

Plastic Moment (M_p):



A → Area of Cross-section

A_1 → Area under compression

A_2 → Area under tension

By equilibrium Condition

$$\Sigma F_{axial} = 0$$

$$F_{comp} - F_{tensile} = 0$$

$$F_{comp} = F_{Tensile}$$

$$f_{comp} \times A_{comp} = f_{tensile} \times A_{Tensile}$$

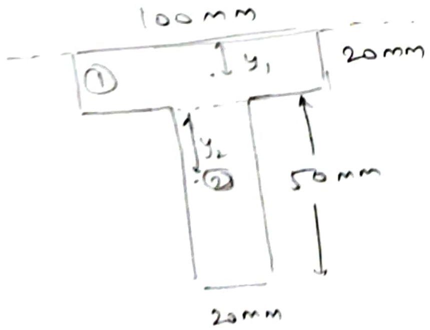
$$f_y \times A_1 = f_y \times A_2$$

$$A_1 = A_2$$

* That at plastic state the section is divided into two equal parts.

Note: If section has a symmetry then elastic N.A may coincide with Plastic N.A.

Eg:



Determine ~~ele~~ the gap between elastic N.A and Plastic N.A

$A_1 = 100 \times 20 = 2000\ mm^2$

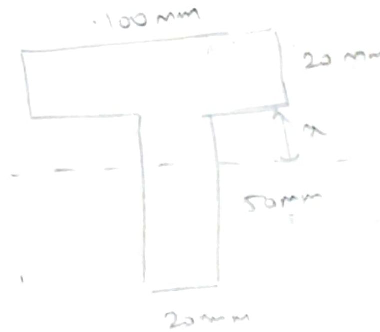
$A_2 = 50 \times 20 = 1000\ mm^2$

$y_1 = \frac{20}{2} = 10mm$

$y_2 = \frac{50}{2} + 20 = 45mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{2000 \times 10 + 1000 \times 45}{300} = 21.62mm.$$

Plastic N.A.



For Plastic

$A_1 = A_2$

$2000 + 20 \times x = (50 - x) \times 20$

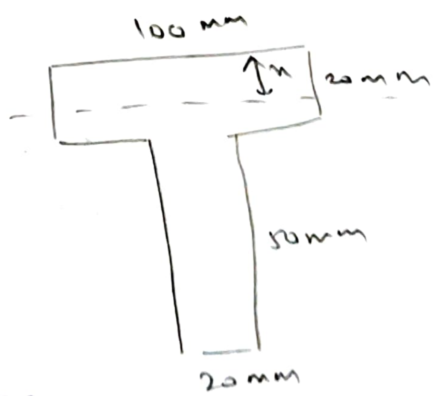
$2000 + 20x = 1000 - 20x$

$1000 = -40x$

$x = -25$ mm.

-ve · the plastic N.A should be above.

So



$A_1 = A_2$

$\Rightarrow 100 \times x = (20 - x) \times 100 + 1000$

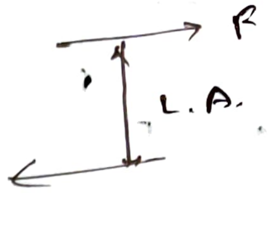
$\Rightarrow 100x = 2000 - 100x + 1000$

$\Rightarrow 200x = 3000$

$x = 15$ mm.

Plastic moment (M_p)

$\Rightarrow$ force $\times$ Lever Arm.



$\bar{y}_1$ = Distance of C.G of Area under compression to Plastic N.A.

$\bar{y}_2$ = Distance of C.G of Area under tension to Plastic N.A.

$$M_p = F_{comp} \times (\bar{y}_1 + \bar{y}_2)$$

$$= f_y . A_1 (\bar{y}_1 + \bar{y}_2)$$

$$\boxed{M_p \Rightarrow f_y . \frac{A}{2} (\bar{y}_1 + \bar{y}_2)}$$

As per elastic theory

$M_y = f_y Z_e$

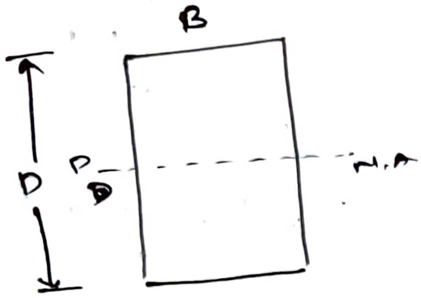
Similarly $M_p = f_y Z_p$

where $Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$

Plastic section modulus.

Plastic section Modulus (Z_p).

① Rectangle.



$A = B.D$

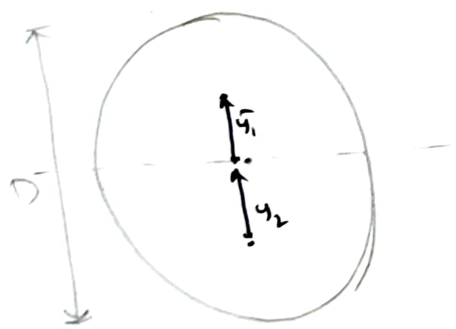
$\bar{y}_1 = D/4$

$\bar{y}_2 = D/4$

$$Z_p = \frac{A}{2}\left(\frac{D}{4} + D/4\right)$$

$$= \frac{A}{2}\left(\frac{D}{2}\right) = \frac{BD^2}{4}.$$

② Circle



$A = \frac{\pi}{4} D^2$

C.G of semi circle

$\frac{4r}{3\pi}$

$\bar{y}_1 = \frac{4R}{3\pi}$

$\bar{y}_2 = \frac{4R}{3\pi}$

$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

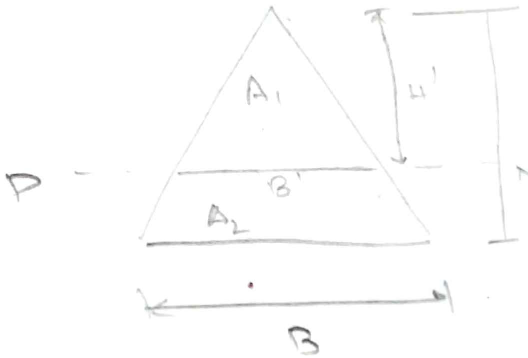
$$= \frac{\pi}{4} D^2 \left(\frac{4R}{3\pi} + \frac{4R}{3\pi} \right)$$

$$= \frac{1}{2} \cdot \frac{\pi}{4} D^2 \left(\frac{8R}{3\pi} \right)$$

$R = D/2$

$$\Rightarrow \frac{D^3}{6}$$

③ Triangle



$$A_1 = A_2$$

$$\frac{1}{2} \times B' \times H' = \frac{1}{2} BH - \frac{1}{2} B'H'$$

$$\boxed{B'H' = \frac{BH}{2}} \quad — ①$$

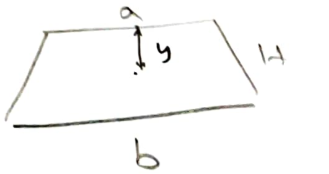
By Similar Δles.

$$\boxed{\frac{B'}{B} = \frac{H'}{H}} \quad — ②$$

$$B' = \frac{BH'}{H} \Rightarrow \frac{B}{H} \cdot \frac{BH}{2B'}$$

$$B'^2 = \frac{B^2}{2} \Rightarrow B' = \frac{B}{\sqrt{2}}$$

$$H' = \frac{BH}{2B'} = \frac{BH}{2\frac{B}{\sqrt{2}}} = \frac{H}{\sqrt{2}}$$



$$\bar{y} = \frac{2b+a}{a+b} \times \frac{H}{3}$$

$$A = \frac{1}{2} BH$$

$$\bar{y}_1 = \frac{H'}{3}, \quad \bar{y}_2 = \text{C.G of trapezium}$$

$$= \frac{2B+B'}{B+B'} \times \frac{(H-H')}{3}$$

$$Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{1}{4} BH \left[\frac{H'}{3} + \frac{2B+B'}{B+B'} \times \frac{(H-H')}{3}\right]$$

$$\Rightarrow \frac{BH}{4}\left[\frac{H}{3\sqrt{2}} + \frac{2B+\frac{B}{\sqrt{2}}}{B+\frac{B}{\sqrt{2}}} \times \frac{H-\frac{H}{\sqrt{2}}}{3}\right]$$

$$\Rightarrow \frac{BH}{4}\left[\frac{H}{3\sqrt{2}} + \frac{2\sqrt{2}+1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}H-H}{3}\right] \Rightarrow \boxed{Z_P = 0.097\, BH^2}$$

Shape Factor:

It is the ratio of Plastic Moment and Elastic moment.

i.e $$\boxed{S = \frac{M_p}{M} = \frac{\text{Plastic Moment}}{\text{Elastic moment}}}$$

Shape factor represents the strength of section.

$$= \frac{M_p - M}{M}$$

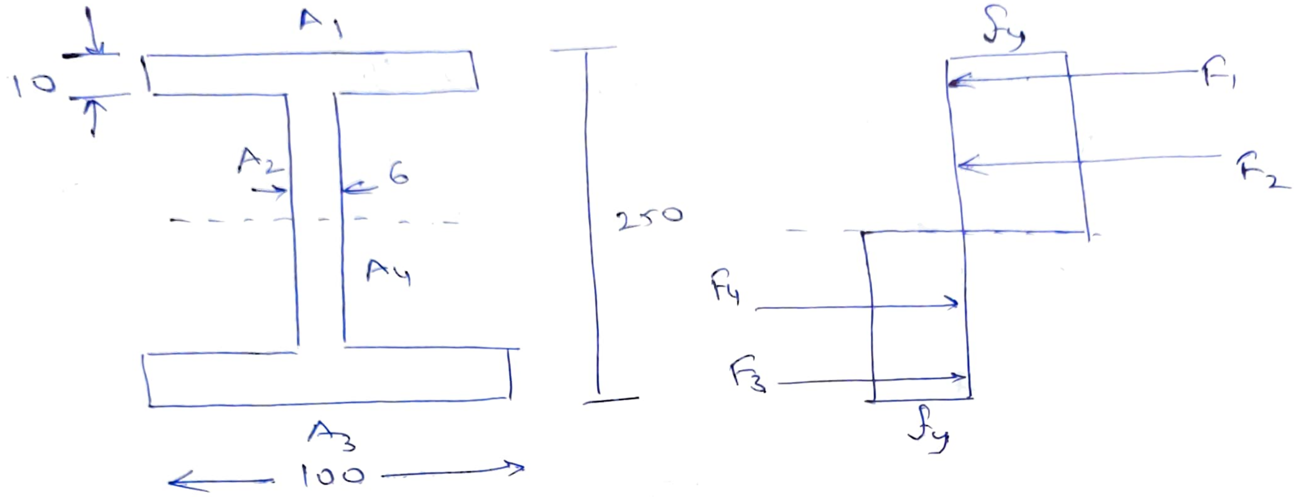
$$(\%) = \left(\frac{M_p - M}{M}\right) \times 100$$

$$= \left(\frac{M_p}{M} - 1\right) \times 100$$

$$= (S-1) \times 100$$

Problems

(1) Fe 250 N/mm^2



To find yield moment M_y.

Due to symmetry, elastic neutral axis is at mid-depth

$$\therefore I = \frac{1}{12} \times 100 \times 10^3 + 100 \times 10(125-5)^2 + \frac{1}{12} \times 6 \times (250-20)^3$$

$$+ \frac{1}{12} \times 100 \times 10^3 + 100 \times 10 (125-5)^2$$

$$= 34900167 \text{ mm}^4.$$

$$\therefore M_y = f_y . \frac{I}{y_{max}} = 250 \times \frac{34900167}{125}$$

$$= 69800333 \text{ N-mm}.$$

To find the plastic moment M_p:

$\cancel{M_p = f_y \frac{A}{2}(y_1 + y_2 +}$

$$M_p = f_y \left(A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4\right)$$

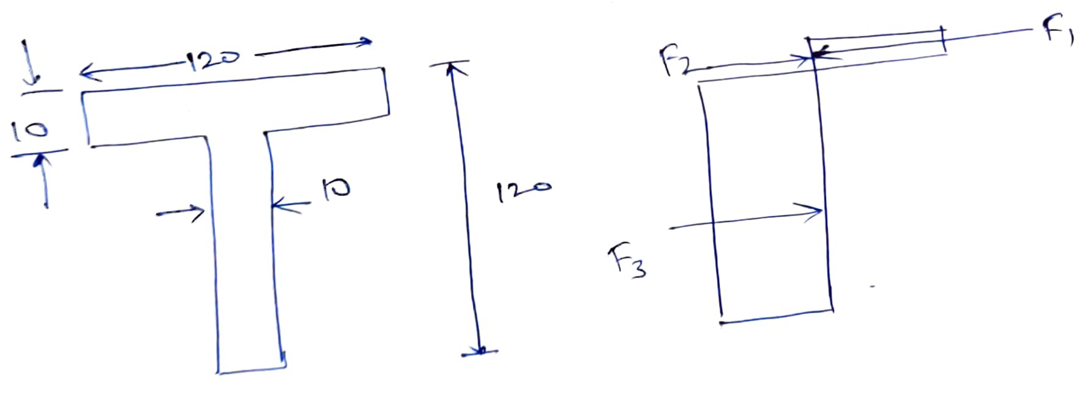
$$\Rightarrow f_y \left[100 \times 10\,(125-5) + 6\,(125-10) \times \frac{125-10}{2}\right] \times 2$$

$$\Rightarrow 79837500 \text{ N-mm.}$$

$$\therefore \text{Shape factor } (S) = \frac{M_p}{M_y} = \frac{79837500}{69800333}$$

$$= \underline{\underline{1.144}}$$

(2)



To find the yield moment M_y:

Elastic neutral axis from top fibre y_t.

$$= \frac{120 \times 10 \times 5 + 110 \times 10 \times (55 + 10)}{120 \times 10 + 110 \times 10}$$

$$= 33.70 \text{ mm.}$$

$$I = \frac{1}{12} \times 120 \times 10^3 + 120 \times 10\,(33.70 - 5)^2 +$$

$$\frac{1}{12} \times 10 \times 110^3 + 10 \times 110\,(65 - 33.70)^2$$

$$= 3185523.7 \text{ mm}^4.$$

$$y_{max} = 120 - 33.70 = 86.3 \text{ mm}.$$

:)

$$M_y = f_y \cdot \frac{I}{y_{max}}$$

$$= f_y \times \frac{3185523.7}{86.3}$$

$$\Rightarrow 36909.08\ f_y \quad \text{N-mm.}$$

To find plastic moment Capacity M_p:

$$A_1 = A_2$$

$$120 \times x = (10 - x) \times 120 + 10 \times 110$$

$$120x = 1200 - 120x + 1100$$

$$240x = 2300$$

$$x = \frac{2300}{240} = 9.583 \text{ mm}.$$

Area under compression:

$$A_1 = 120 \times 9.583 \text{ mm}^2.$$

$$y_1 = \frac{9.583}{2}$$

Area under tension:

$$A_2 = 120 \times (10 - 9.583)$$

$$= 120 \times 0.417$$

$$y_2 = \frac{0.417}{2}$$

$$A_3 = 10 \times 110 = 1100 \text{ mm}^2.$$

$$y_3 = 0.417 + \frac{110}{2} = 55.417 \text{ mm}$$

$$\therefore M_p = f_y (A_1 y_1 + A_2 y_2 + A_3 y_3)$$

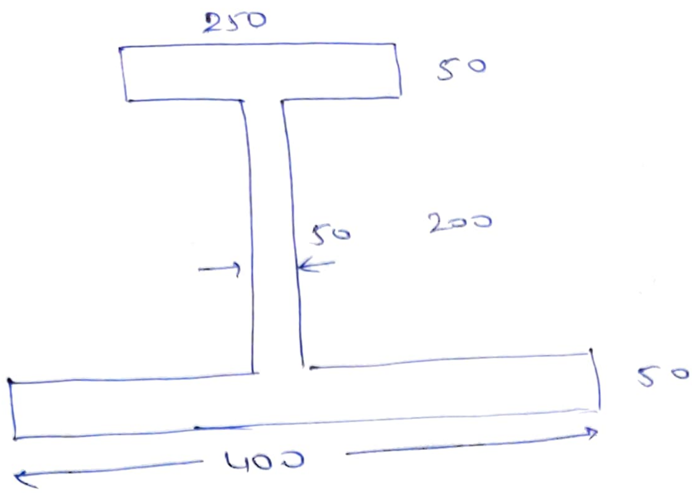
$$= f_y \left(120 \times 9.583 \times \frac{9.583}{2} + 120 \times 0.417 \times \frac{0.417}{2} + 1100 \times 55.417\right]$$

$$= 66479.17 f_y \text{ N-mm.}$$

$$S = \frac{M_p}{M_y} = \frac{66479.17}{36909.08}$$

$$= \underline{\underline{1.801}}$$

(3)



$$y_t = \frac{250\times50\times25 + 200\times50\times(100+50) + 400\times50\,(250+25)}{250\times50 + 50\times200 + 400\times50}$$

$$= \frac{7312500}{42500} = 172.06\ \text{mm.}$$

$$\therefore\ I = \frac{1}{12}\times250\times50^3 + 250\times50\,(172.06-25)^2$$

$$+ \frac{1}{12}\times50\times200^3 + 50\times200\,(172.06-150)^2$$

$$+ \frac{1}{12}\times400\times50^3 + 400\times50\,(275-172.06)^2.$$

$$= 5.27236\times10^8\ \text{mm}^4.$$

$$y_{max} = y_t = 172.06\ \text{mm}$$

$$M_y = f_y\cdot\frac{I}{y_{max}} = f_y\cdot\frac{5.27236\times10^8}{172.06}$$

$$= 3064259.7\, f_y\ \text{N-mm.}$$

Plastic Moment M_p:

Let the plastic neutral axis be at a distance y_p from top fibre. Assuming it fall in the web.

$$\cancel{20\times}\ 250\times 50 + 50(y_p - 50) = \frac{A}{2}$$

$$= \frac{(250+200+400)50}{2}$$

$$= \frac{42500}{2}$$

$$y_p - 50 = 175$$

$$y_p = 225\,mm.$$

Dividing the total ~~rectangle~~ area into four rectangles, two in compression and two in tension.

$$M_p = f_y \Sigma A y.$$

$$= f_y \left(250\times 50(225-25) + 50(225-50)\times\left(\frac{225-50}{2}\right) + 50\times(250-225)\times\left(\frac{250-225}{2}\right) + 50\times 400\left(\frac{275-225}{}\right)\right)$$

$$= 42812500 f_y \ N\text{-}mm$$

$$S = \frac{M_p}{M_y} = \frac{4281250}{3064259.7}$$

$$= \underline{\underline{1.397}}$$

Collapse load

- A structure is said to have collapsed if the entire structure or part of the structure starts undergoing unlimited deformation.

- This happen when the number of static equilibrium equations available are more then the number of reaction components.

- The state at which this condition develops is said to be collapse mechanism and the load carried at this ~~stage~~ state is called collapse load.

- Determining the collapse load of a structure is called Plastic Analysis.

- Structures are permitted to carry only a fraction of collapse load, called working load. The relationship b/w collapse and working load is

Collapse load = Load factor × Working load.

There are three basic theorems :

1. static theorem
2. Kinematic theorem
3. Uniqueness theorem.

Static theorem :-

For a given structure and loading, if there exists any distribution of bending moment throughout the structure which is both safe and statically admissible with a set of loads W, the value of W must be less than or equal to the collapse load W.

$$W \leq W_c$$

Statically admissible means bending moment diagram satisfies equilibrium conditions, the term safe means at no point bending moment is more ~~than~~ than plastic moment capacity of the section.

This theorem is called as lower bound theorem.

Kinematic theorem:-

For a given structure subjected to a set of loads W, the value of W found to correspond to any assumed mechanism must be ~~eigt~~ either greater or equal to the collapse load Wc

$$W \geq W_c$$

This is also called as Upper Bound theorem.

Uniqueness Theorem

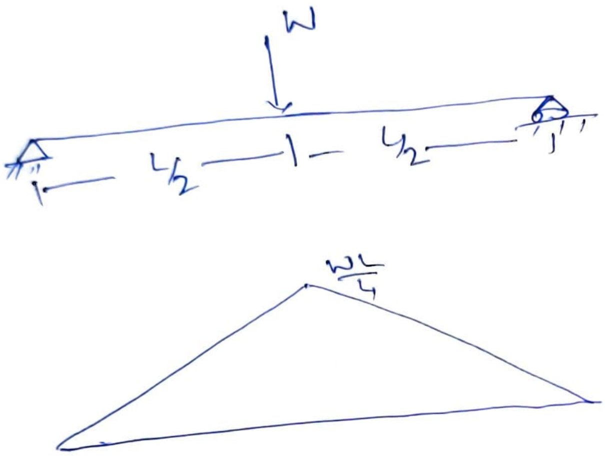
If for a given structure and loading at least one safe and statically admissible bending moment distribution can be found and in this distribution the bending moment is equal to the fully plastic moment at enough cross-sections to cause failure due to unlimited

Statical Method

This method is suitable for the analysis of structure for which the shape of the bending moment diagram is easily known.

Problems

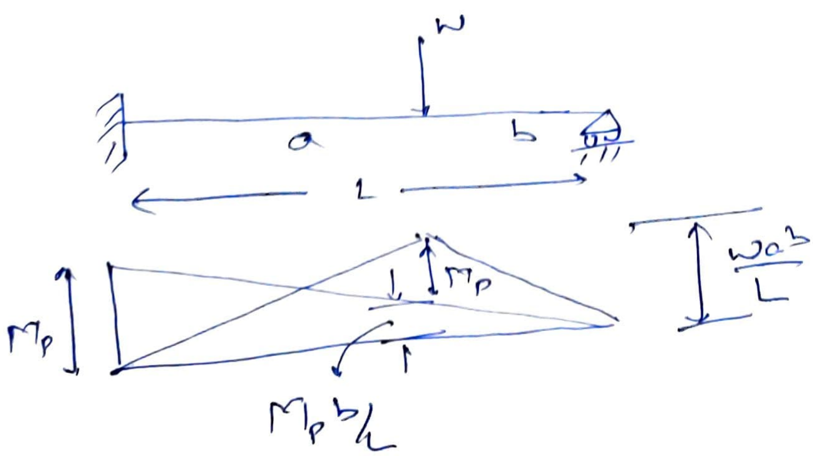
(1)



$$\frac{WL}{4} = M_p$$

$$W_c = \frac{4M_p}{L}$$

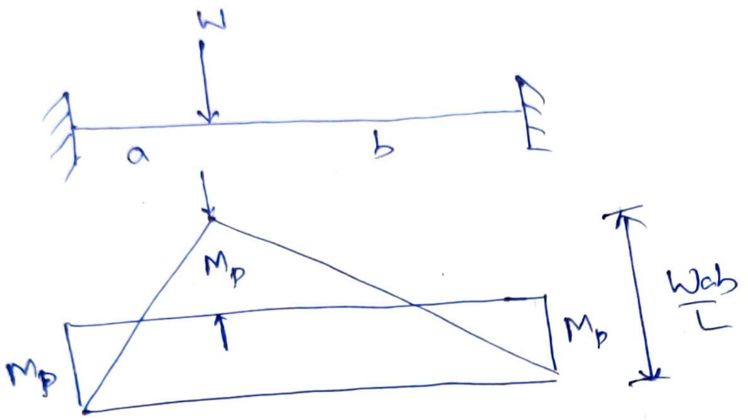
(2)



$$\therefore \quad \frac{W_c ab}{L} = M_p + M_p \frac{b}{L}$$

$$W_c = \frac{L+b}{ab} M_p$$

3



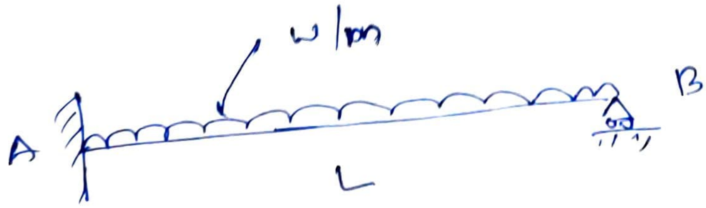
$$\frac{W_c ab}{L} = M_p + M_p$$

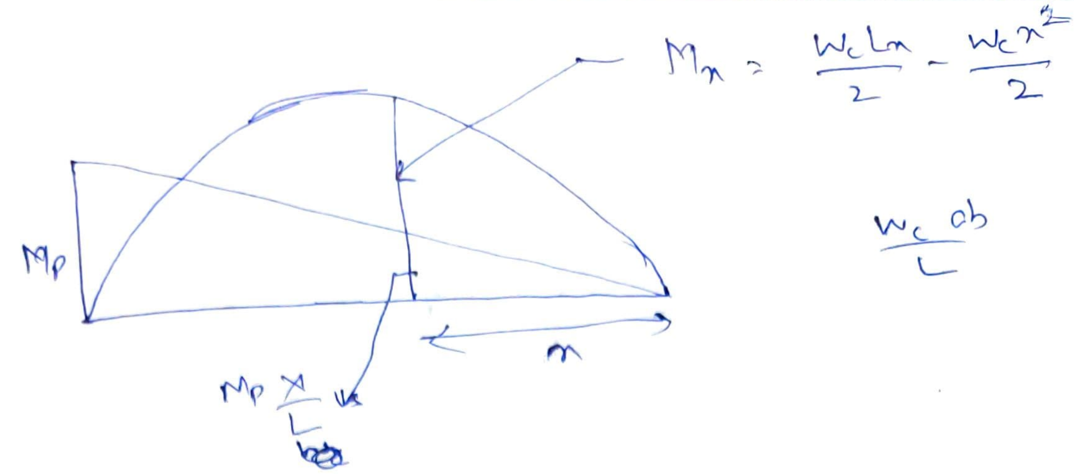
$$W_c = 2M_p \cdot \frac{L}{ab}$$

Note: IF $a = b = L/2$

$$W_c = \frac{8M_p}{L}$$

4





For the formation of collapse mechanism, plastic hinge has to develop at fixed end and at any point in the portion AB. Since bending moment at any point should not exceed plastic moment M_p, the interior hinge will be at the point of maximum bending moment. ~~And~~ At any section distance x, from propped end,

$$M_x = \frac{W_c L x}{2} - \frac{W_c x^2}{2} - M_p \frac{x}{L}$$

For M_x to be max.

$$\frac{dM_x}{dx} = 0 = \frac{W_c L}{2} - \frac{2W_c x}{2} - \frac{M_p}{L}$$

$$M_p = \frac{W_c L(L-2x)}{2}$$

Since hinge will be formed at x.

$$M_x = M_p$$

i.e $$M_p = \frac{W_c L}{2} x - \frac{W_c x^2}{2} - M_p \frac{x}{L}$$

$$M_p\left(1+\frac{x}{L}\right) = \frac{W_c}{2}\left(Lx-x^2\right)$$

Substituting the value of M_p from (i), we get

$$\frac{W_c L}{2}(L-2x)\left(1+\frac{x}{L}\right) = \frac{W_c}{2}\left(Lx-x^2\right)$$

$$(L-2x)(L+x) = (Lx-x^2)$$

$$L^2 - Lx - 2x^2 = Lx - x^2$$

$$x = \frac{-2L \pm \sqrt{4L^2+4L^2}}{2}$$

$$= -L \pm L\sqrt{2}$$

$$= L(\sqrt{2}-1)$$

$$= 0.414L$$

$$\therefore M_p = W_c\frac{L}{2}(L-2x)$$

$$= W_c\frac{L^2}{2}(1-2\times 0.414)$$

$$W_c = 11.655\frac{M_p}{L^2}$$

Thus, in propped cantilever subjected to udl, hinge occurs at 0.414L from propped end and its collapse load is $11.655\frac{M_p}{L^2}$.

Kinematic Method

This method starts with an assumed collapse mechanism.
After collapse mechanism is formed, there can be no changes of curvature at any cross-section except where plastic hinges are formed.

Hence, if a virtual displacement is given to the structure just after collapse mechanism is formed, the internal work is done only at plastic hinges, where plastic moment (Mp) is acting.

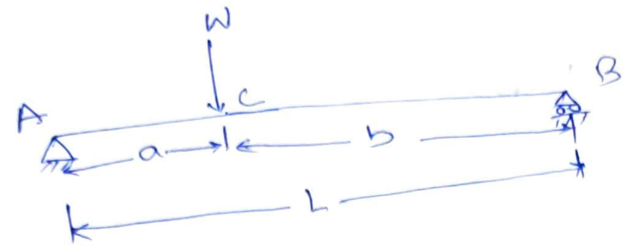
By equating internal work done by plastic moments at plastic hinges to ~~ex~~ external work done by loads, we can get collapse load.

Kinematic method Applied to Beams

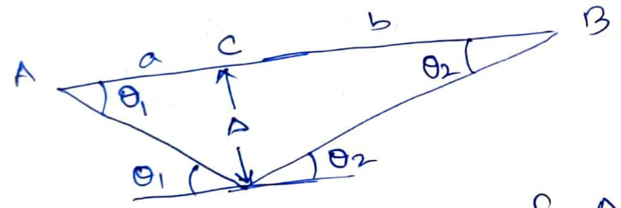
For any span of a beam for collapse, there should be natural hinges or plastic hinges at the support and one plastic hinge in the span of the beam. For a set of concentrated loads, maximum moment is always under a load.

Problems

1. Determine the collapse load in the simply supported beam.



Since B.M is maximum under the load, interior hinge will be under the load. The collapse mechanism is shown with virtual displacement Δ at load point.



Let θ_1 and θ_2 be the rotations of AC and CB. There is only one plastic hinge and rotation at this hinge $(=\theta_1+\theta_2)$

$$a\theta_1 = \Delta = b\theta_2$$

$$\theta_2 = \frac{b}{a}\theta_1$$

$\therefore$ Internal work done $= M_p(\theta_1+\theta_2)$

$$= M_p\left(\theta_1 + \frac{b}{a}\theta_1\right)$$

$$= M_p\left(\frac{L}{a}\theta_1\right) \quad (a+b=L)$$

External work done $= W_c\Delta$

$$= W_c\, a\theta_1$$

Equating external work done to internal work done, we get.

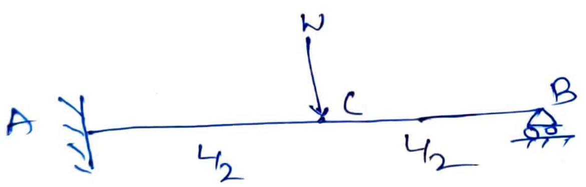
$$W_c a\phi_1 = M_p \frac{L}{b}\phi_1$$

$$W_c = \frac{L}{ab} M_p$$

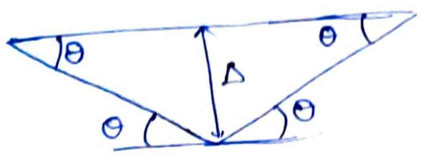
If $a = b = L/2$

$$W_c = \frac{L}{(L/2)(L/2)} \cdot M_p = \frac{4M_p}{L}$$

(2) Determine the collapse load in a propped cantilever of span L. subjected to central concentrate Load.



This beam develops collapse mechanism when plastic hinge is formed at fixed end and ~~and~~ another under the load.



Let the vertical displacement Δ be given at central hinge. Since the hinge is at mid span, rotations at two ends are equal, say 'θ'.

$$\text{Internal work done} = M_p\theta + M_p(\theta+\theta).$$

$$= 3M_p\theta$$

$$\text{External work done} = W_c\,\Delta$$

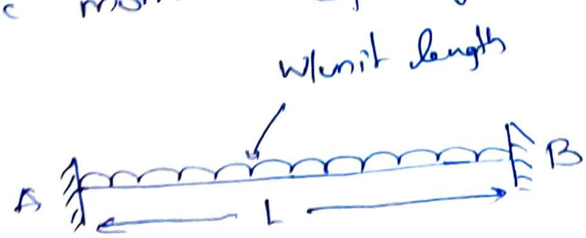
$$= W_c\,\frac{L}{2}\,\theta.$$

Equating internal work to external work, we get.

$$3M_p\theta = W_c\,\frac{L}{2}\,\theta$$

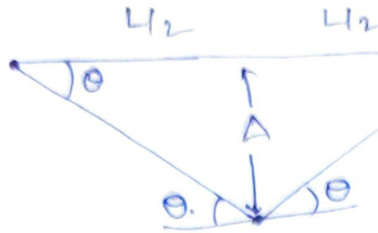
$$W_c = \frac{6M_p}{L}$$

(3) Find the collapse load in a fixed beam of span L carrying uniformly distributed load over the entire span. Plastic moment capacity is M_p throughout.



At collapse, hinge should be formed at supports and another hinge at mid-span, where positive movement is maximum. Let the virtual displacement Δ be given to

Central hinge. The deformed shape is shown in fig.



Internal work done $= M_p\theta + M_p(\theta+\theta) + M_p\theta$.

$= 4M_p\theta$.

External work done = Total load × Average distance moved.

$= W_c L \times \frac{\Delta}{2}$

$= \frac{W_c L}{2} \times \frac{L}{2}\theta$

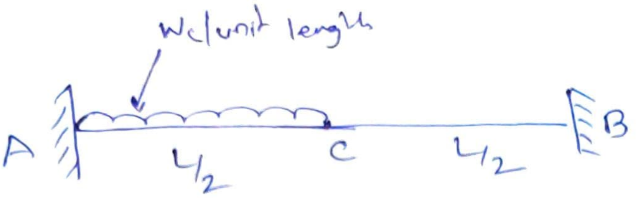
$\Rightarrow \frac{W_c L^2}{4}\theta$.

Equating external work to internal work,

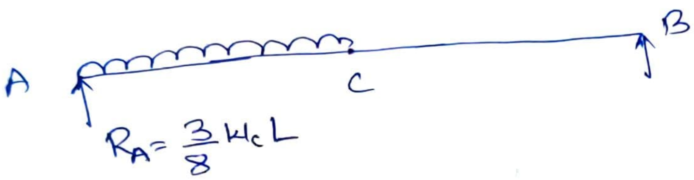
$4M_p\phi = \frac{W_c L^2}{4}\phi$.

$W_c = \frac{16M_p}{L^2}$.

(4) Determine the collapse load in a fixed beam of span L carrying uniformly distributed load under one half of span. The plastic moment capacity is Mp throughout.



At collapse, plastic hinges are formed at ends. Hence, end moments diagram is as shown in figure below. The free moment diagram is also shown in this fig.



The moment at any interior point in the portion AC is given by $M_x =$ Free moment − End moment.

Taking simply supported beam for free moment calculations

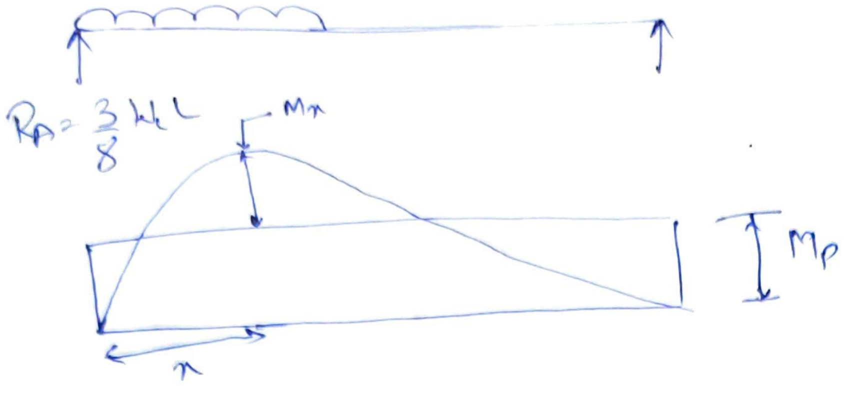
$$R_A L = W_c \times \frac{L}{2} \times \frac{3L}{4}$$

$$R_A = \frac{3 W_c L}{8}$$

∴ In portion AC, free moment $= R_A x - W_c \frac{x^2}{2}$

$$= \frac{3 W_c L}{8} x - W_c \frac{x^2}{2}$$

$$M_x = \frac{3}{8} W_c L x - W_c \frac{x^2}{2} - M_p$$

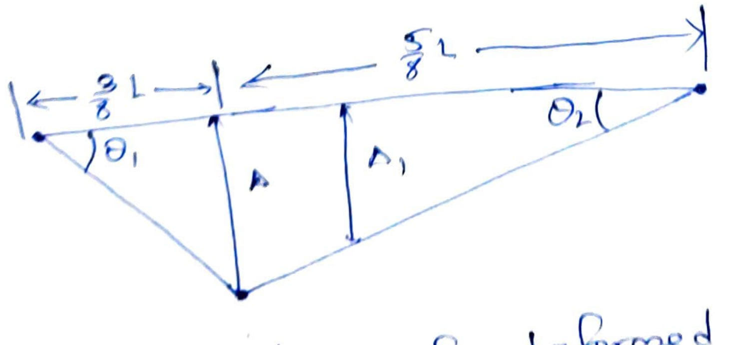


For Max. M_x.

$$\frac{dM_x}{dx} = 0 = \frac{3}{8} W_c L - W_c x.$$

$$x = \frac{3}{8} L$$

Hence, interior hinge will be at $x = \frac{3}{8} L$ from end A as shown in the figure below. Let virtual displacement given to interior hinge be Δ. Rotations at A and B be θ_1 and θ_2 respectively.



From the geometry of deformed shape,

$$\frac{3}{8} L \theta_1 = \Delta = \frac{5}{8} L \theta_2$$

$$\theta_2 = 0.6\, \theta_1$$

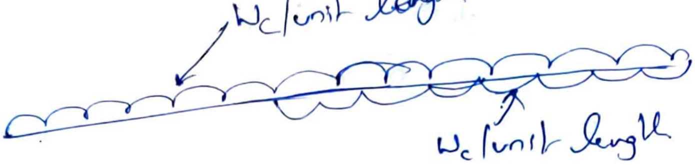
Internal work done $= M_p\theta_1 + M_p(\theta_1+\theta_2) + M_p\theta_2$

$= M_p\theta_1 + M_p(\theta_1 + 0.6\theta_1) + M_p\, 0.6\theta_1$

$= 3.2 M_p\theta_1$

External work done:

Let us take the given load as udl w_c/unit length over entire span minus udl of same intensity in right half.



Let the displacement at mid span be Δ_1.

$$\Delta_1 = \frac{L/2}{\frac{5L}{8}}\Delta$$

$= 0.8\Delta$

∴ External work done

$= w_c L \cdot \frac{\Delta}{2} - w_c \frac{L}{2} \cdot \frac{\Delta_1}{2}$

$= \frac{w_c L\Delta}{2} - \frac{w_c L}{4} \cdot 0.8\Delta$

$\Rightarrow w_c L\Delta(0.5 - 0.2)$

$= 0.3 w_c L\Delta$

$= 0.3 w_c \cdot L \cdot \frac{3}{8}L\theta_1$ $\quad (\Delta = \frac{3}{8}L\theta_1)$

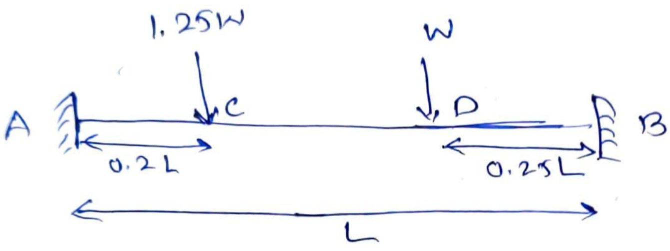
$= \frac{0.9}{8} w_c L^2\theta_1$

Equating internal work to external work,

$$3.2 M_p \theta_1 = \frac{0.9}{8} W_c L^2 . \theta_1$$

$$W_c = 28.44 \frac{M_p}{L^2}$$

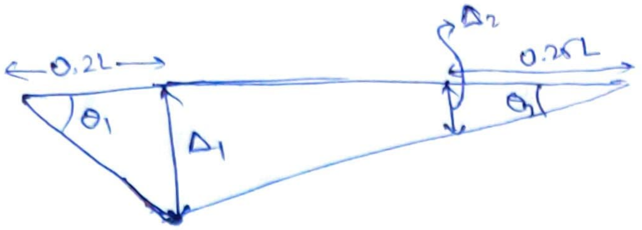
(5) Determine the collapse load in a fixed beam.



At collapse, hinges should form at supports and another hinge in the span. The interior hinge may develop under 1.25W load or under load W. Hence both ~~meth~~ mechanisms will be investigated and the one giving less W_c will be selected as the real collapse mechanism.

Mechanism I : Interior hinge under the load 1.25W.

Let Δ_1 be virtual displacement under the load 1.25W and Δ_2 be the displacement under the load W. Rotations at ends A and B be θ_1 and θ_2 respectively.



$$\Delta_2 = \frac{0.25L}{0.8L}\Delta_1 = \frac{0.25}{0.8}\Delta_1$$

$$0.2L\theta_1 = \Delta_1 = 0.8L\theta_2$$

$$\theta_1 = 4\theta_2$$

Internal work done $= M_p\theta_1 + M_p(\theta_1+\theta_2) + M_p\theta_2$

$= 2M_p(\theta_1+\theta_2)$

$\Rightarrow 10M_p\theta_2$

External work done $= 1.25W_c\Delta_1 + W_c\Delta_2$

$= 1.25W_c\Delta_1 + W_c \cdot \frac{0.25}{0.8}\Delta_1$

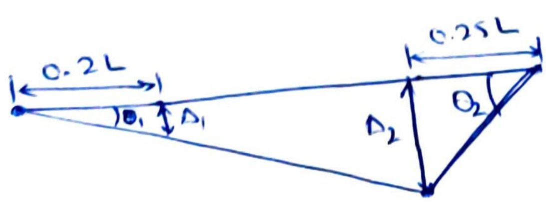
$= W_c\, 1.25\, L\,\theta_2$ $\quad(\Delta_1 = 0.8L\theta_2)$

$\therefore$ Equating internal work done to external work done,

$$10M_p\cancel{\theta_2} = 1.25W_cL\cancel{\theta_2}$$

$$W_c = \frac{10M_p}{1.25L} = \frac{8M_p}{L}$$

Mechanism II: (Internal hinge under load W).



$$\Delta_1 = \frac{0.2L}{0.75L}\Delta_2 = \frac{0.2}{0.75}\Delta_2$$

and $0.75L\theta_1 = \Delta_2 = 0.25L\theta_2$

$\theta_2 = 3\theta_1$

Internal Workdone =

$$M_p\theta_1 + M_p(\theta_1 + \theta_2) + M_p\theta_2$$

$$\Rightarrow 2M_p(\theta_1 + \theta_2) = 2M_p(\theta_1 + 3\theta_1)$$

$$= 8M_p\theta_1$$

External Workdone $= 1.25W_c\Delta_1 + W_c\Delta_2$

$$= 1.25W_c \cdot \frac{0.2}{0.75}\Delta_2 + W_c\Delta_2$$

$$\Rightarrow W_c\left(1.25 \times \frac{0.2}{0.75} + 1\right)\Delta_2$$

$$= W_c\left(1.25 \times \frac{0.2}{0.75} + 1\right) 0.75L\theta_1$$

$(\Delta_2 = 0.75L\theta_1)$ $\Rightarrow W_cL\theta_1$

Equating internal work done to External work done

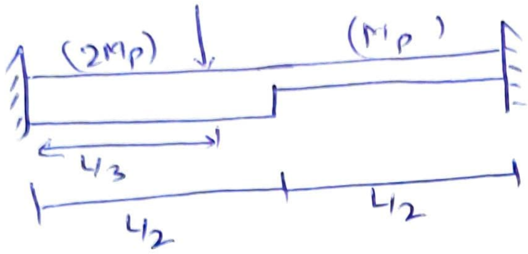
$$8M_p\theta_1 = W_cL\theta_1$$

$$W_c = \frac{8M_p}{L}$$

In this case, We find both mechanisms give same collapse load $\frac{8M_p}{L}$. Hence the two mechanisms occur simultaneously and $W_c = \frac{8M_p}{L}$

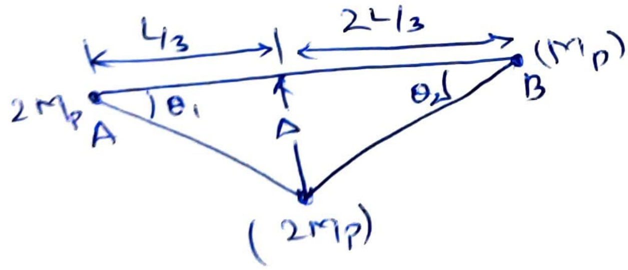
⑥ Determine collopse load in the fixed beam. (10)

Plastic moment capacity is $2M_p$ in one half and M_p in the other half.



At collopse, plastic hinge will form at the ends and the third hinge may for under the load where positive moment is high or it may form at mid-span where, though moment is less then that under the load, plastic moment capacity is less. Hence, both mechanism are to be investigated.

Mechanism 1



$$\cancel{L}/\cancel{3}\,\theta_1 = \frac{2\cancel{L}}{\cancel{3}}\theta_2$$

$$\theta_1 = 2\theta_2$$

Internal work done $= 2M_p\theta_1 + 2M_p(\theta_1+\theta_2) + M_p\theta_2$

$$\Rightarrow 2M_p(2\theta_2) + 2M_p(2\theta_2+\theta_2) + M_p\theta_2)$$

$$= 4M_p\theta_2 + 6M_p\theta_2 + M_p\theta_2$$

$$= 11M_p\theta_2$$

External work done $= kl_c \Delta$

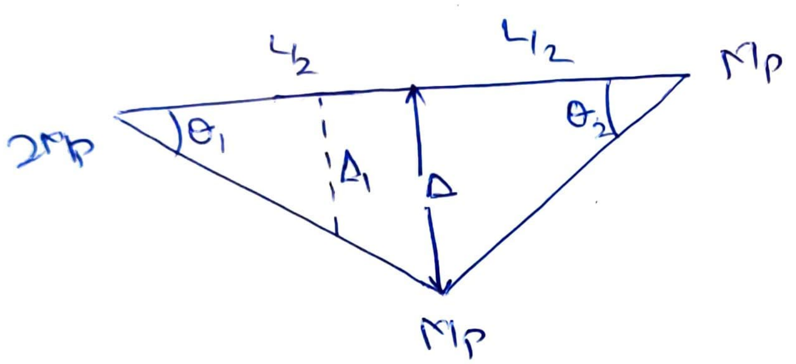
$= kl_c \times \frac{2L}{3}\theta_2$

Equating Internal work done to External work done.

$11 M_p \theta_2 = kl_c \times \frac{2L}{3}\theta_2$

$kl_c = \frac{33 M_p}{2L} \rightarrow (1)$

Mechanism II



It is having hinge at mid span. Let virtual displacement at this point be Δ and the rotations at ends be θ_1 and θ_2 respectively. Then.

$\frac{L}{2}\theta_1 = \Delta = \frac{L}{2}\theta_2$

$\theta_1 = \theta_2 = \theta$

Displacement under the load $= \Delta_1 = \frac{L/3}{L/2}\Delta = \frac{2}{3}\Delta$

$= \frac{2}{3} \cdot \frac{L}{2} \cdot \theta$

$= \frac{L}{3}\theta.$

Internal work done

$= 2M_p\theta + M_p(\theta + \theta) + M_p\theta = 5M_p\theta$

Note: When there is a sudden change in the section, the hinge will form on weaker side and the necessary rotations take place.

External Work done $= W_c \cdot \Delta_1$

$= W_c \times \frac{L}{3} \theta$

Equating Internal work to External work

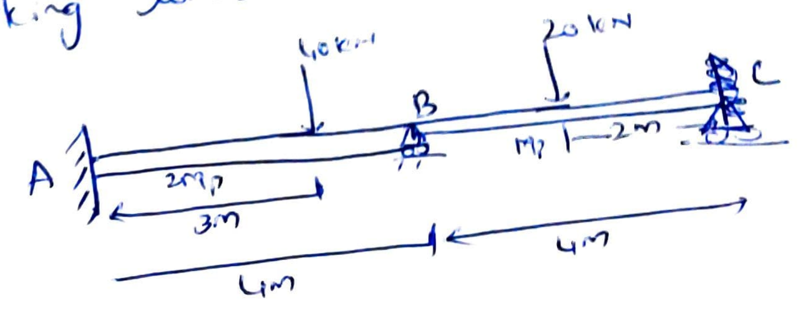
$$5M_p \theta = W_c \cdot \frac{L}{3} \theta$$

$$W_c = \frac{15 M_p}{L} \longrightarrow (2)$$

From (1) and (2), we conclude Mechanism II is the real mechanism and the collapse load is

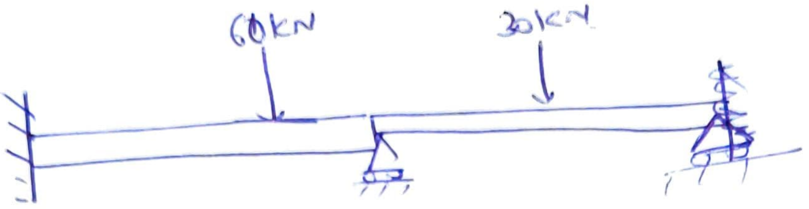
$$\boxed{W_c = \frac{15 M_p}{L}}$$

(7) In the continuous beam, plastic moment capacity of AB is to be kept twice that of BC. Determine the plastic moment capacity of the beam if the load shown are working loads. Take load factor as 1.5.

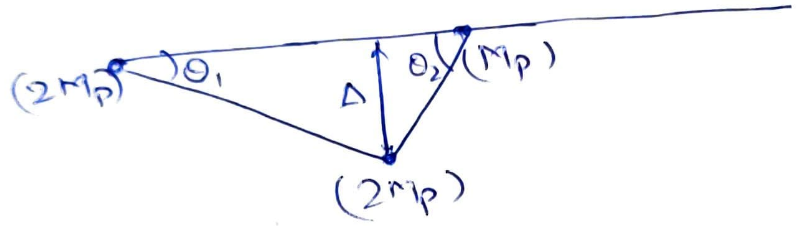


Let plastic moment capacity of BC be Mp. Hence plastic moment capacity of AB is 2Mp. The given load wrt

Two mechanisms are possible, one for AB and another for BC.



Collapse Mechanism of AB:



Internal work done = $2M_p\theta_1 + 2M_p(\theta_1+\theta_2) + M_p\theta_2$

But $3\theta_1 = \Delta = 1\times\theta_2$

$\theta_2 = 3\theta_1$

$\therefore$ Internal work done = $2M_p\theta_1 + 2M_p(\theta_1+3\theta_1) + M_p(3\theta_1)$

$= 2M_p\theta_1 + 2M_p(4\theta_1) + 3M_p\theta_1$

$\Rightarrow 13M_p\theta_1$

External work done = $60\times\Delta$

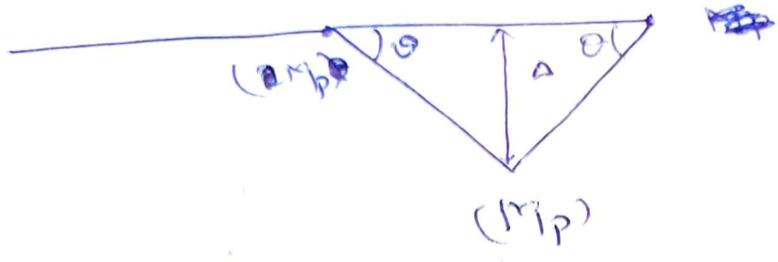
$= 60\times 3\theta_1$

$= 180\,\theta_1$

$\therefore\ 13M_p\theta_1 = 180\,\theta_1 \Rightarrow M_p = 13.846$ kN-m $\rightarrow$ (1)

Collapse mechanism of BC:

~~Let the~~



~~Let the~~ Internal work done $= 2M_p\theta + M_p(\theta+\theta)$

$= 3M_p\theta.$

External work done $= 30\times 2\theta = 60\theta.$

Equating Internal to External work done

$60\theta = 3M_p\theta$

$M_p = 20\ kN.m \longrightarrow (2)$

From (1) & (2), we conclude $M_p = 20\ KN.m.$

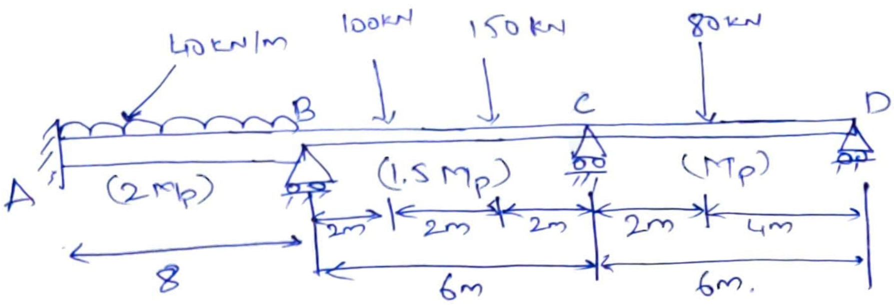
Hence, plastic moment capacity of AB required $= 2M_p$

$= 2\times 20\ kN.m$

$= 40\ kN.m$

and that of BC $= M_p = 20\ KN.m.$

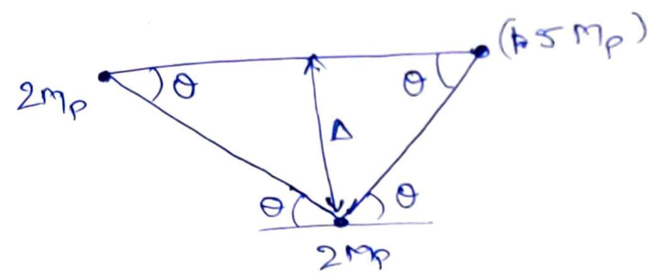
(7) Find the required value of plastic moment capacity in the continuous beam. The loads shown in the figure are the collapse loads.



There are four possible mechanisms for the collapse of the continuous beam.

$4\theta_1 = \Delta = 4\theta_2$

(1) Beam mechanism AB.

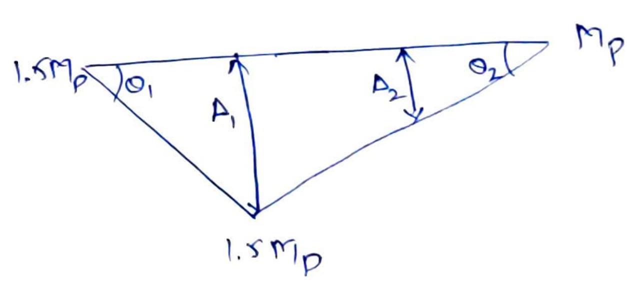


Equating External work done to internal work done

$$2M_p\theta + 2M_p(\theta+\theta) + 1.5M_p\theta = 40 \times 8 \times \frac{1}{2}(4\theta)$$

$$M_p = 85.333 \text{ kN-m}$$

(2) First Beam Mechanism in BC.



$2\theta_1 = \Delta_1 = 4\theta_2$

$\theta_1 = 2\theta_2$

$\Delta_2 = \frac{2}{4}\Delta_1 = \frac{\Delta_1}{2}$

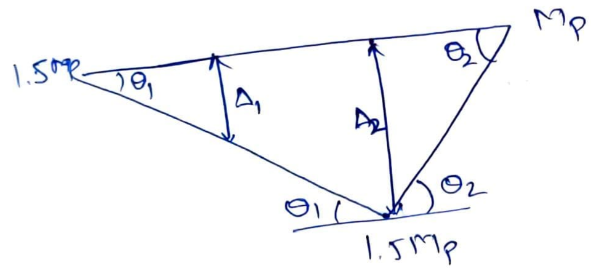
$\therefore$ The equilibrium equation is

$1.5M_p\theta_1 + 1.5M_p(\theta_1+\theta_2) + M_p\theta_2 = 100 \times 4\theta_2 + 150 \times 2\theta_2$

$M_p(1.5\times 2 + 1.5(2+1) + 1]\theta_2 = 700\theta_2$

$M_p = \frac{700}{8} = 82.352$ KN-m

(3) Second Beam mechanism in BC.



$4\theta_1 = \Delta_2 = 2\theta_2$

$\theta_2 = 2\theta_1$

$\Delta_1 = \frac{2}{4}\Delta_2$

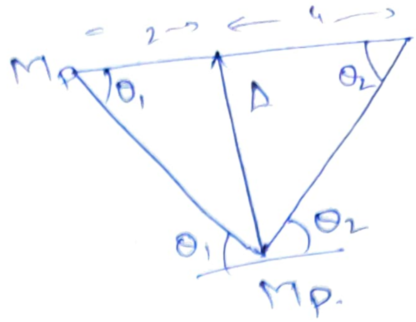
$\therefore$ Equilibrium equation is

$1.5M_p\theta_1 + 1.5M_p(\theta_1+\theta_2) + M_p(\theta_2) = 100\Delta_1 + 150\Delta_2$

$M_p[1.5 + 1.5(3) + 2]\theta_1 = 100\times 2\theta_1 + 150\times 4\theta_1$

$M_p = \frac{800}{8} = 100$ kN-m

(4). Beam Mechanism in CD.



$$2\theta_1 = \Delta = 4\theta_2$$

$$\theta_1 = 2\theta_2$$

Equilibrium equation.

$$M_p\theta_1 + M_p(\theta_1 + \theta_2) = 80 \times \Delta$$

$$2M_p\theta_2 + M_p(2\theta_2 + \theta_2) = 80 \times 4\theta_2$$

$$2M_p\theta_2 + 3M_p\theta_2 = 320\theta_2$$

$$5M_p\theta_2 = 320\theta_2$$

$$M_p = \cancel{80\,kN\text{-}m}\ 64\ kN\text{-}m.$$

Hence mechanism 3 is the real mechanism and required

Value of $M_p = 100$ kN-m

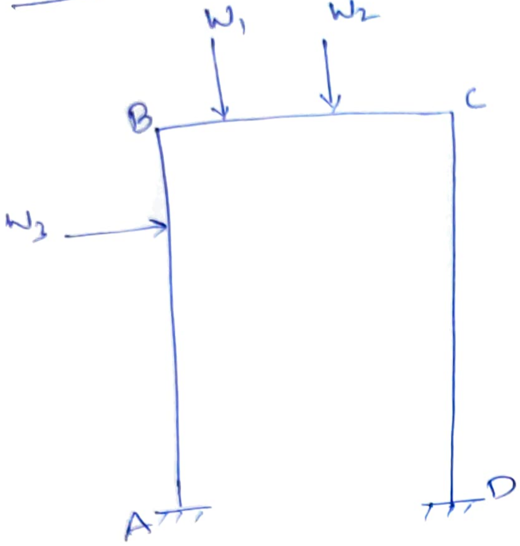
$\therefore$ Plastic moment Capacity for various spans required is

for AB $= 2M_p = 2 \times 100 = 200$ kN-m

BC $= 1.5M_p = 1.5 \times 100 = 150$ kN-m

CD $= M_p = 100$ kN-m.

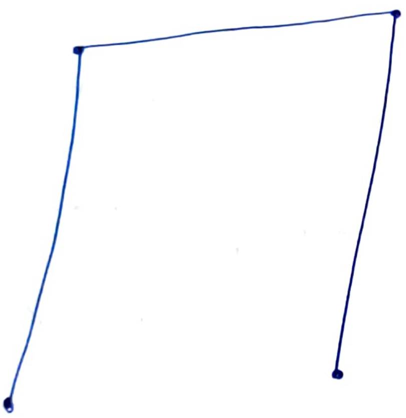
KINEMATIC METHOD APPLIED TO FRAMES



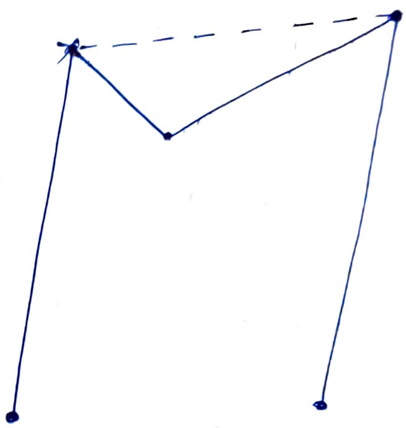
(1)

(2)

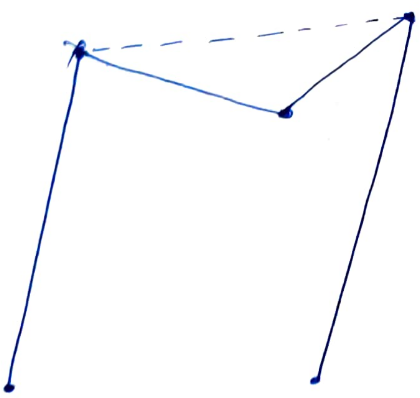
(3)



(4)



(5)



(6)

The frame may fail by forming any one of the following mechanisms. These mechanisms may be grouped into three.

1. Beam Mechanism
2. Sway Mechanism
3. Combined Mechanism.

Beam mechanism:

If Concentrated loads are acting, possible position of intermediate hinge is under any of those loads. Hence in a member with concentrated loads, possible beam mechanisms to be tried is equal to the number of Concentrated loads.

If both ends of a member are continuous or Simply supported and load is uniformly distributed over the entire span, intermediate hinge will develop at mid-span.

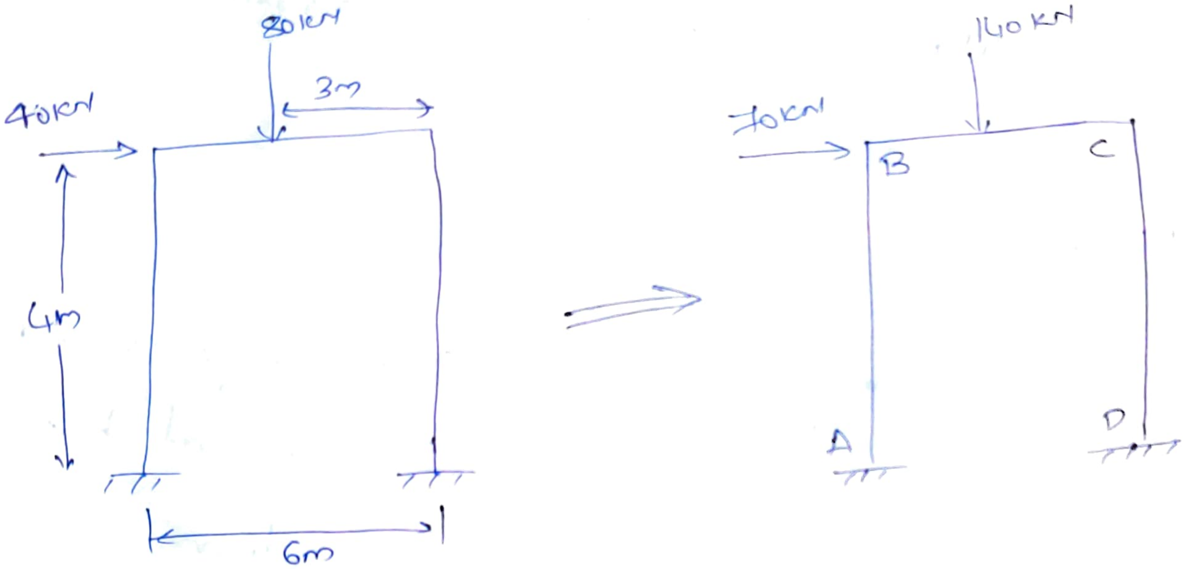
If one end is continuous / fixed and the other end is hinged / simply supported and the load is uniformly distributed over the entire span, intermediate hinge develops at 0.414L from hinged / simply supported end.

Sway Mechanisms

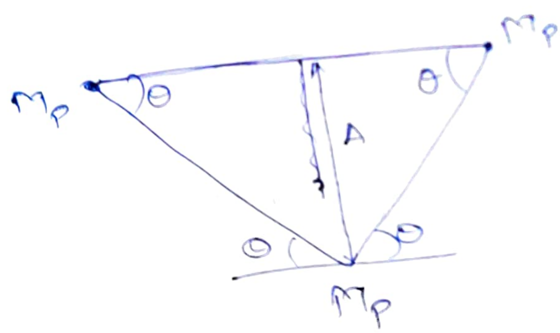
Due to lateral forces, a frame may sway considerably and form collapse mechanism. Two hinges at bottom and two hinges at the top of the columns are necessary for this type of collapse. Sometimes, hinges at bottom may be mechanical hinges.

Work is done by plastic moments acting at plastic hinges.

① Determine the plastic moment capacity of the section required for the frame. The loads shown are working loads. Take load factor $\lambda = 1.75$.



① Beam Mechanism



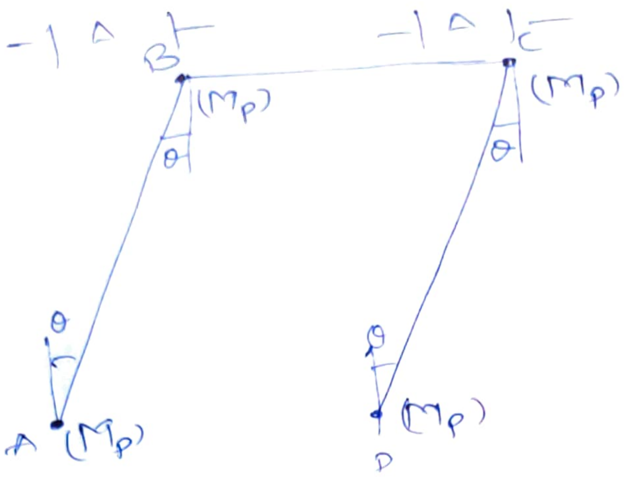
$$M_p\theta + M_p(\theta+\theta) + M_p\theta = 140\Delta$$

$$4M_p\theta = 140 \times 3\theta$$

$$M_p = \frac{\cancel{140}^{\cancel{70}\,35} \times 3}{\cancel{4}_{2}} = 105 \text{ kN-m.}$$

→ ①

② Sway Mechanism

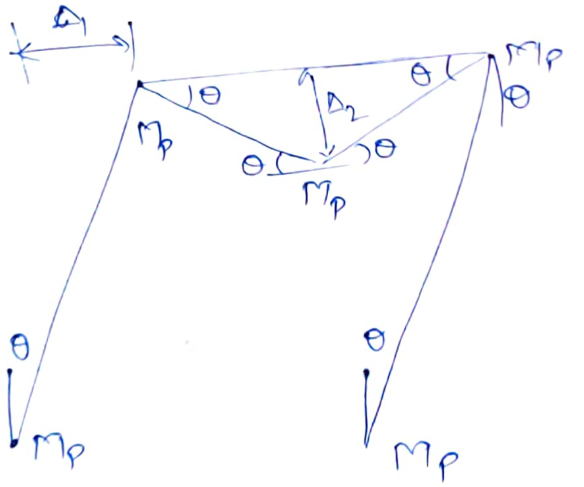


$M_p\theta + M_p\theta + M_p\theta + M_p\theta = 70\,\Delta$

$4M_p\cancel{\theta} = 70 \times 4\cancel{\theta}$

$M_p = 70$ kN-m. → ②

③ Combined Mechanism.



$M_p\theta + \cancel{M_p\theta} + M_p(\theta+\theta) + M_p(\theta+\theta) + M_p\theta = 70\,\Delta_1 + 140\,\Delta_2$

$6M_p\theta = 70 \times 4\theta + 140 \times 3\theta$

$6M_p\theta = 700\,\theta \Rightarrow M_p = \frac{700}{6} = 116.67$

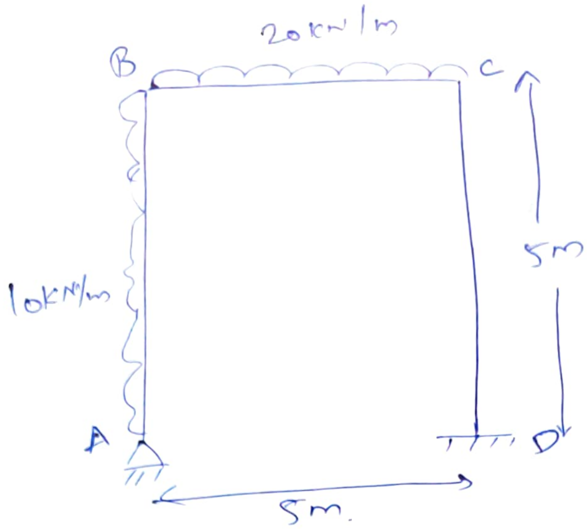
→ ③

From ①, ② & ③

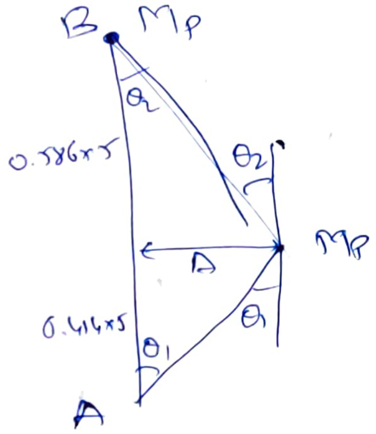
$M_p = 116.67$ KN

and Combined mechanism is the real mechanism.

(2)



(1) Beam Mechanism of AB.



$0.414 \times 5 \times \theta_1 = \Delta = 0.586 \times 5 \times \theta_2$

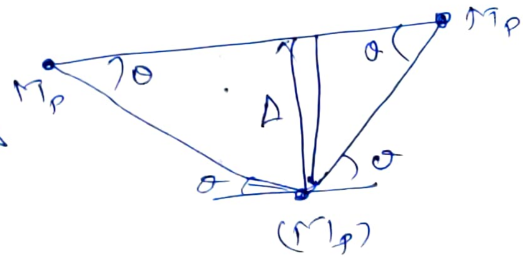
$\theta_1 = 1.415\,\theta_2$

$M_p(\theta_1+\theta_2) + M_p\theta_2 = 10 \times 5 \times \frac{\Delta}{2}$

$M_p(1.415\,\theta_2+\theta_2) + M_p\theta_2 = 10 \times 5 \times \frac{1}{2} \times 0.586 \times \theta_2$

$M_p = 21.449$ kN-m

(2) Beam Mechanism of BC.

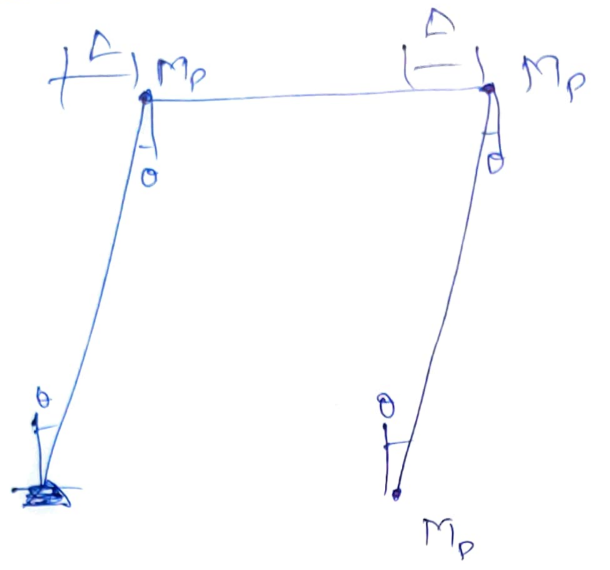


$M_p\theta + M_p(\theta+\theta) + M_p\theta = 20 \times 5 \times \frac{1}{2} \times \Delta$

$\Rightarrow 4 M_p\theta = 100 \times \frac{1}{2} \times 2.5\,\theta$

$M_p = 31.25$ kN-m.

③ Sway mechanism

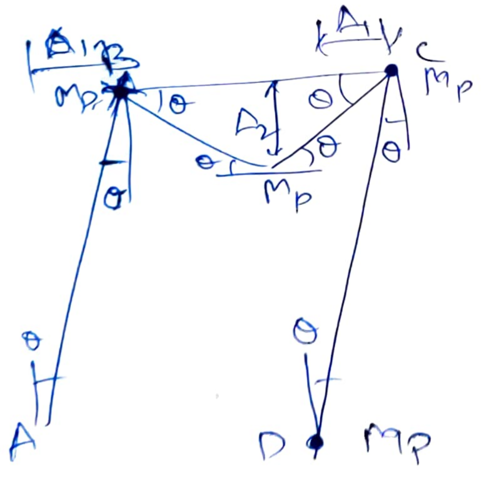


$$M_p\theta + M_p\theta + M_p\theta = 10 \times 5 \times \frac{\Delta}{2}$$

$$3M_p\theta = 50 \times \frac{5}{2}\theta$$

$$M_p = 41.667 \text{ KN-m}$$

④ Combined mechanism.



$$\cancel{M_p(\theta+\theta)} + M_p(\theta+\theta) + M_p(\theta+\theta) + M_p\theta = 10 \times 5 \times \frac{\Delta_1}{2} + 20 \times 5 \times \frac{\Delta_2}{2}$$

$$5M_p\theta = 10 \times 5 \times \frac{1}{2} \times 5\theta + 20 \times 5 \times \frac{1}{2} \times 2.5\theta$$

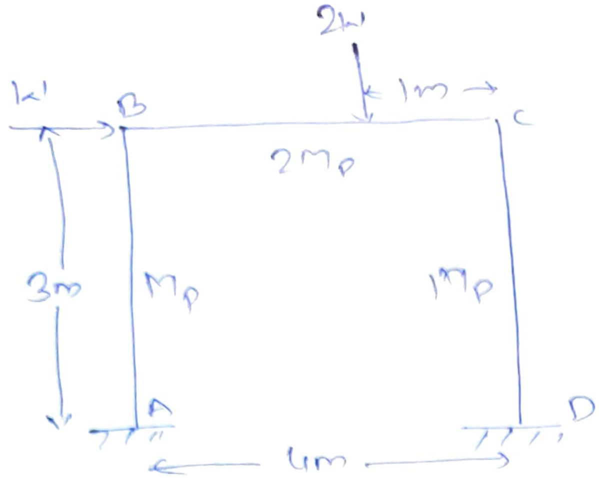
~~25~~ + ~~50~~ 75/4

50 + 50 100/2

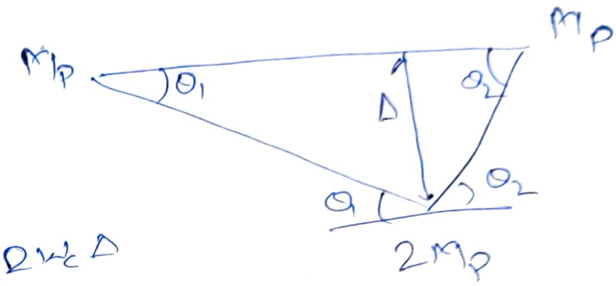
$M_p = \underline{50 \text{ kN-m}}$

∴ M_p required is 50 kN-m

③



① Beam Mechanism



$3\theta_1 = \Delta = \theta_2$

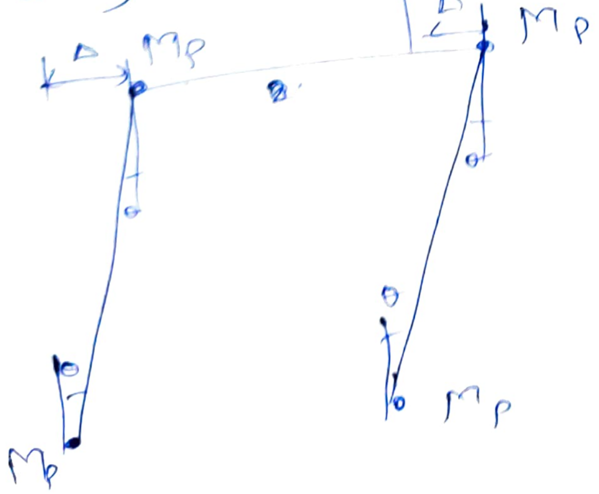
$M_p\theta_1 + 2M_p(\theta_1+\theta_2) + M_p\theta_2 = 2W_c\Delta$

$M_p[\theta_1 + 2(\theta_1+3\theta_1) + 3\theta_1] = 2W_c \times 3\theta_1$

$M_p = \frac{6W_c}{12}$

$W_c = 2M_p$

② Sway mechanism

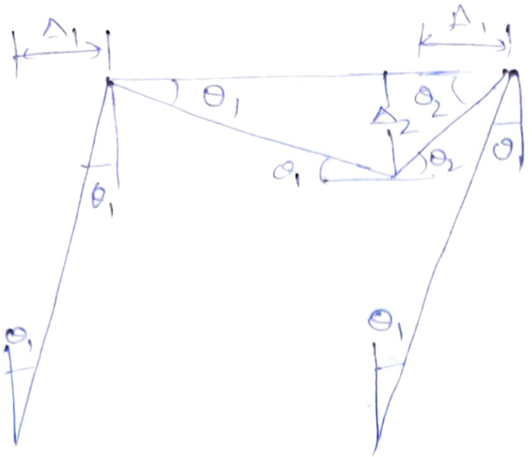


$M_p\theta + M_p\theta + M_p\theta + M_p\theta = W\Delta$

$\Rightarrow 4M_p\theta = W \times 3\theta$

$W_c = \frac{4M_p}{3} = 1.333M_p$

Combined Mechanism.



$$3\theta_1 = \Delta_2 = \theta_2$$

$$M_p\theta_1 + 2M_p(\theta_1+\theta_2) + M_p(\theta_2+\theta_1) + M_p\theta_1$$

$$= W_c\, 3\theta_1 + 2W_c \times 3\theta_1$$

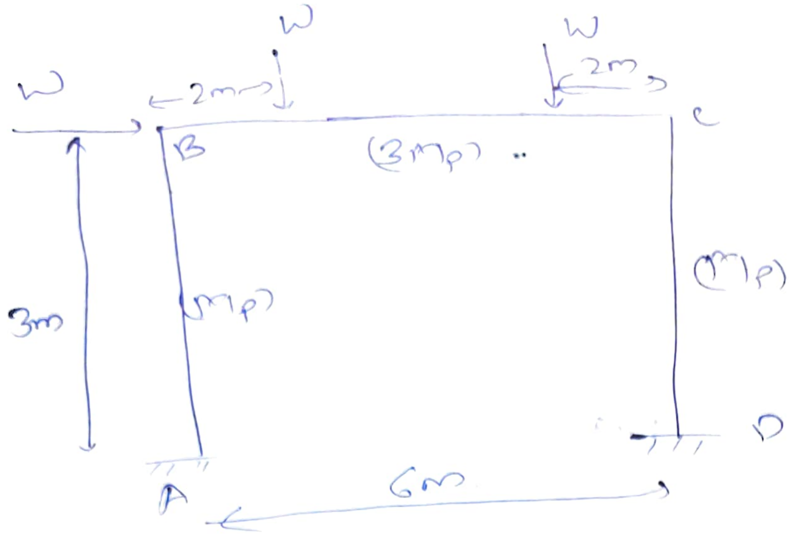
$$M_p\left[1 + 2(1+3) + (1+3) + 1\right] = 9 W_c$$

$$W_c = \frac{14}{9} M_p = 1.556 M_p$$

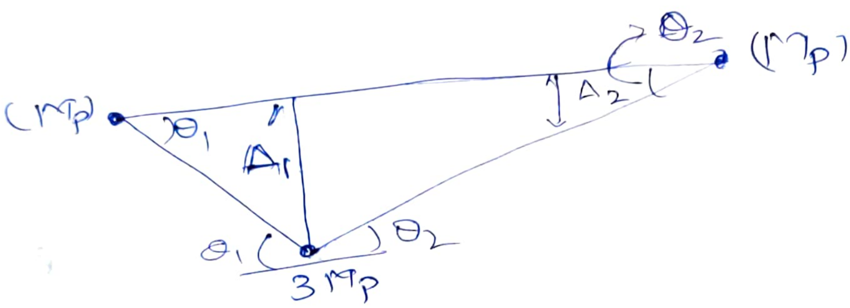
From ①, ②, ③, we can conclude that real

Collapse load is $W_c = 1.333 M_p$

(4)



(1) Beam mechanism (1) in BC



$$2\theta_1 = 4\theta_2 = \Delta_1$$

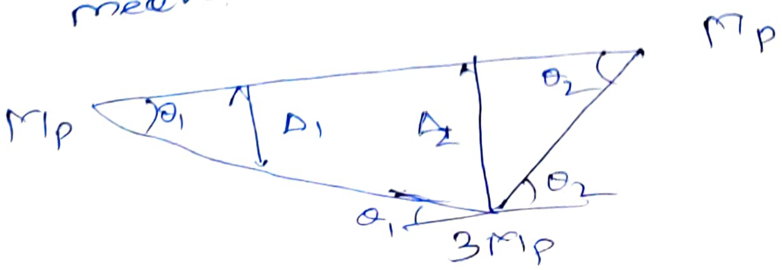
$$\theta_1 = 2\theta_2 \quad ; \quad \Delta_2 = 2\theta_2$$

$$M_p\theta_1 + 3M_p(\theta_1+\theta_2) + M_p\theta_2 = W_c \times 4\theta_2 + W_c\, 2\theta_2$$

$$M_p\,(2 + 3(2+1) + 1]\theta_2 = 6W_c\theta_2$$

$$W_c = 2M_p$$

(2) Beam mechanism (2) in BC



$4\theta_1 = \Delta_2 = 2\theta_2$

$\theta_2 = 2\theta_1$

$\Delta_1 = 2\theta_1$

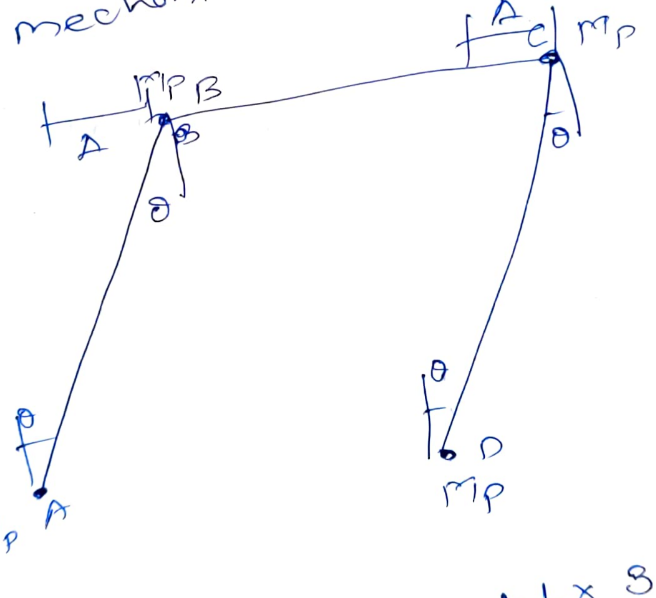
$M_p\theta_1 + 3M_p(\theta_1+\theta_2) + M_p\theta_2 = \frac{wl}{c} \times 2\theta_1 + \frac{wl}{c} \times 4\theta_1$

$M_p\,[1+3(1+2) + 1\times 2]\,\theta_1 = 6\frac{wl}{c}\theta_1$

$12M_p = 6\frac{wl}{c}$.

$\frac{wl}{c} = 2M_p$

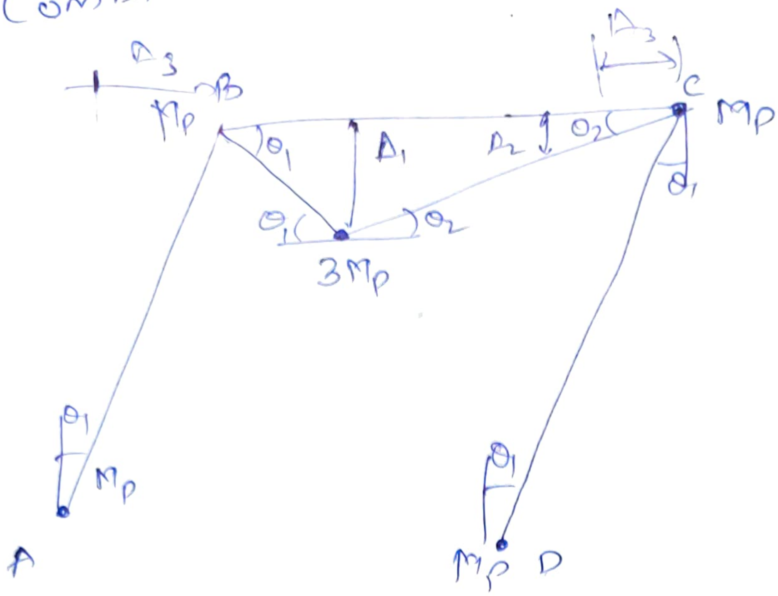
③ Sway mechanism.



$M_p\theta + M_p\theta + M_p\theta + M_p\theta = \frac{wl}{c} \times 3\theta$

$\frac{wl}{c} = \frac{4}{3}M_p = 1.333M_p$

④ Combined mechanism I



$2\theta_1 = \Delta_1 = 4\theta_2$ $\qquad$ $2\theta_2 = \Delta_2$

$\theta_1 = 2\theta_2$ $\qquad$ $\Delta_3 = 3\theta_1 = 6\theta_2$

$M_p\theta_1 + 3M_p(\theta_1+\theta_2) + M_p(\theta_1+\theta_2) + M_p(\theta_1) = W_c\Delta_1 + W_c\Delta_2 + W_c\Delta_3$

$\Rightarrow M_p[2+3(2+1) + (2+1) + 2]\theta_2 = W_c \times 4\theta_2 + W_c \times 2\theta_2 + W_c \times 6\theta_2]$

$16M_p = 12W_c$

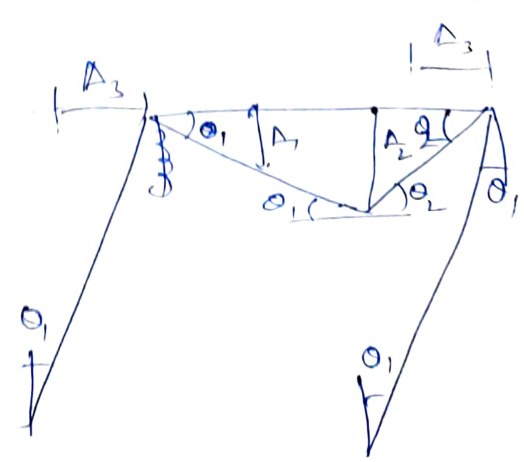
$W_c = \frac{16}{12}M_p = 1.333M_p$

⑤ Combined Mechanism II

$4\theta_1 = \Delta_2 = 2\theta_2$

$\theta_2 = 2\theta_1$

$2\theta_1 = \Delta_1$, $\Delta_3 = 3\theta_1$



$M_p\theta_1 + 3M_p(\theta_1+\theta_2) + M_p(\theta_1+\theta_2) + M_p(\theta_1) = W_c\Delta_1 + W_c\Delta_2 + W_c\Delta_3$

$M_p\theta_1[1+3(1+2)+(1+2)+1] = W_c(2\theta_1)+W_c(4\theta_1)+W_c(3\theta_1)$

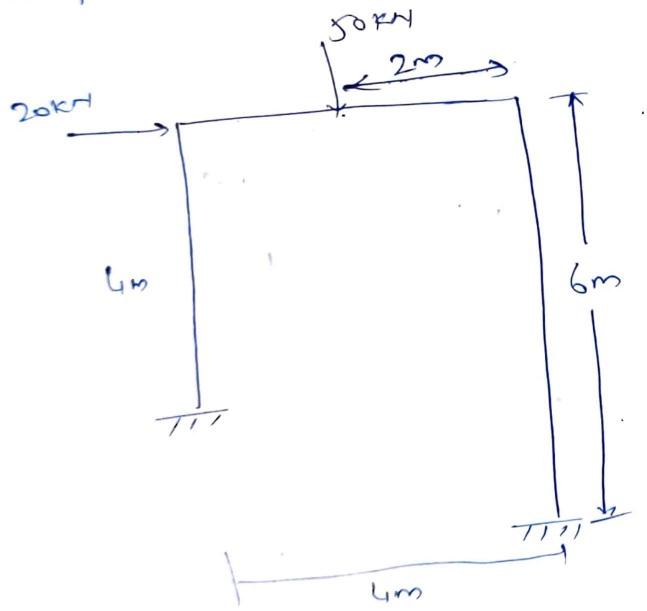
$14M_p\theta_1 = 9W_c\theta_1$

$W_c = \frac{14}{9}M_p = 1.556M_p$

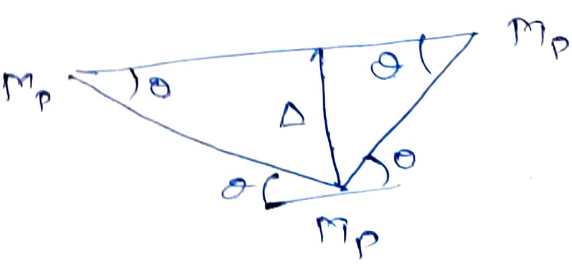
$\therefore W_c = 1.333M_p$

and Sway and Combined mechanism will be developed Simultaneously.

(5)



(1) Beam mechanism



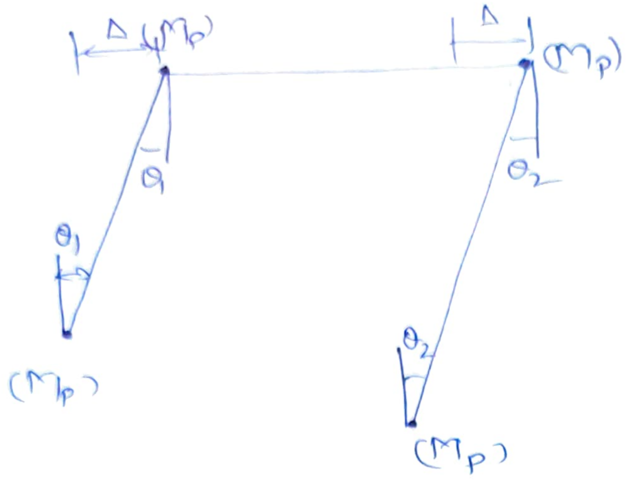
$\Delta = 2\theta$

$M_p\theta + M_p(\theta+\theta) + M_p\theta = 50\times 2\theta$

$4M_p\theta = 100\theta$

$M_p = 25\ KN\text{-}m \rightarrow (1)$

② Sway mechanism.



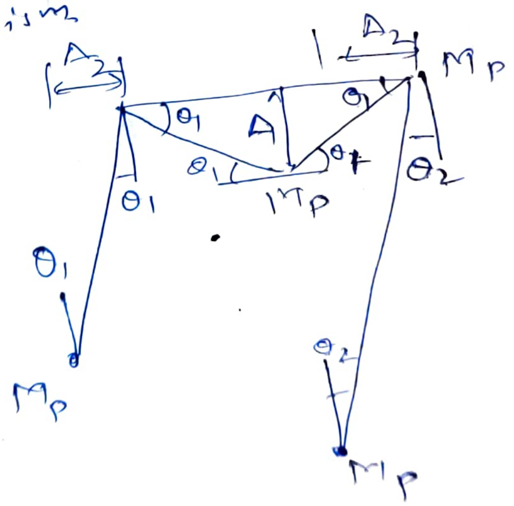
$$4\theta_1 = \Delta = 6\theta_2$$

$$M_p\theta_1 + M_p\theta_1 + M_p\theta_2 + M_p\theta_2 = 20 \times \Delta$$

$$M_p(1.5+1.5+1+1)\theta_2 = 20 \times 6\theta_2$$

$$M_p = \frac{20 \times 6}{5} = 24 \text{ kN-m.}$$

③ Combined mechanism



$$4\theta_1 = \Delta_2 = 6\theta_2$$

$$\theta_1 = 1.5\theta_2$$

$$\Delta_1 = 2\theta_1 = 2 \times 1.5\theta_2$$

$$= 3\theta_2$$

$$M_p\theta_1 + M_p(\theta_1+\theta_1) + M_p(\theta_1+\theta_2) + M_p\theta_2 = 20 \times 6\theta_2 + 50 \times 3\theta_2$$

$$M_p = 33.75 \text{ kN-m.}$$

∴ Plastic moment capacity required

$$M_p = 33.75 \text{ kN-m.}$$